

# Linking star formation to black hole spectra: simulations and inference for the Einstein Telescope era



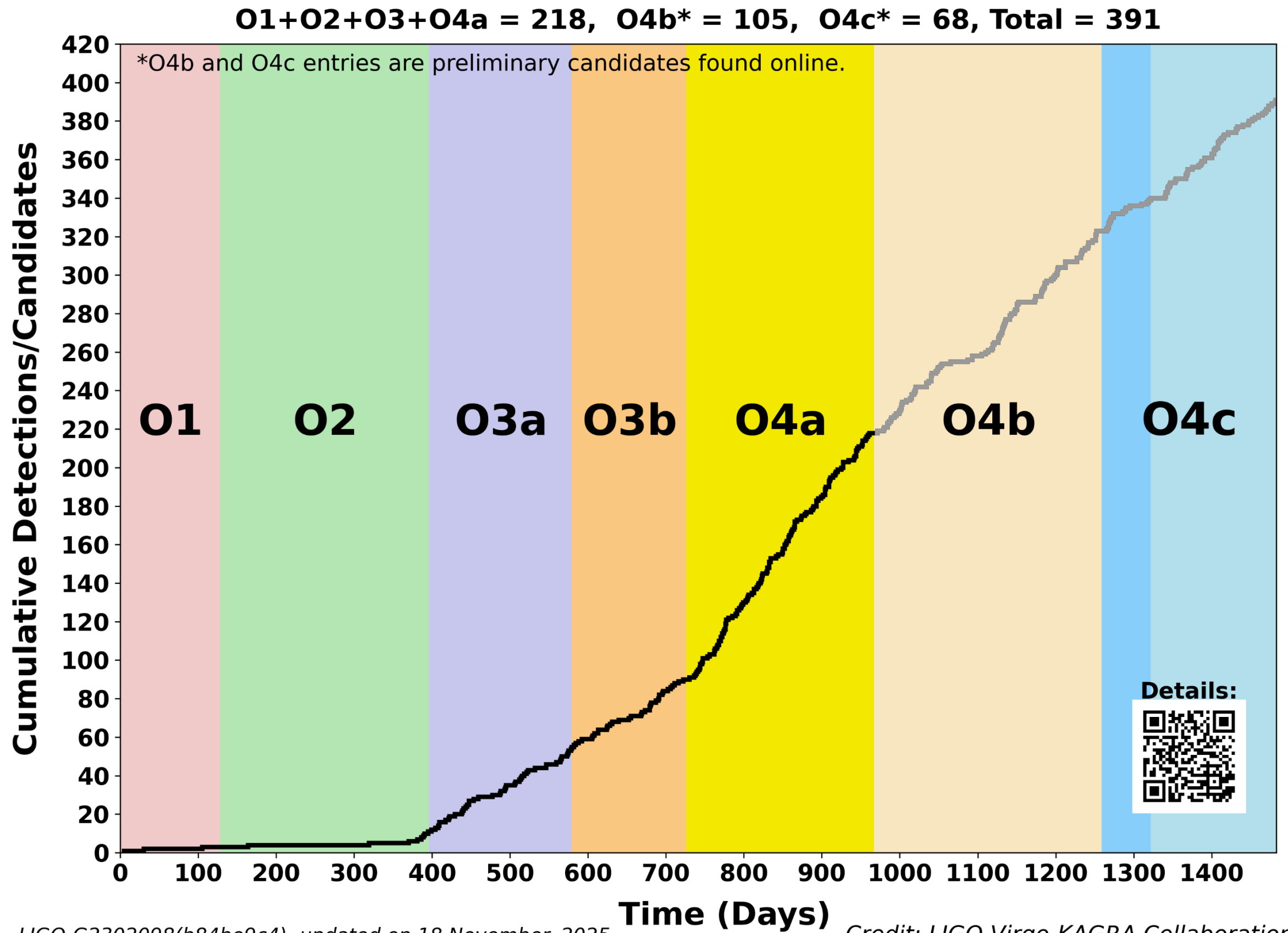
**SISSA**



Giovanni Antinozzi, Astrophysics & Cosmology Ph.D.  
student @ Scuola Internazionale Superiore di Studi Avanzati,  
Trieste, Italy

INAF - IASF Milano Seminars – 27/05/2026

# **1 : Introduction**



LIGO-G2302098(b84be9c4), updated on 18 November, 2025

Credit: LIGO-Virgo-KAGRA Collaboration

# Masses in the Stellar Graveyard

Solar Masses

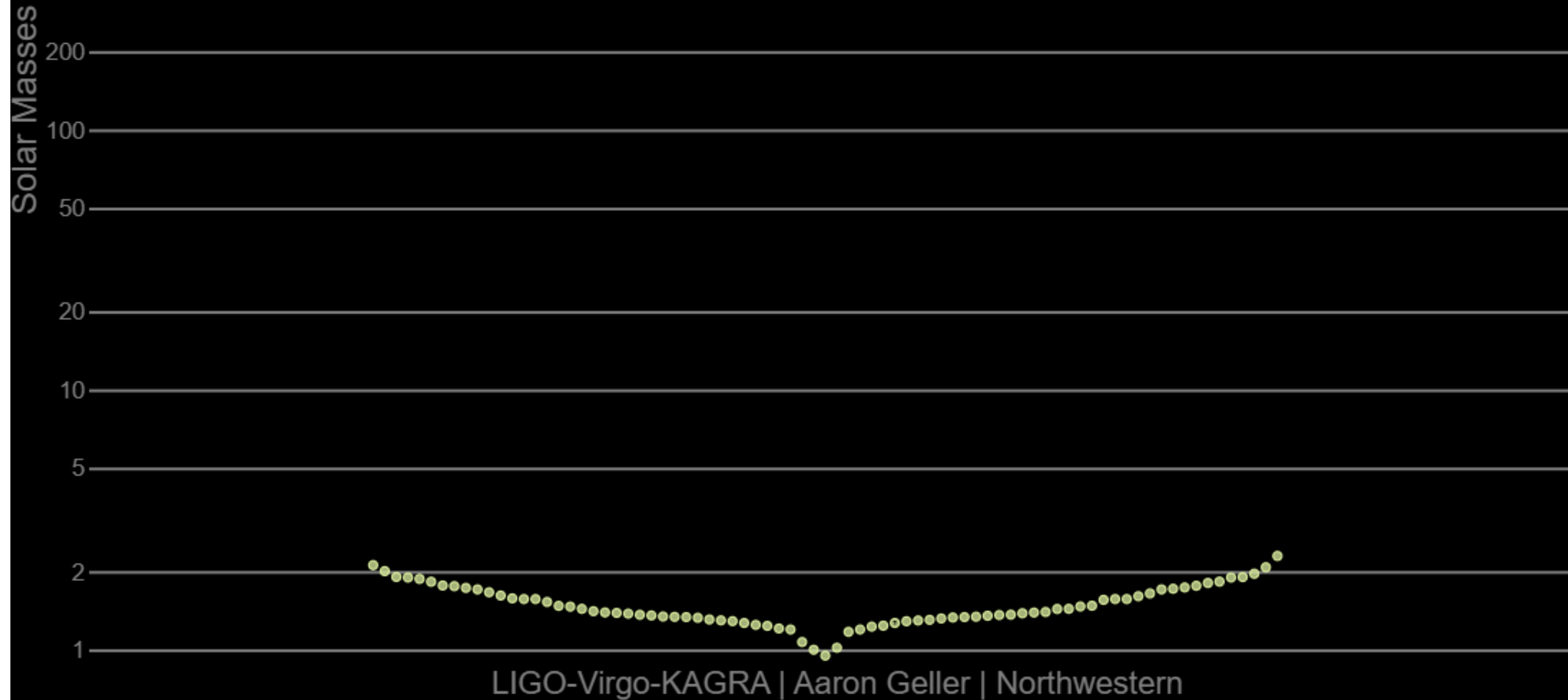
200  
100  
50  
20  
10  
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2  
1

LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

1

# Masses in the Stellar Graveyard

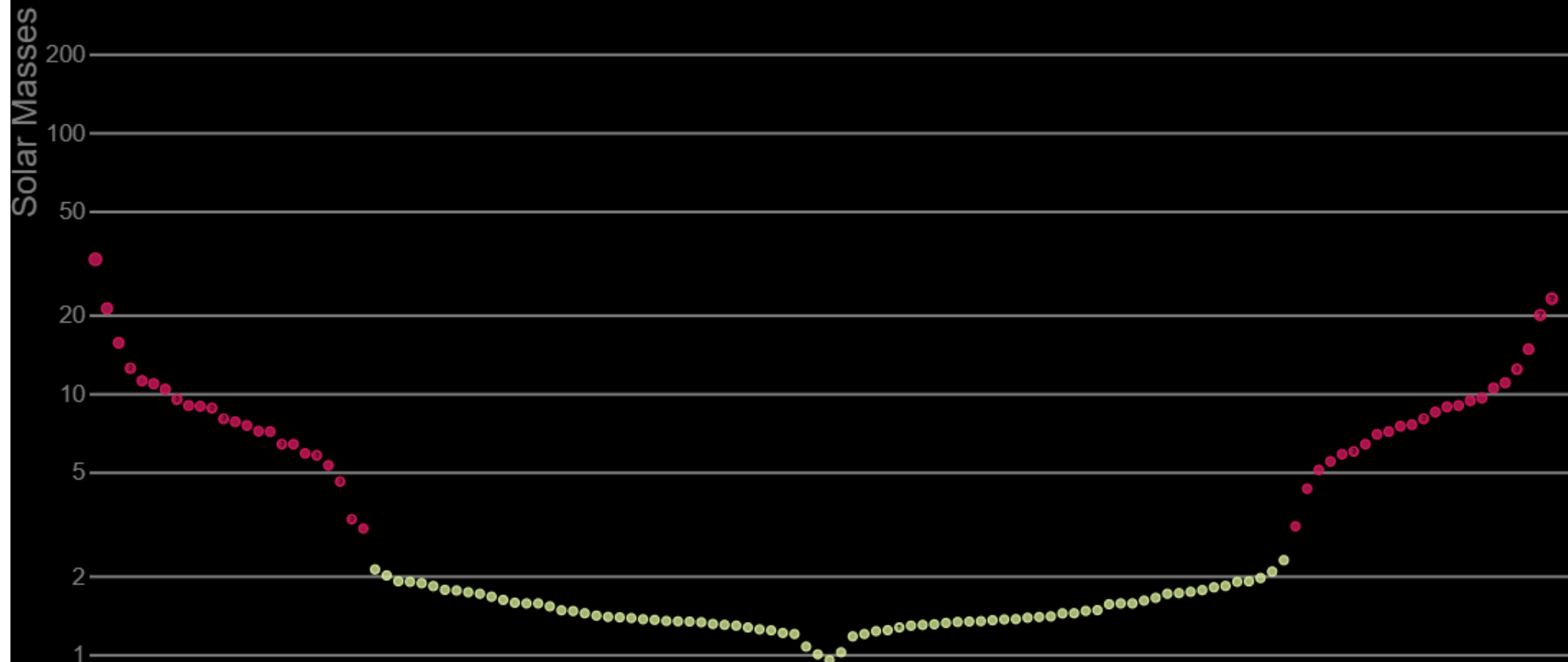
*EM Neutron Stars*



1

# Masses in the Stellar Graveyard

*EM Black Holes* *EM Neutron Stars*

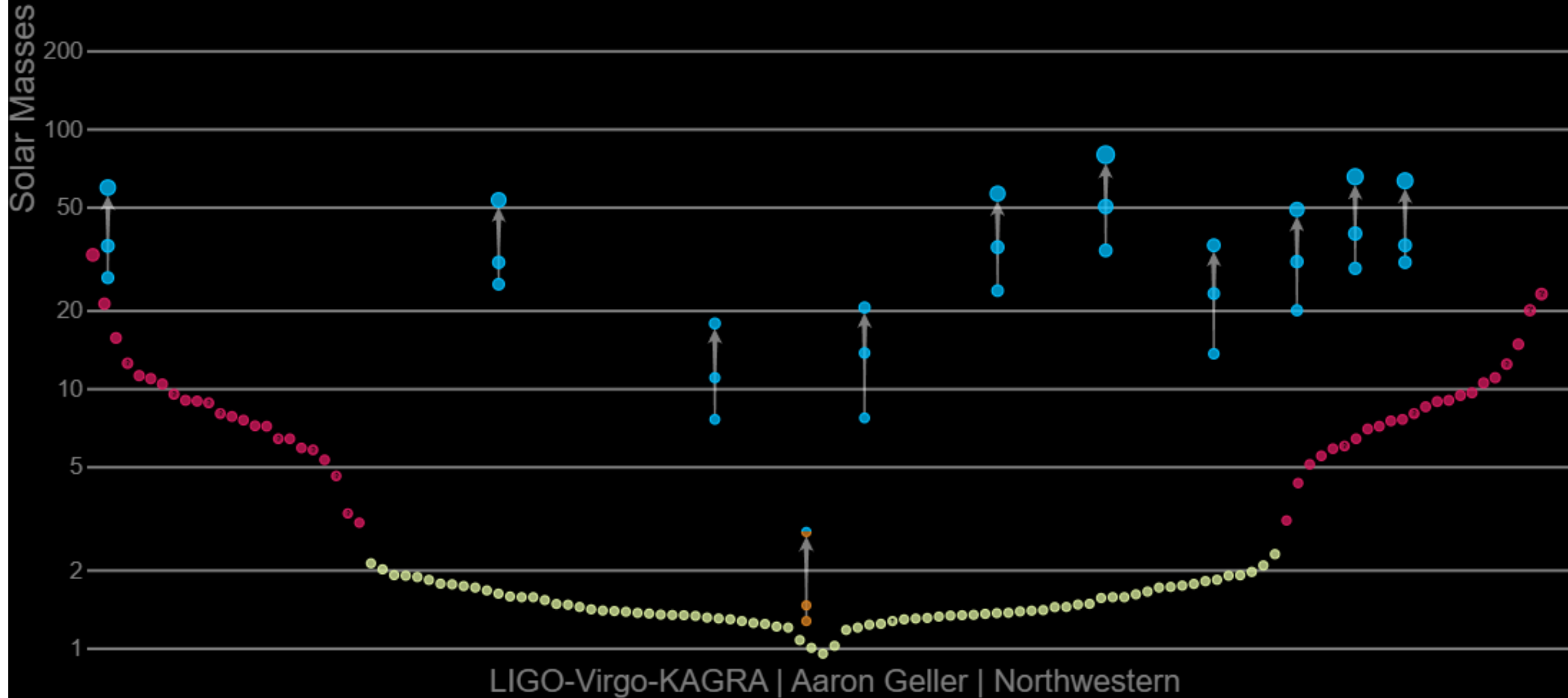


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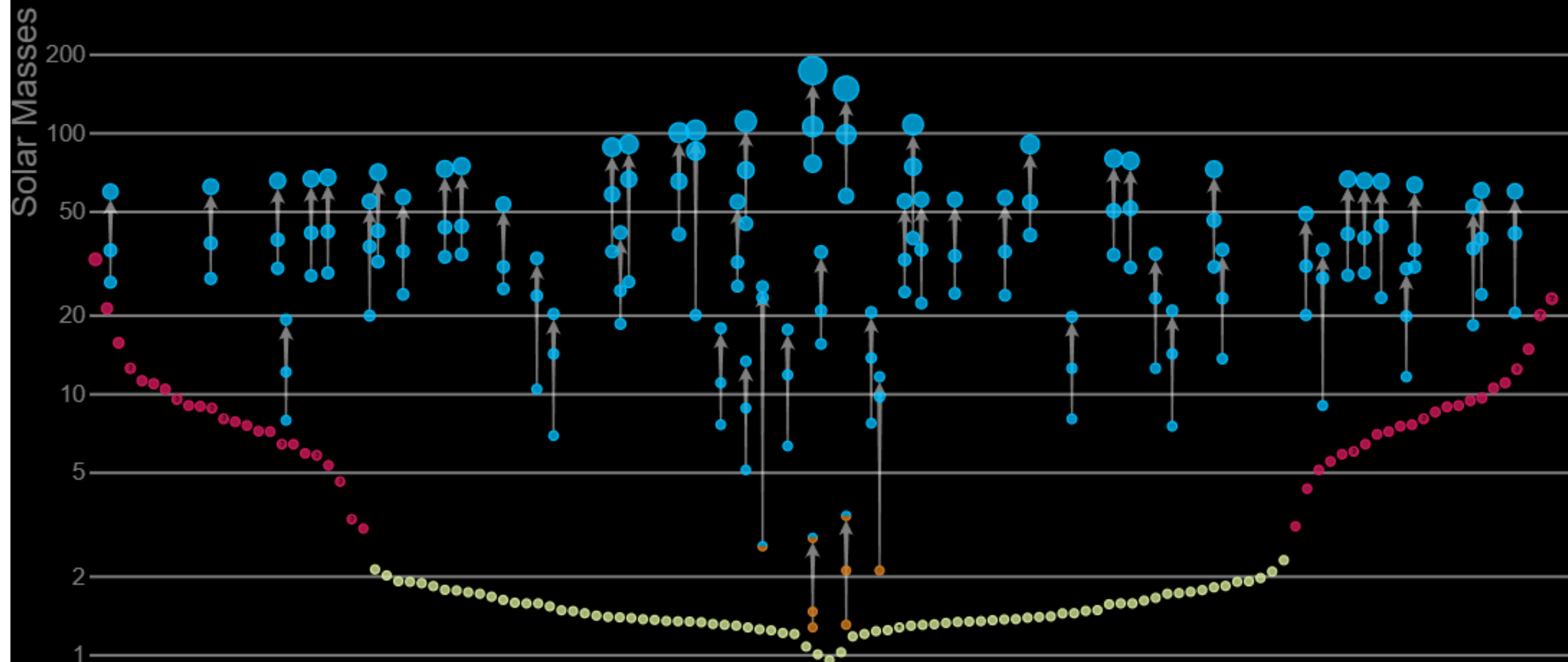
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LIGO-Virgo-KAGRA Black Holes LIGO-Virgo-KAGRA Neutron Stars EM Black Holes EM Neutron Stars



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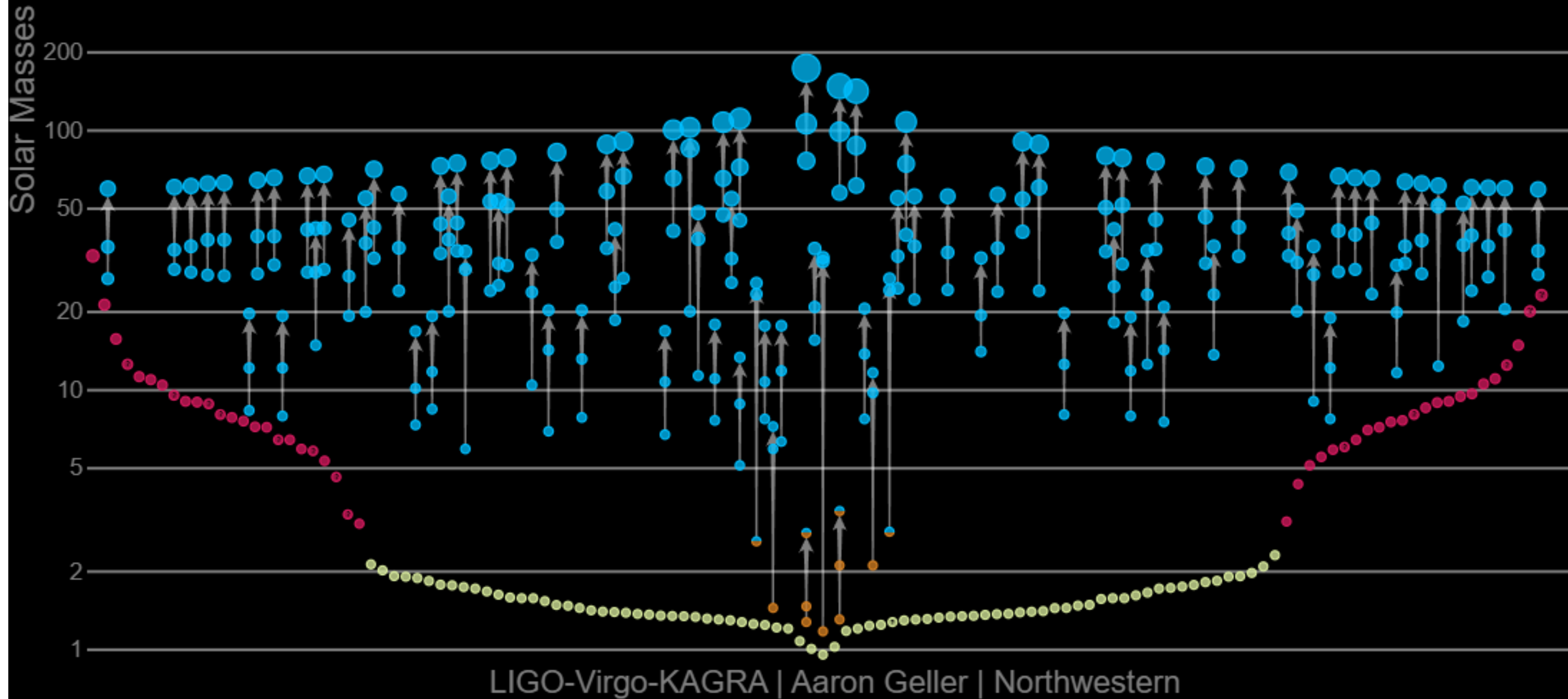
LIGO-Virgo-KAGRA Black Holes LIGO-Virgo-KAGRA Neutron Stars EM Black Holes EM Neutron Stars



LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

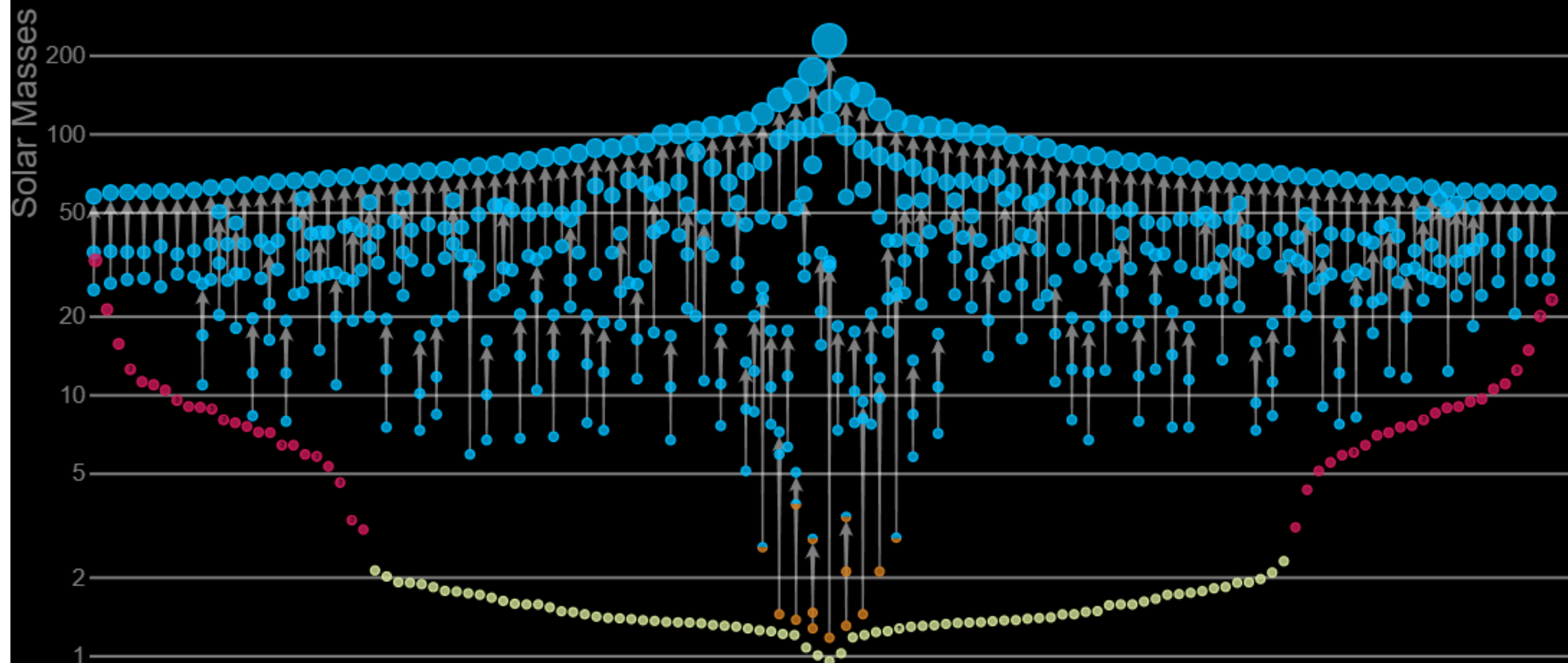
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LIGO-Virgo-KAGRA Black Holes LIGO-Virgo-KAGRA Neutron Stars EM Black Holes EM Neutron Stars



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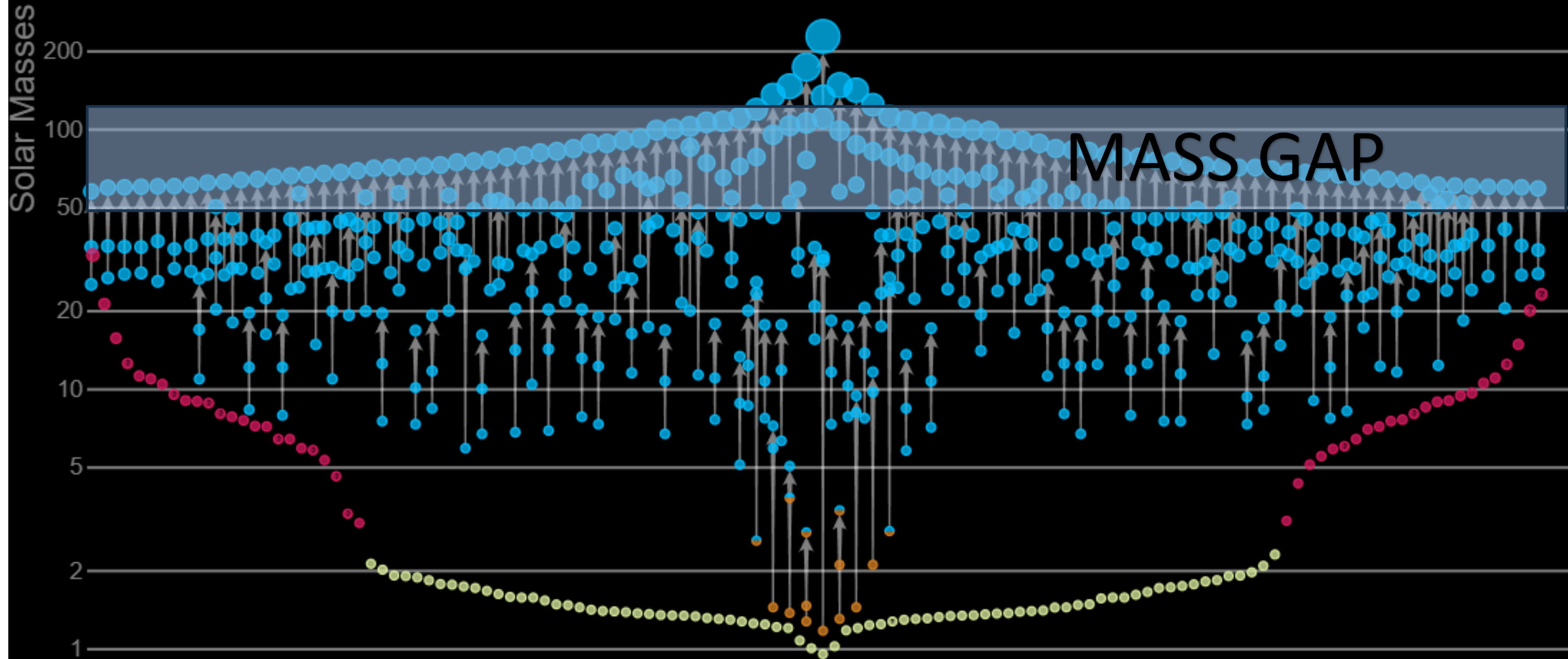
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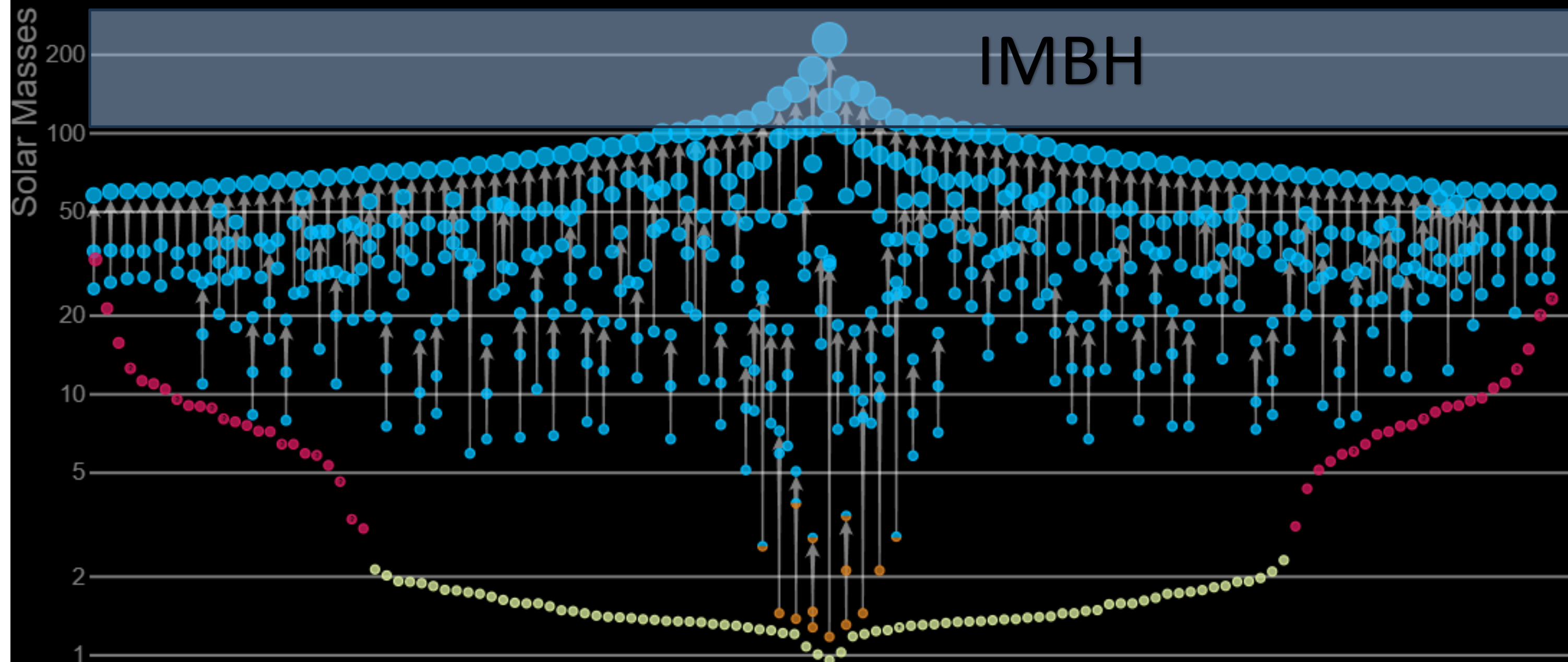
LIGO-Virgo-KAGRA Black Holes LIGO-Virgo-KAGRA Neutron Stars EM Black Holes EM Neutron Stars



LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

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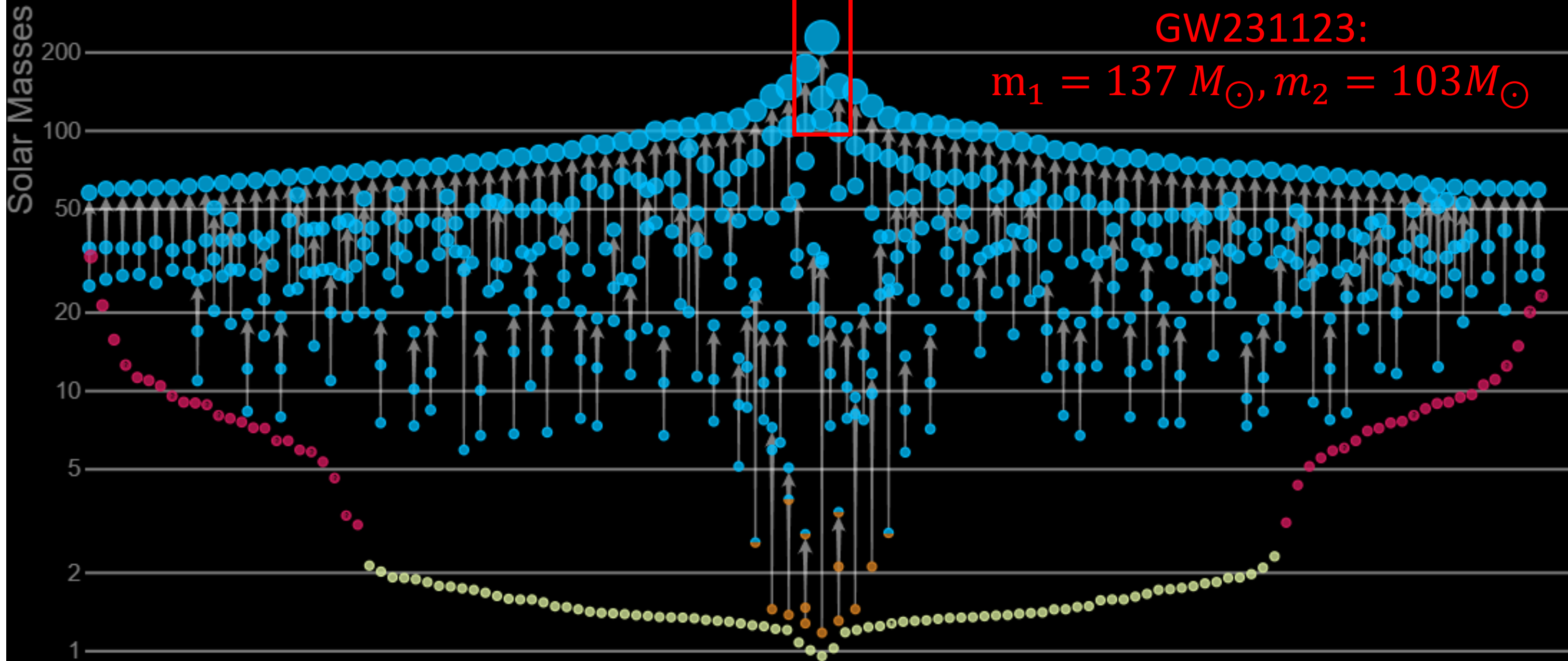


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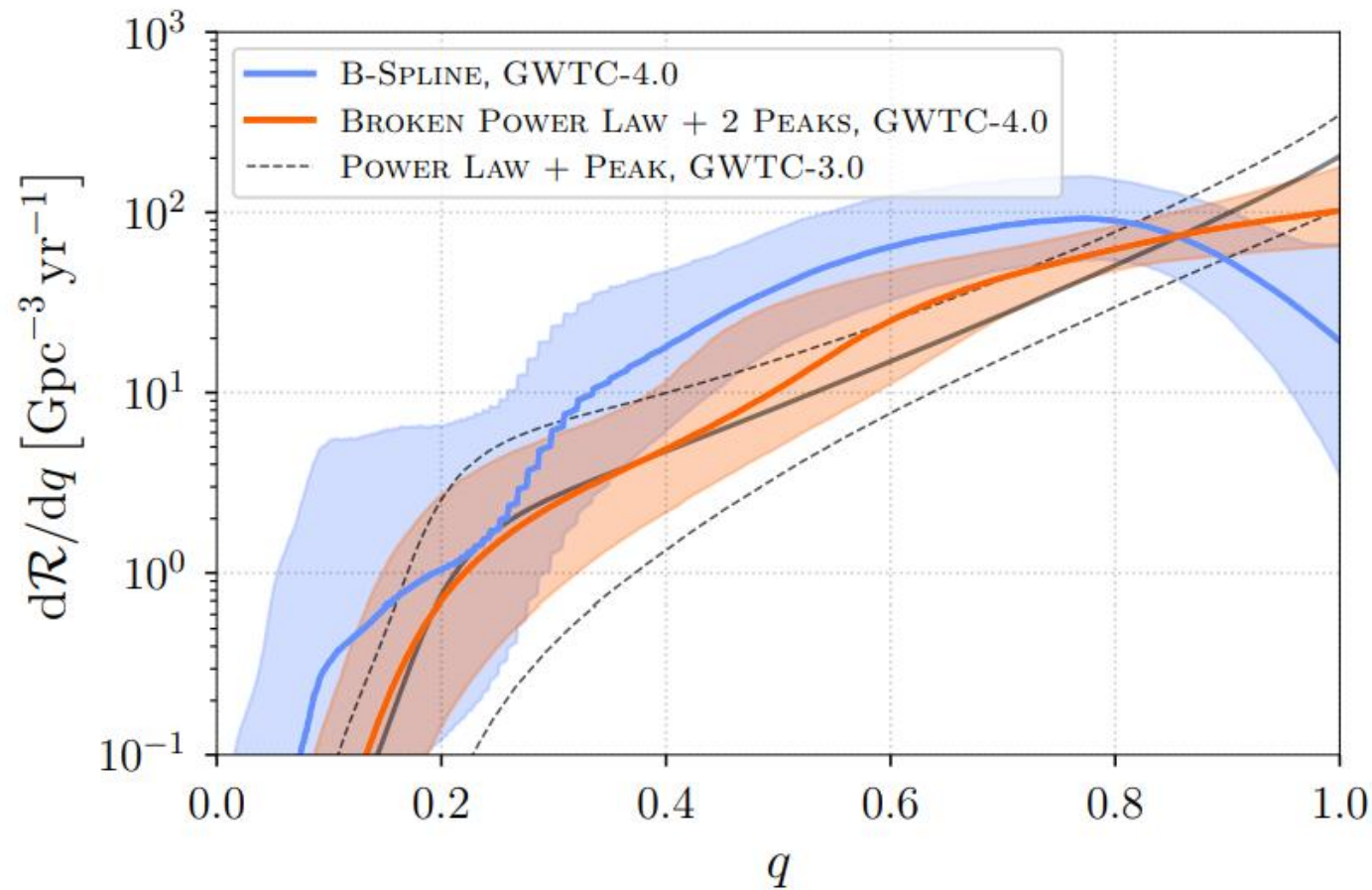
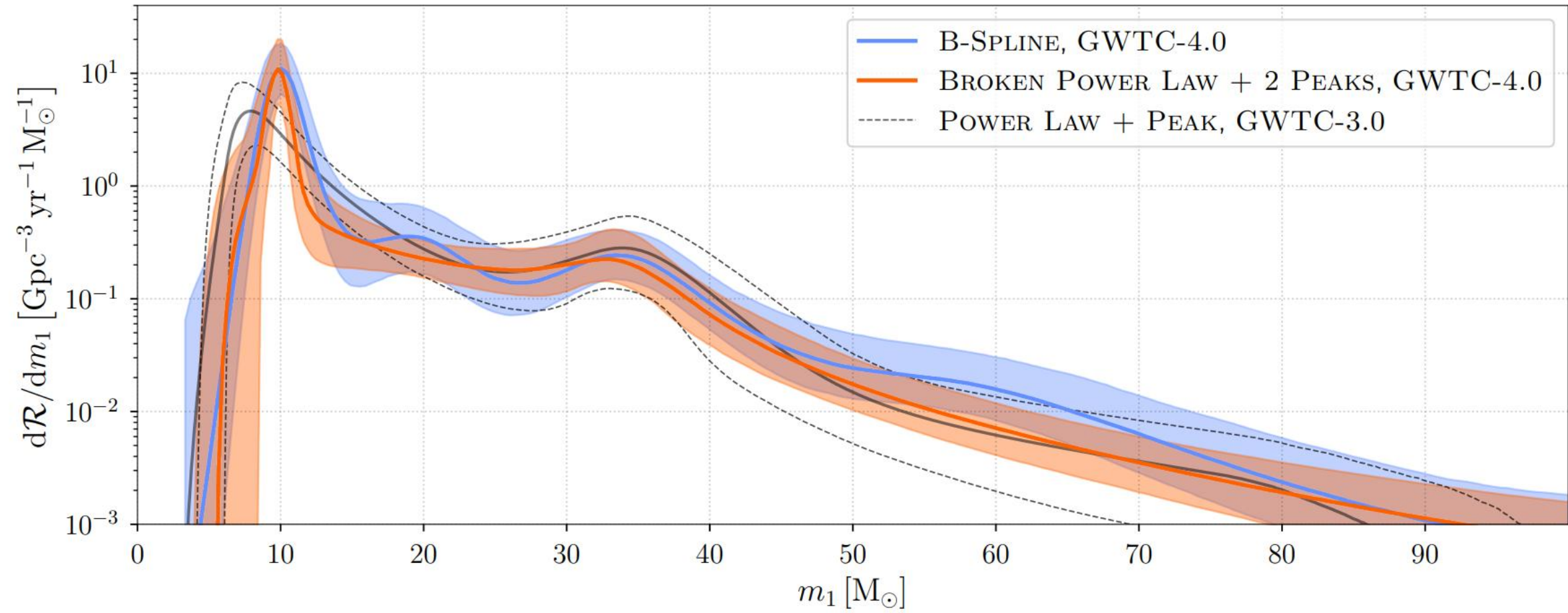
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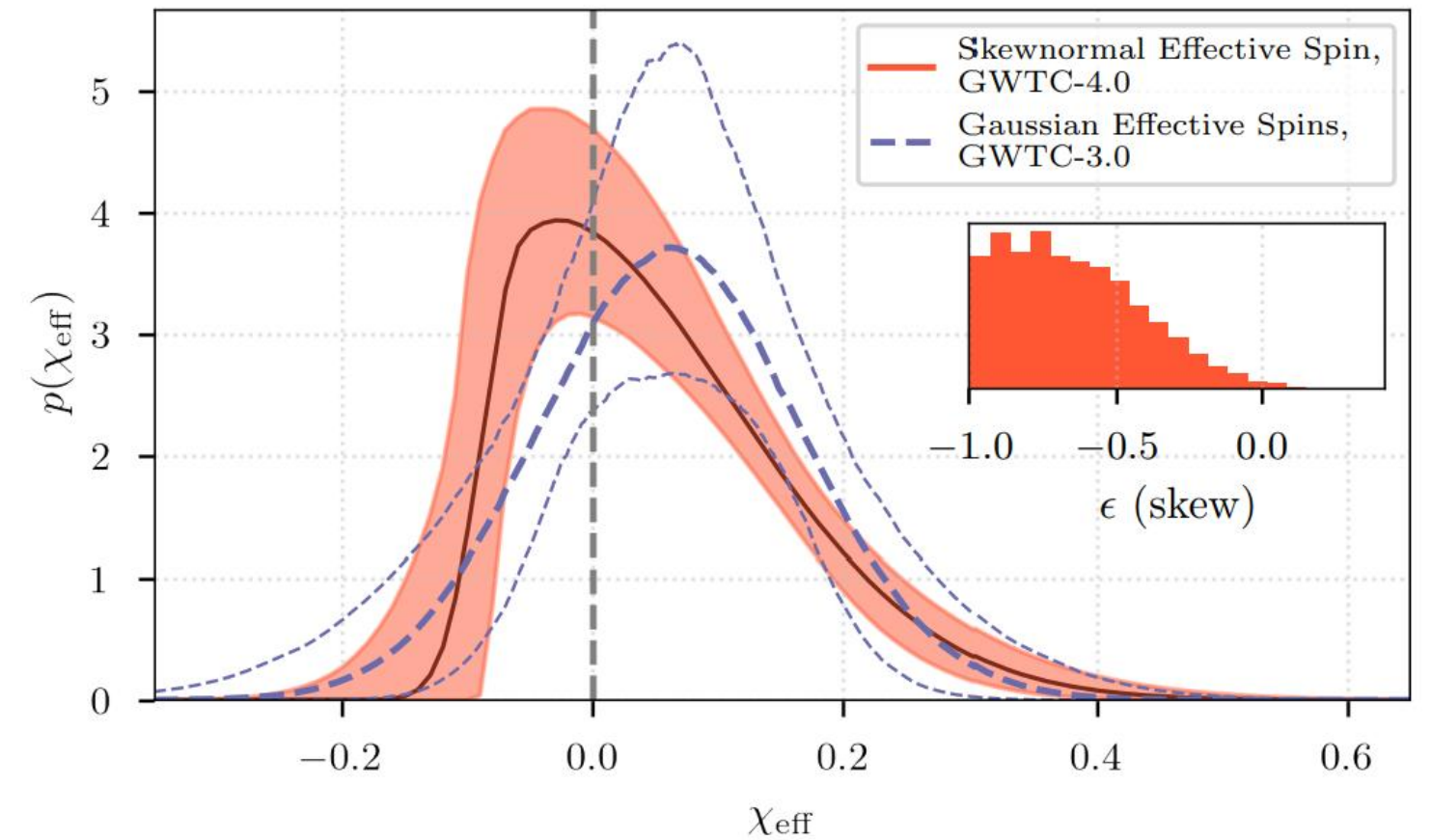
# Population properties inferred from observations



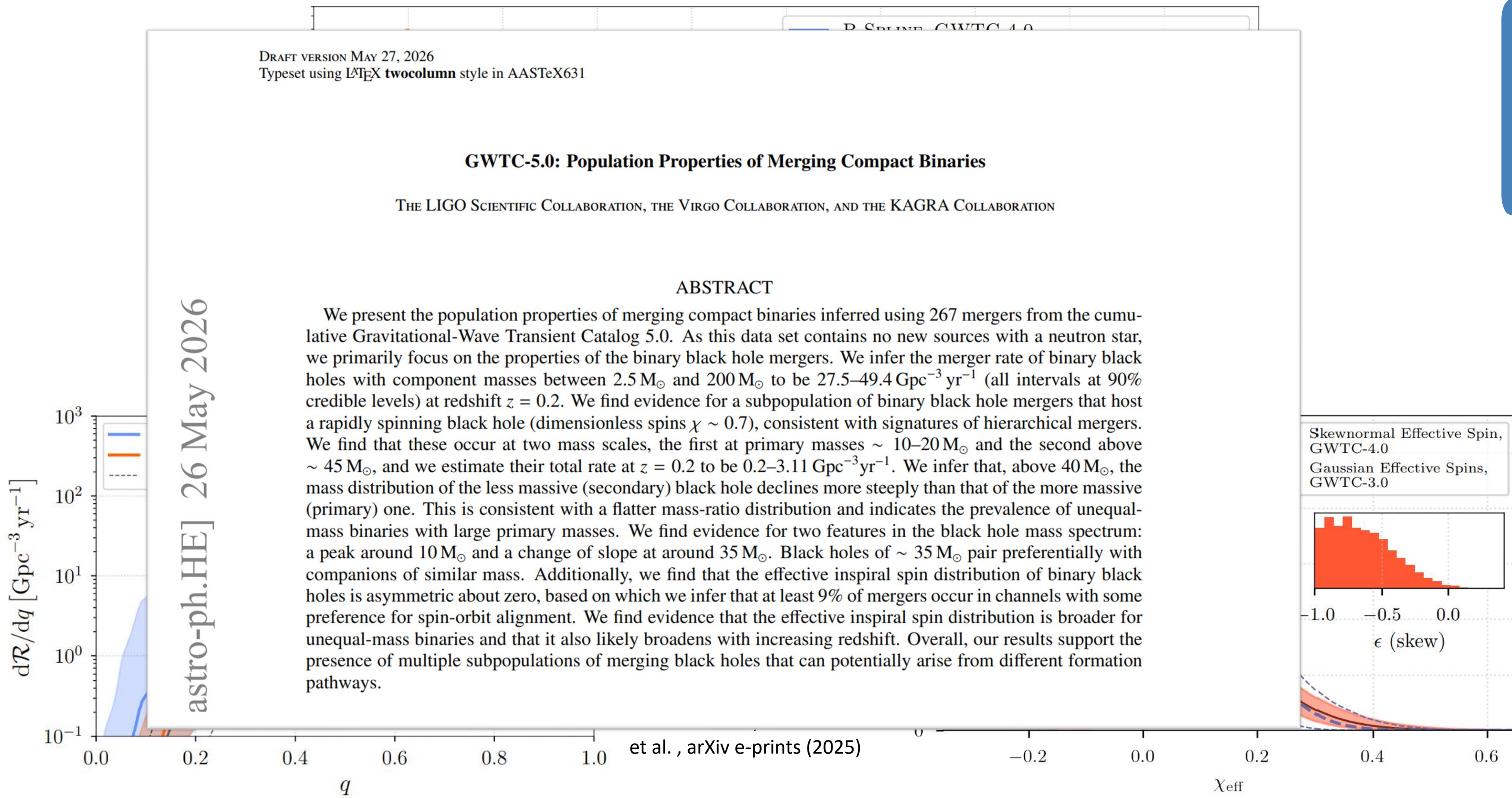
$$q = \frac{m_2}{m_1}, m_1 > m_2$$

$$\chi_{\text{eff}} = \left( \frac{m_1 \vec{\chi}_1 + m_2 \vec{\chi}_2}{m_1 + m_2} \right) \cdot \vec{L}$$

The LIGO Scientific Collaboration, the Virgo Collaboration, the KAGRA Collaboration, A.G. Abac, I. Abouelfettouh, F. Acernese et al., arXiv e-prints (2025)

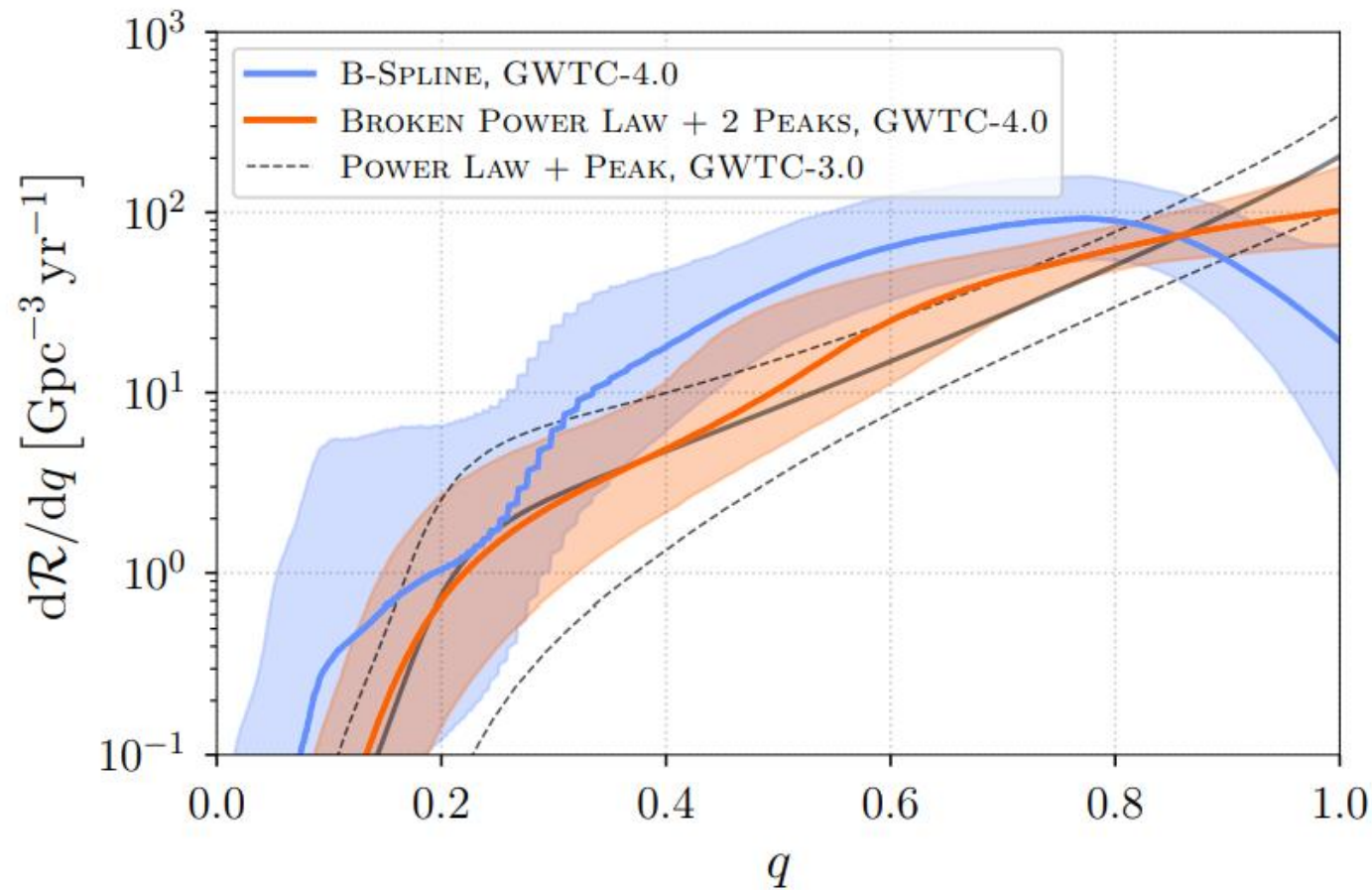
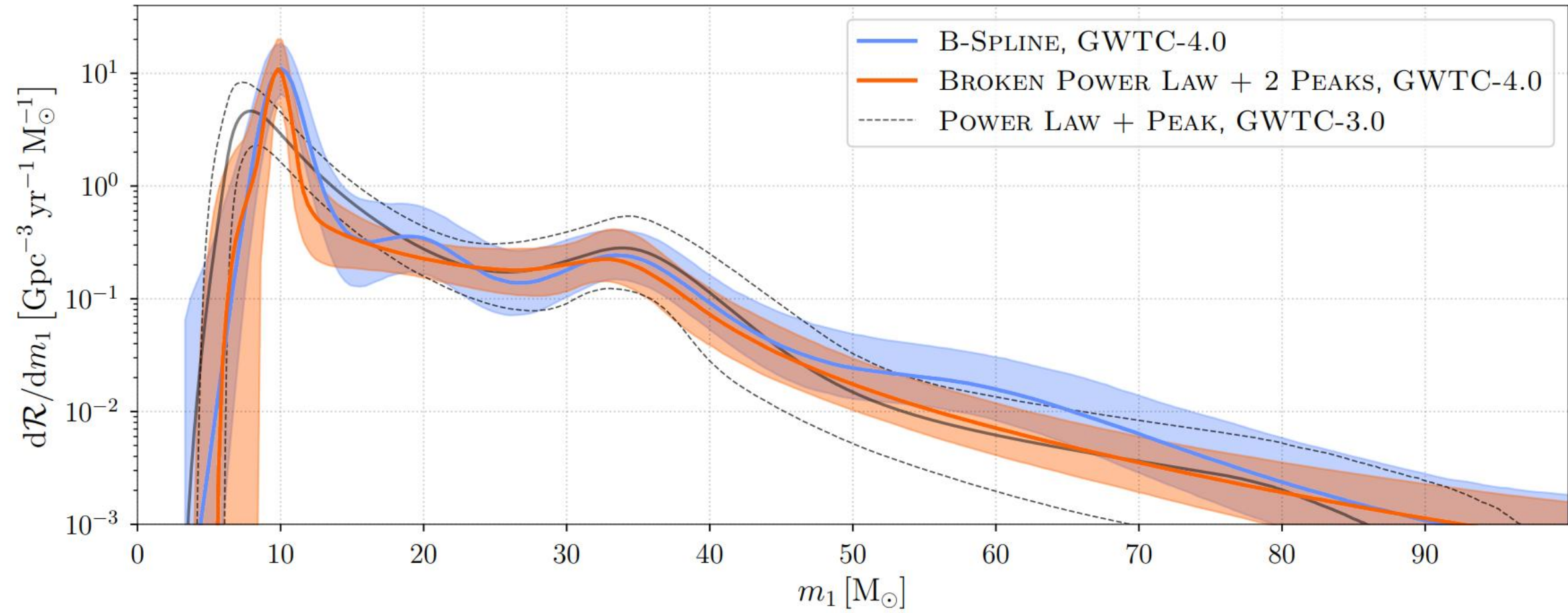


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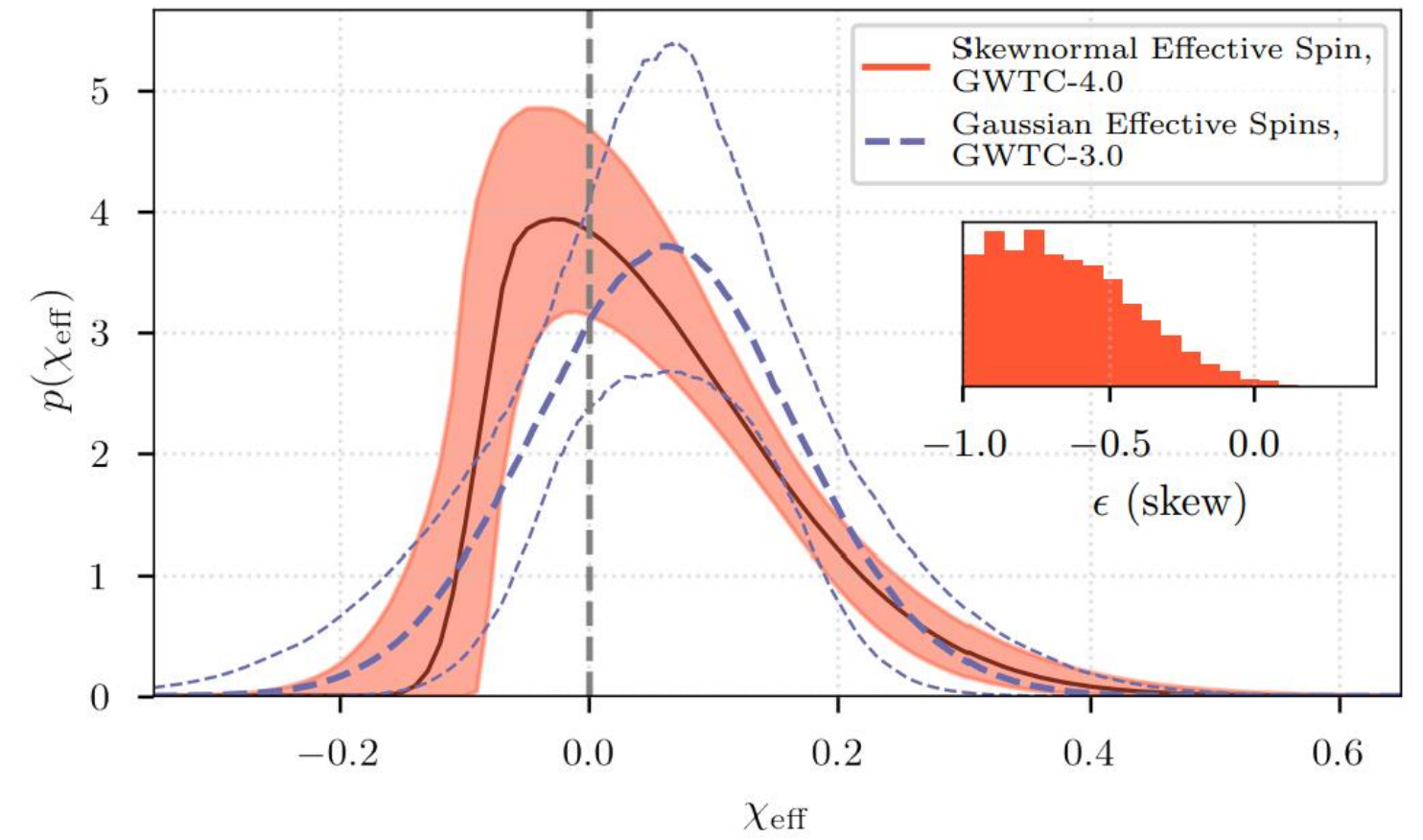
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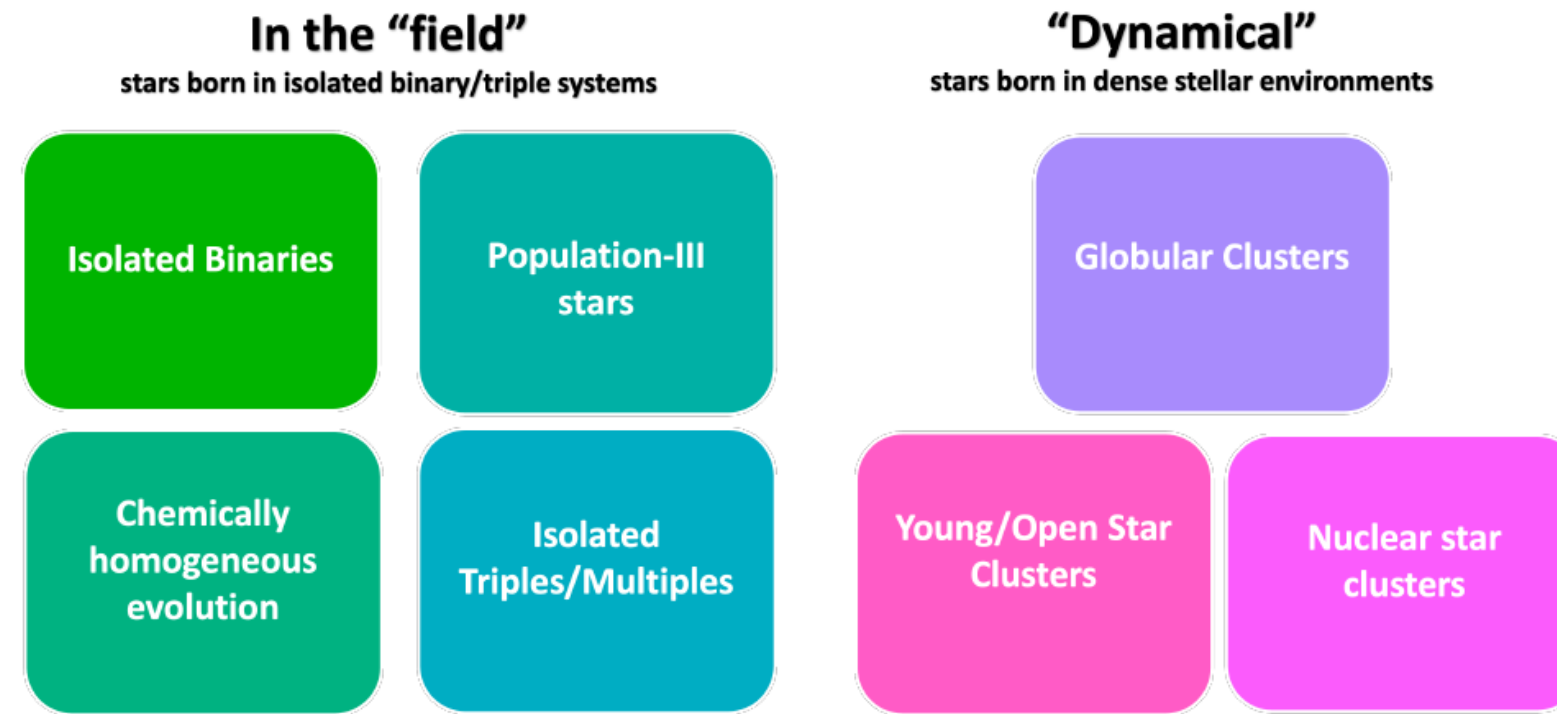
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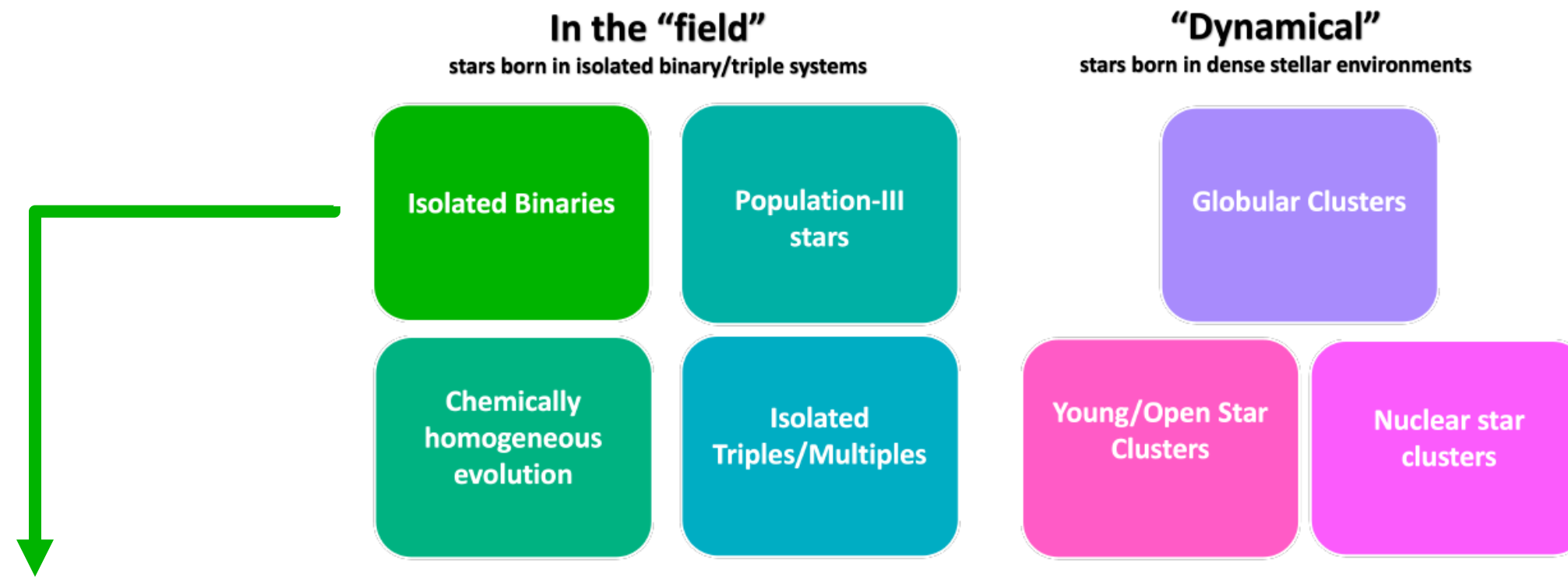


# Formation channels

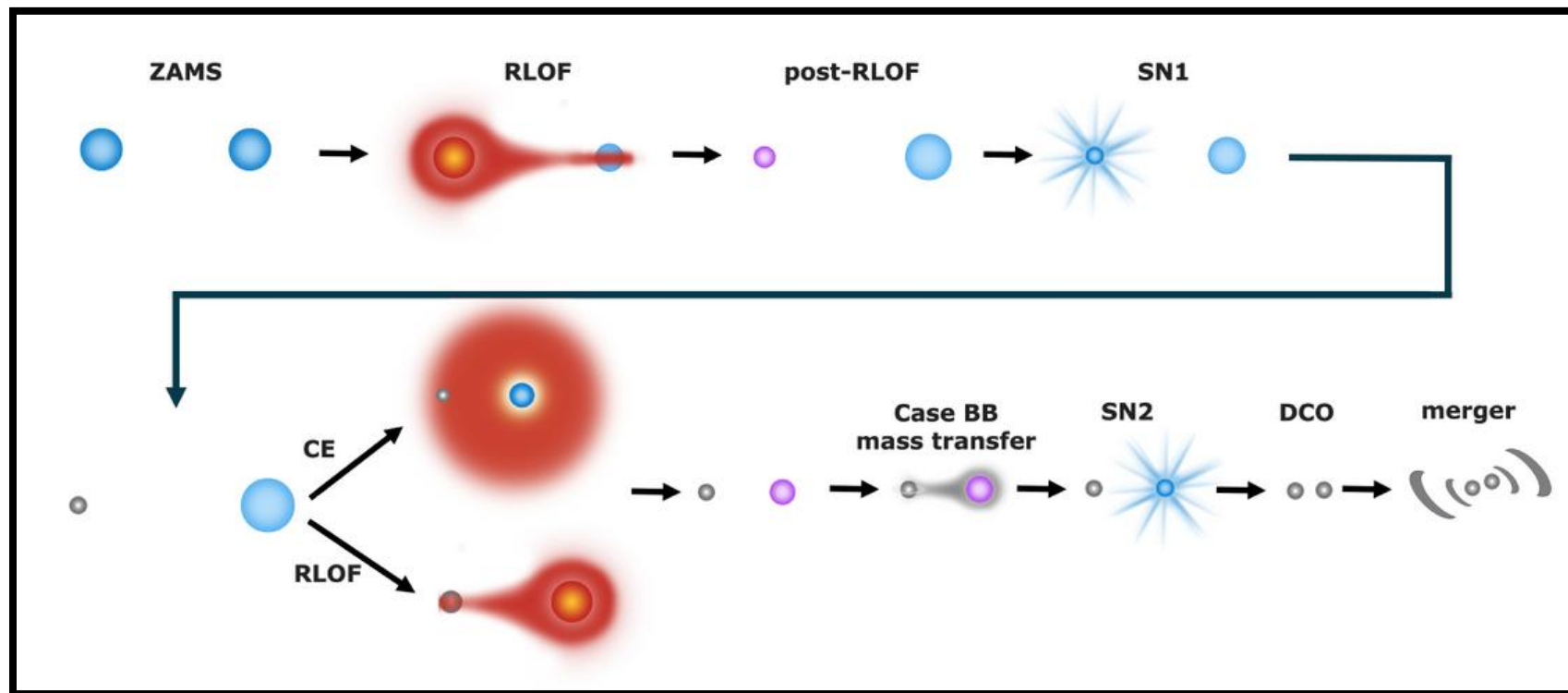


Costa G., et al., arXiv:2311.15778

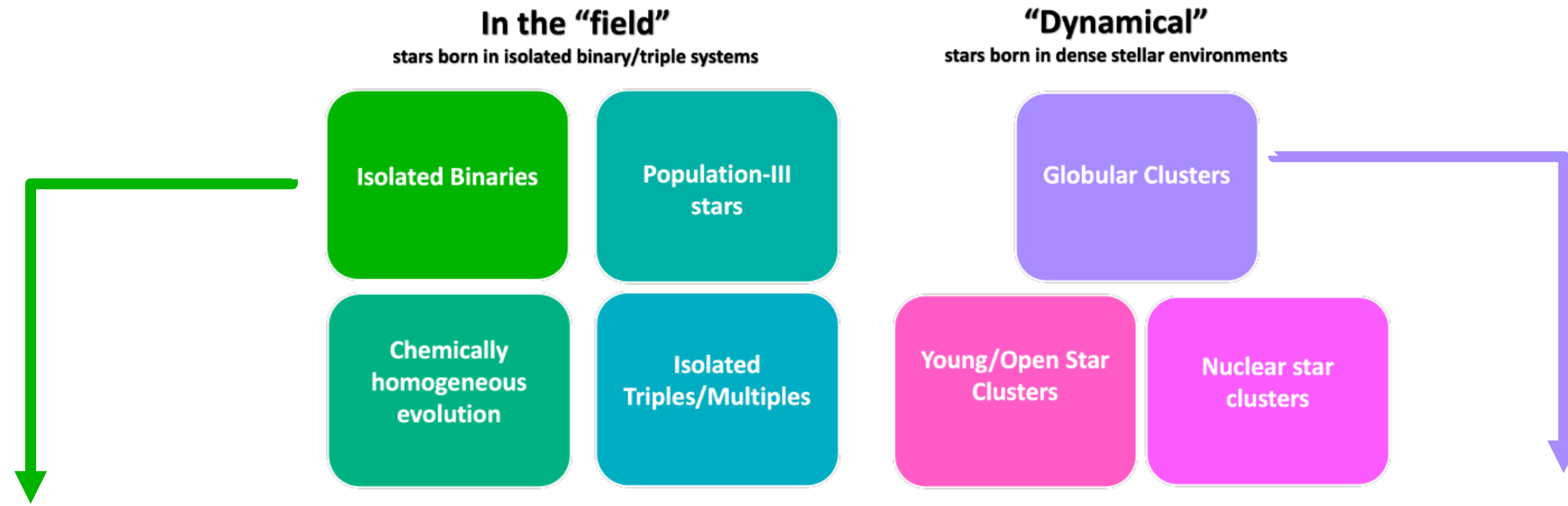
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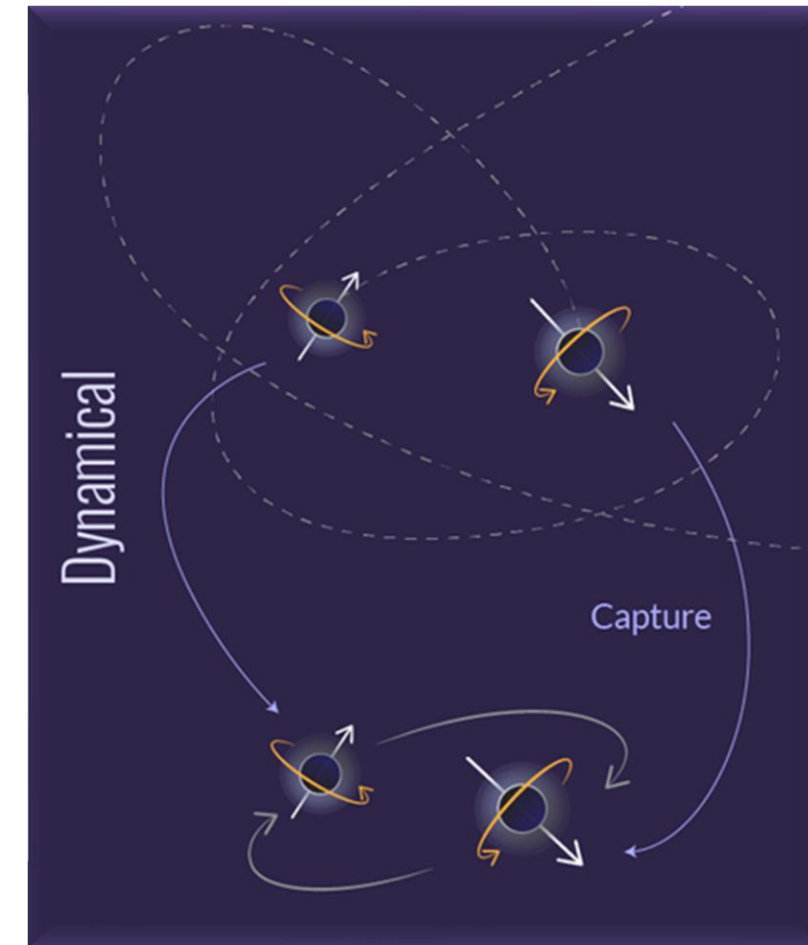
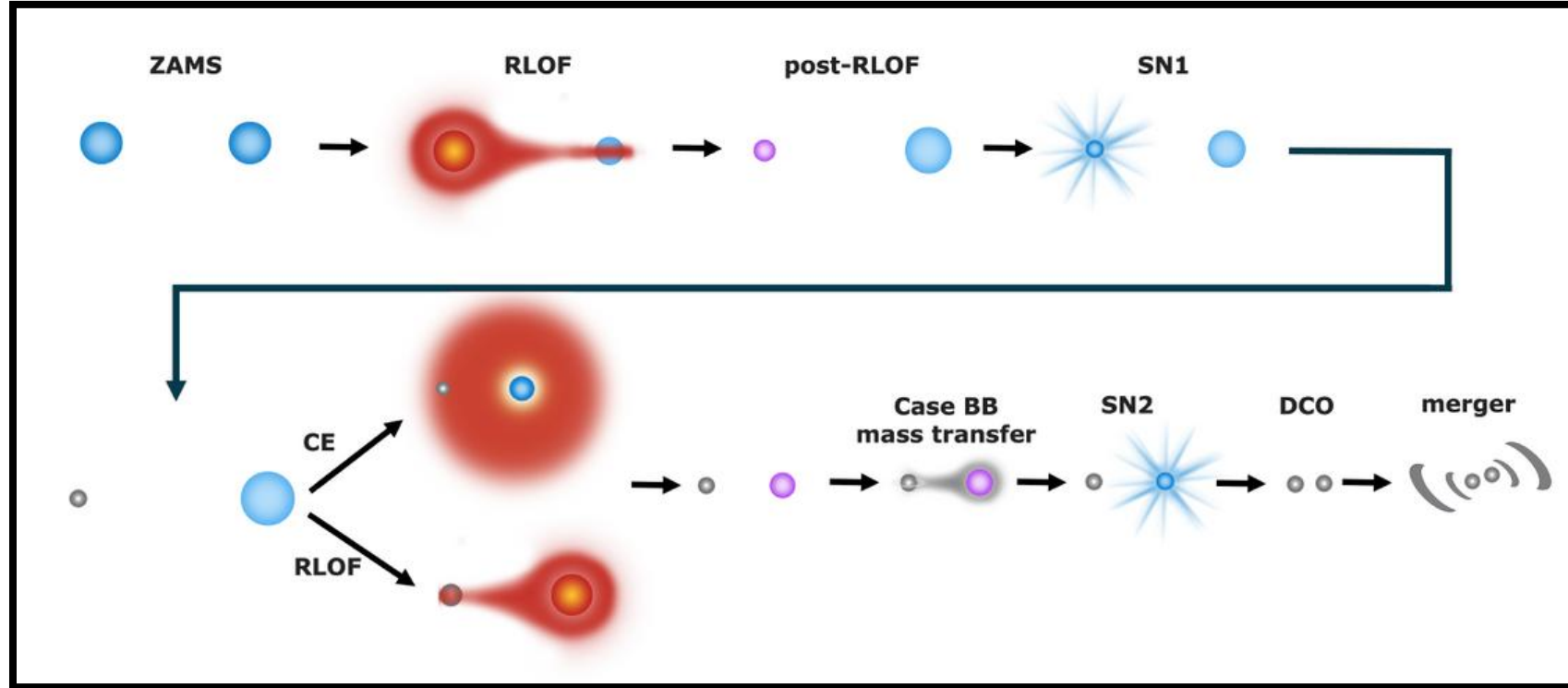
Costa G., et al., arXiv:2311.15778



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Costa G., et al., arXiv:2311.15778



# Formation channels

## In the "field"

stars born in isolated binary/triple systems

Isolated Binaries

Population-III stars

Chemically homogeneous evolution

Isolated Triples/Multiples

## "Dynamical"

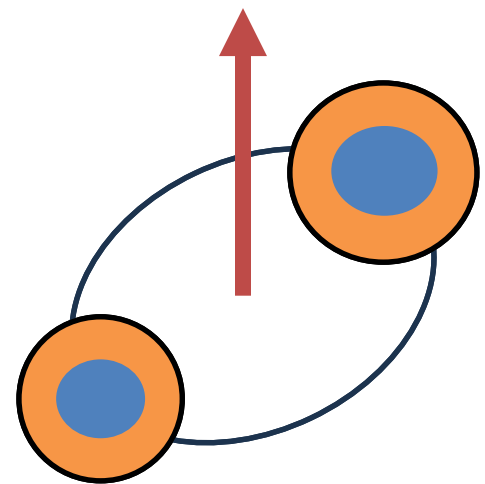
stars born in dense stellar environments

Globular Clusters

Young/Open Star Clusters

Nuclear star clusters

# CHE



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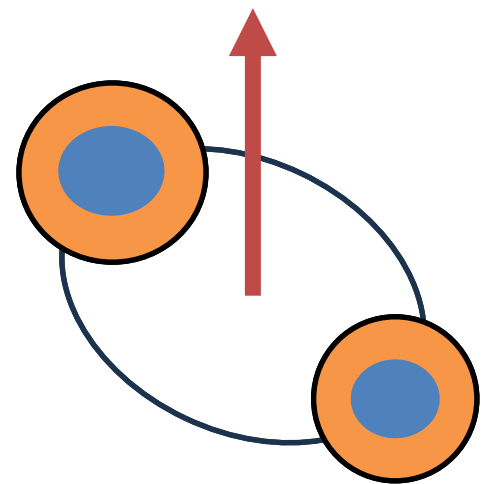
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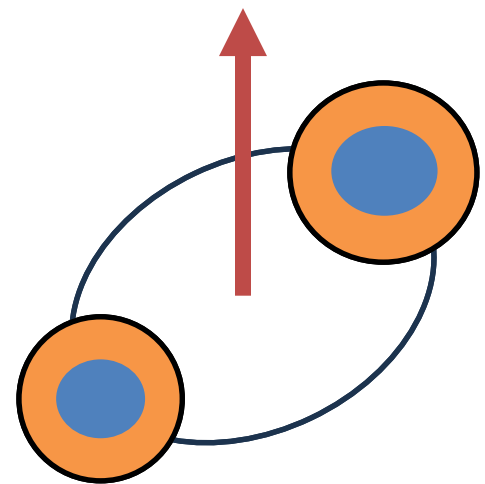
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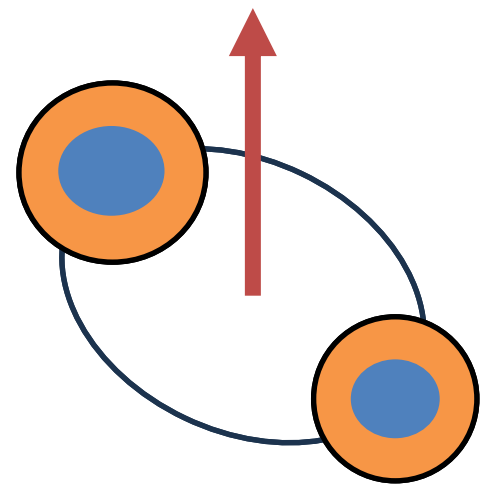
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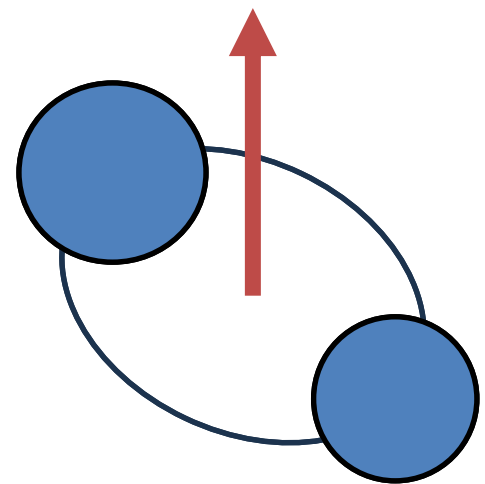
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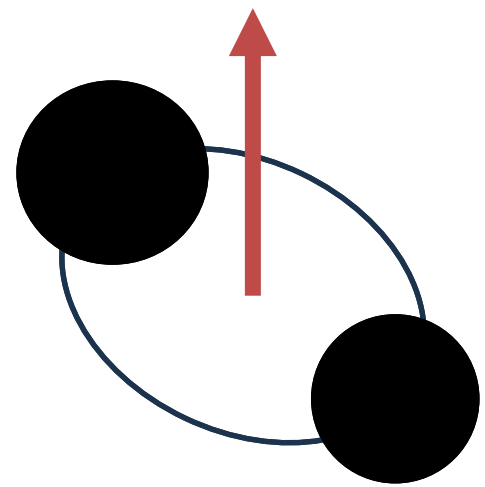
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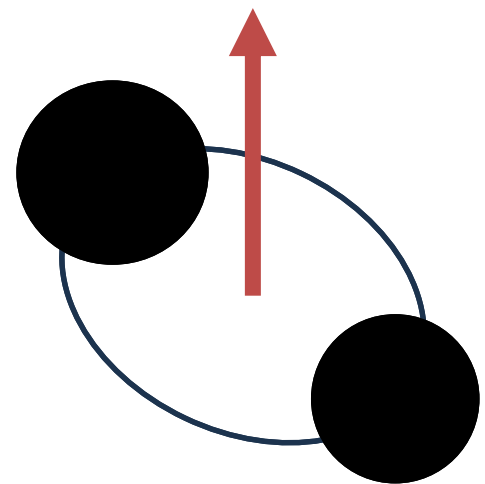
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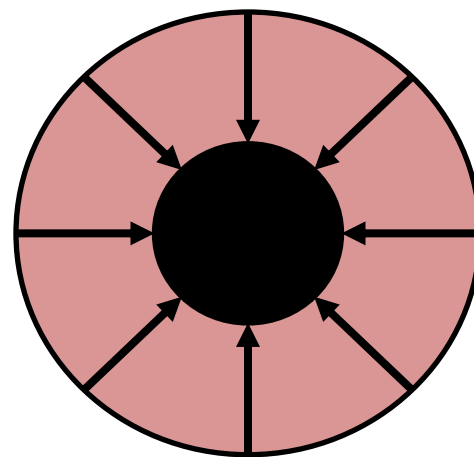
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# Pop III



# Formation channels

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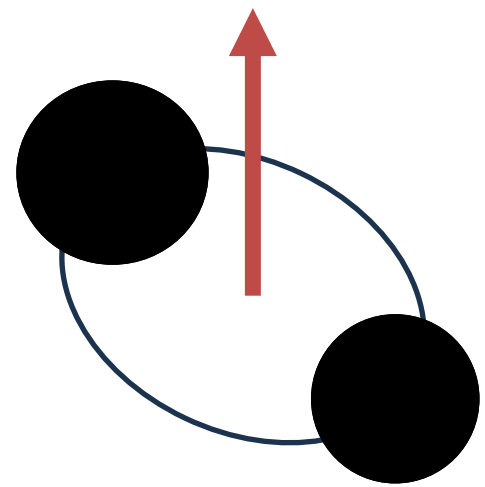
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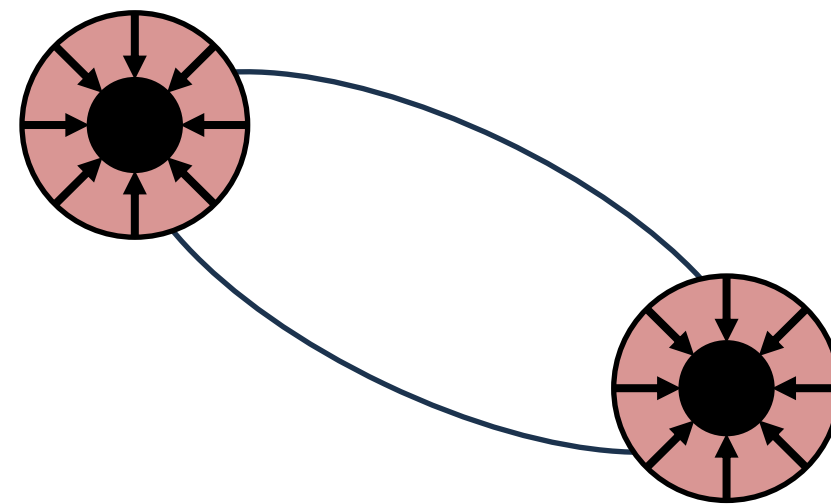
Young/Open Star Clusters

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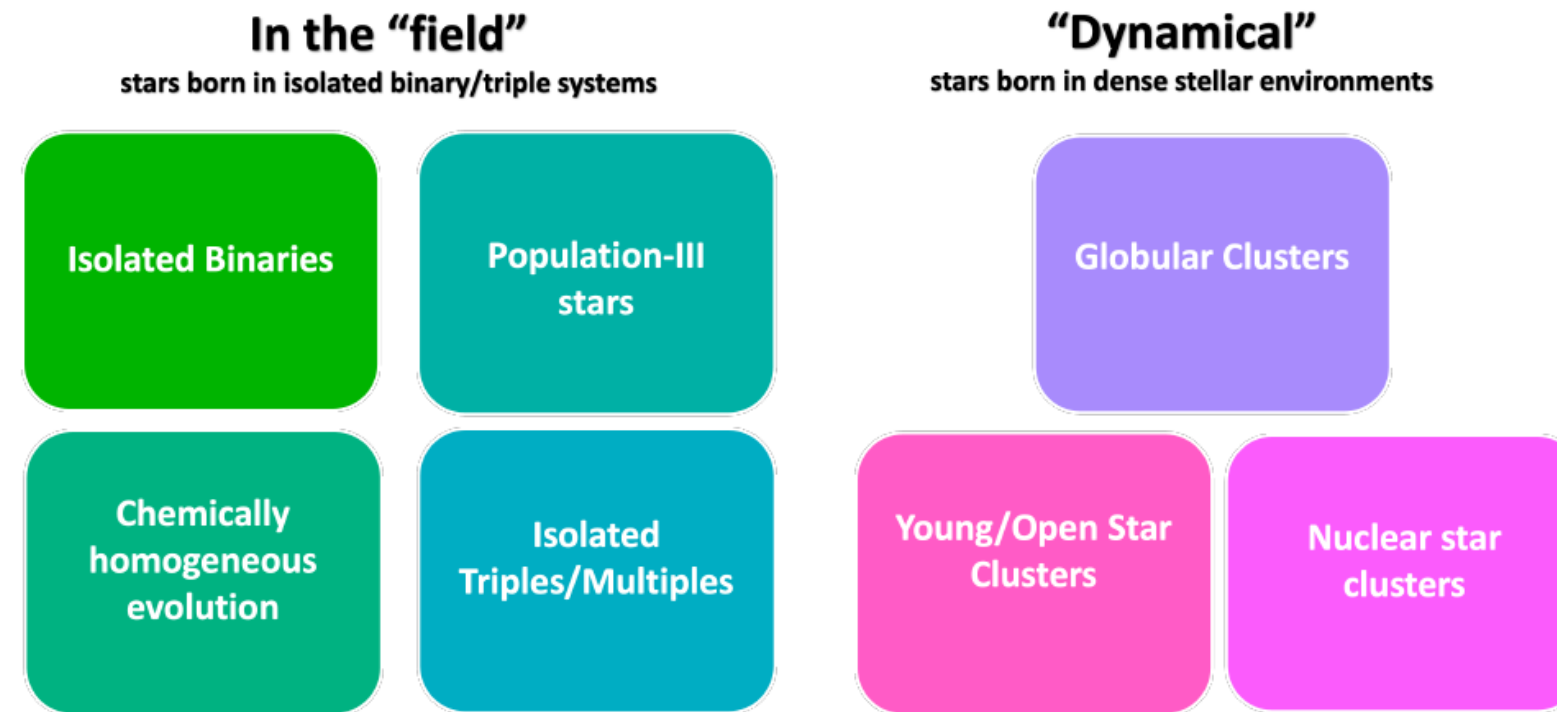
# CHE



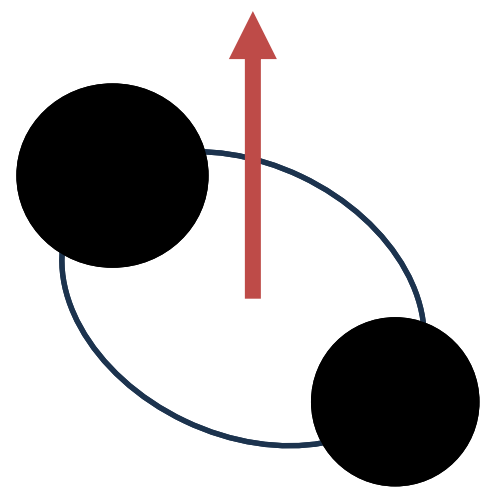
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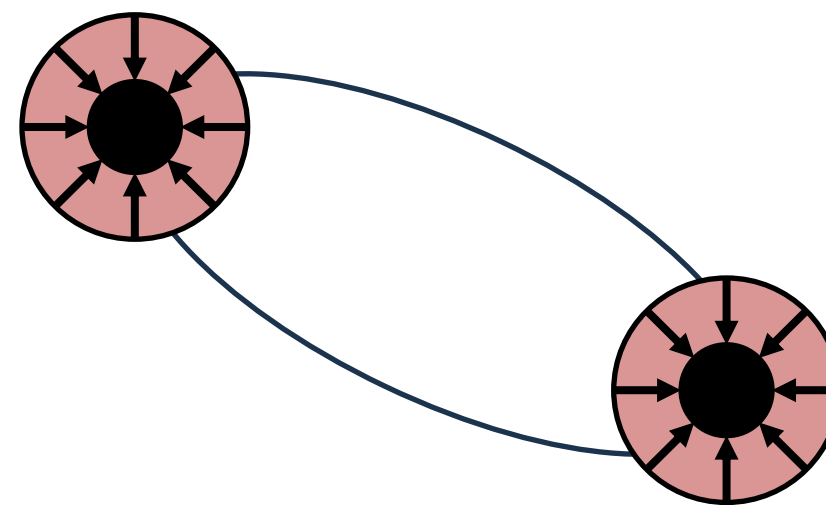
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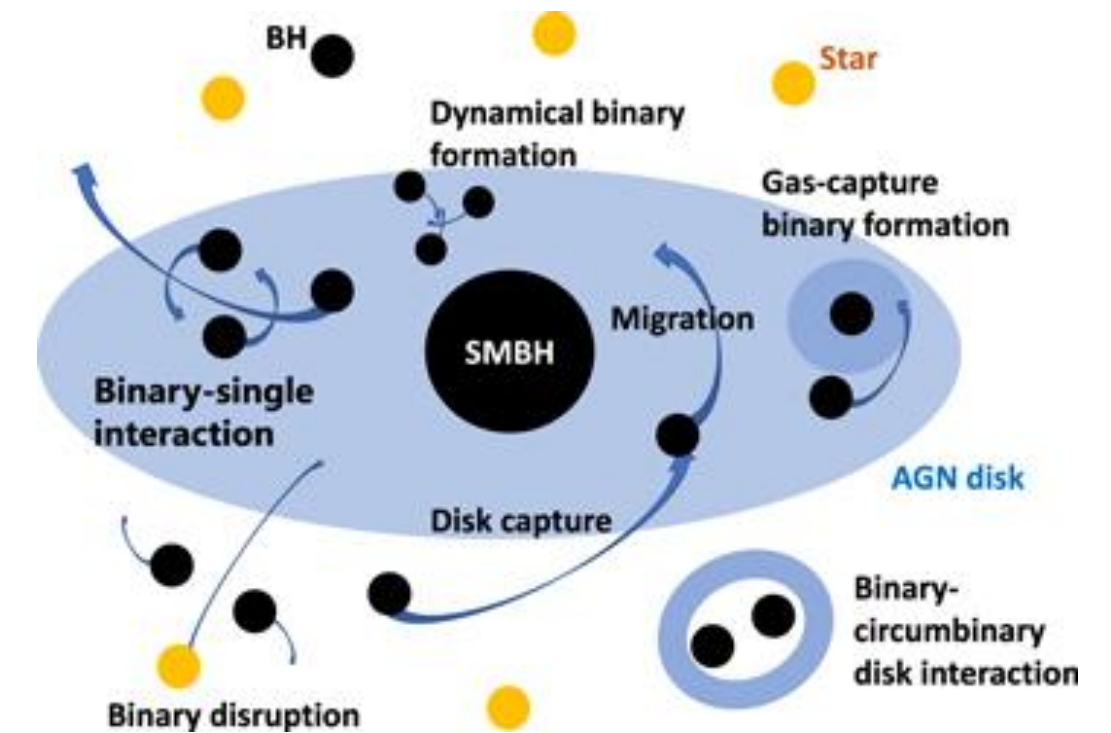
## CHE



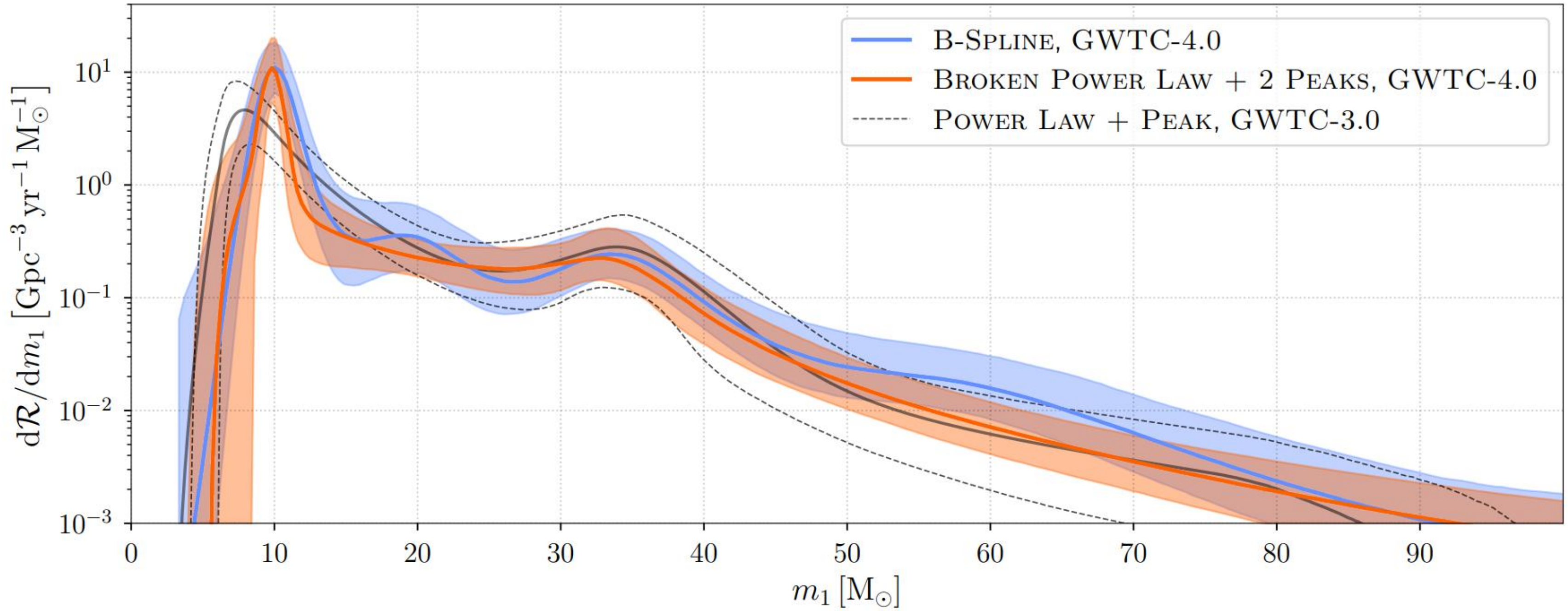
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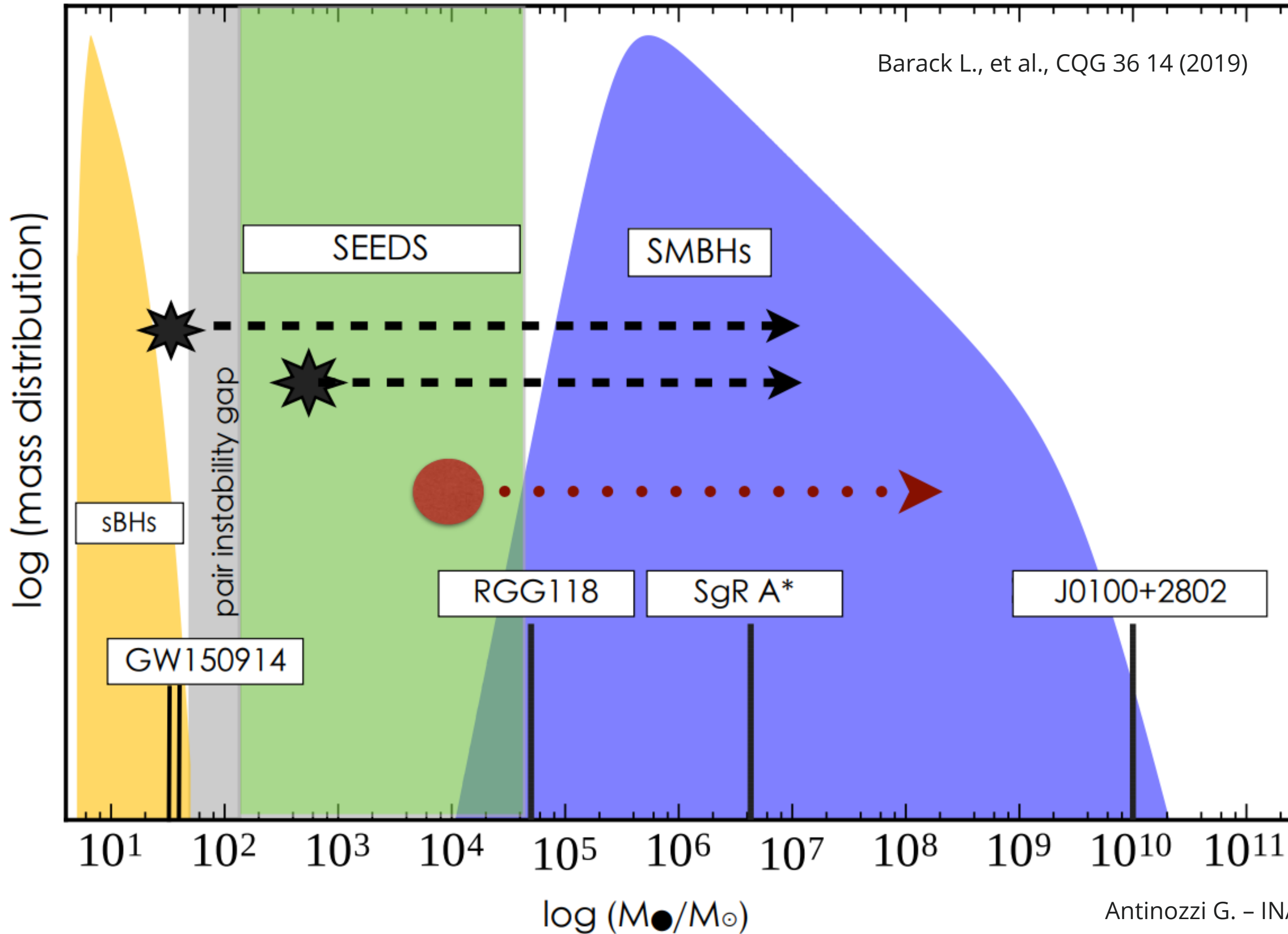
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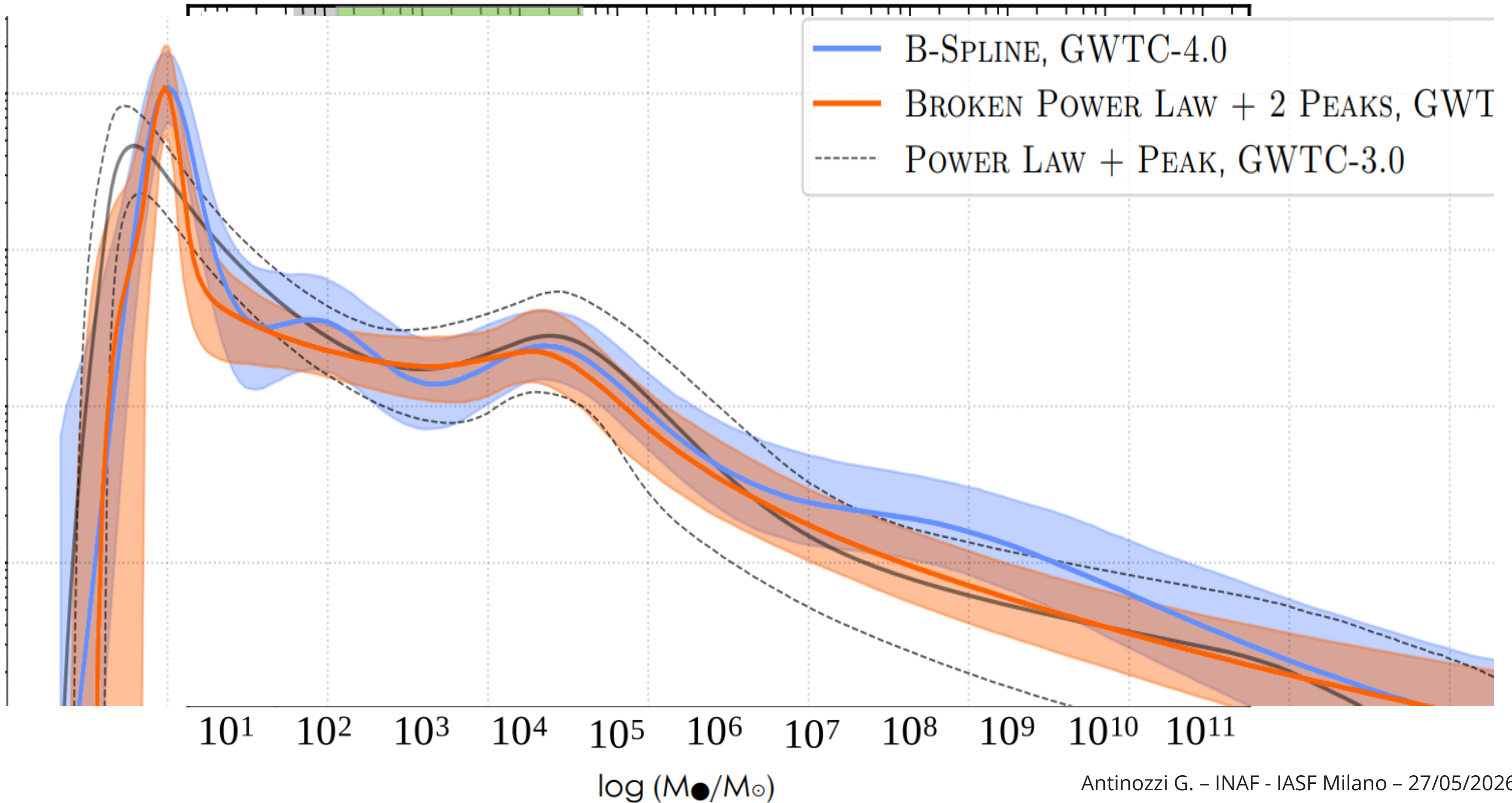
# BH Mass regimes



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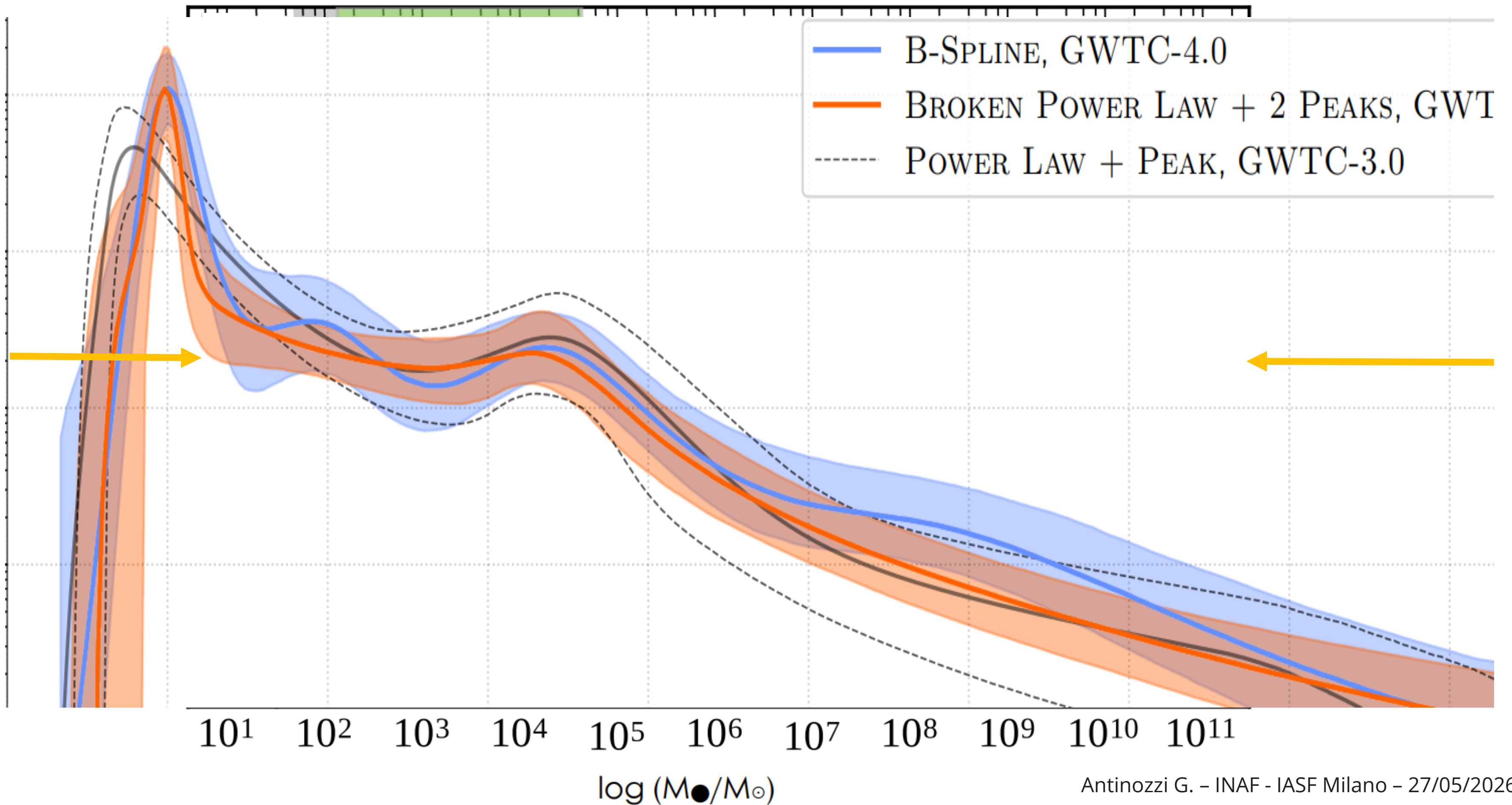


# BH Mass regimes

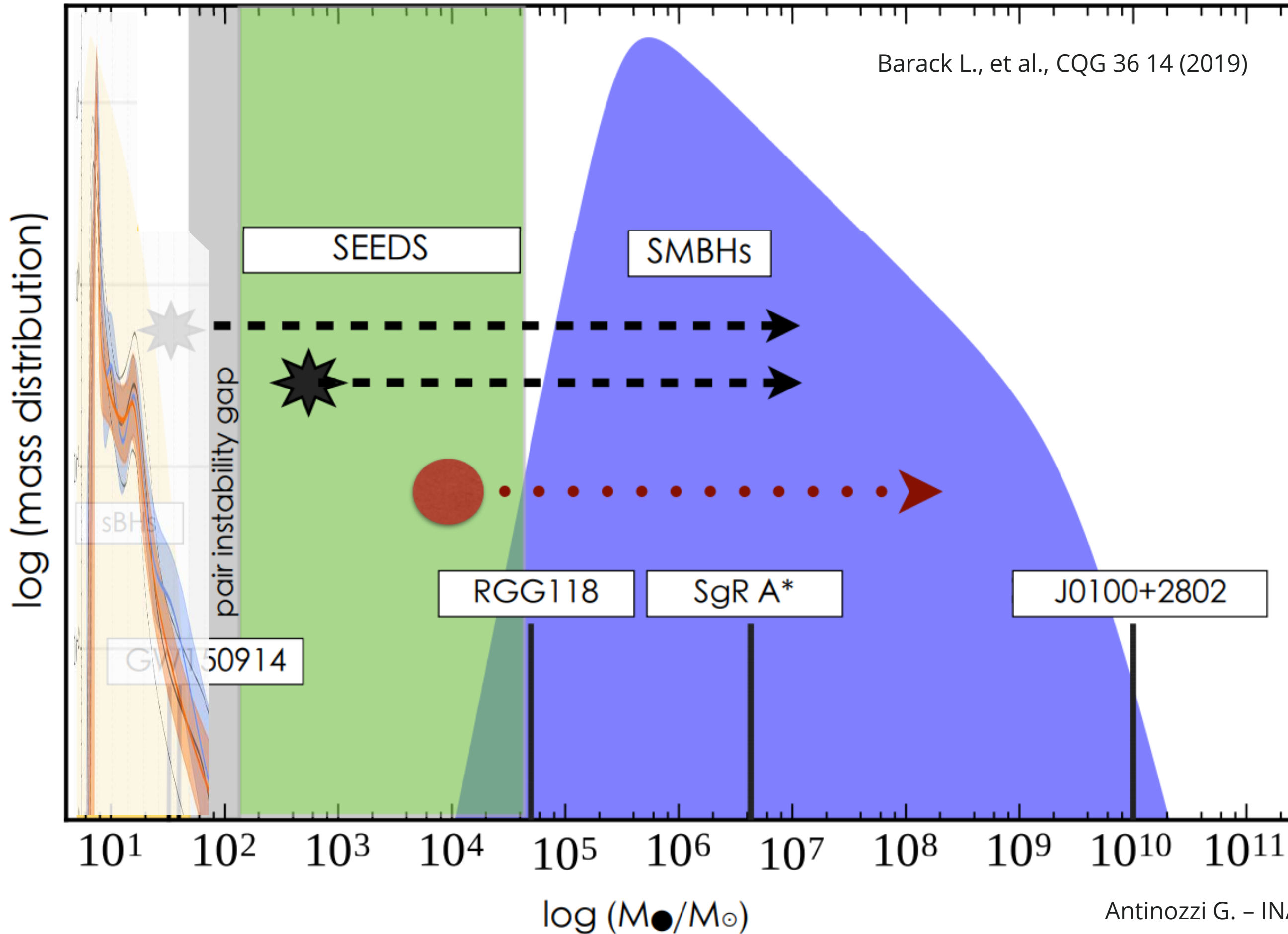


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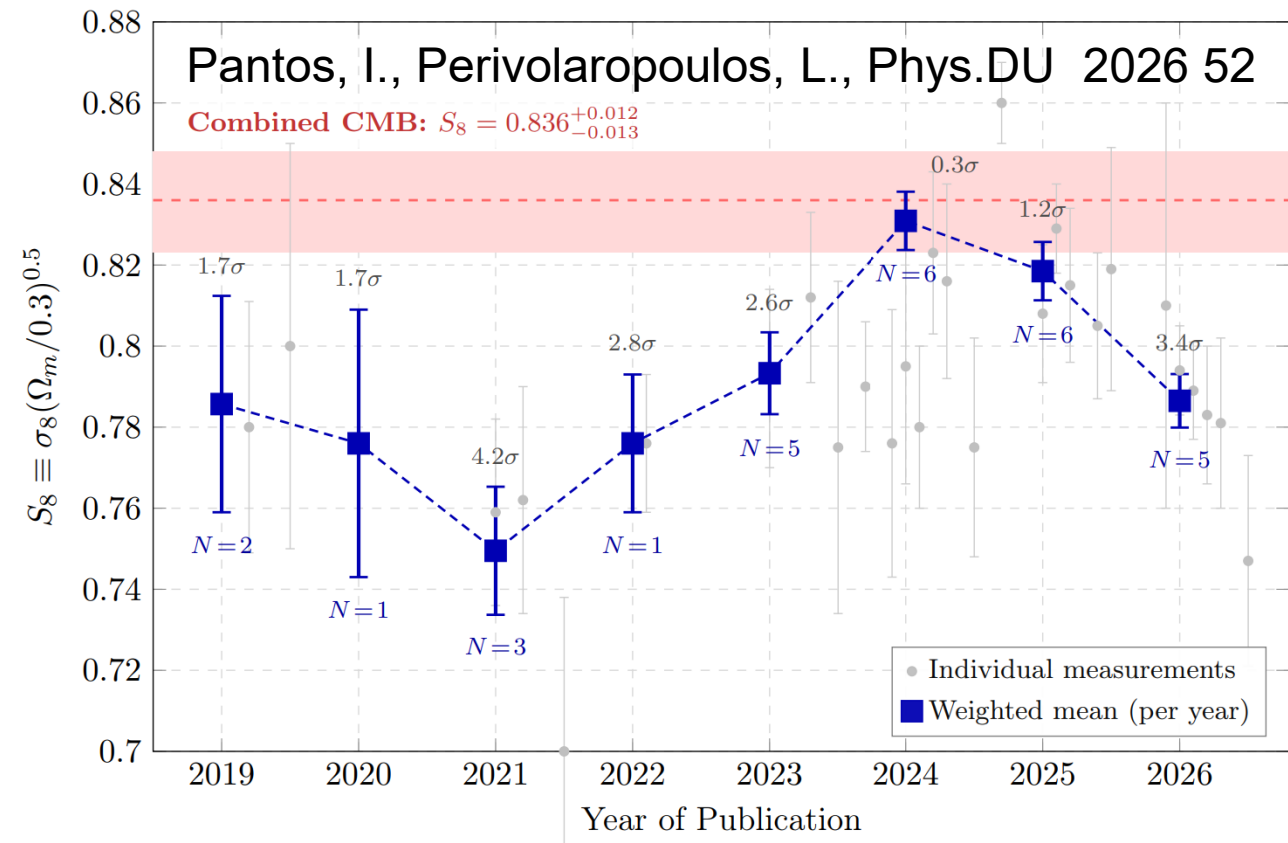
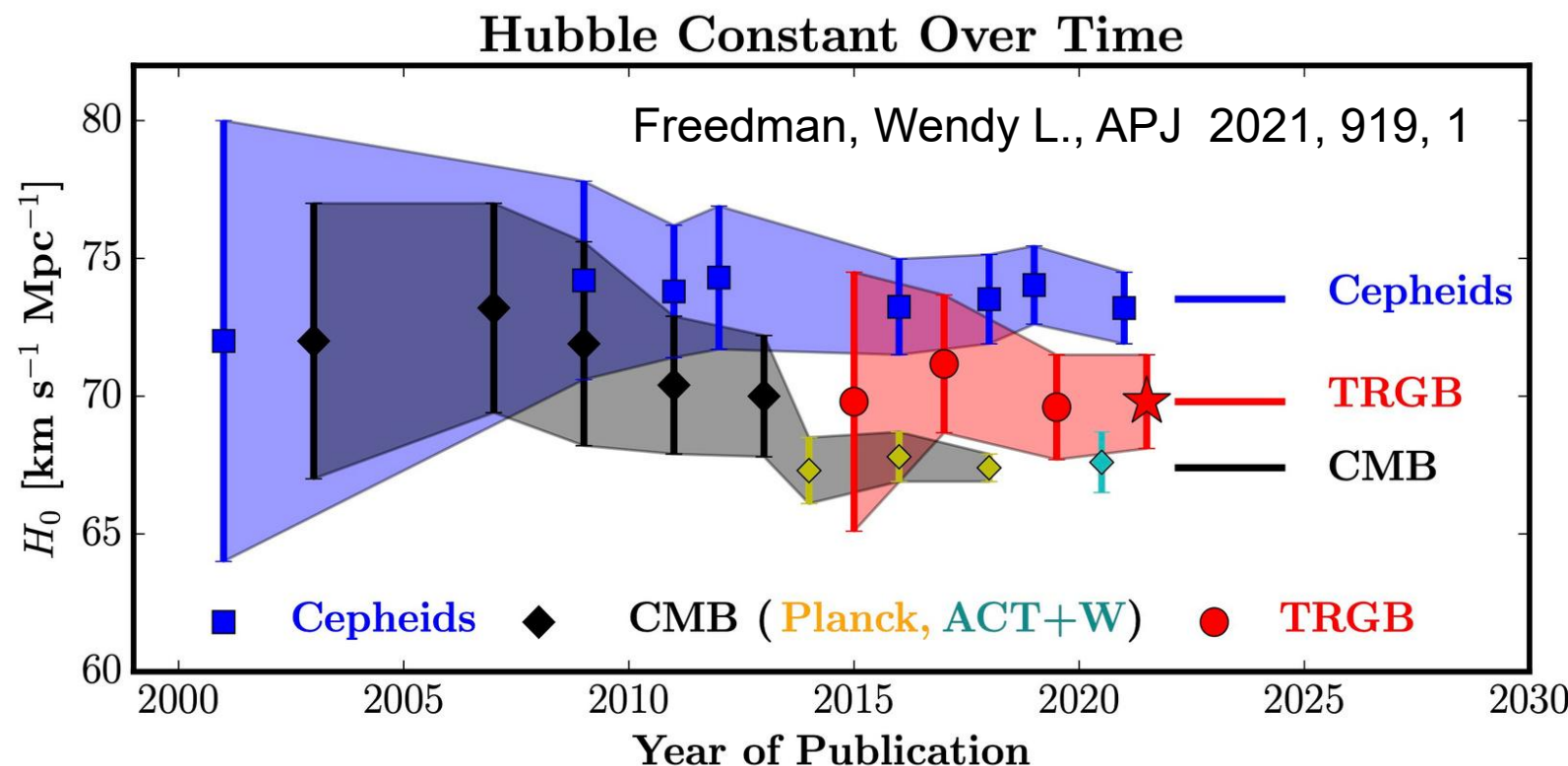


# Cosmological tensions and anomalies

Tensions ( $> 5\sigma$ ):

a) Hubble tension  $H_0$

b) Growth tension  $S_8 = \sigma_8 \left( \frac{\Omega_m}{0.3} \right)^{0.5}$



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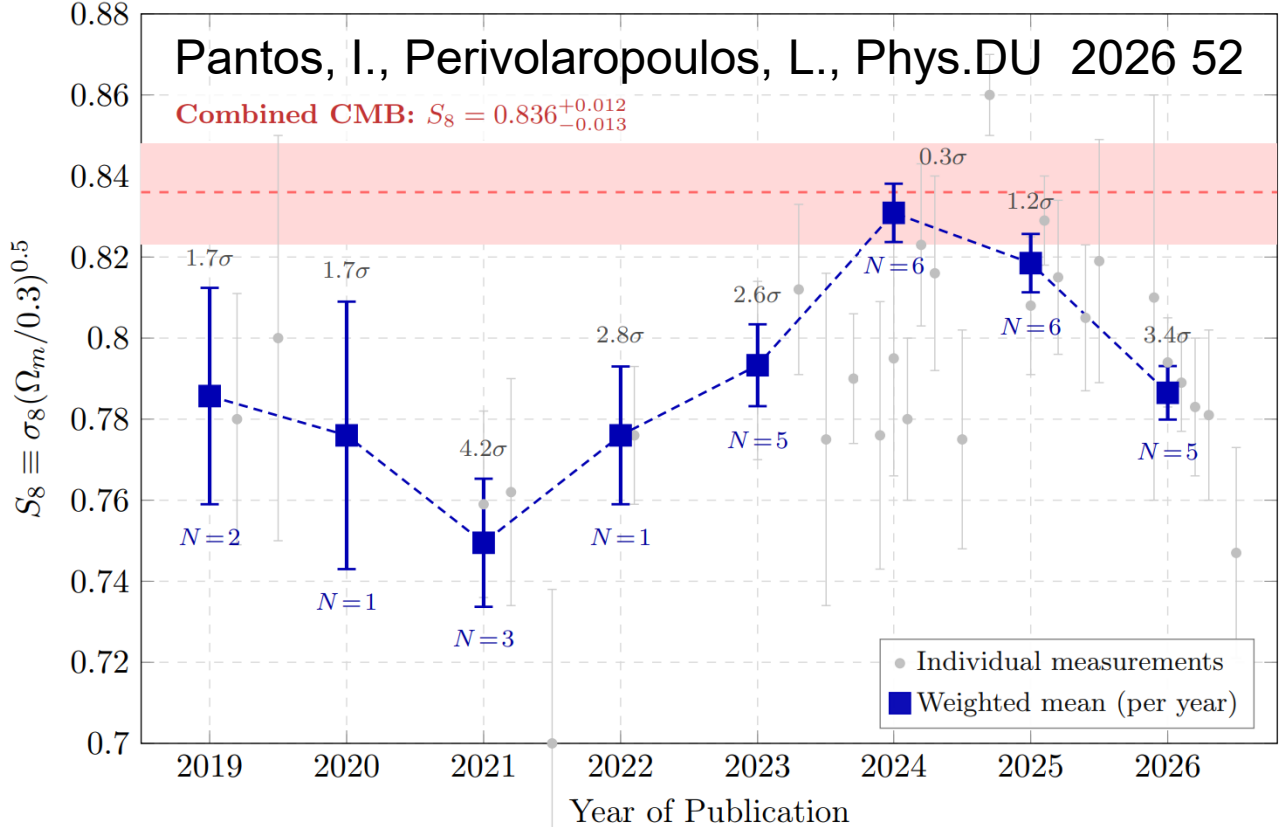
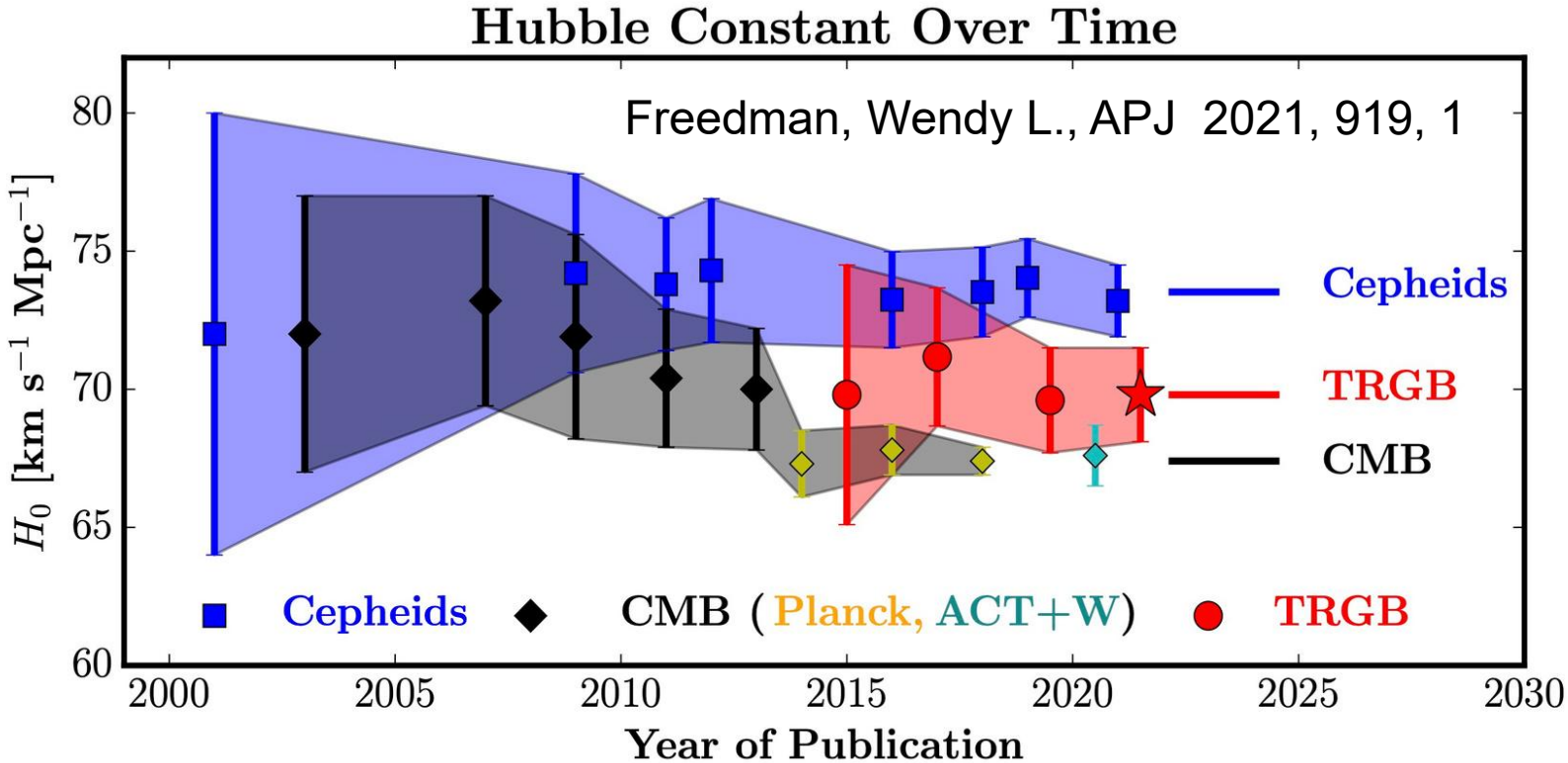
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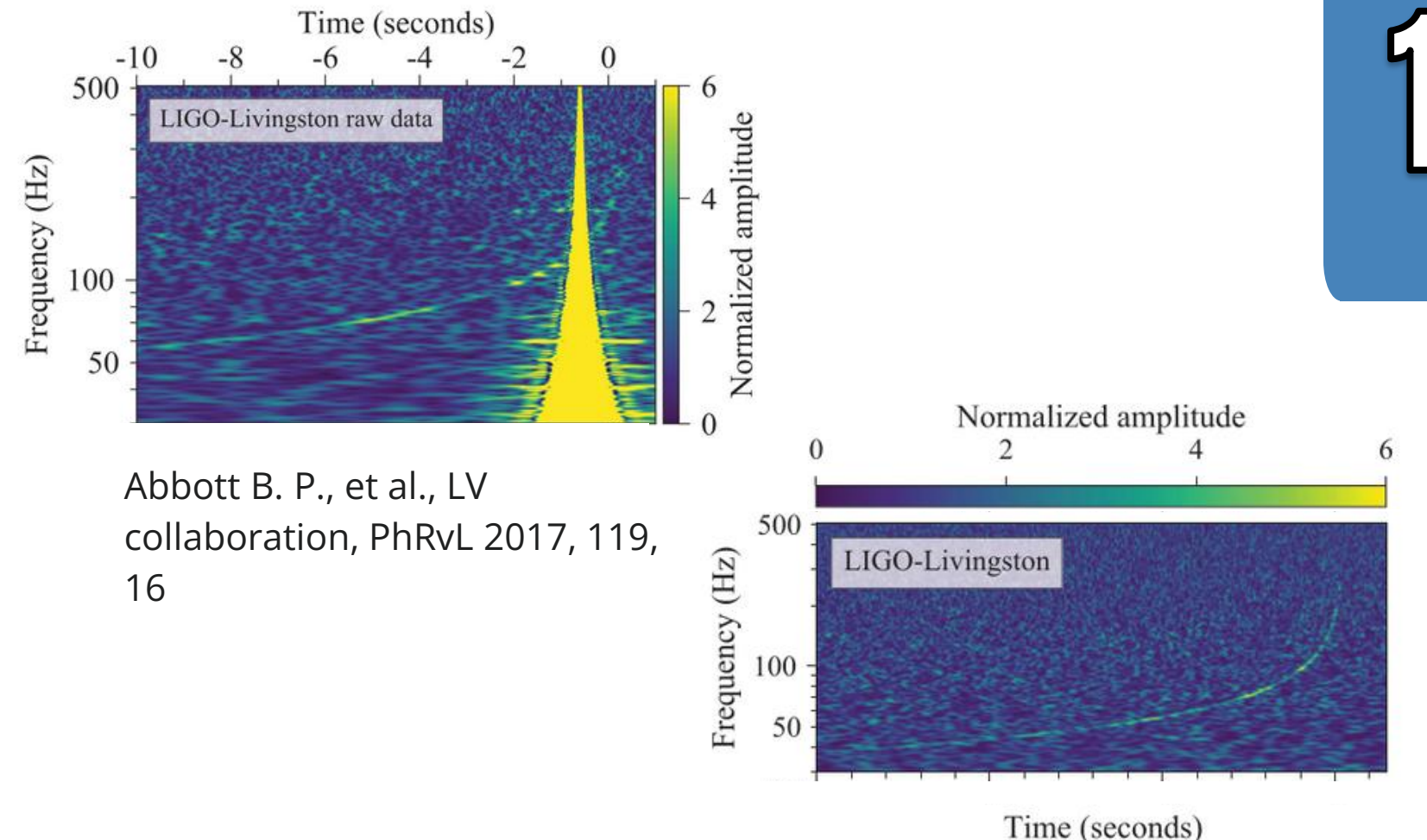
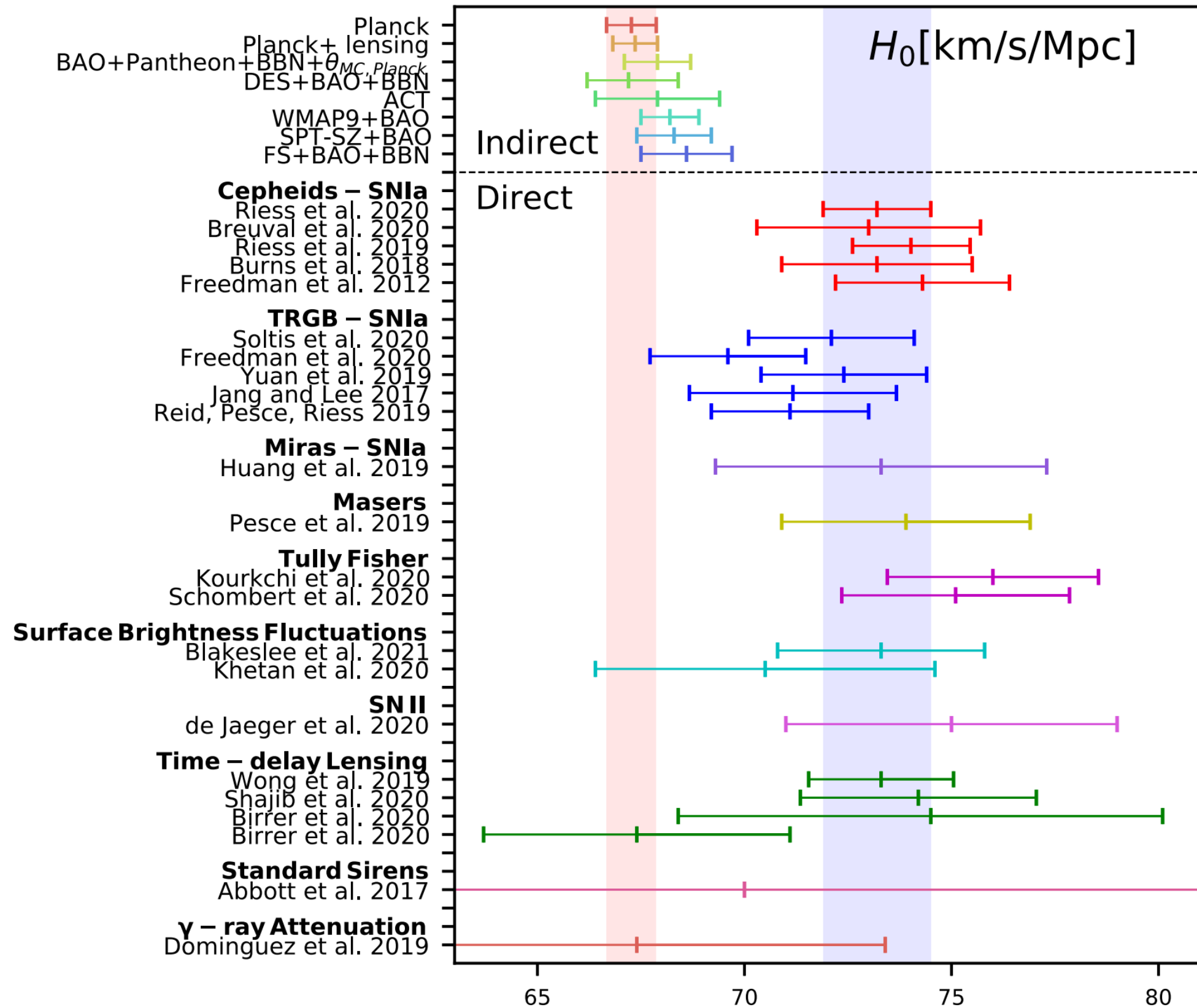
Anomalies/Curiosities ( $3 - 5 \sigma$ ):

- BBN Lithium anomaly
- Core-Cusp problem
- ...



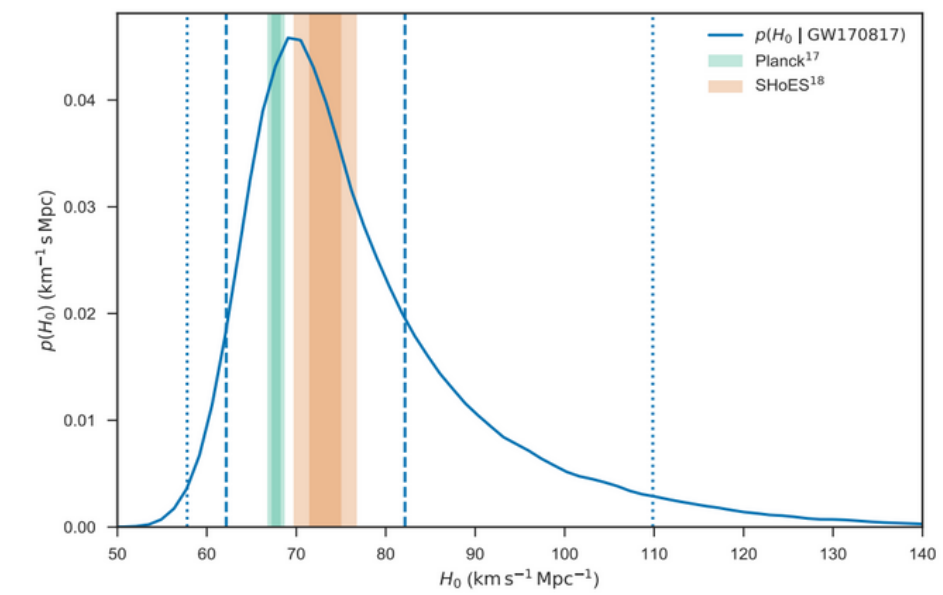
# Hubble tension and standard sirens

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Abbott B. P., et al., LV collaboration, PhRvL 2017, 119, 16

## GW170817 + GRB170817A (+ Kilonova) = $H_0$

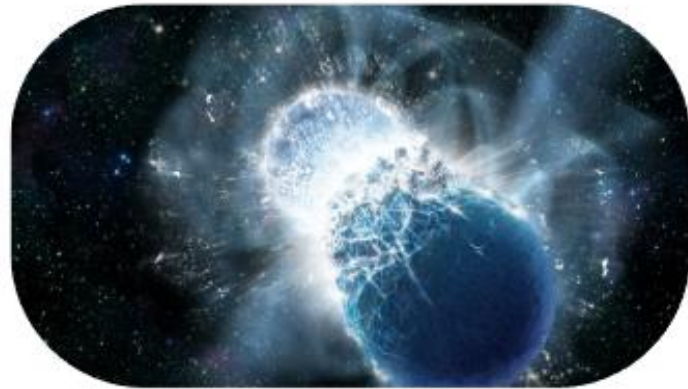


Abbott B. P. et al., LV collaboration, Nature 2017, 551, 7678

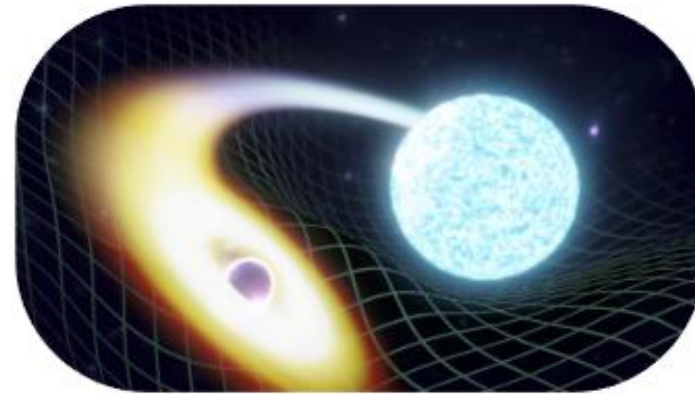
Di Valentino, E., MNRAS 2021, 502, 2

# Cosmology with Standard Sirens

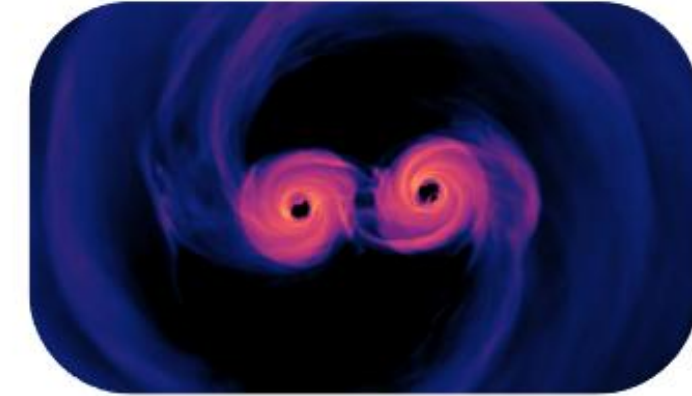
**Binary Neutron  
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**Black Hole Neutron  
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**Binary Black Hole  
(BBH)**



1

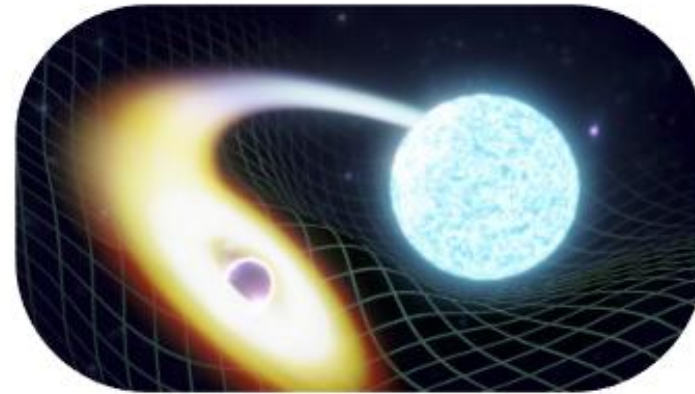
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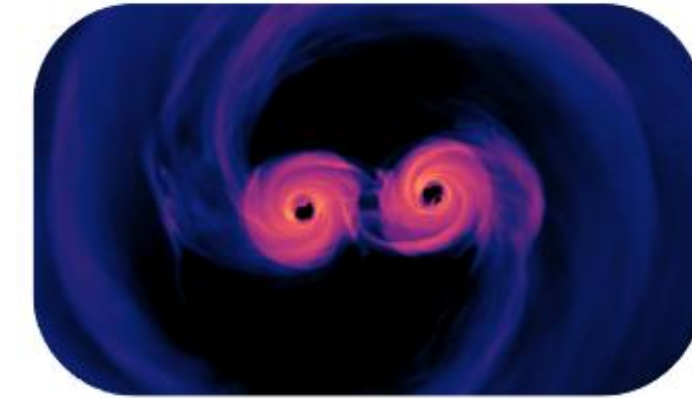
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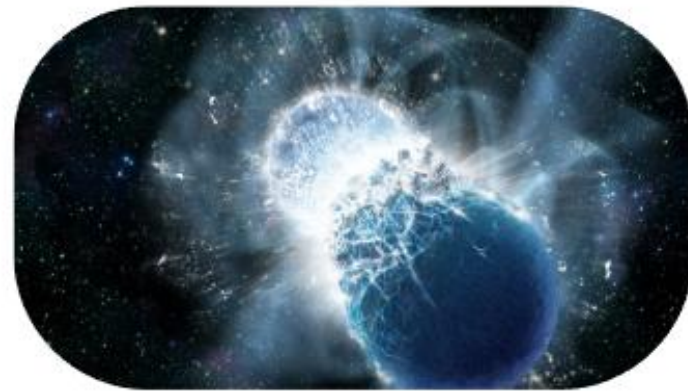
## GW + Kilonova: Bright Siren

Abbott B. P. et al., LV collaboration,  
Nature 2017, 551, 7678

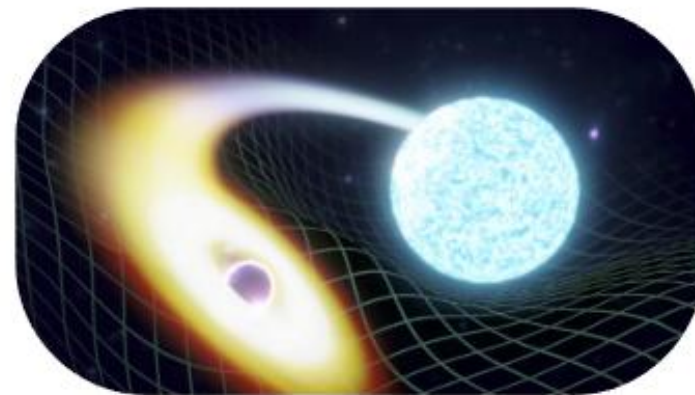
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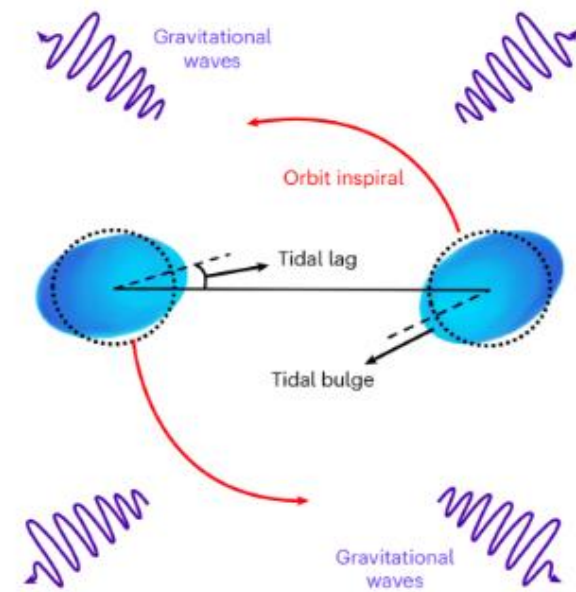
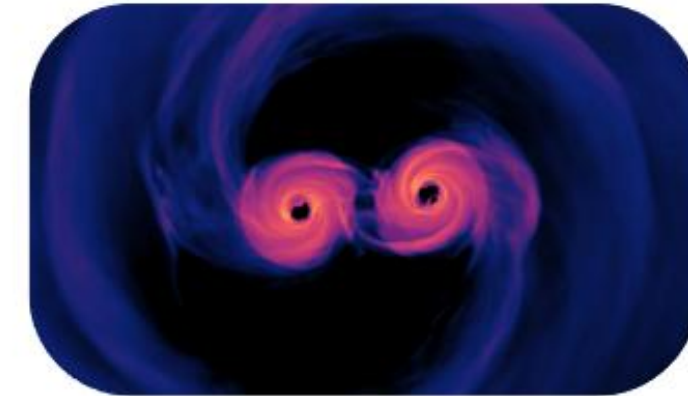
## Binary Neutron Star (BNS)



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## Binary Black Hole (BBH)



GW + Kilonova:  
Bright Siren

NS Tidal  
deformation + EoS:  
Love Siren

Abbott B. P. et al., LV collaboration,  
Nature 2017, 551, 7678

C. Messenger and J. Read, Phys. Rev. Lett. 108, 091101

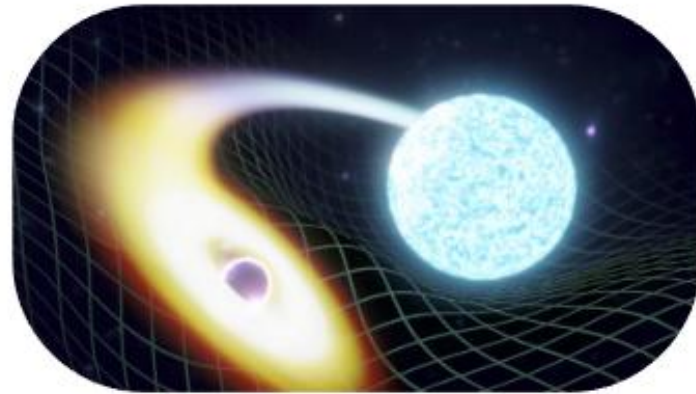
Antinozzi G. – INAF - IASF Milano – 27/05/2026 – 8/30

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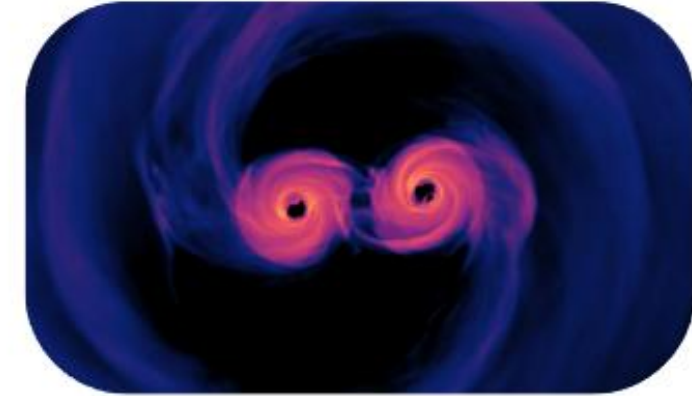
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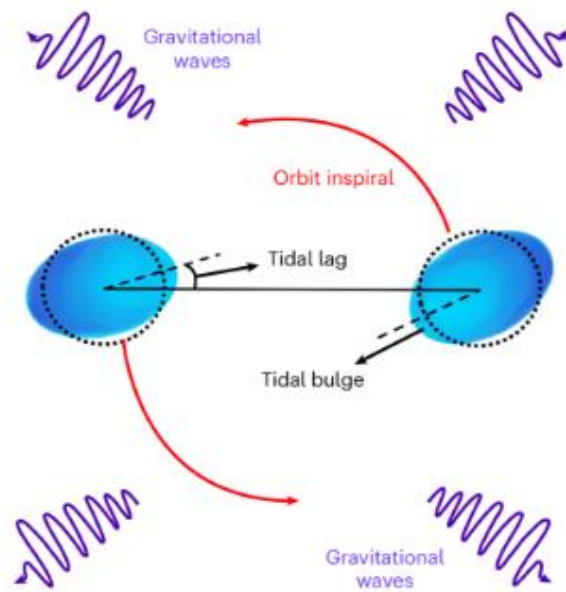


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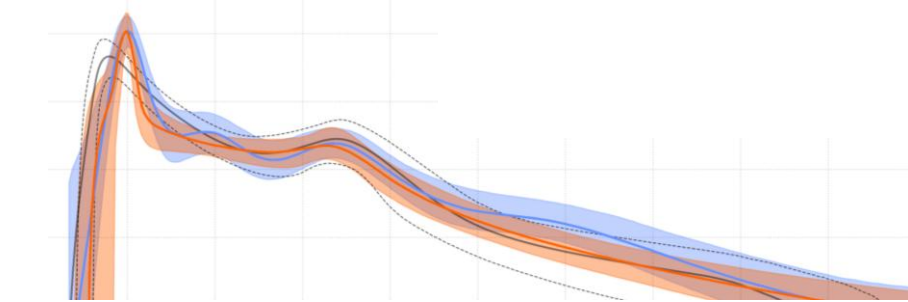
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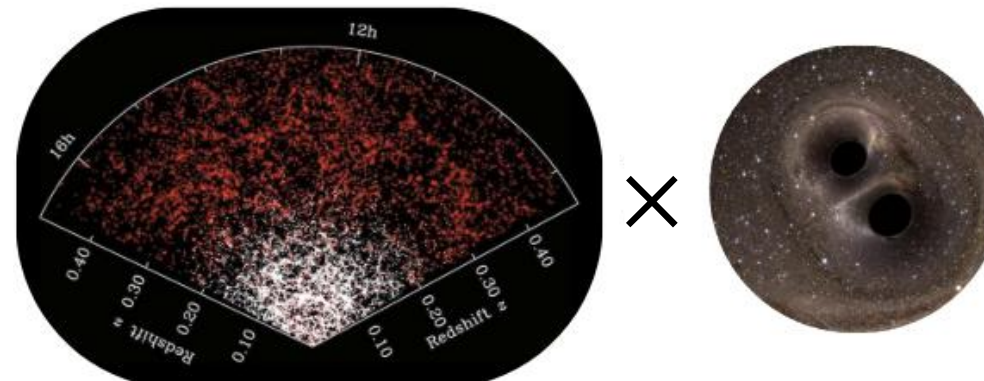


NS Tidal deformation + EoS:  
Love Siren

C. Messenger and J. Read, Phys. Rev. Lett. 108, 091101



S.R. Taylor and J.R. Gair, Phys. Rev. D 86 (2012) 023502



W. Del Pozzo, Phys. Rev. D 86 (2012) 043011

BH or BH+NS mass spectrum: Spectral Siren  
Cross-correlation Sirens:

× Galaxies

× LSS

M. Oguri, 2016, Phys. Rev. D, 93, 083511

× HI

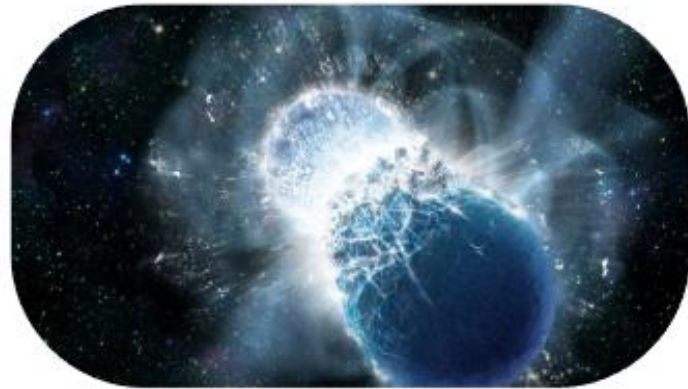
U. Dupletsa, 2026, arXiv

...

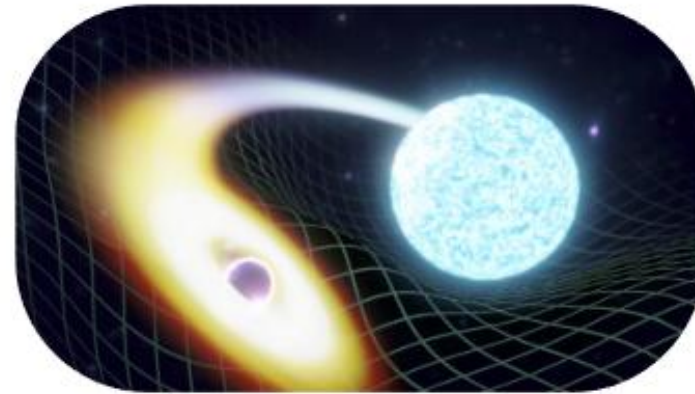
# Cosmology with Standard Sirens

1

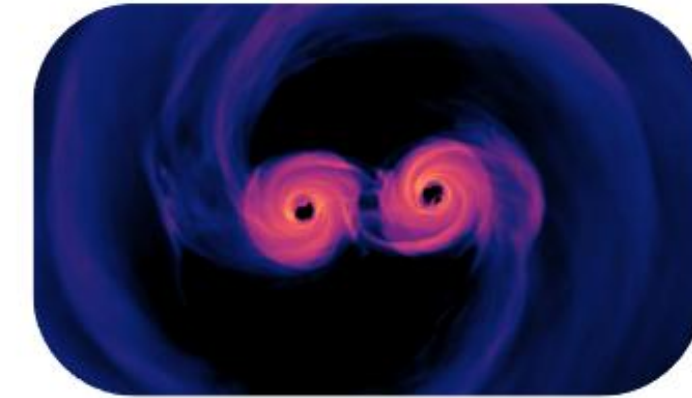
## Binary Neutron Star (BNS)



## Black Hole Neutron Star (BHNS)

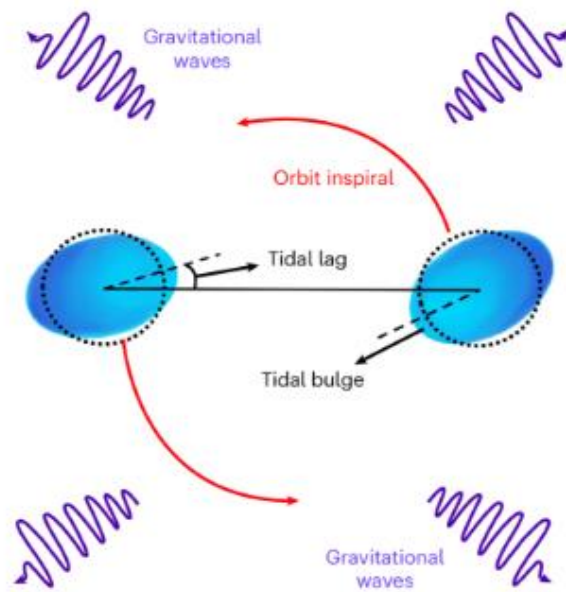


## Binary Black Hole (BBH)



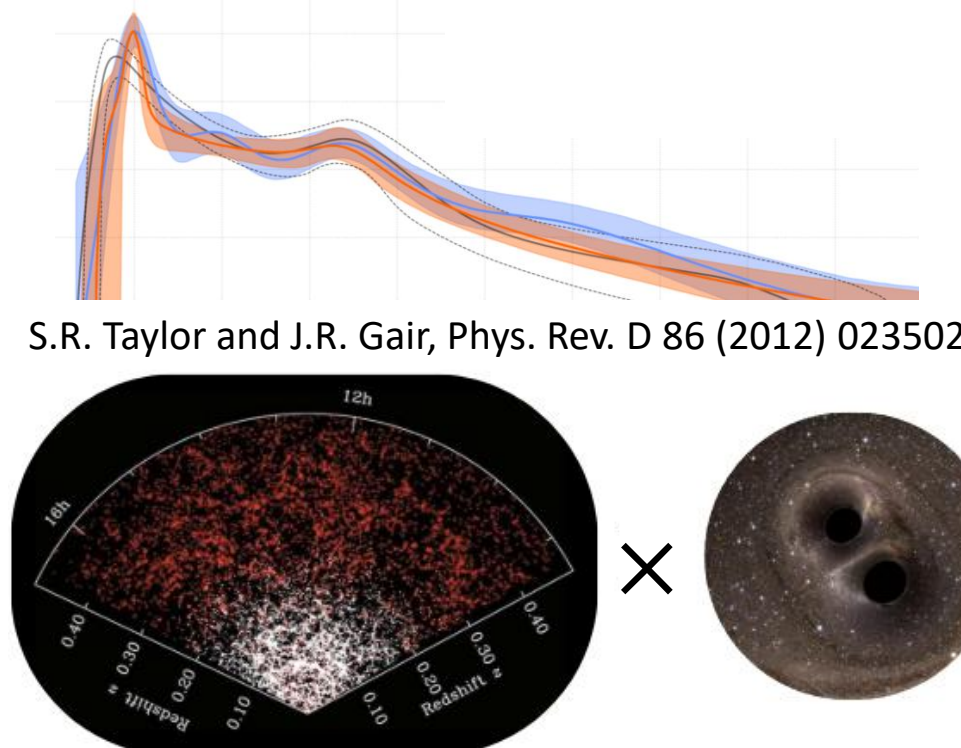
GW + Kilonova:  
Bright Siren

Abbott B. P. et al., LV collaboration, Nature 2017, 551, 7678



NS Tidal deformation + EoS:  
Love Siren

C. Messenger and J. Read, Phys. Rev. Lett. 108, 091101



W. Del Pozzo, Phys. Rev. D 86 (2012) 043011

BH or BH+NS mass spectrum: **Spectral Siren**  
Cross-correlation Sirens:

× Galaxies

× LSS

× HI

...

M. Oguri, 2016, Phys. Rev. D, 93, 083511

U. Dupletsa, 2026, arXiv

# Spectral Sirens

$$h_{+, \times}(t) \propto \frac{\mathcal{M}_d}{d_L(z|H_0, \Omega_0)}$$

$$\mathcal{M}_d = \mathcal{M}_S(1 + z)$$

# Spectral Sirens

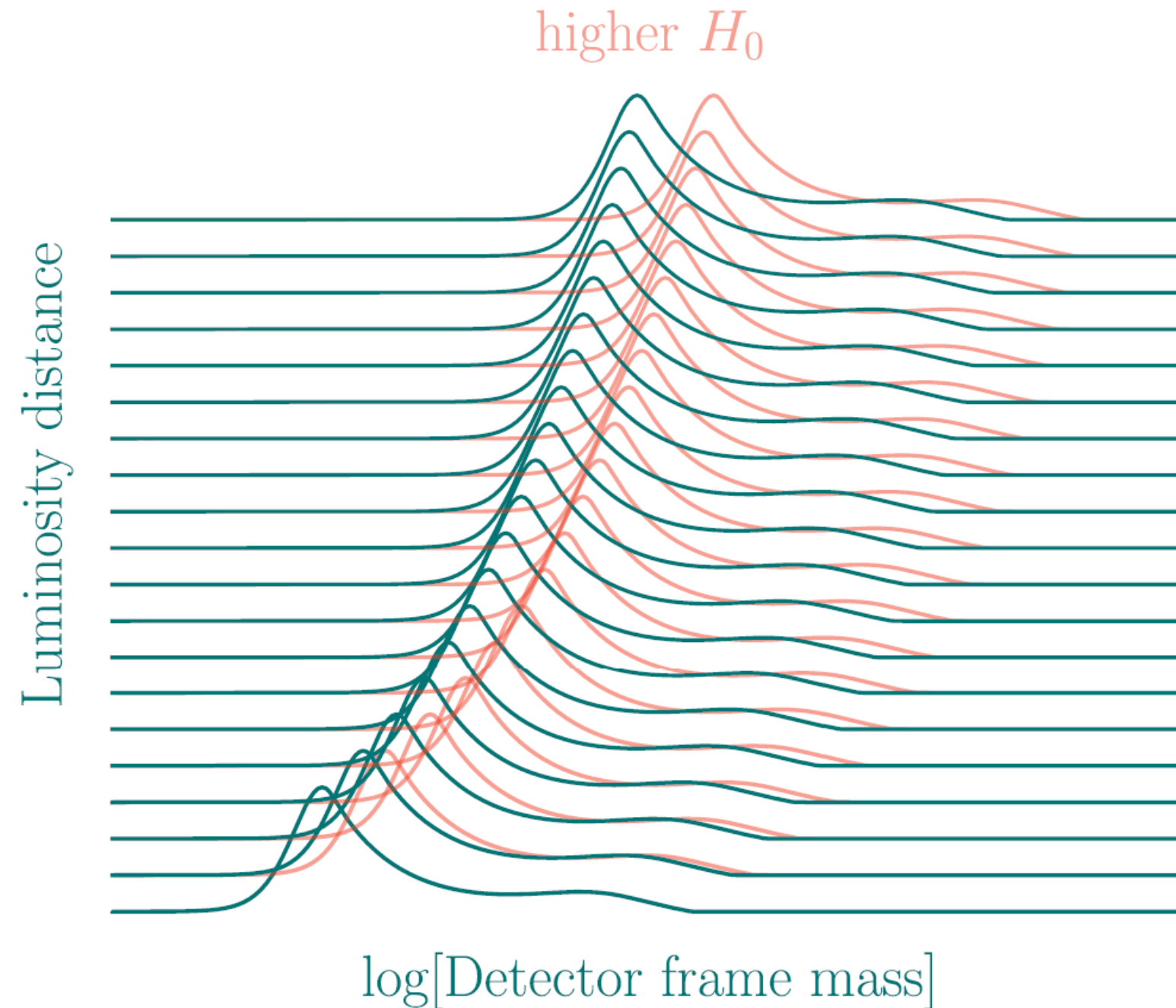
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**Assumption:** Source-frame mass spectrum does not evolve with redshift



A standard “ruler” to extract redshift information indirectly



Chen, H-Y., Ezquiaga, J.M., Gupta I., CQG 2024 41 12

# Spectral Sirens

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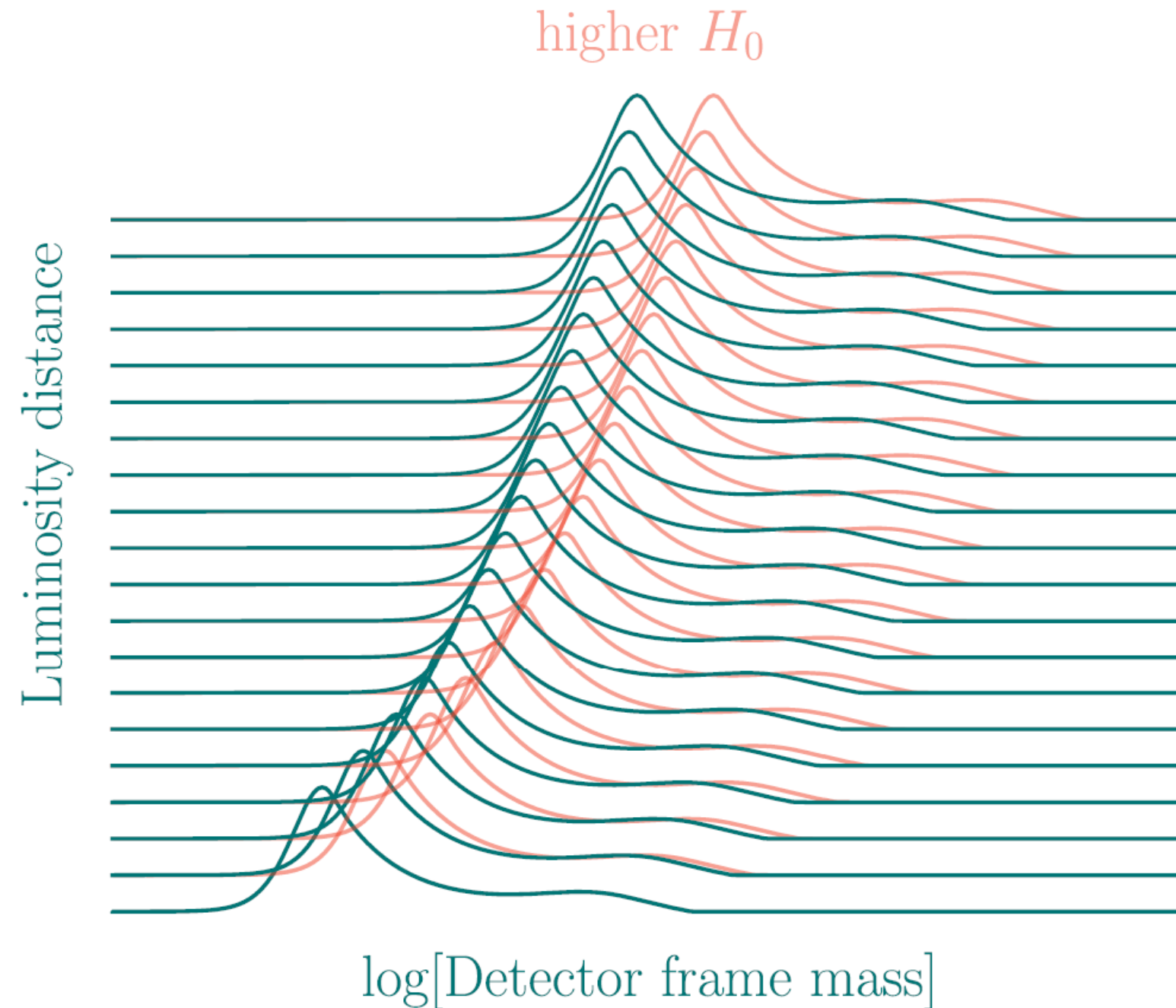
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A standard “ruler” to extract redshift information indirectly

Recent work from Agapito A., De Renzi V., Mancarella M. arXiv 2026 suggest a weak redshift evolution (**systematic effect**) within the current redshift reach.



Chen, H-Y., Ezquiaga, J.M., Gupta I., CQG 2024 41 12

# XG Forecasts

1

## Motivations:

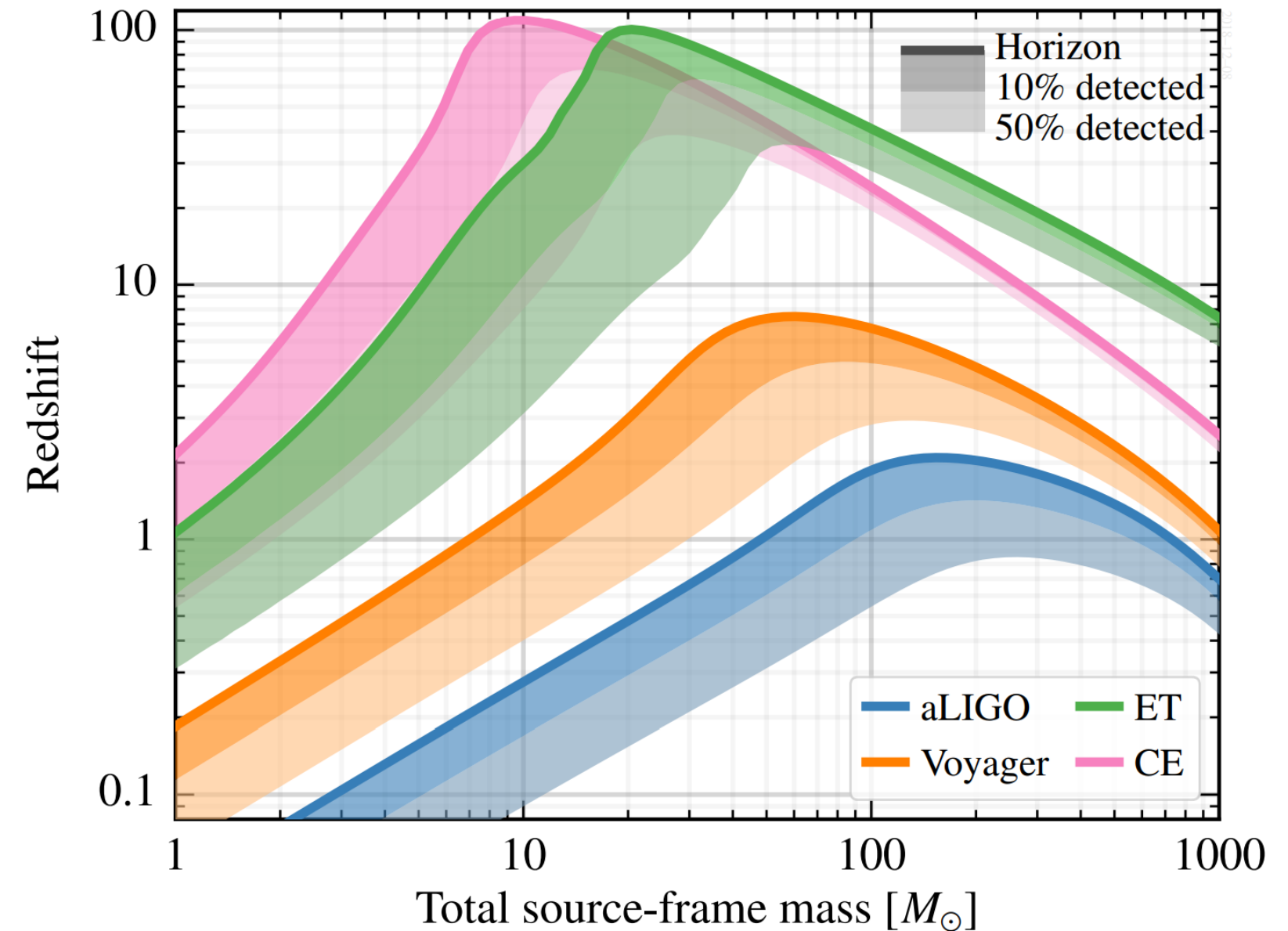
- a) Test hints of redshift dependence
- b) Progenitor-Remnant relation across the history of the Universe
- c) New Physics?

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1

## Motivations:

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Hall Evan D., Evans M., Classical and Quantum Gravity 2019, 36, 22



# 2 : Model

# Theoretical model

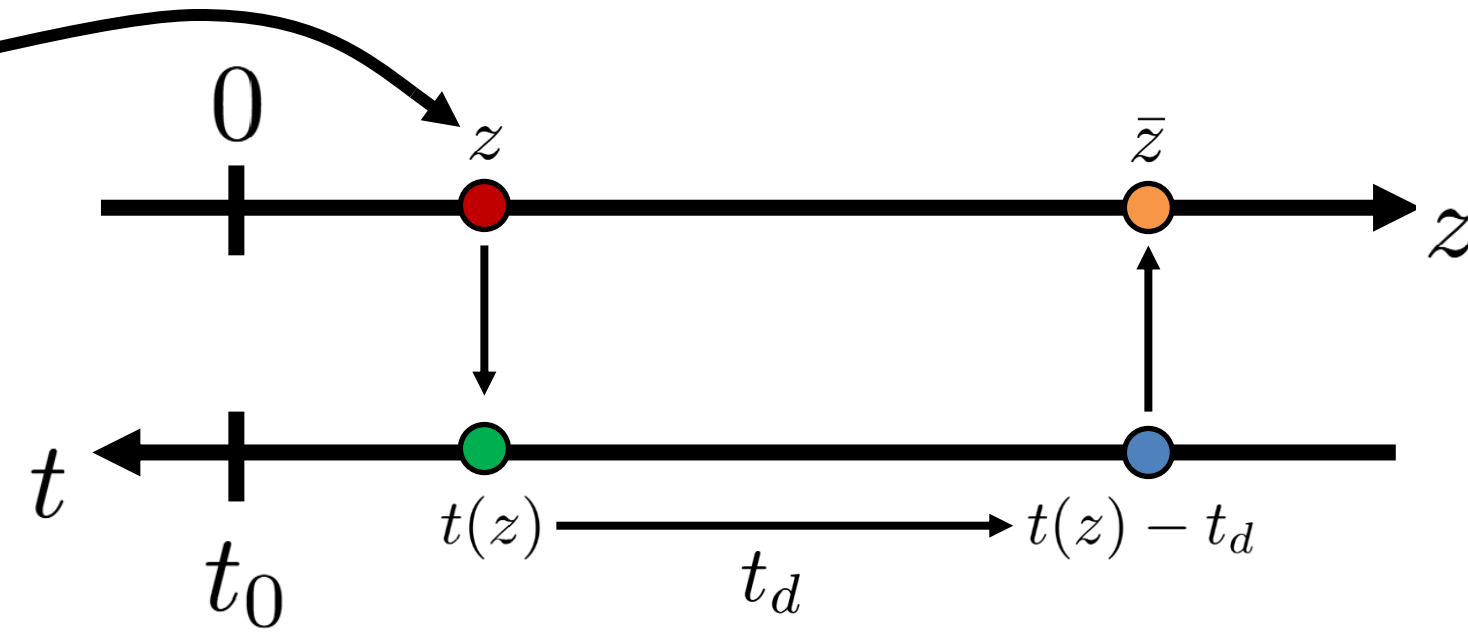
Binary Black Hole Merger  
Rate Density :  $\frac{d\dot{N}}{dV}(z)$

# Theoretical model

**Binary Black Hole Merger  
Rate Density :**  $\frac{d\dot{N}}{dV}(z) :=$  Convolution of Cosmic Star Formation Rate Density  
and Merger Distribution

# Theoretical model

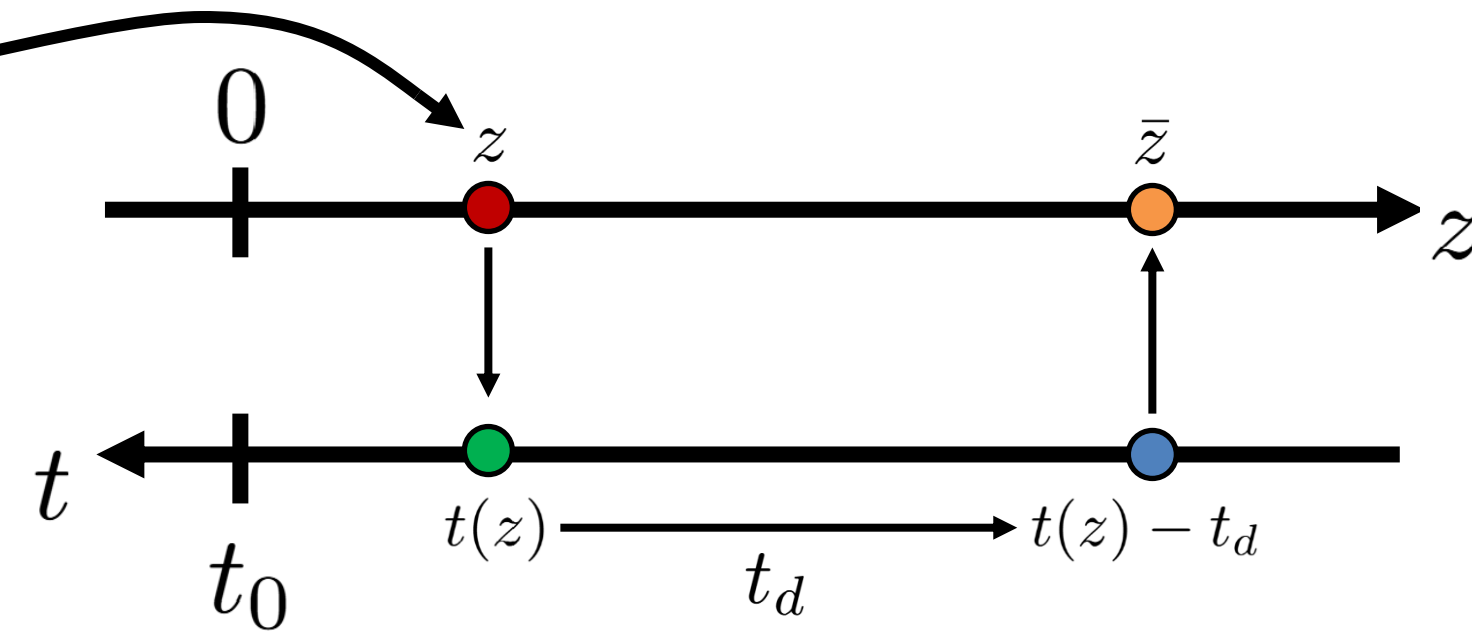
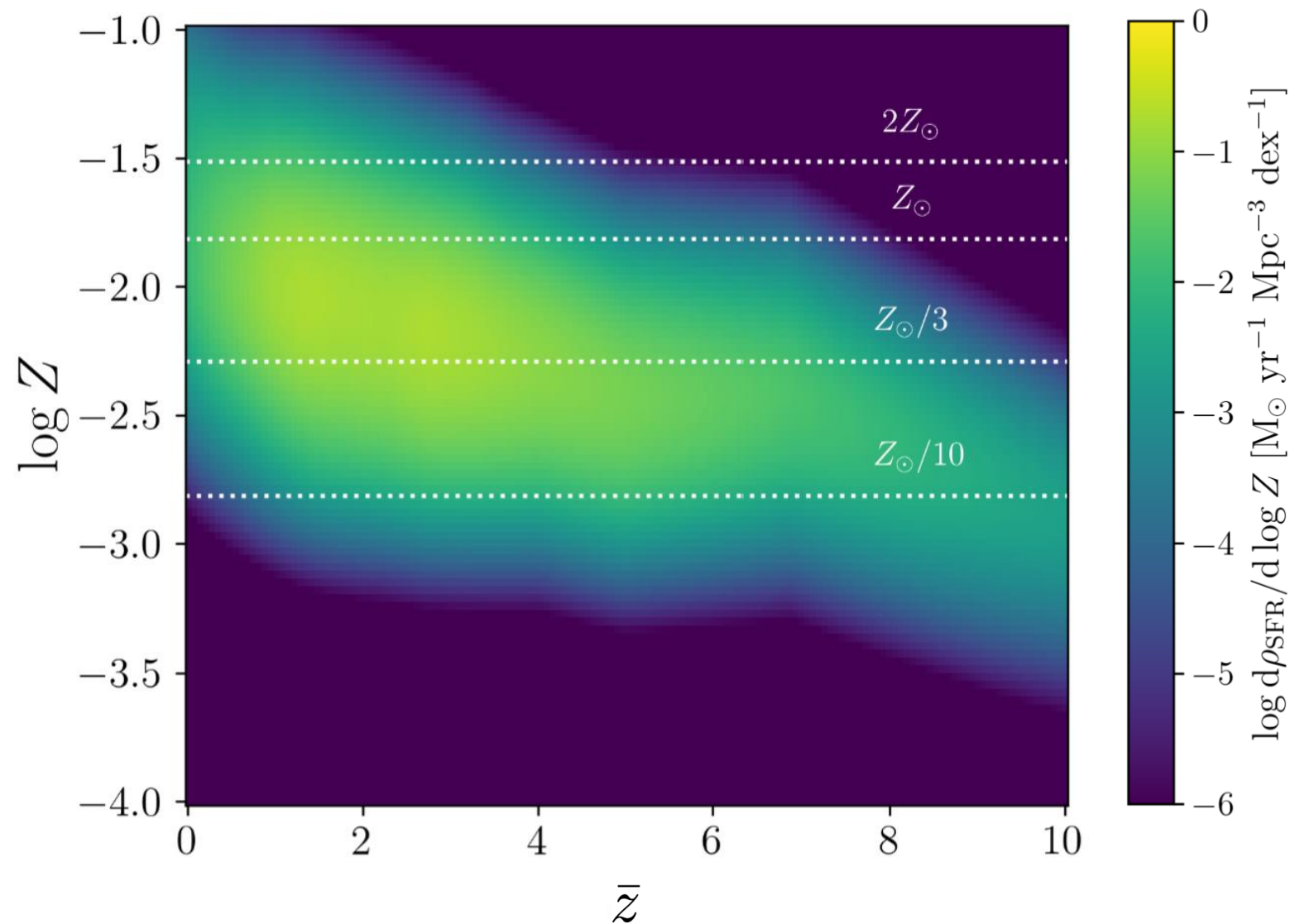
Binary Black Hole Merger Rate Density :  $\frac{d\dot{N}}{dV}(z) :=$  Convolution of Cosmic Star Formation Rate Density and Merger Distribution



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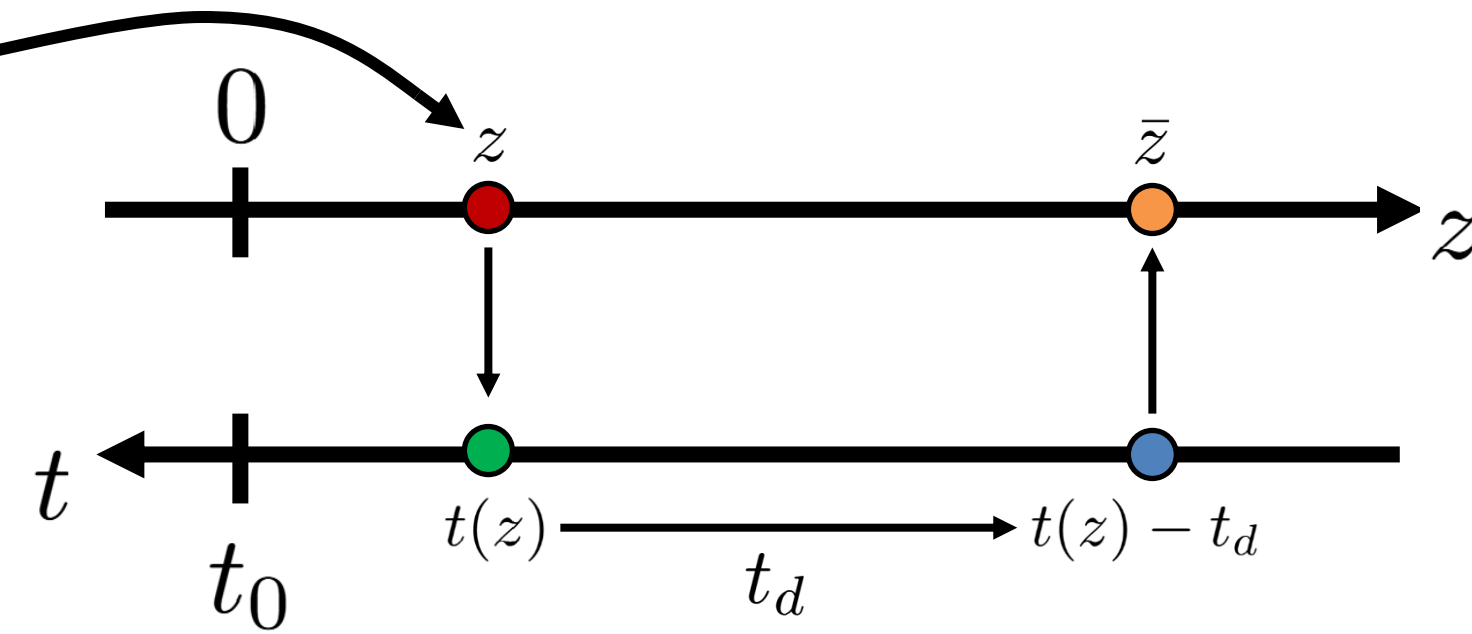
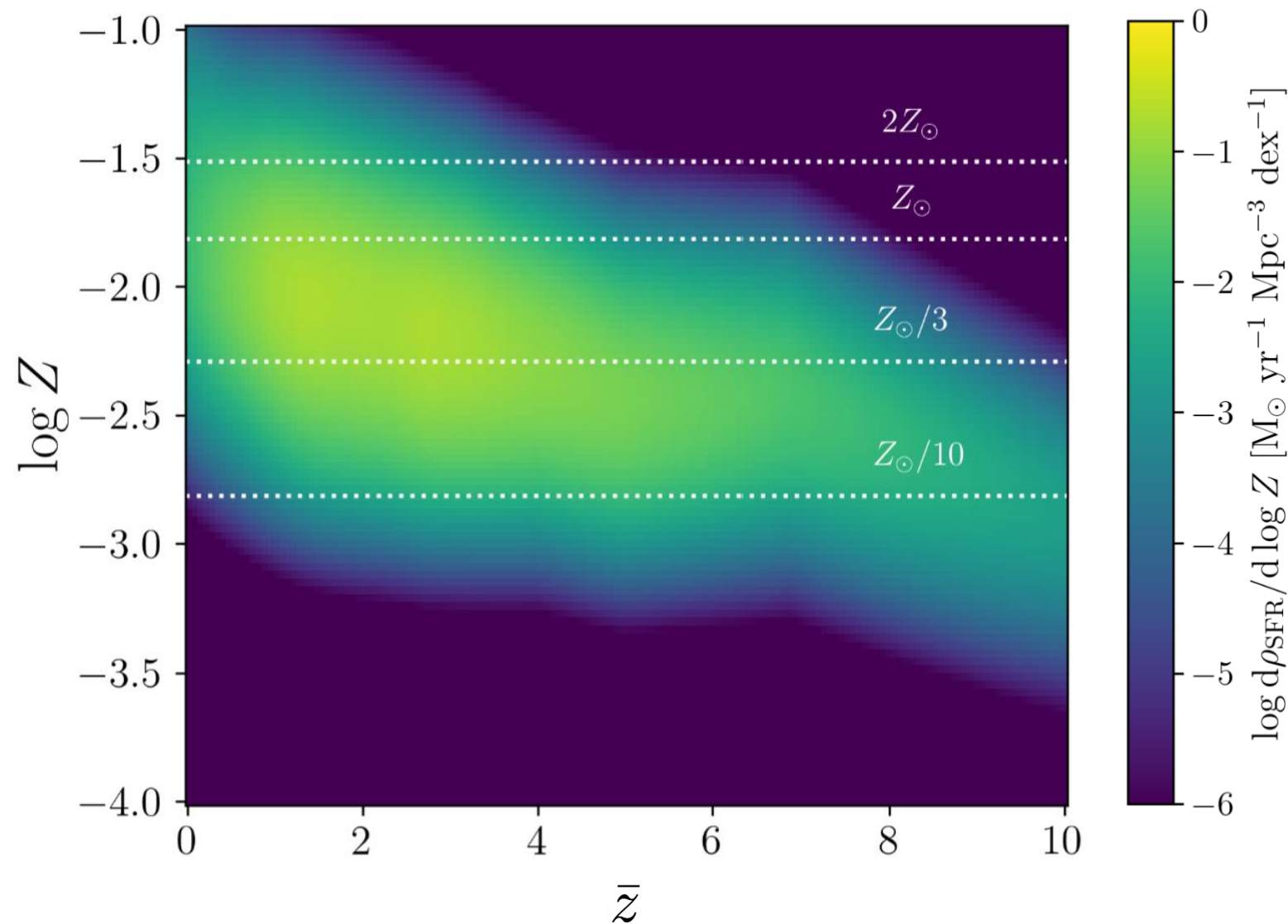
Data driven Star Formation Rate Density per unit metallicity



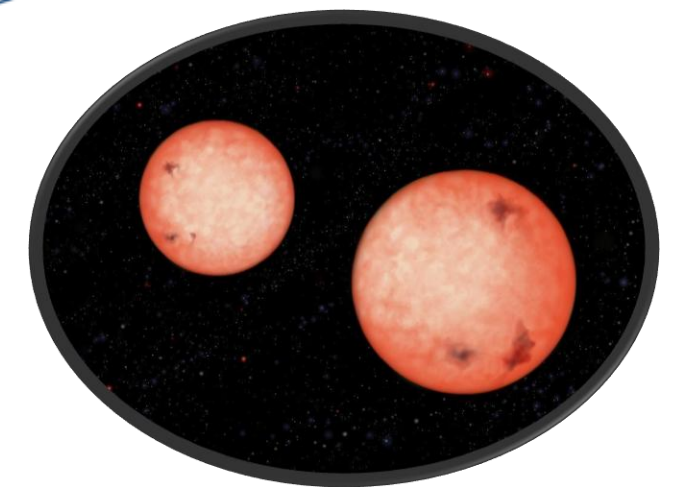
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Data driven Star Formation Rate Density per unit metallicity



Population synthesis of isolated binaries

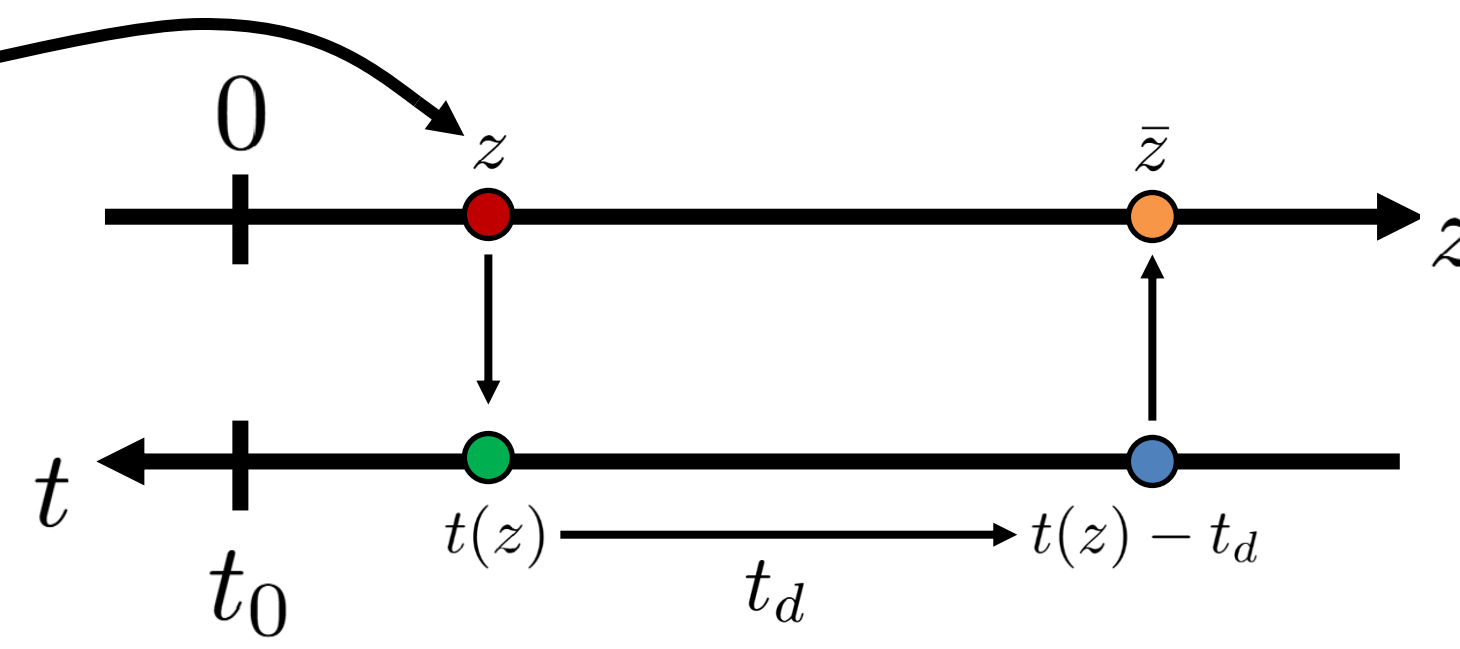
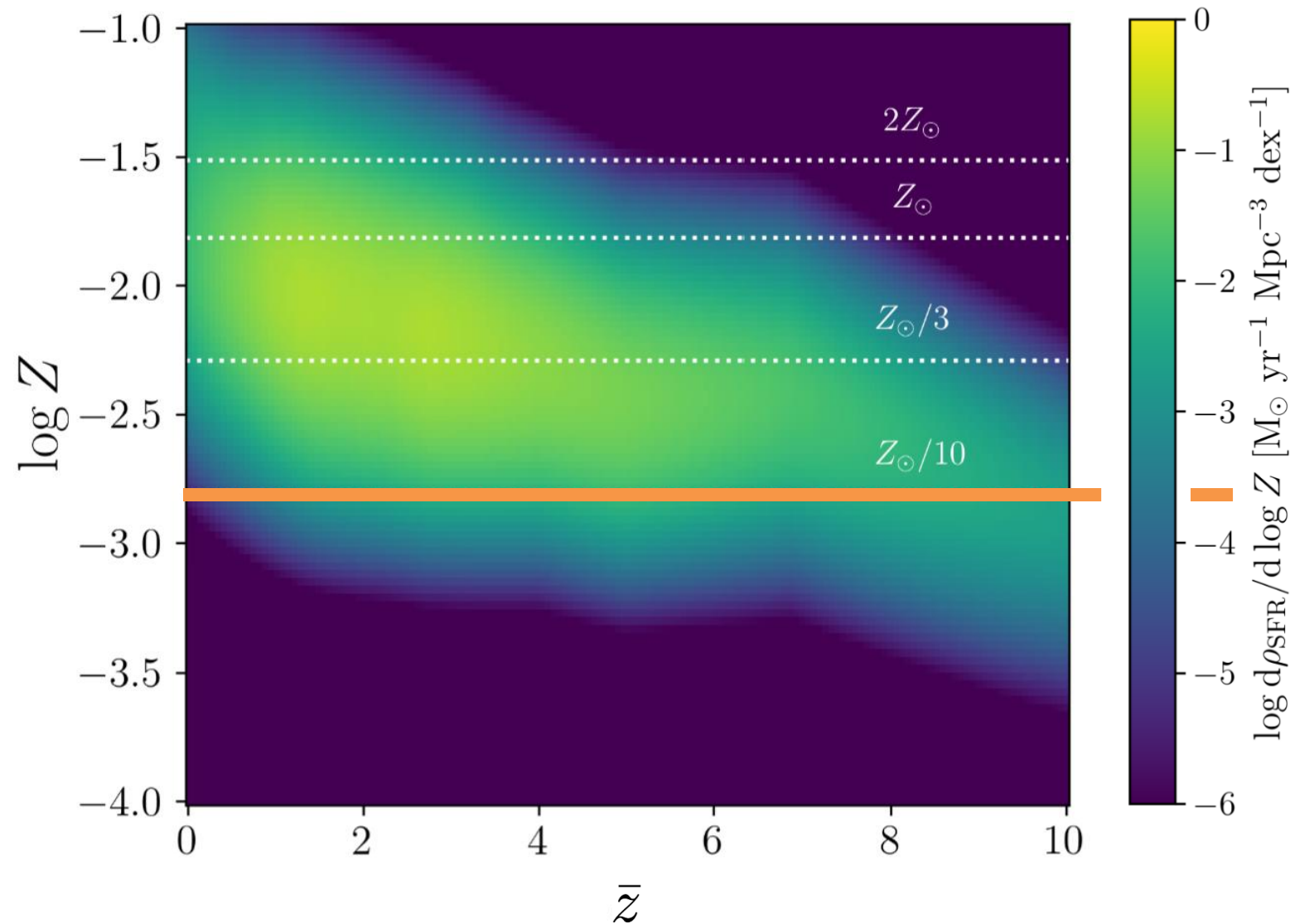


2

# Theoretical model

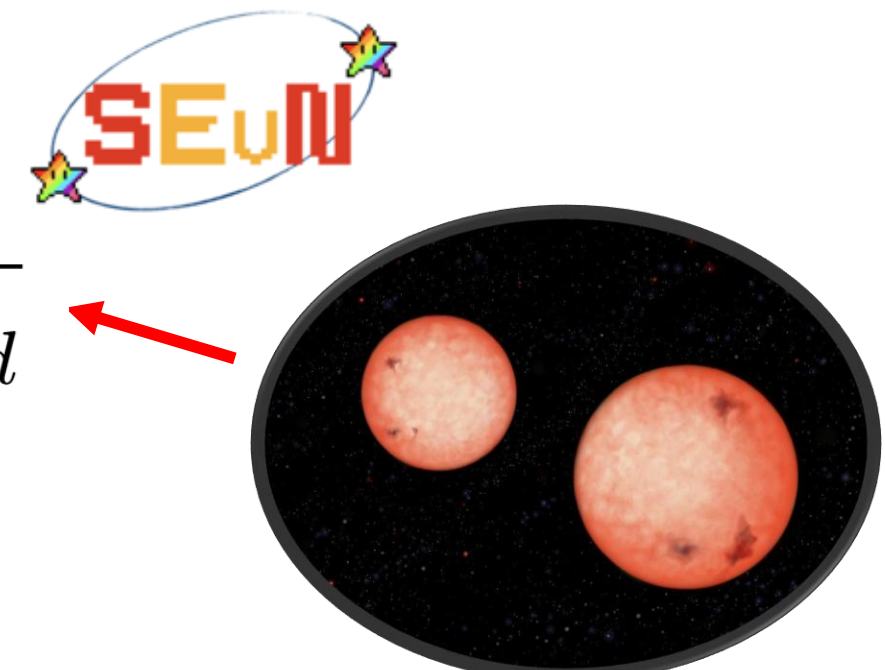
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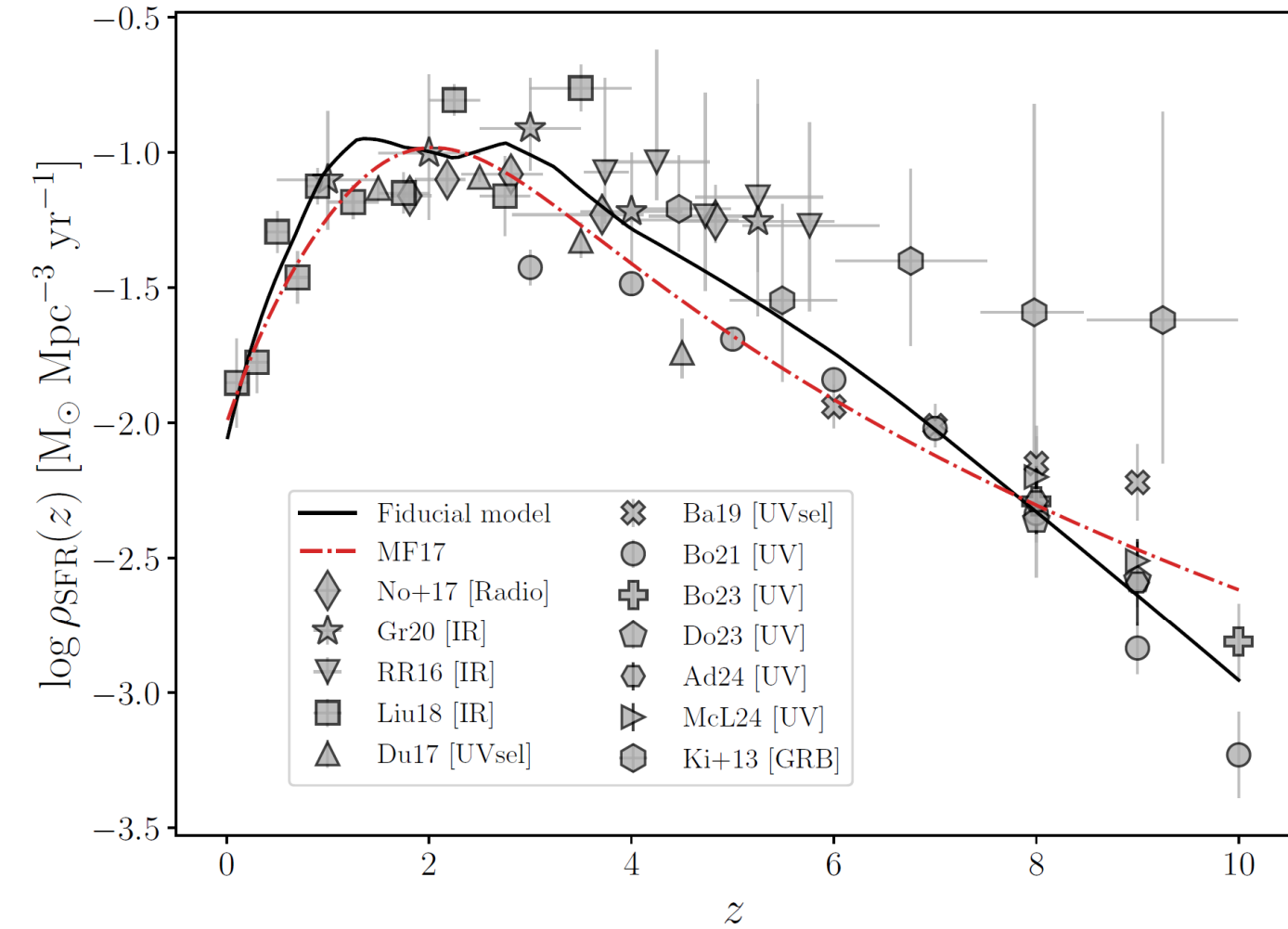
$$\int dt_d \left( \right) \times \frac{dp}{d \log t_d}$$

Population synthesis of isolated binaries



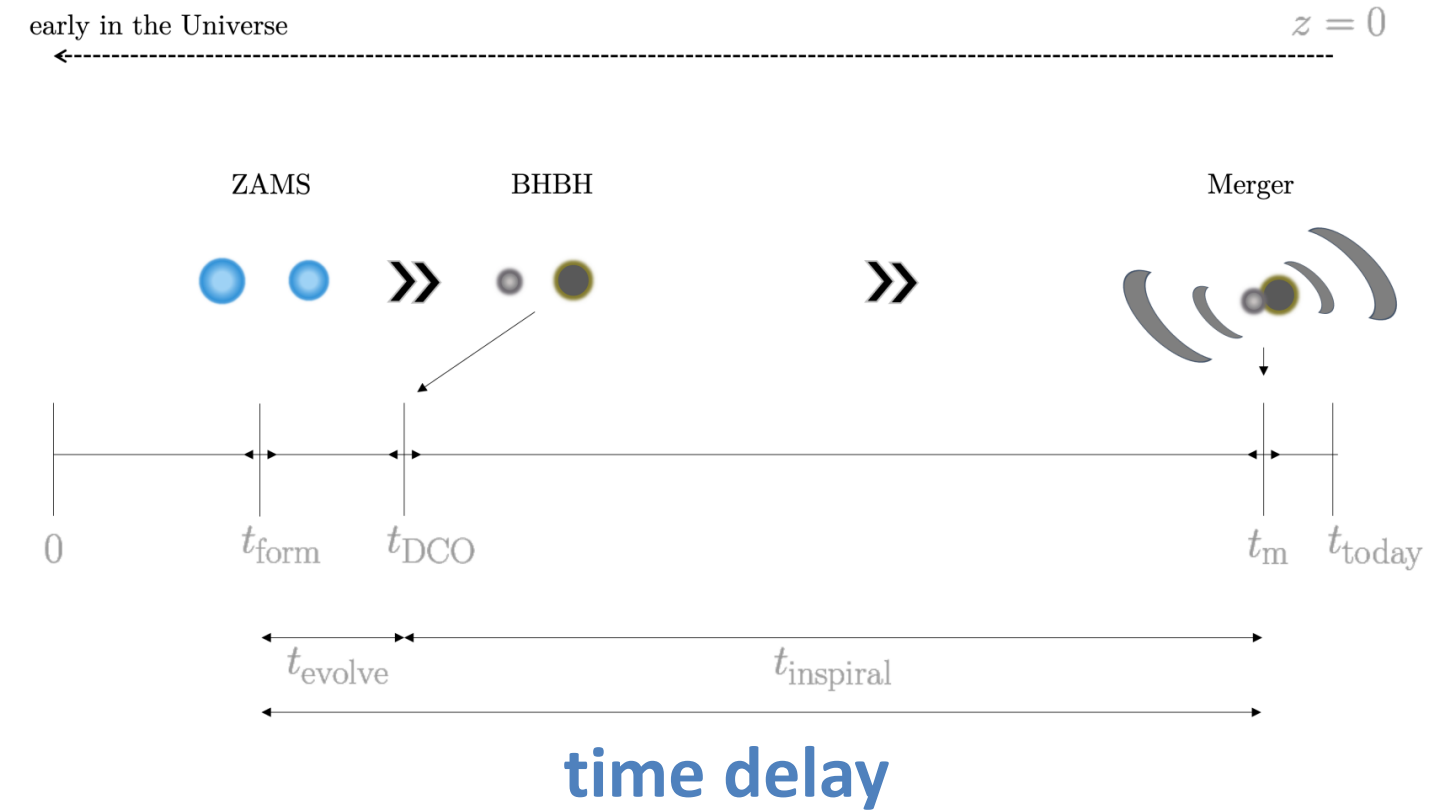
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1. **SFRD computed with empirical relations from latest galaxy survey data** (more flexible than cosmological simulations and consistent with recent JWST data)



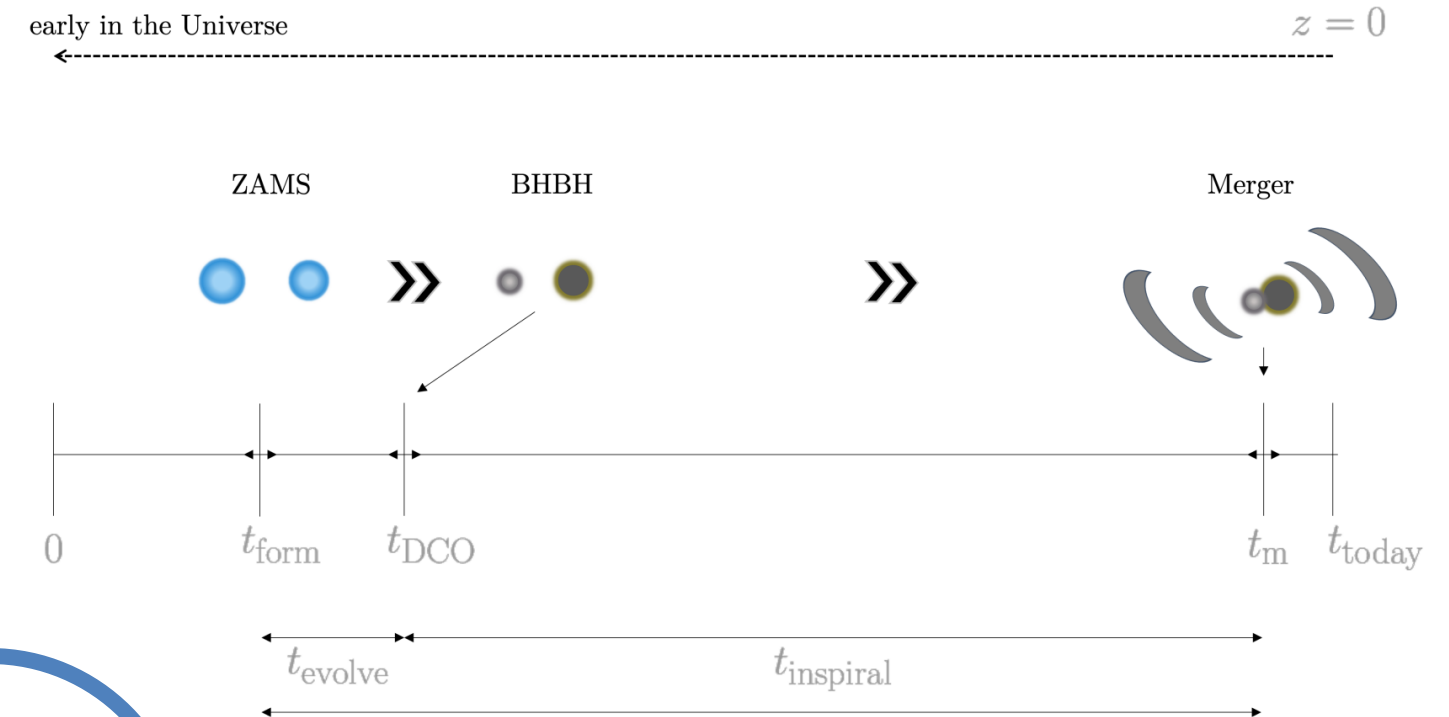
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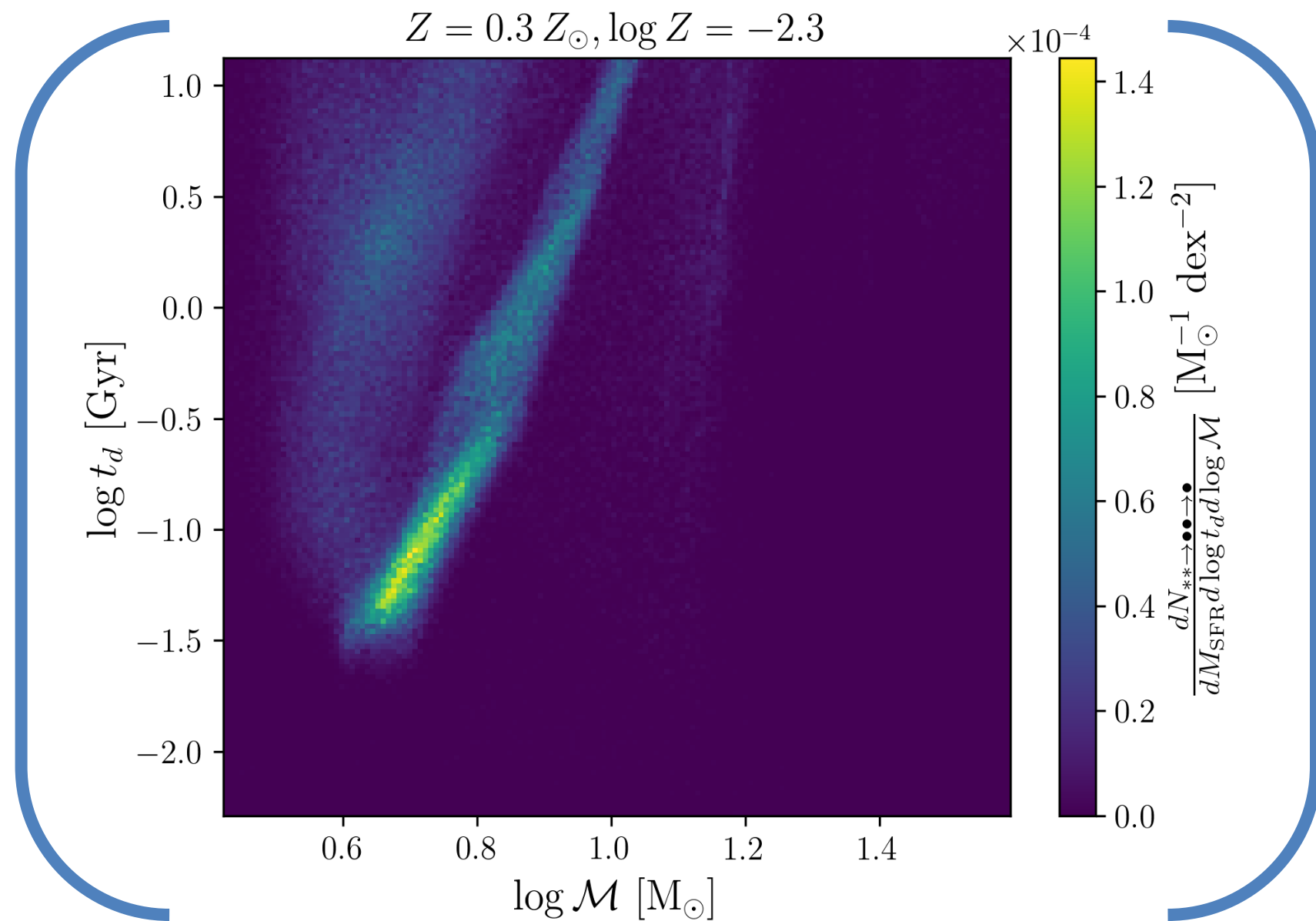


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*An Example of construction:*



$$\int d \log \mathcal{M}$$

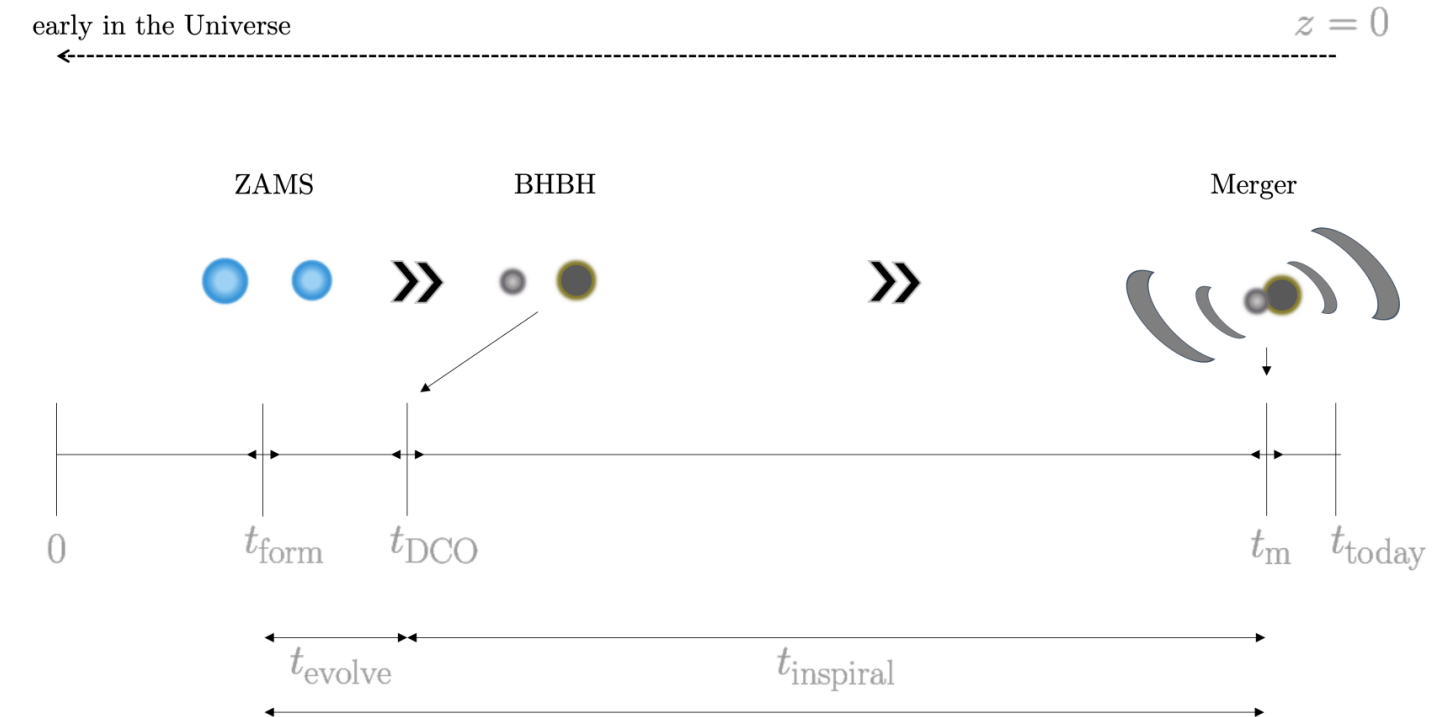
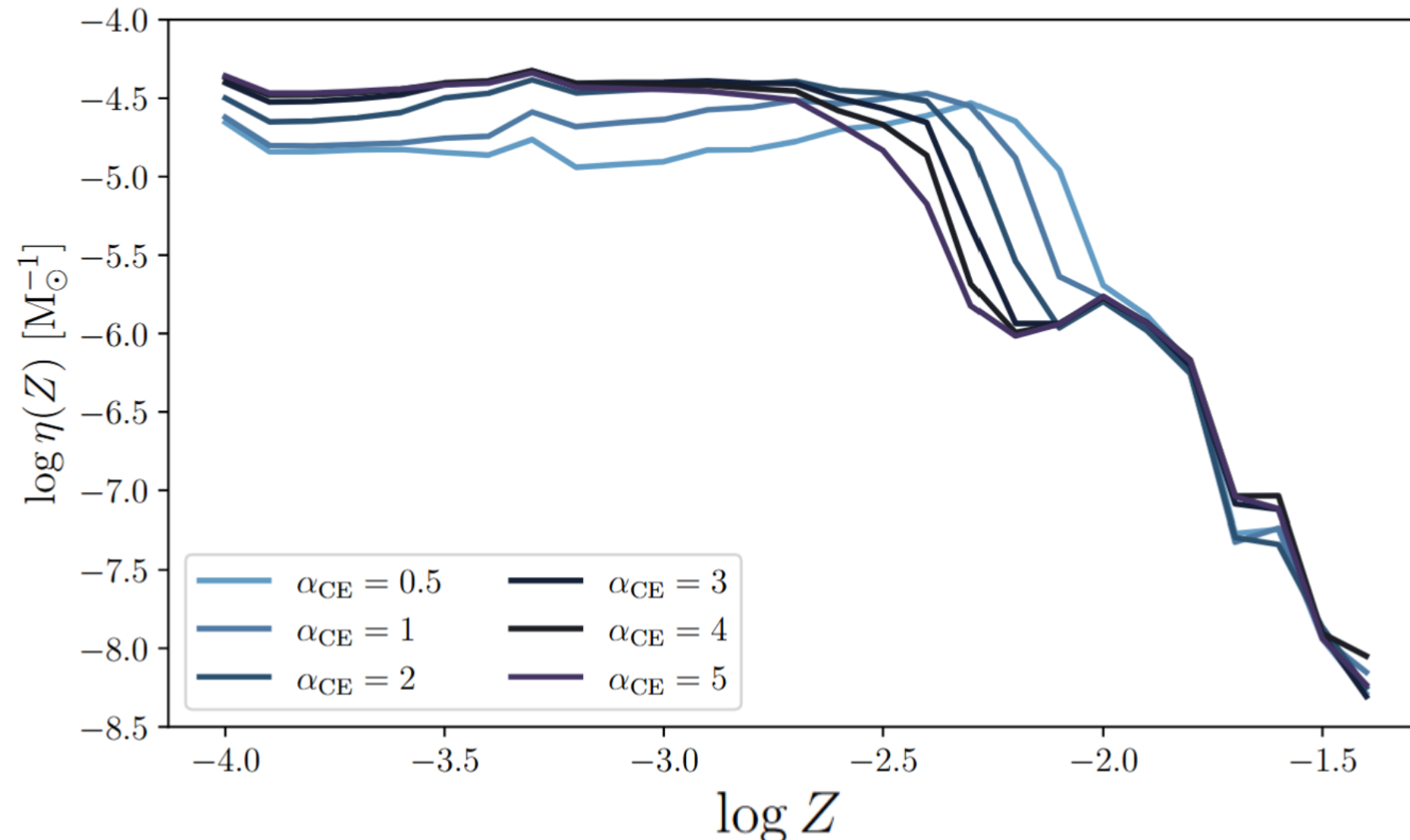


$$= \eta(Z) \frac{dp}{d \log t_d}$$

**Merger efficiency:** Number of merging BBH within a Hubble time per unit star mass formed

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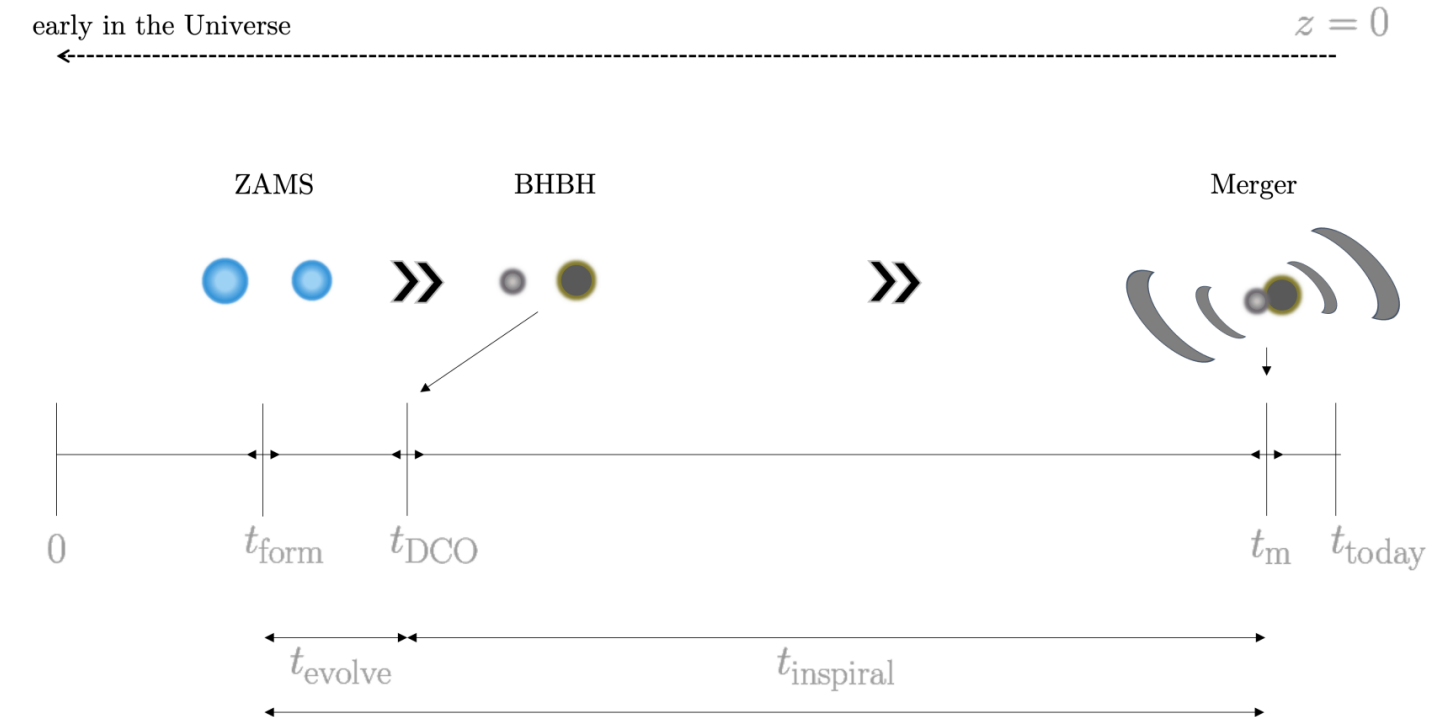
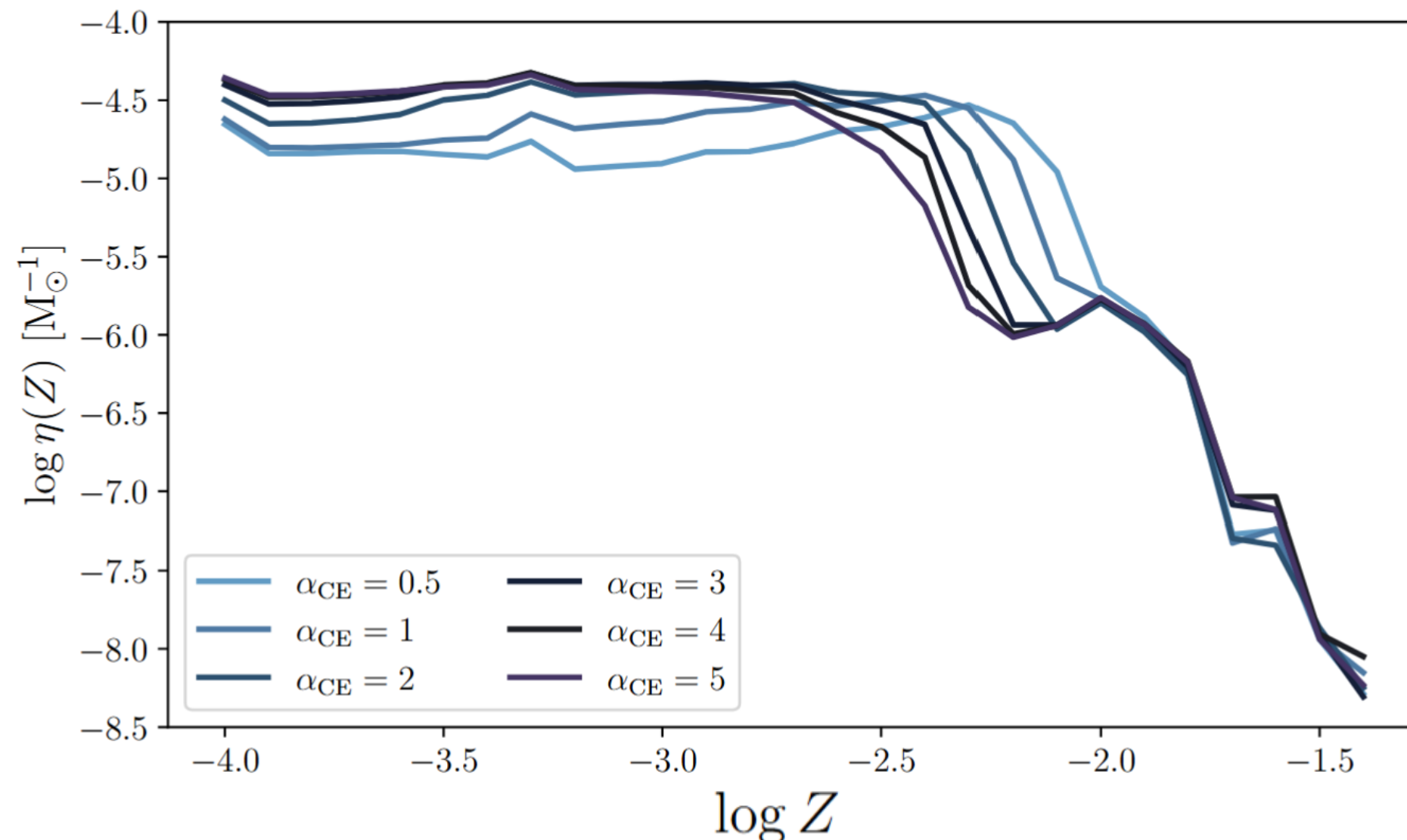
**time delay**

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**Merger efficiency:** varies with **metallicity** (e.g. winds) and with **common envelope phase energy efficiency (CE hardening)**, **formation channel (IB, YC, GC, NC)**

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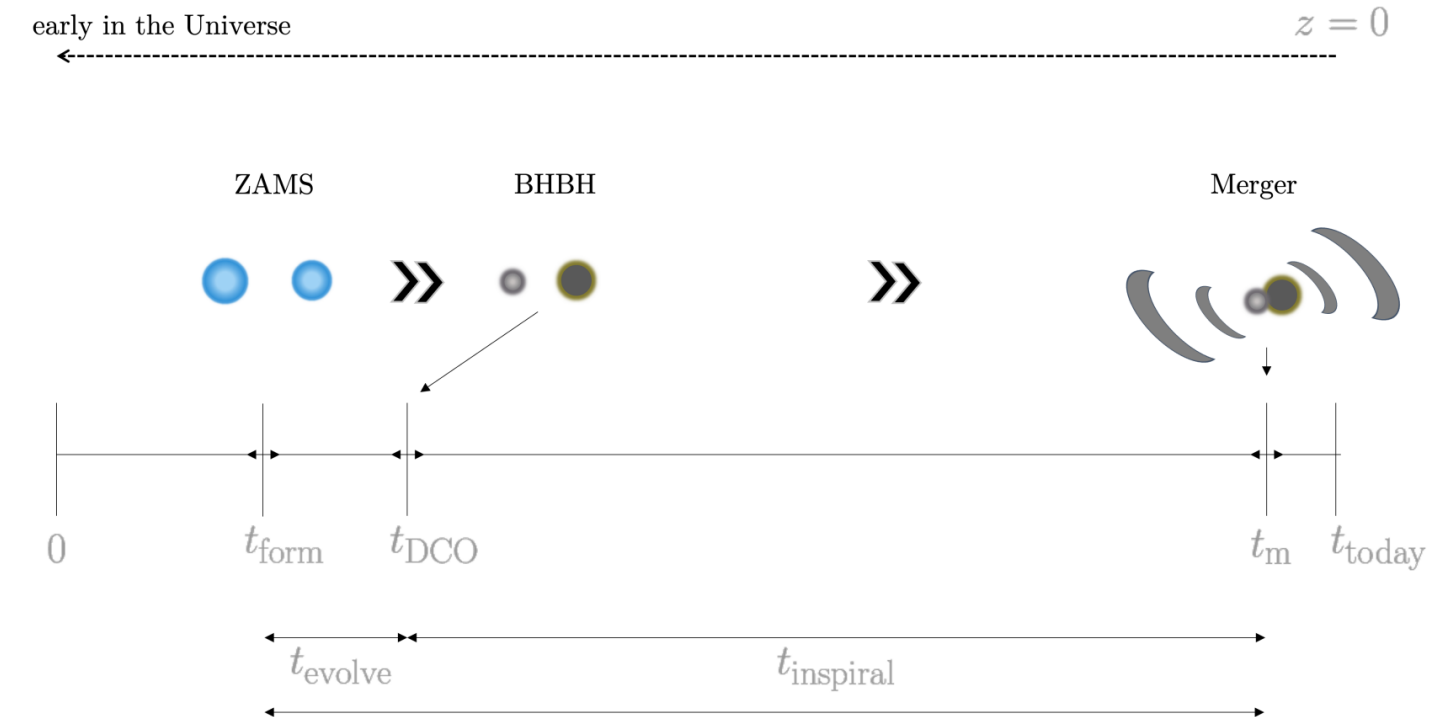
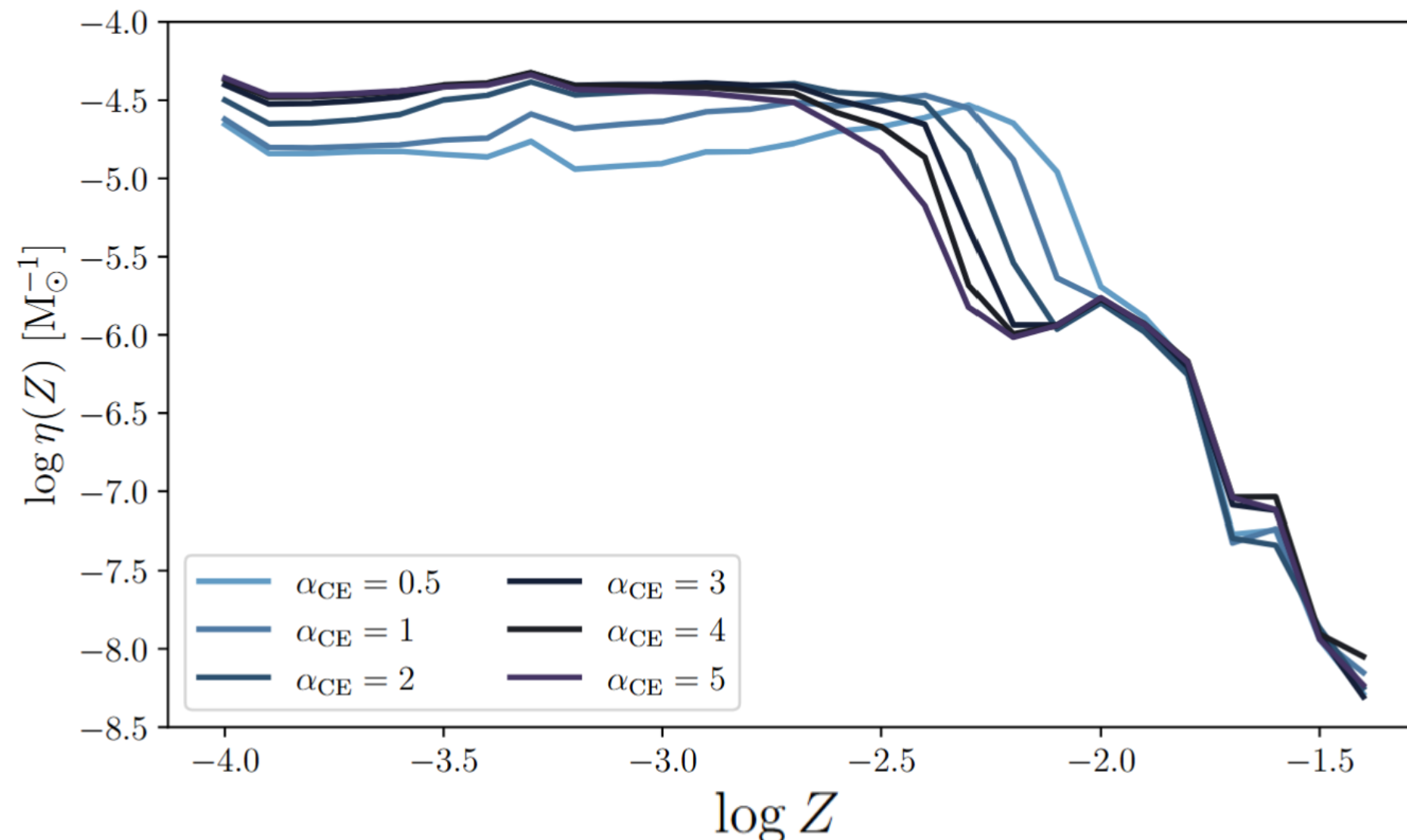
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**Caveat: only isolated channel**

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3. **High speed population model computation with JAX** (necessary for population inference)



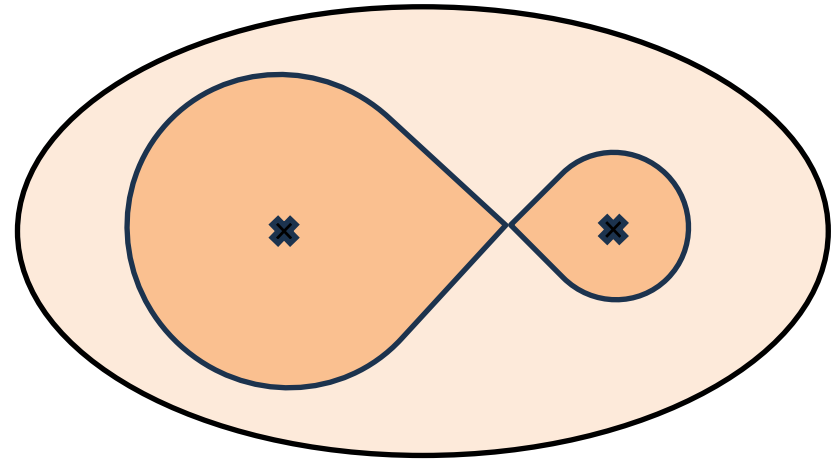
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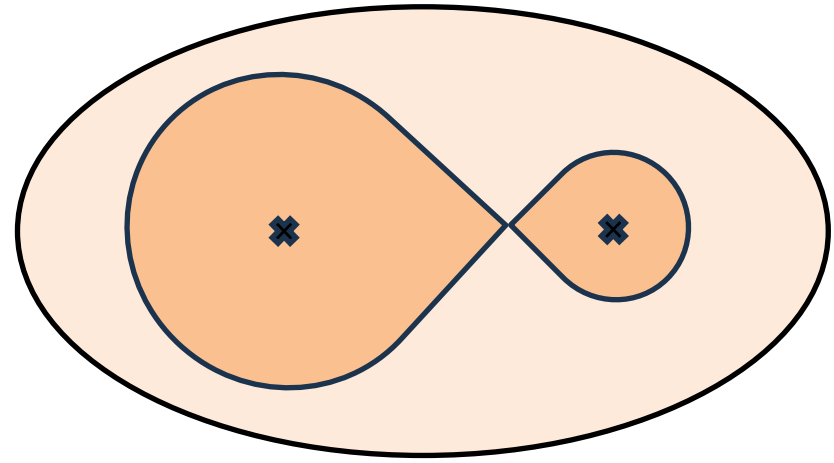
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# Astro-cosmo parameters

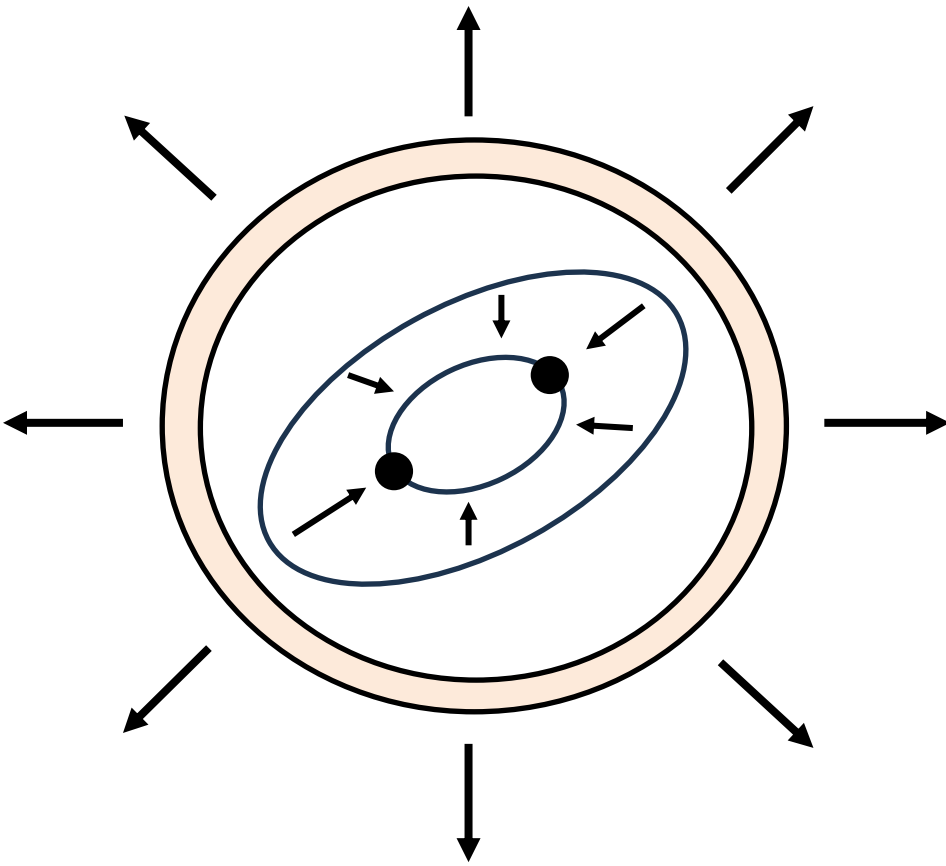


$$E_{\text{bind},i} = \alpha_{\text{CE}} \Delta E_{\text{orb}}$$

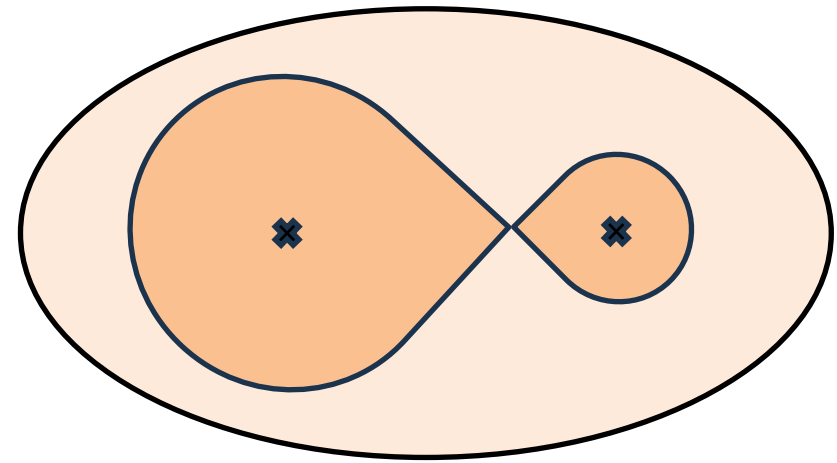
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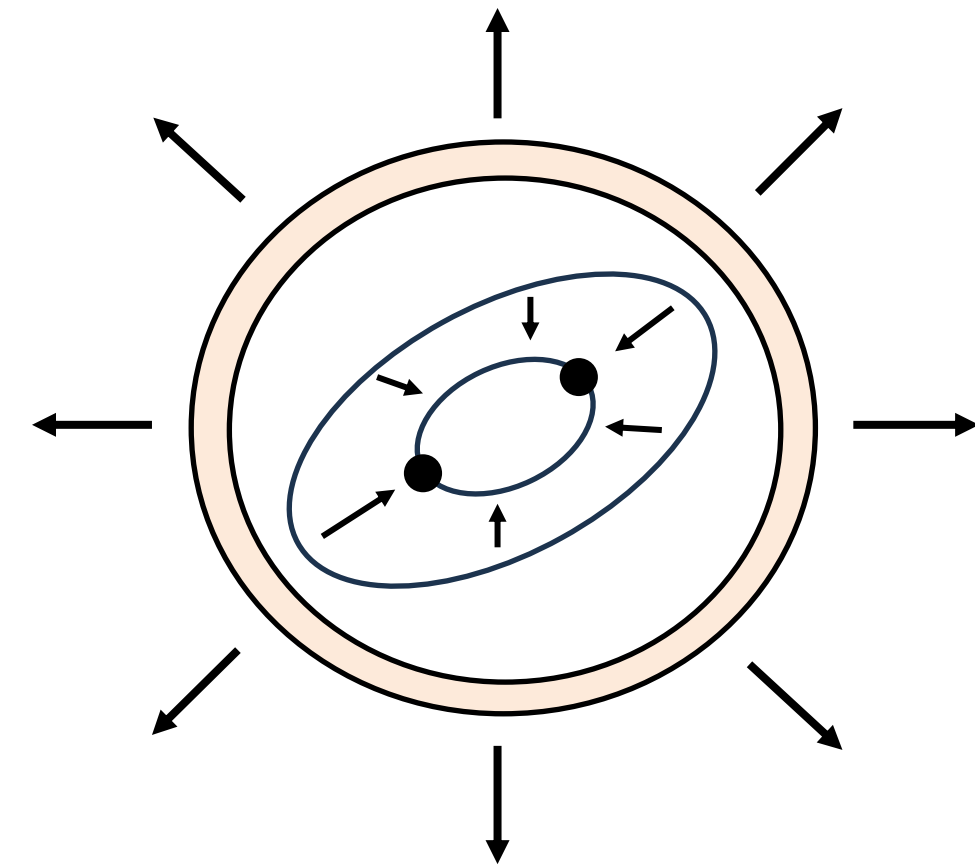
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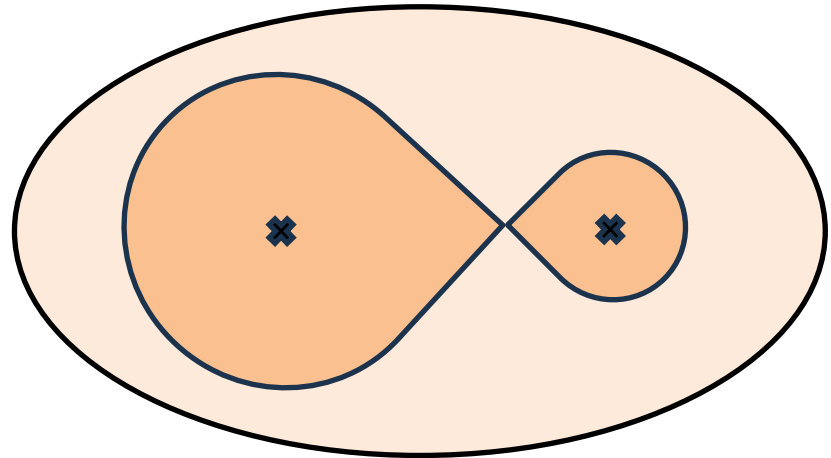
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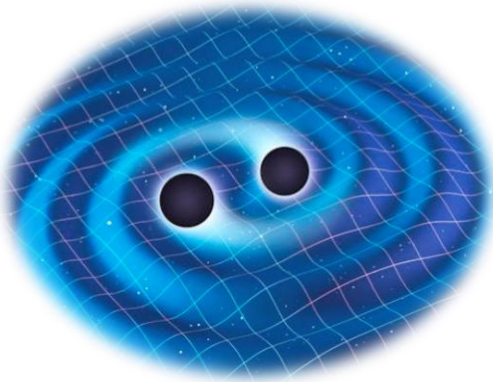
$$\alpha \nearrow \Rightarrow N_{\text{det}} \searrow$$

Larger  $\alpha$  : lower gas kinetic energy : more loose binaries than lower  $\alpha$ s :  
longer GW merger time : less are observed within a Hubble time

# Astro-cosmo parameters



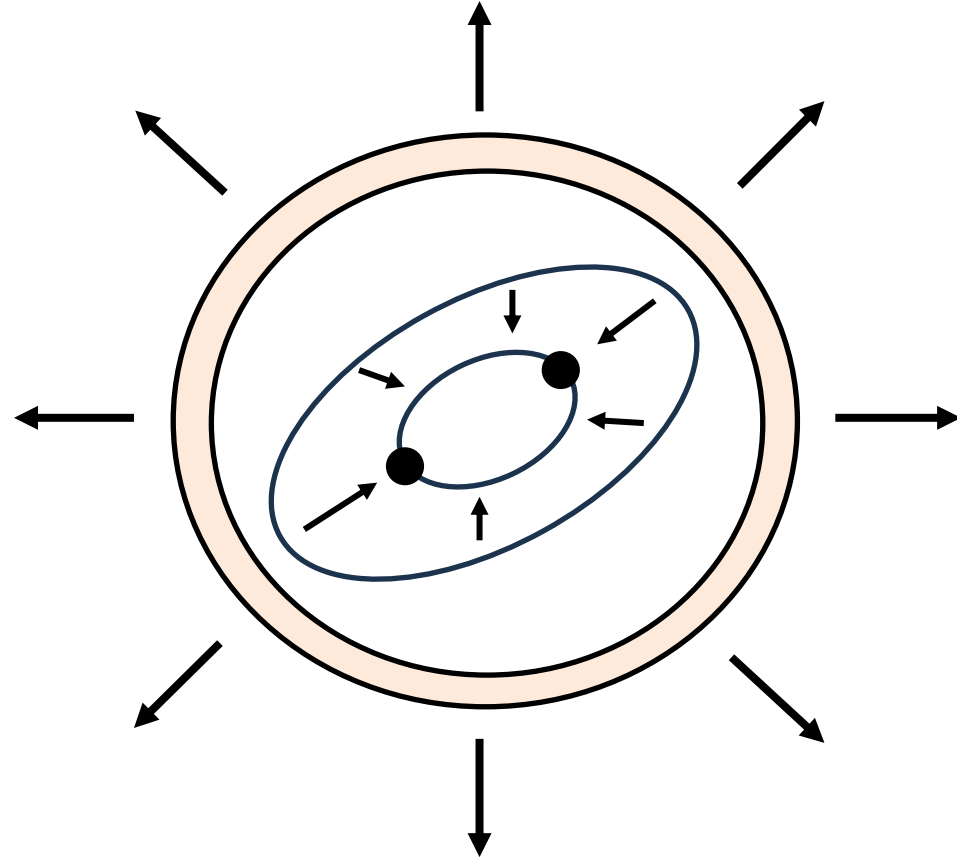
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$d_L$

$$h_{+,\times} \propto 1/d_L \propto H_0$$

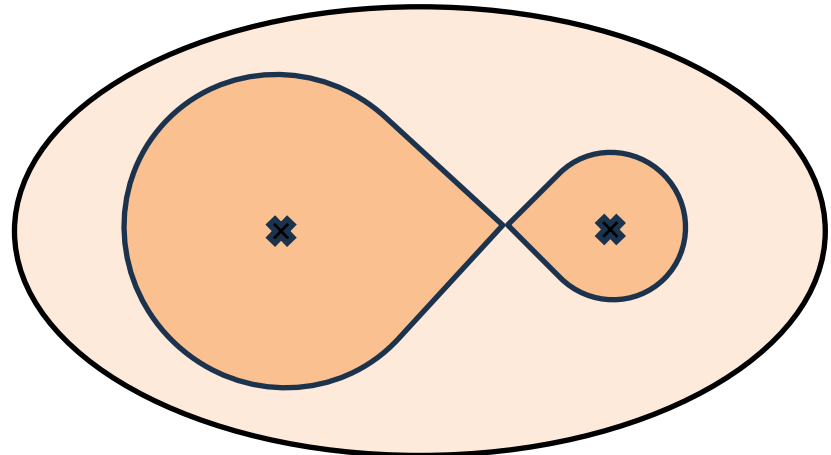
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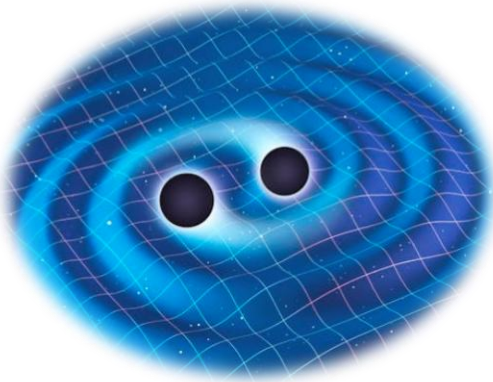
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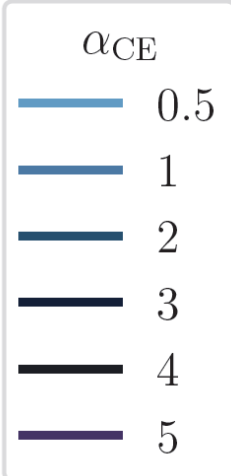
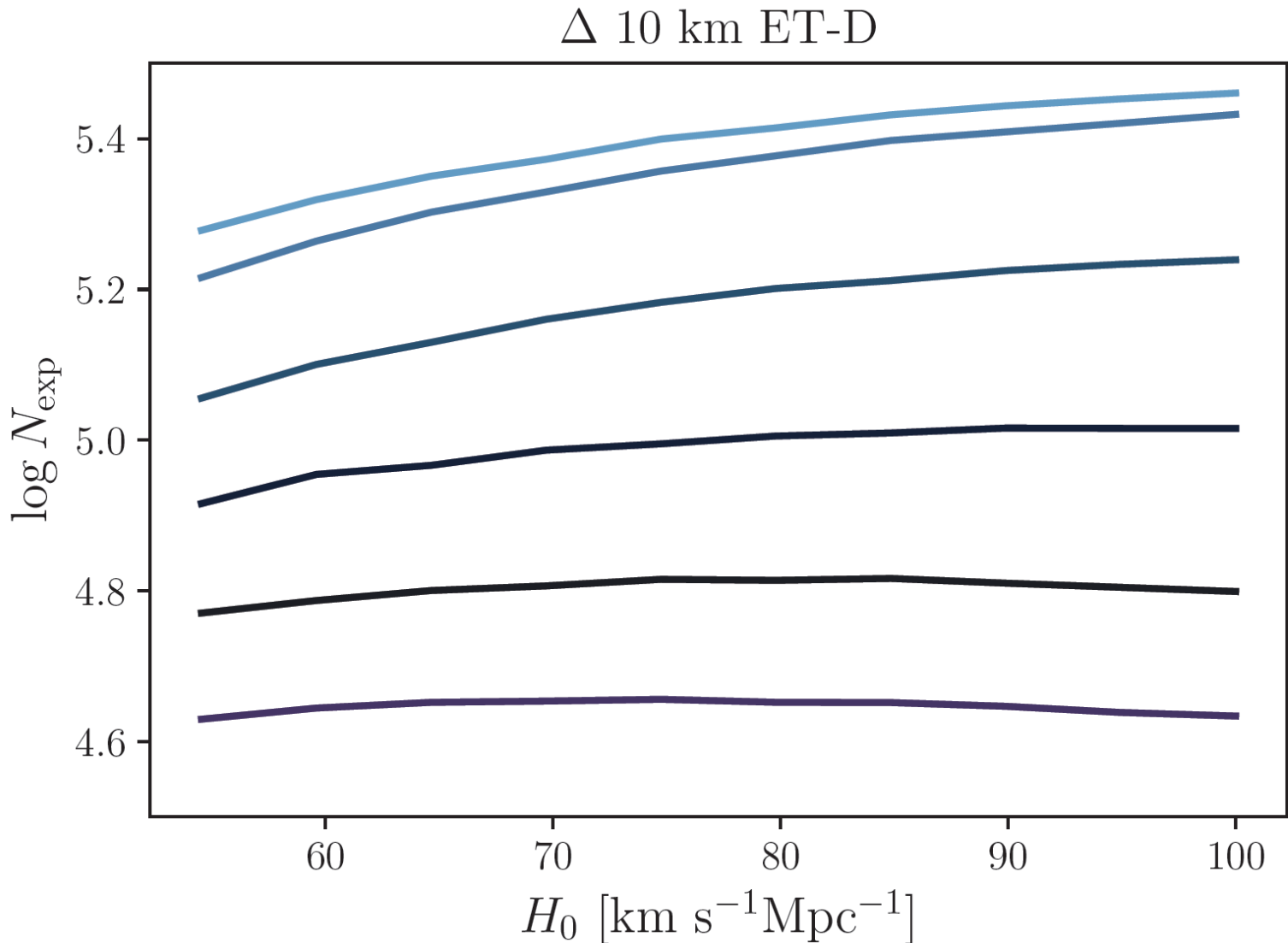
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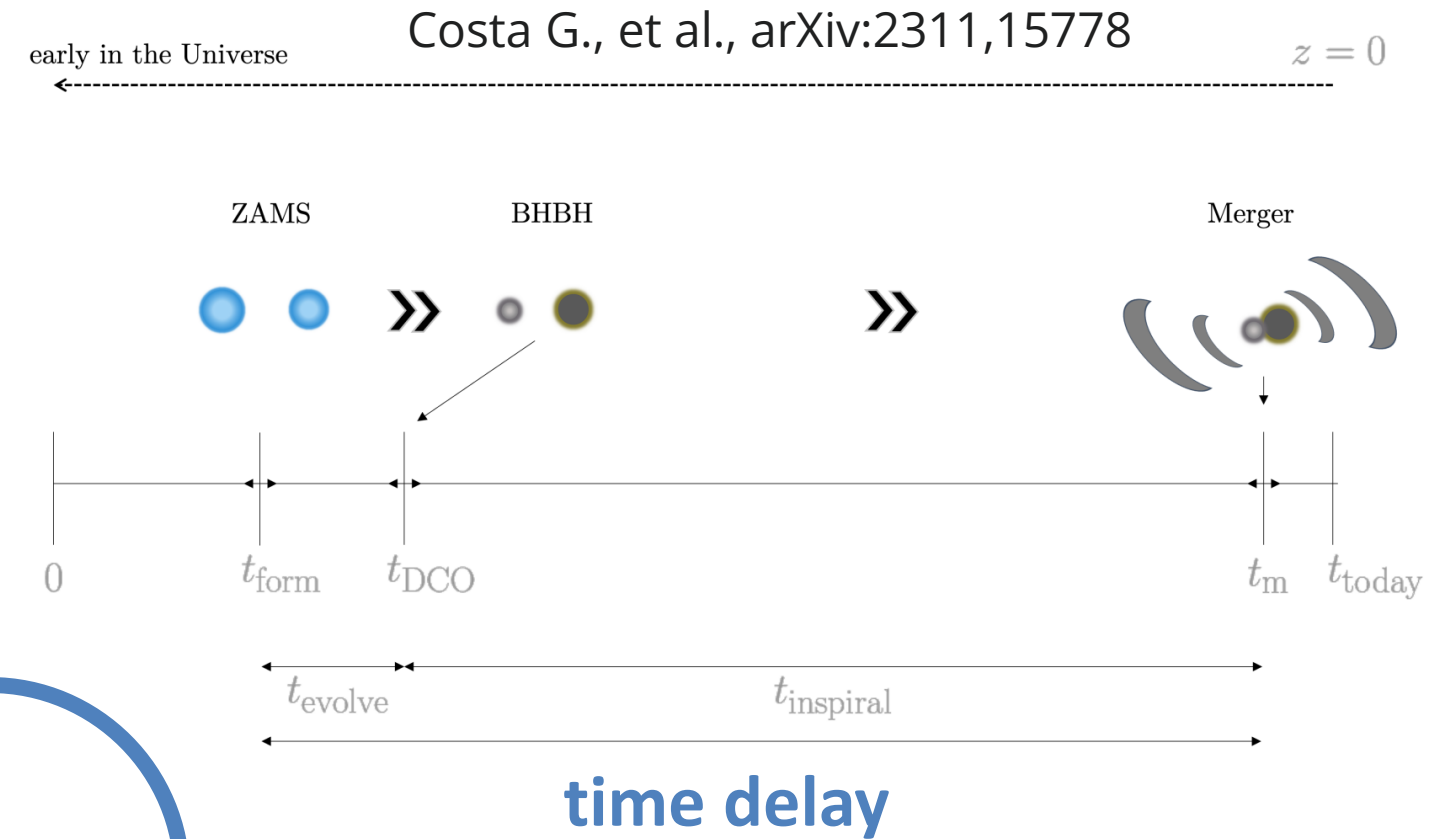
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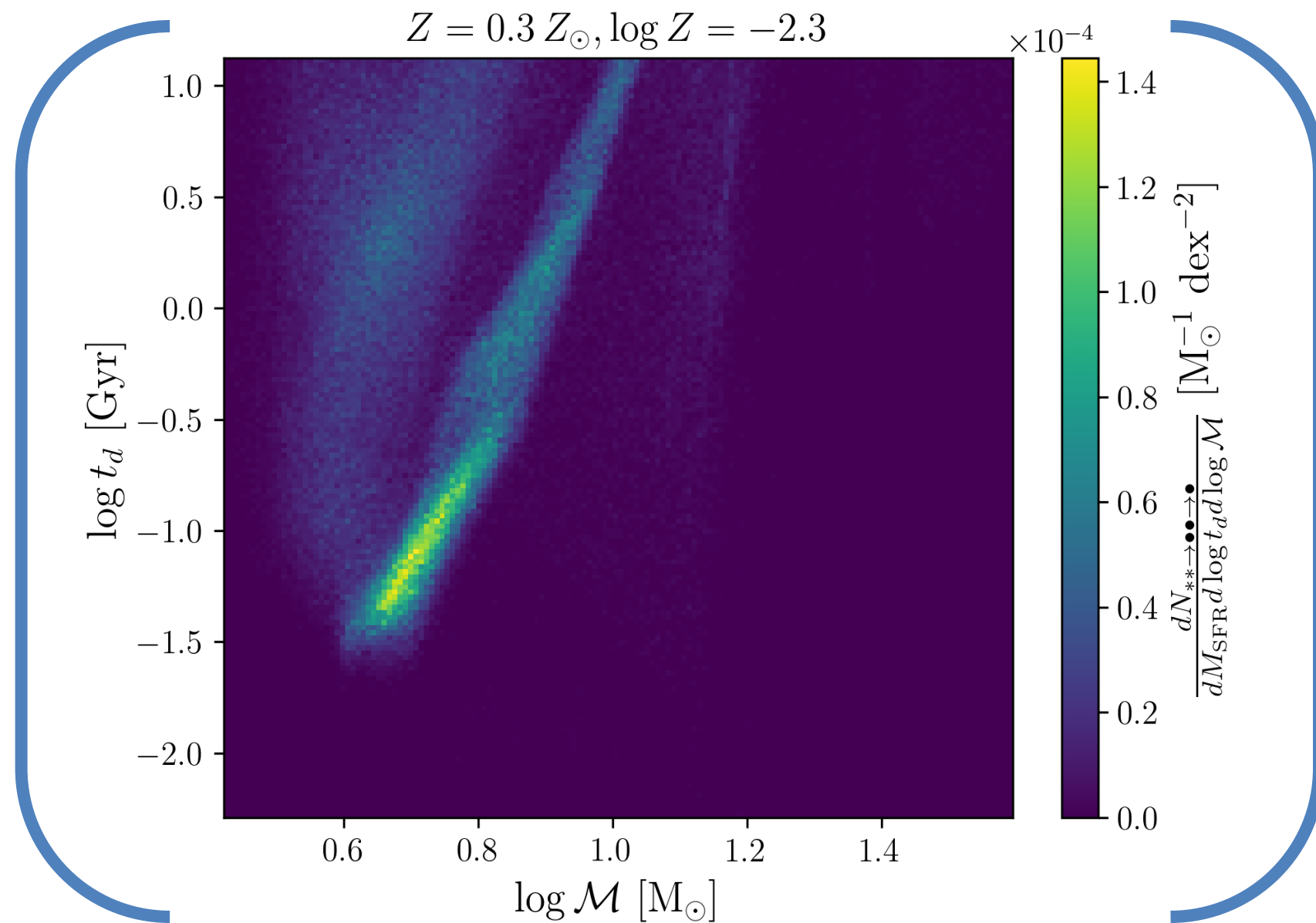
# 3.1 : Results

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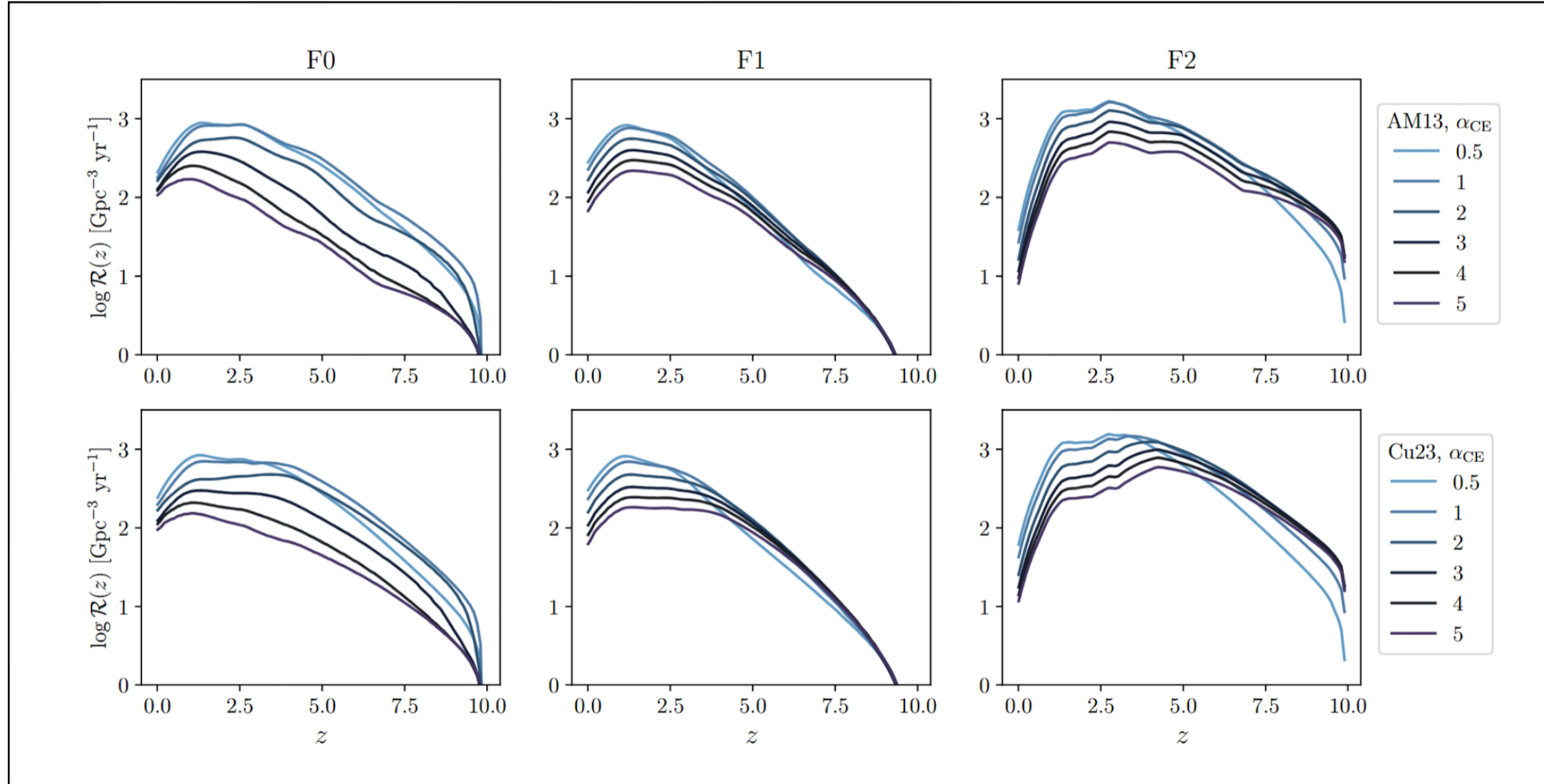
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# Resulting models and ET forecast

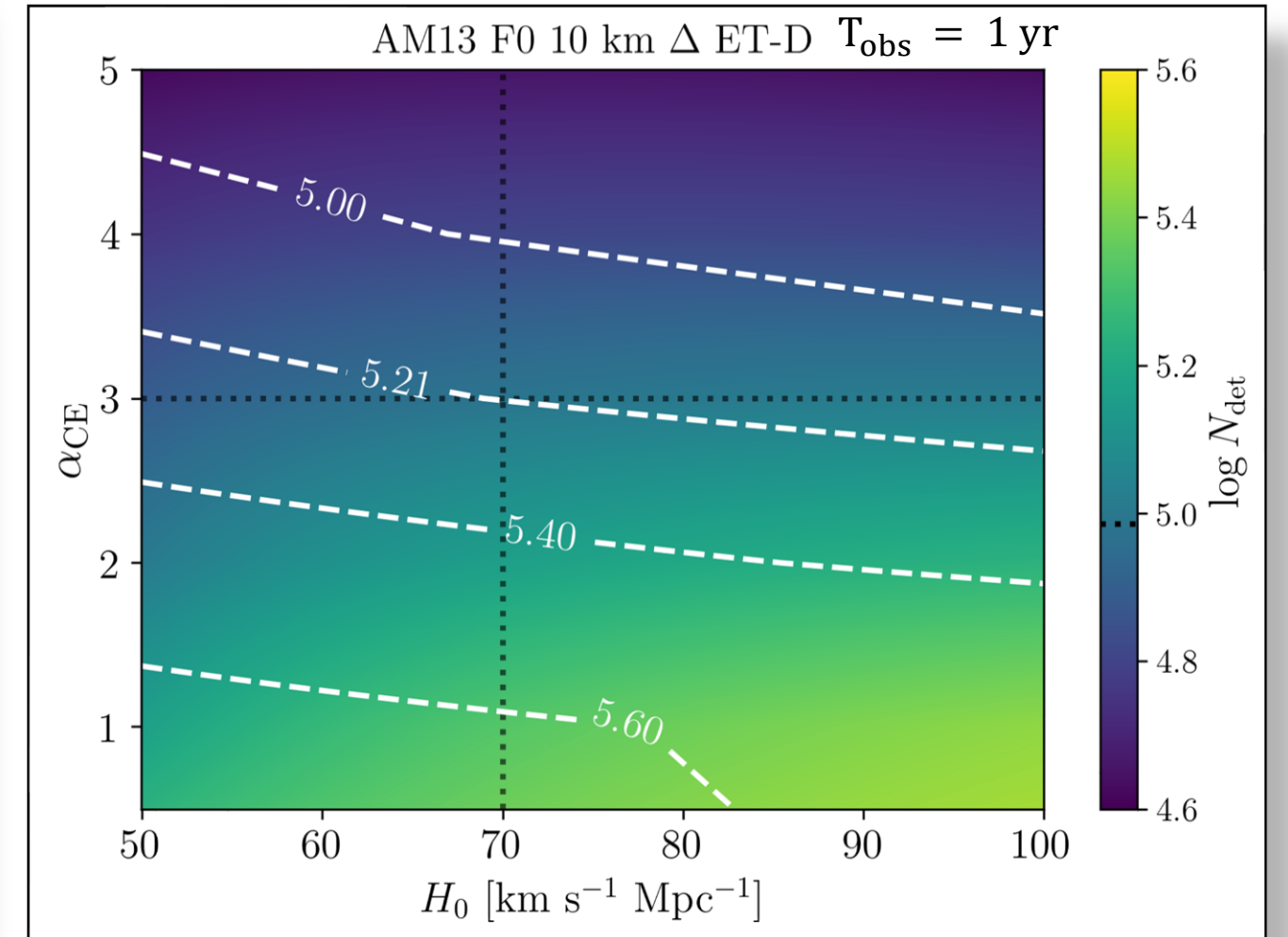
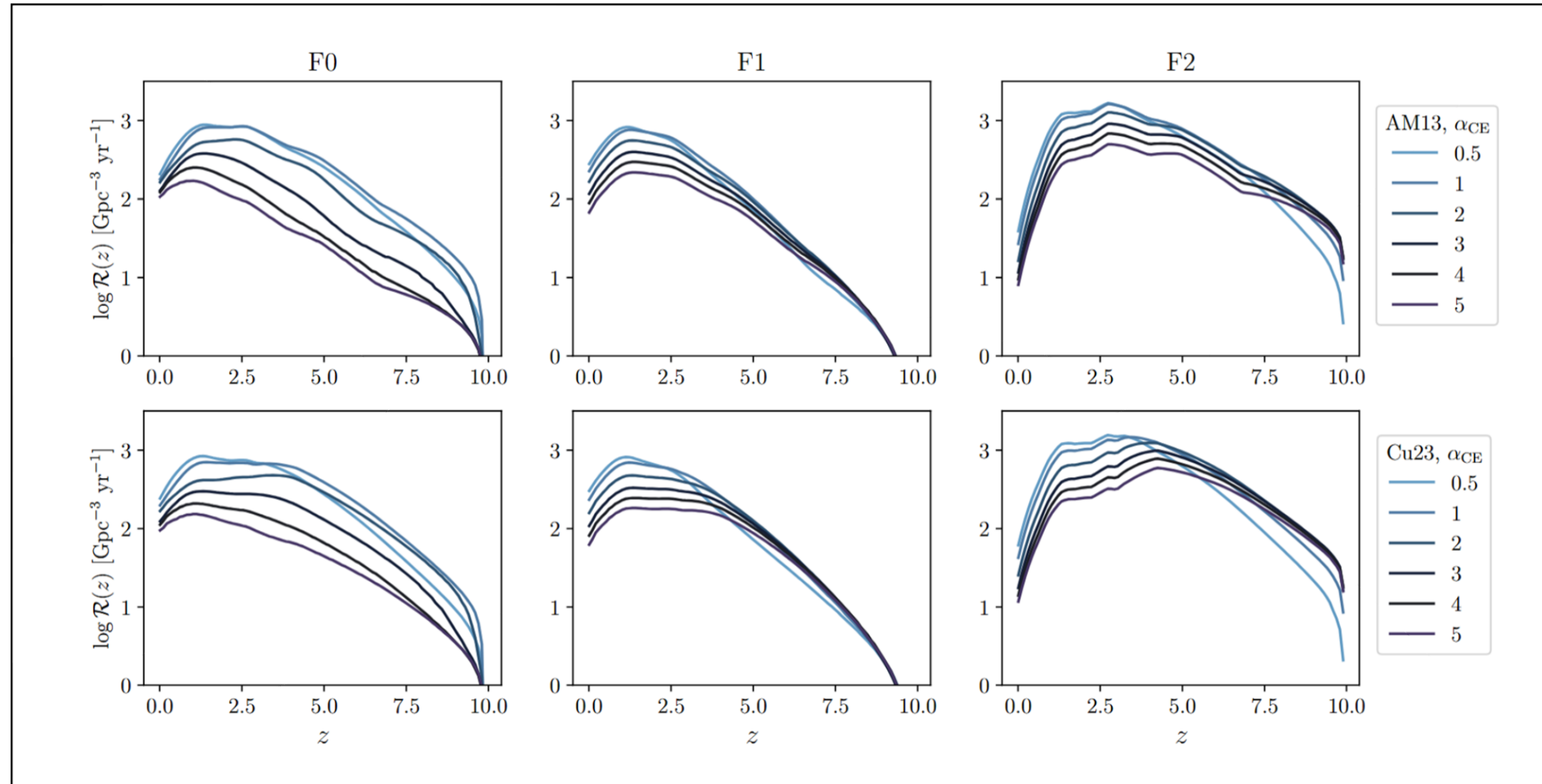
$$\frac{dp}{dq d\mathcal{M}_s d \log \tau}(q, \mathcal{M}_s, \tau|Z, \alpha_{\text{CE}}) = \begin{cases} \text{No factorization} & \text{F0} \\ \frac{dp}{dq d\mathcal{M}_s}(q, \mathcal{M}_s|Z, \alpha_{\text{CE}}) \frac{dp}{d \log \tau}(\tau|Z, \alpha_{\text{CE}}) & \text{F1} \\ C(\alpha_{\text{CE}}) \frac{dp}{dq d\mathcal{M}_s}(q, \mathcal{M}_s|Z, \alpha_{\text{CE}}) & \text{F2} \end{cases}$$

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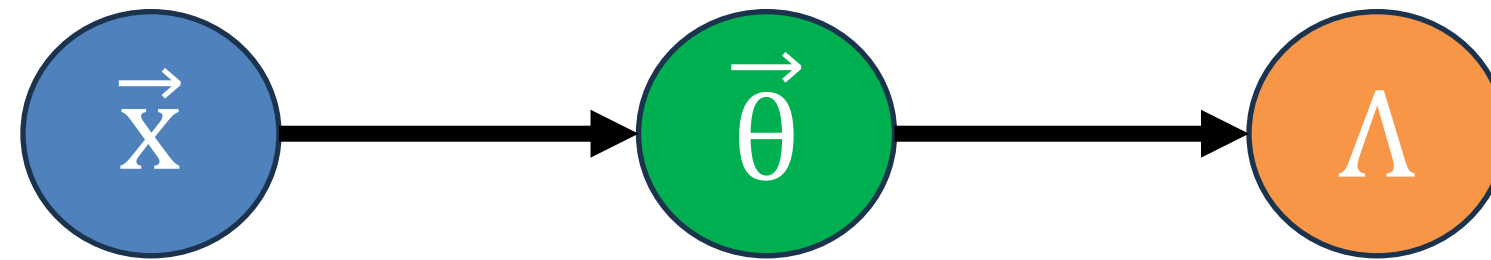
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$$N_{\text{det}}(\Lambda) = T_{\text{obs}} \int d\theta p_{\text{det}}(\theta, \Lambda) \frac{dN}{dt_d d\theta}(\theta | \Lambda)$$

Einstein Telescope **detection function**: selects parameters that can produce detectable events

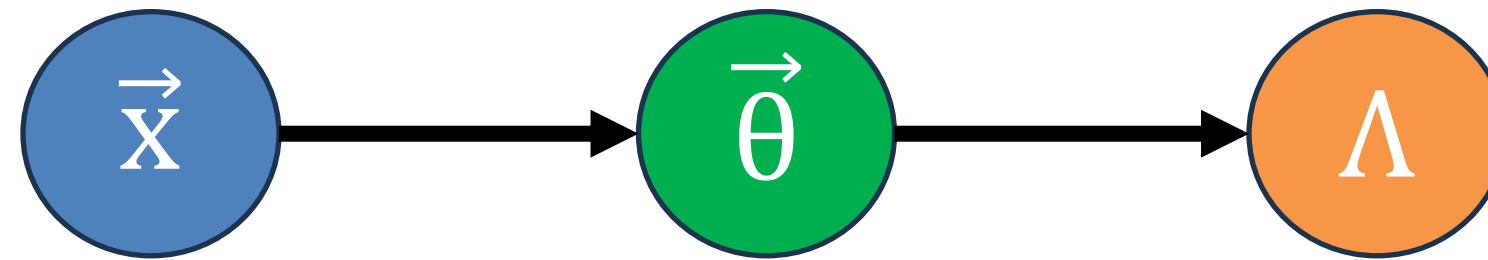
# Hierarchical/population inference

Our population models on source-frame mass parameters  $\vec{\theta} = (m_1, m_2)$  or  $(q, \mathcal{M})$  (other parameters could be included e.g. spin) depends on a set of **hyper**-parameters  $\Lambda = (\alpha_{\text{CE}}, H_0)$ . Because of noise, the measured parameters  $\vec{x}$  will differ from  $\vec{\theta}$ , hence a hierarchy links measurements to hyper-parameters.



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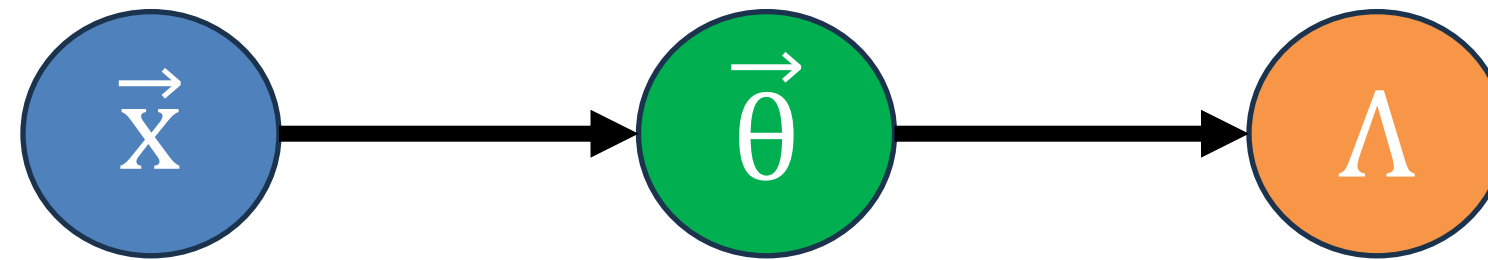


**Hierarchical inference** is the approach to tackle this problem (Vitale S., Gerosa D., Farr W.M. Taylor S.R. HGWA Springer 2022 45):

$$\mathcal{L}(\{\vec{x}_i\}, N_{\text{obs}}|\Lambda) \propto e^{-N_{\text{det}}} N_{\text{det}}^{N_{\text{obs}}} \prod_{i=1}^{N_{\text{obs}}} \frac{\int d\vec{\theta} p(\vec{x}_i|\vec{\theta}) p(\vec{\theta}|\Lambda)}{\int d\vec{\theta} p(\text{det}|\vec{\theta}) p(\vec{\theta}|\Lambda)}$$

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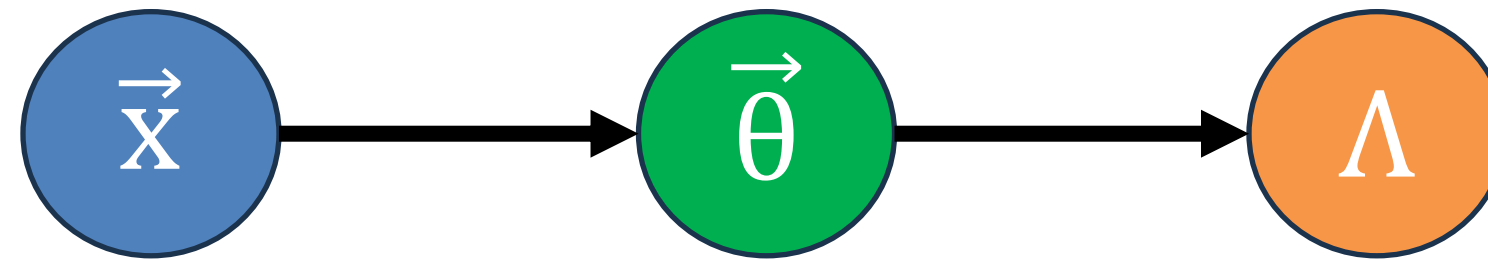
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The *hierarchical likelihood* is composed of the **poissonian** occurrence of  $N_{\text{obs}}$  events and the contribution from the **shape of the spectrum** modified by **selection effects**  $p(\text{det}|\vec{\theta})$ .

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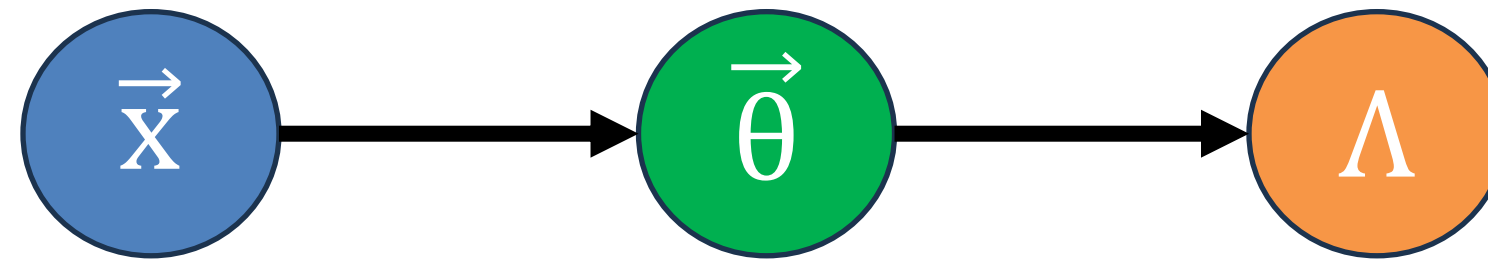
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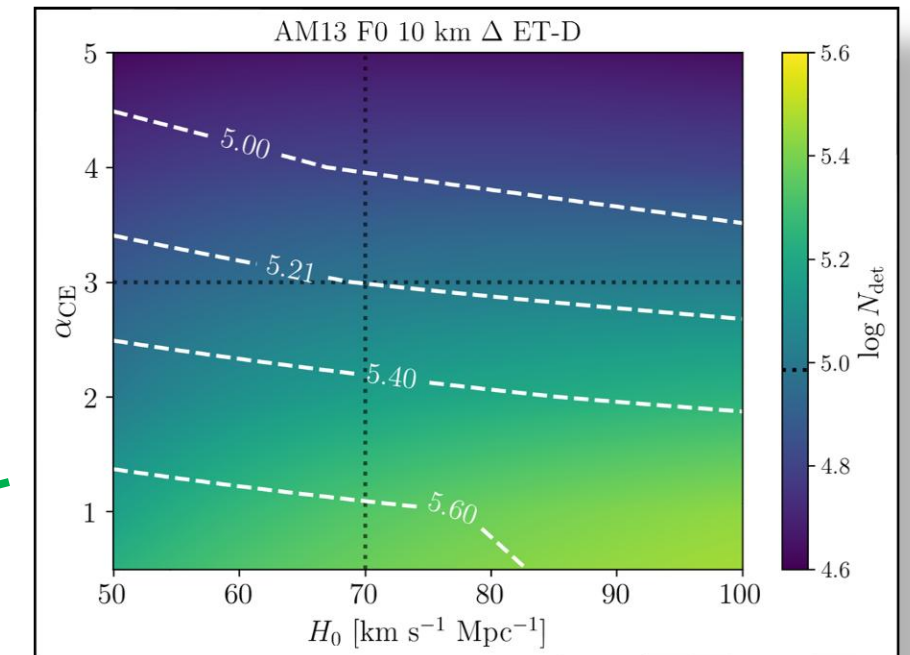
# Hierarchical/population inference

Our population models on source-frame mass parameters  $\vec{\theta} = (m_1, m_2)$  or  $(q, \mathcal{M})$  (other parameters could be included e.g. spin) depends on a set of **hyper-parameters**  $\Lambda = (\alpha_{\text{CE}}, H_0)$ . Because of noise, the measured parameters  $\vec{x}$  will differ from  $\vec{\theta}$ , hence a hierarchy links measurements to hyper-parameters.



**Hierarchical inference** is the approach to tackle this problem:

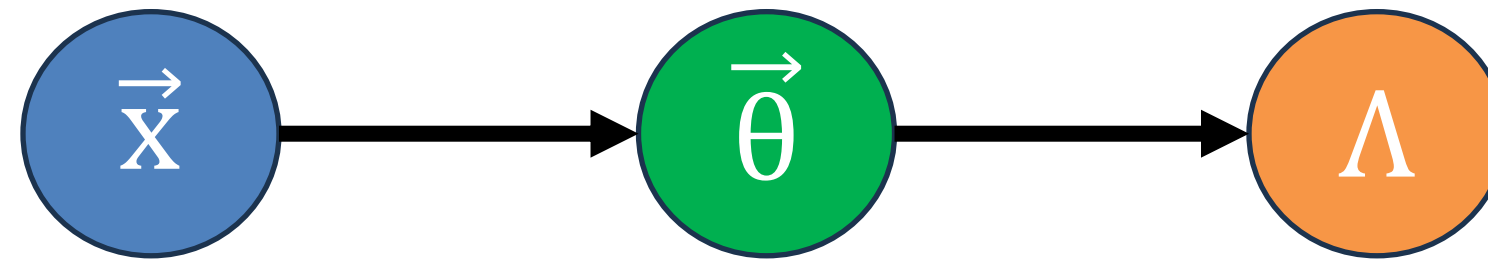
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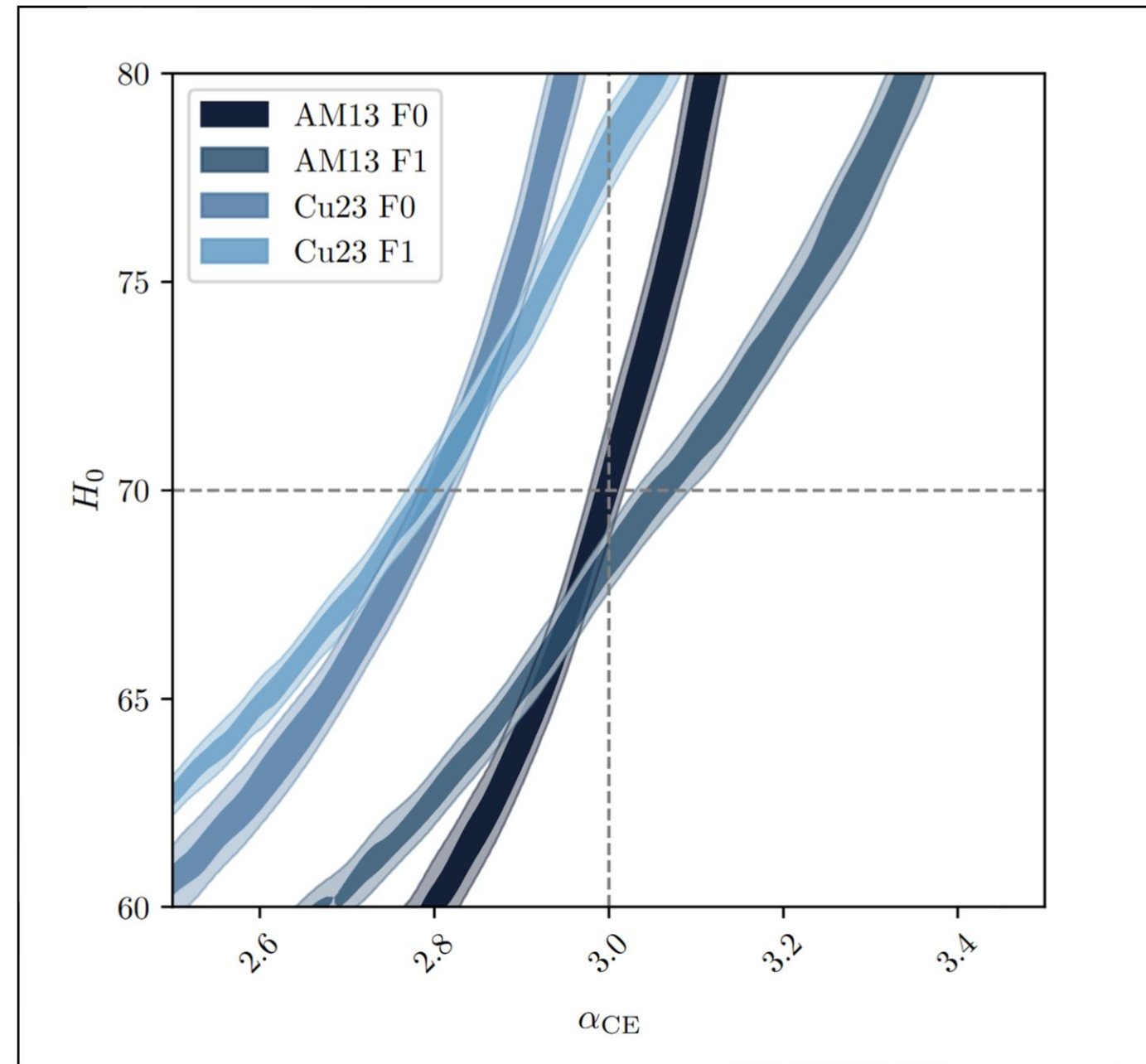
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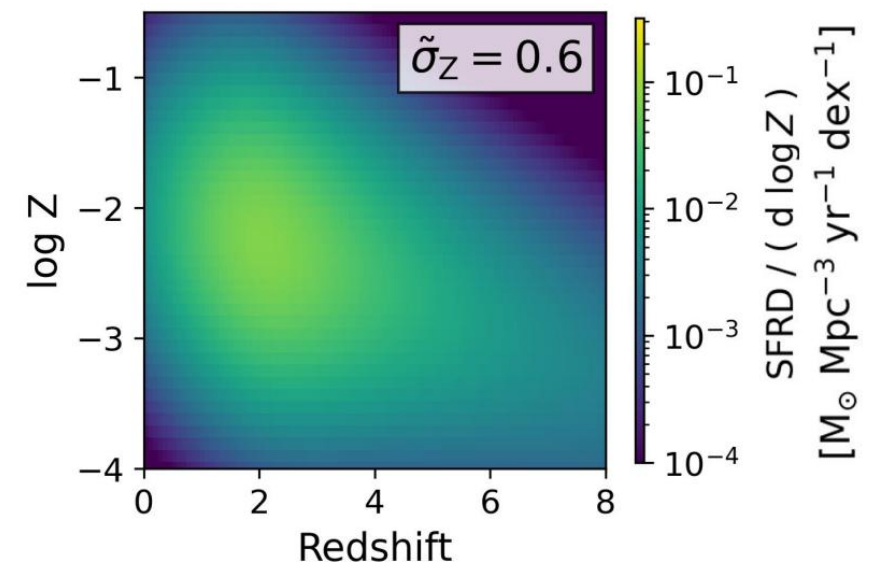
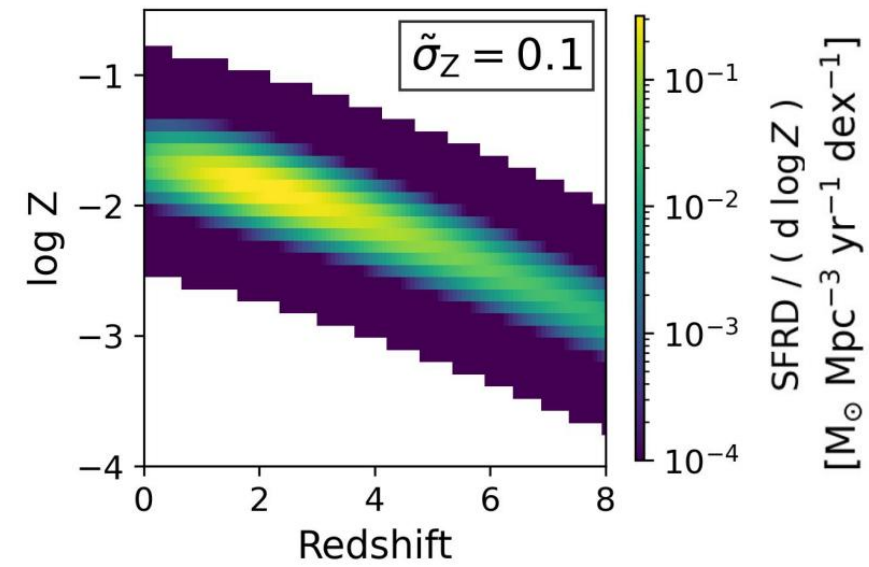
The largest bias is given by an incorrect FMR relation (AM13 or Cu23) and less by the factorizations F0-1, unless the correct factorization is F2.

The strong dependence on the factorization is a strong dependence on the time delay model which can determine a concerning bias in the estimates of the hyperparameters.

# Alternative SFRD model

$$\rho_{\text{SFR}}^{\text{MF}}(z) = 0.01 \frac{(1+z)^{2.6}}{1 + [(1+z)/3.2]^{6.2}}$$

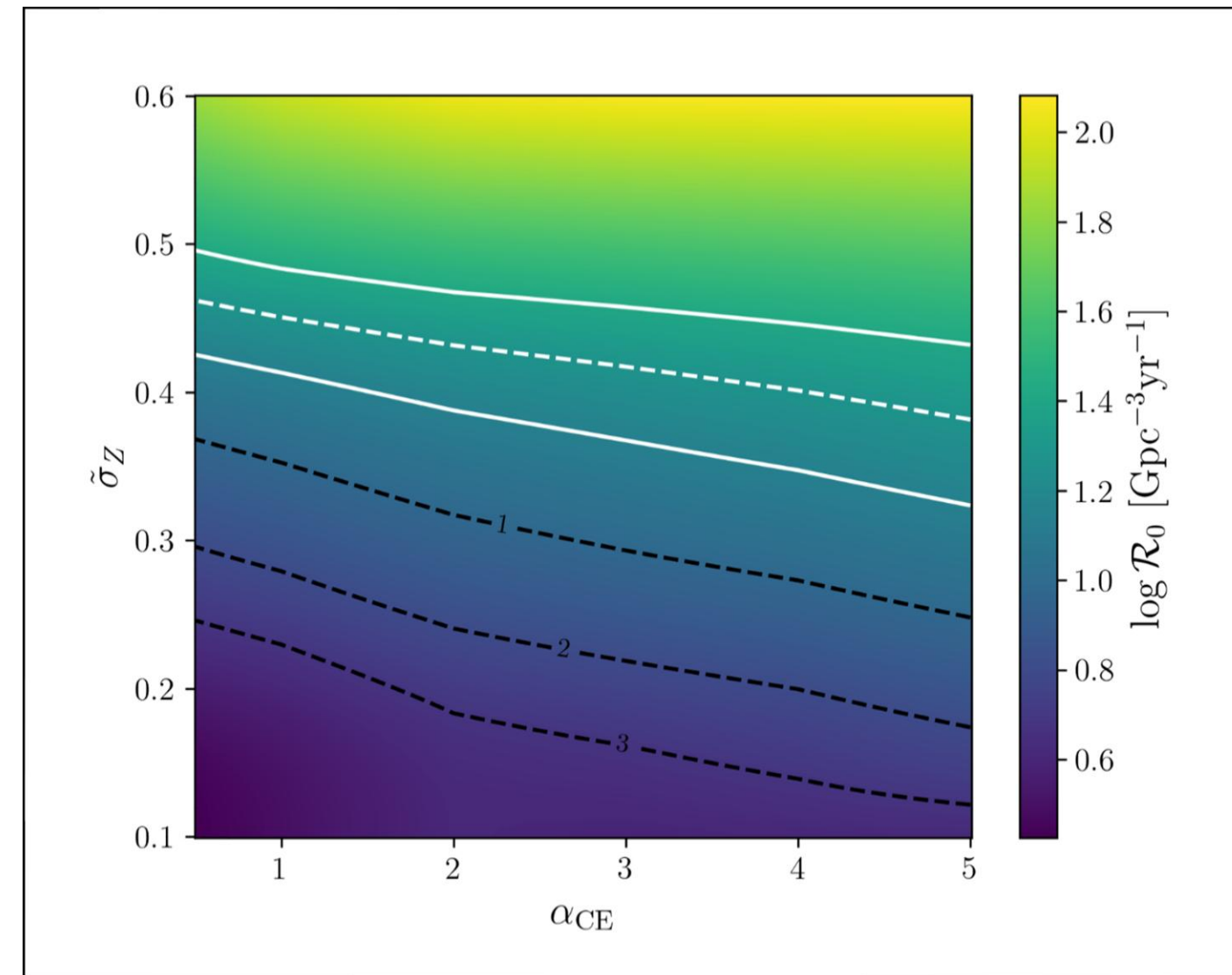
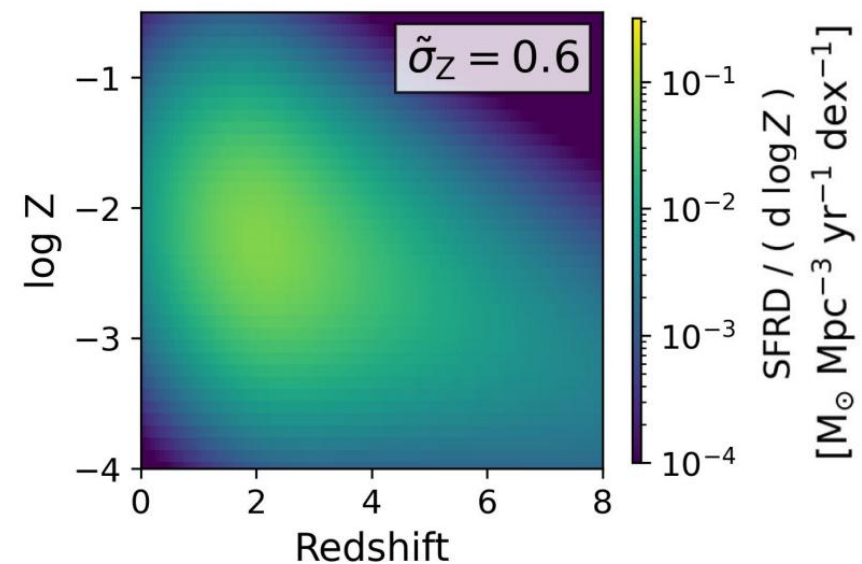
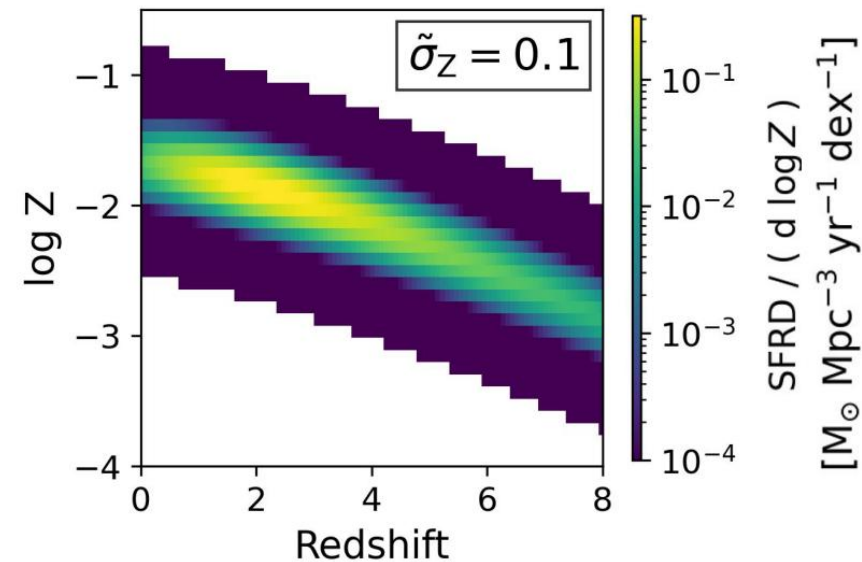
$$\frac{dp}{d \log Z}(Z|z) = \frac{1}{\sqrt{2\pi\tilde{\sigma}_Z^2}} e^{-\frac{[\log(Z/Z_\odot) - \langle \log(Z/Z_\odot) \rangle]^2}{2\tilde{\sigma}_Z^2}}$$



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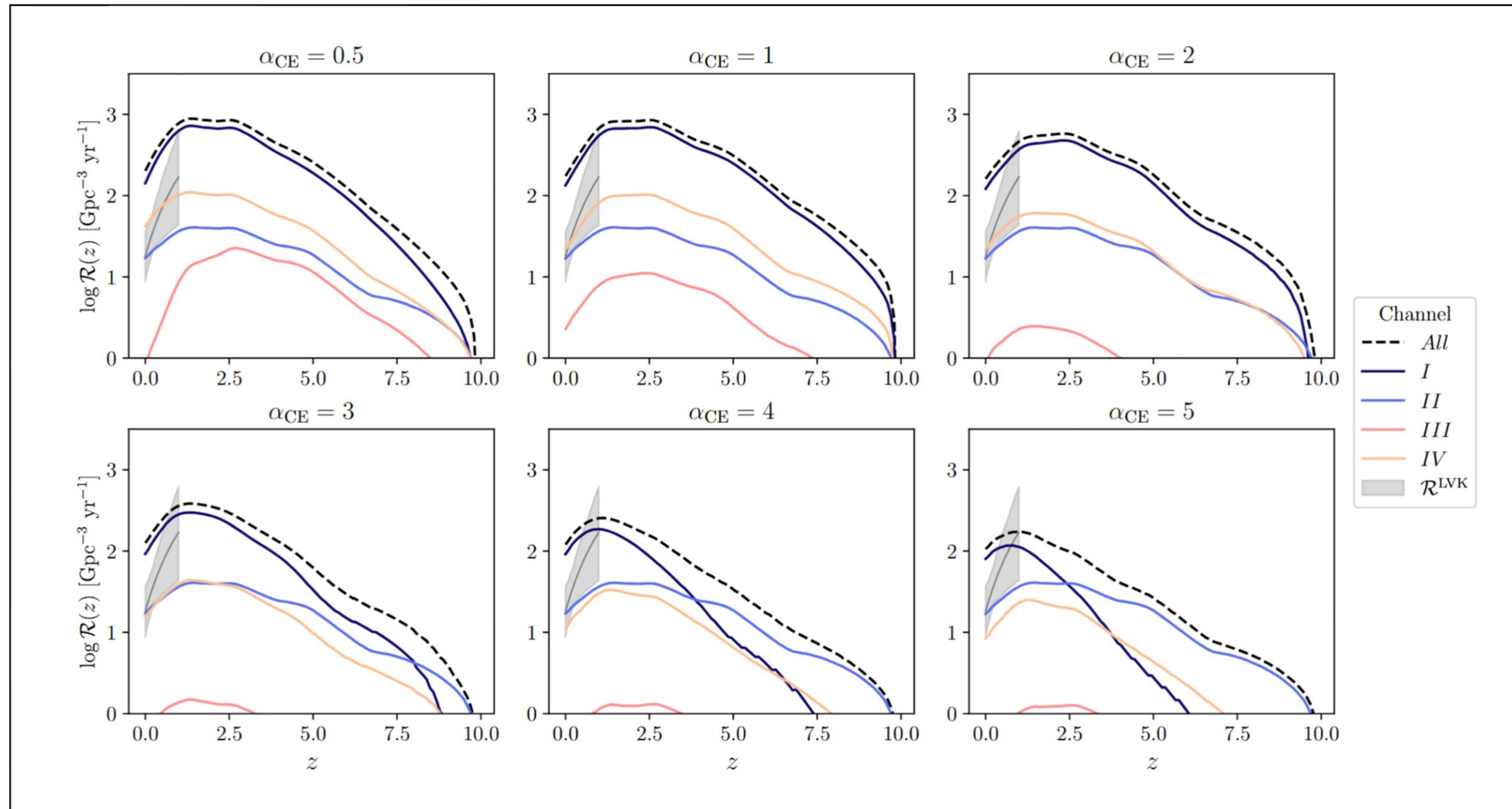
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We find  $\tilde{\sigma}_Z \leq 0.5$  from matching the local merger rate from LVK to our models. If other channels are added to isolate binary, value up to 0.15 can be reached depending on  $\alpha_{CE}$ .

This GW constrain is unfortunately in tension with galaxy formation requirements (M. Chruslinska, Annalen der Physik 536 (2024)) of  $\tilde{\sigma}_Z > 0.5$ ...

# Local rates and stable mass transfer



**Channel I:** SMT before Compact Object formation, then  $> 1$  CE  
**Channel II:** only SMT  
**Channel III:** CE  $> 1$  before Cof, only one stripped star  
**Channel IV:** Same but two stripped stars

F.S. Broekgaarden, et al., MNRAS 516 (2022) 5737

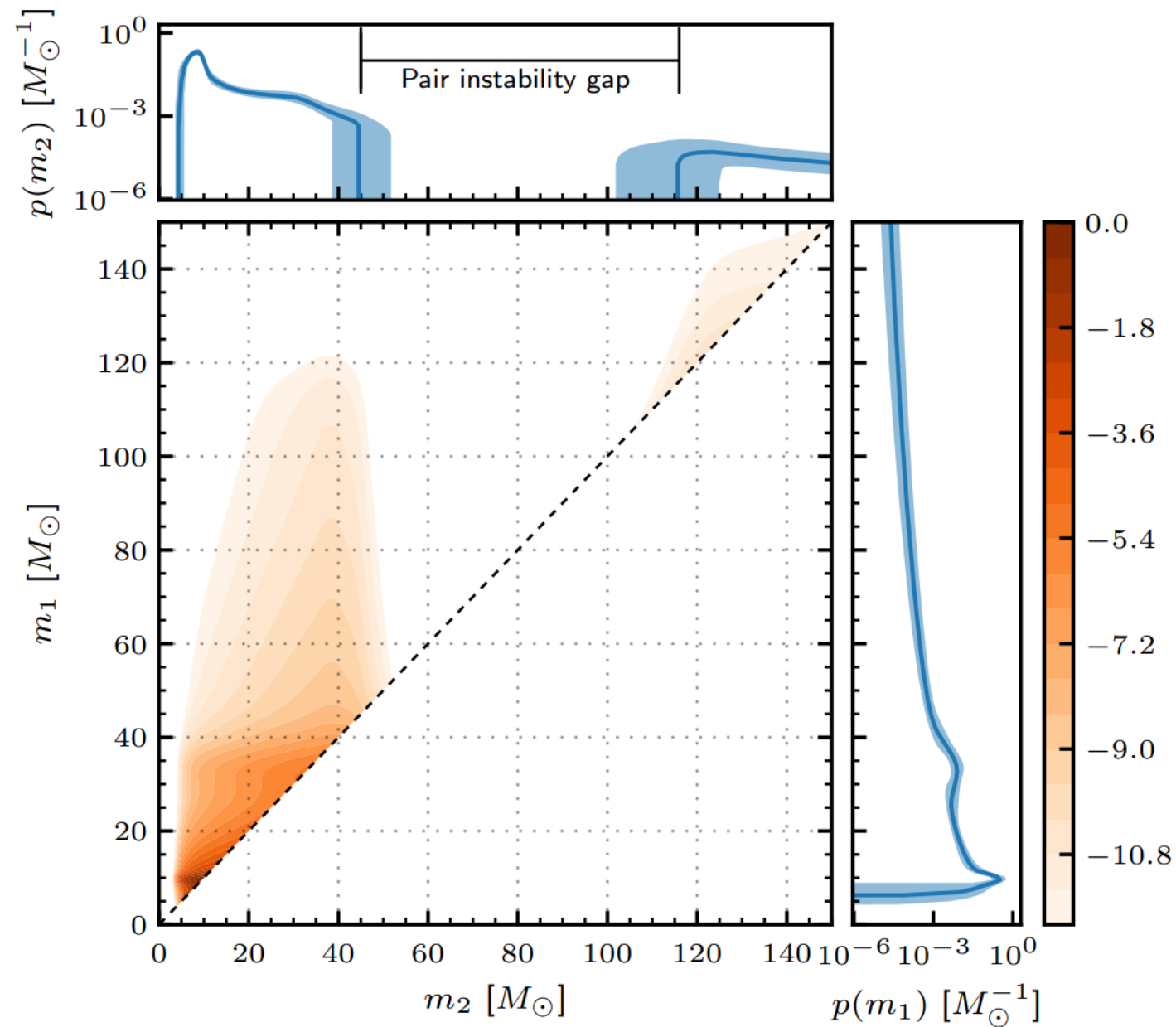
$\alpha_{CE} > 1$  are required, especially when adding additional formation channels.

Less CE systems and same order of SMT systems could be more appropriate to explain the merger rates.

The strong dependence on time-delay distribution suggests a revision of the criteria for unstable mass transfer.

# 3.2 : Open questions

# $m_2$ PISN gap

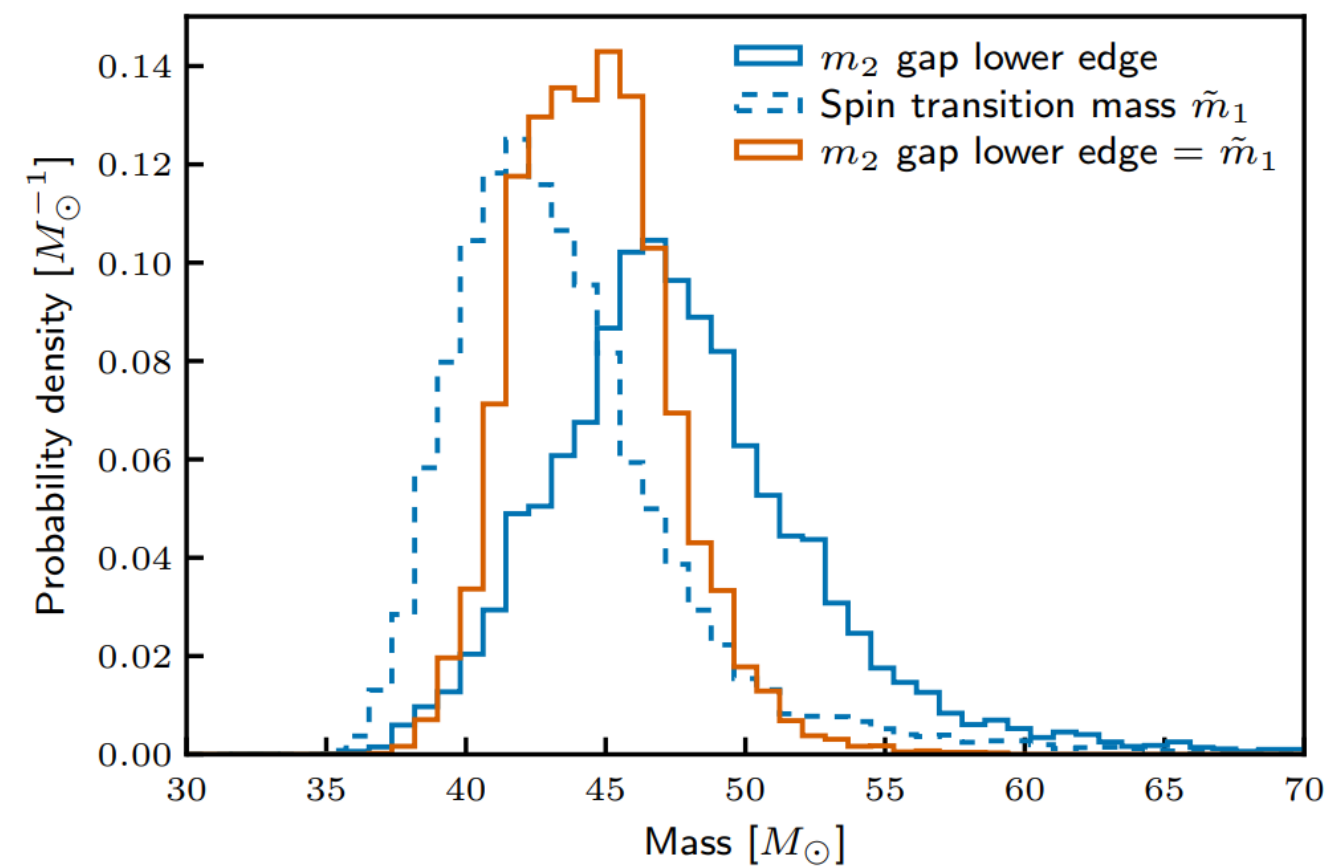


The recent work from Tong H., et al., Nature 652 8111 2026 showed the presence of a PISN gap in the secondary mass spectrum.

## Motivation:

Primary BH PISN gap filled with 2G BHs, but mergers of 2G+2G are very rare, hence if the 2G+1G are the only possibility and the 1G is the least massive, then  $m_2$  spectrum should show signs of PISN gap.

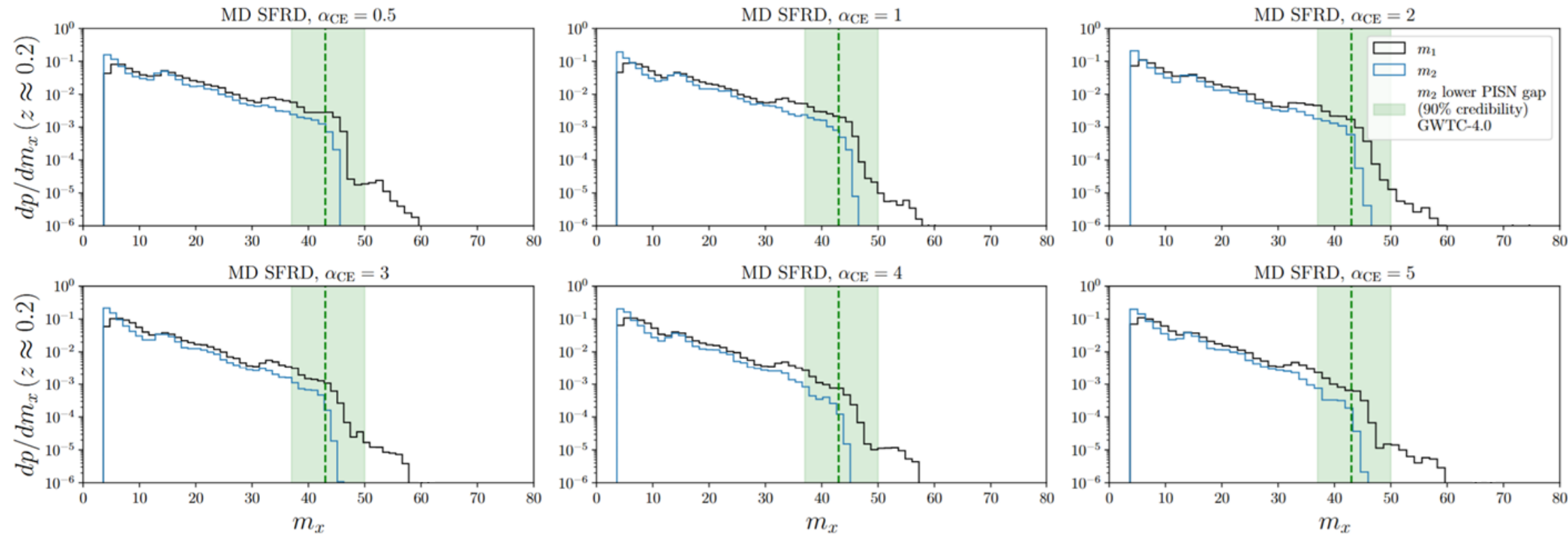
2G should have very high spin wrt 1G, therefore there is a spin transition mass that informs on the 1G->2G transition; this is compatible with the  $m_2$  PISN gap lower limit



# How does cosmology affect the PISN gap?

Can the  $m_2$  PISN gap depend on the cosmic SFR model?

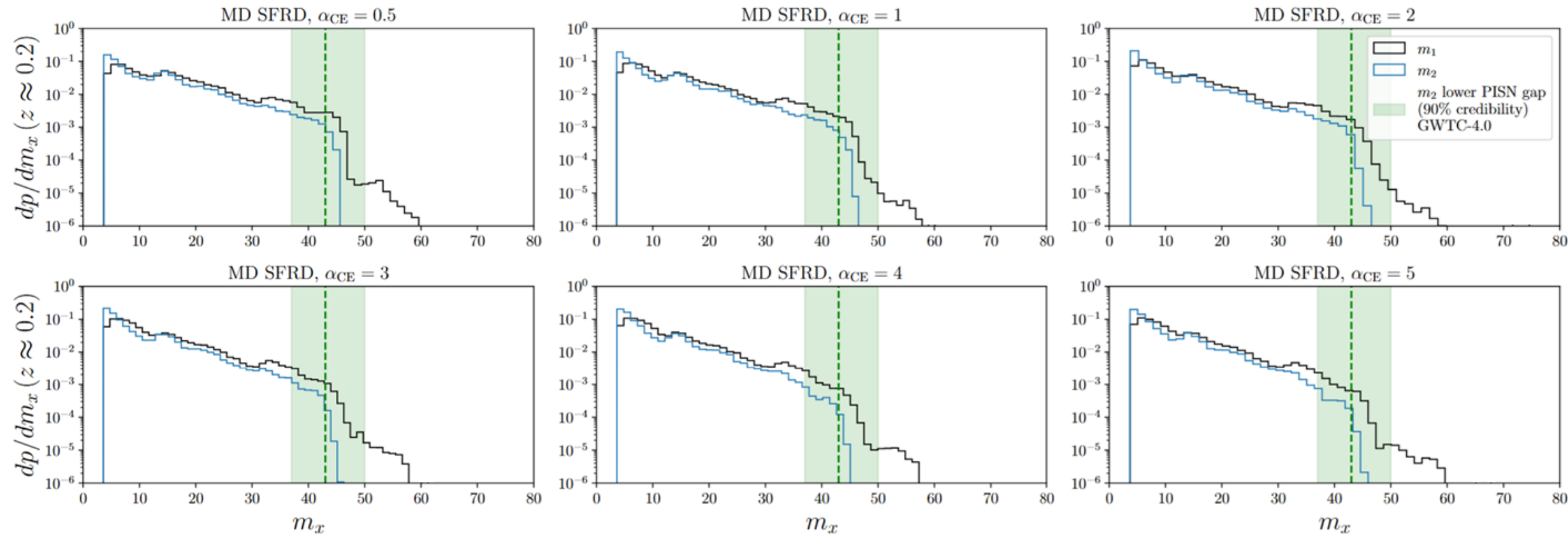
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1. The standard Madau Dickinson model reproduces the expected PISN cut from O4

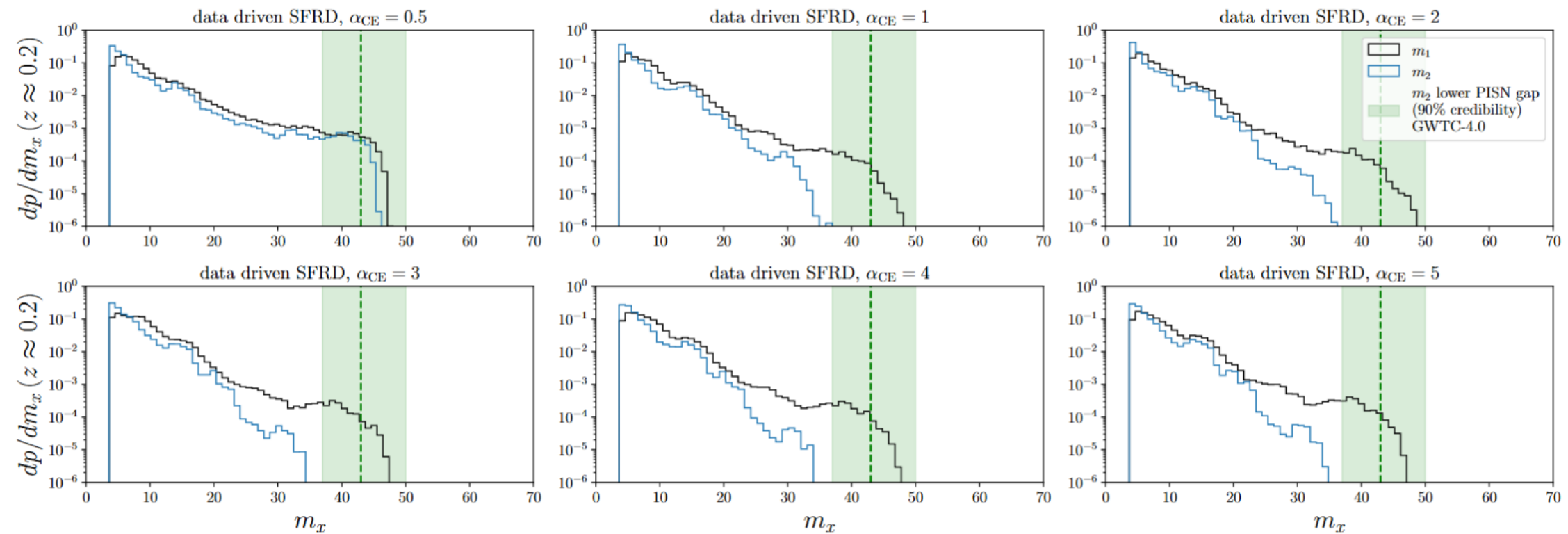
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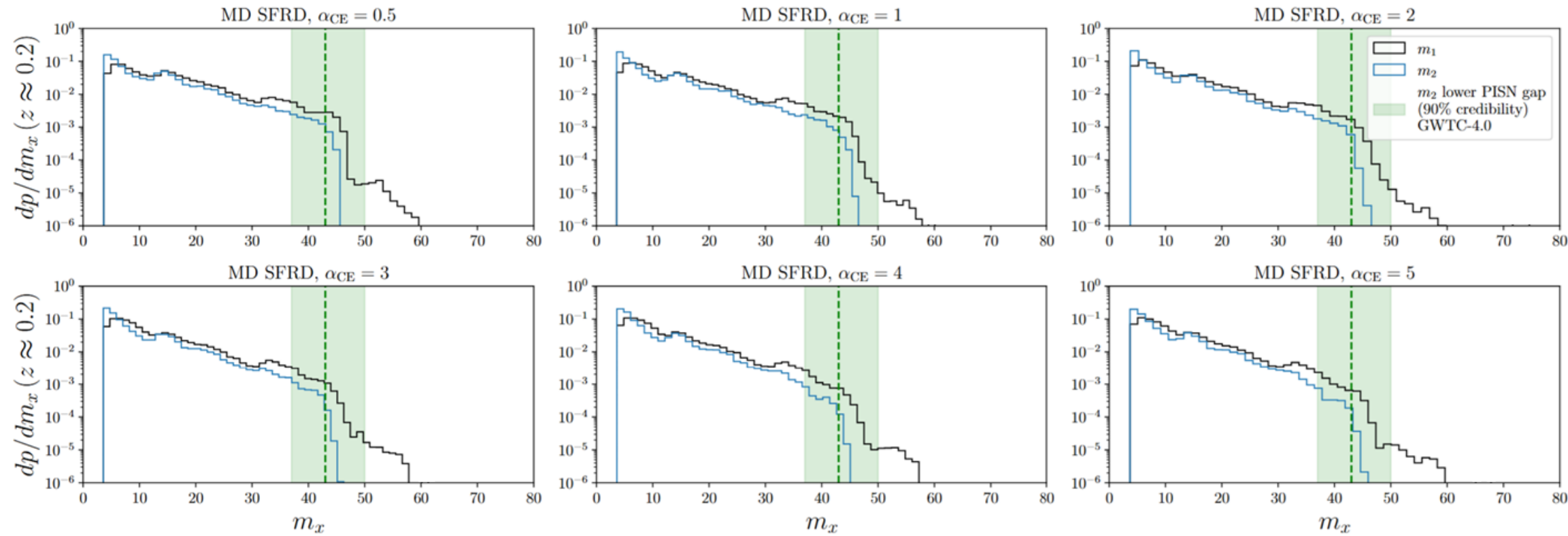
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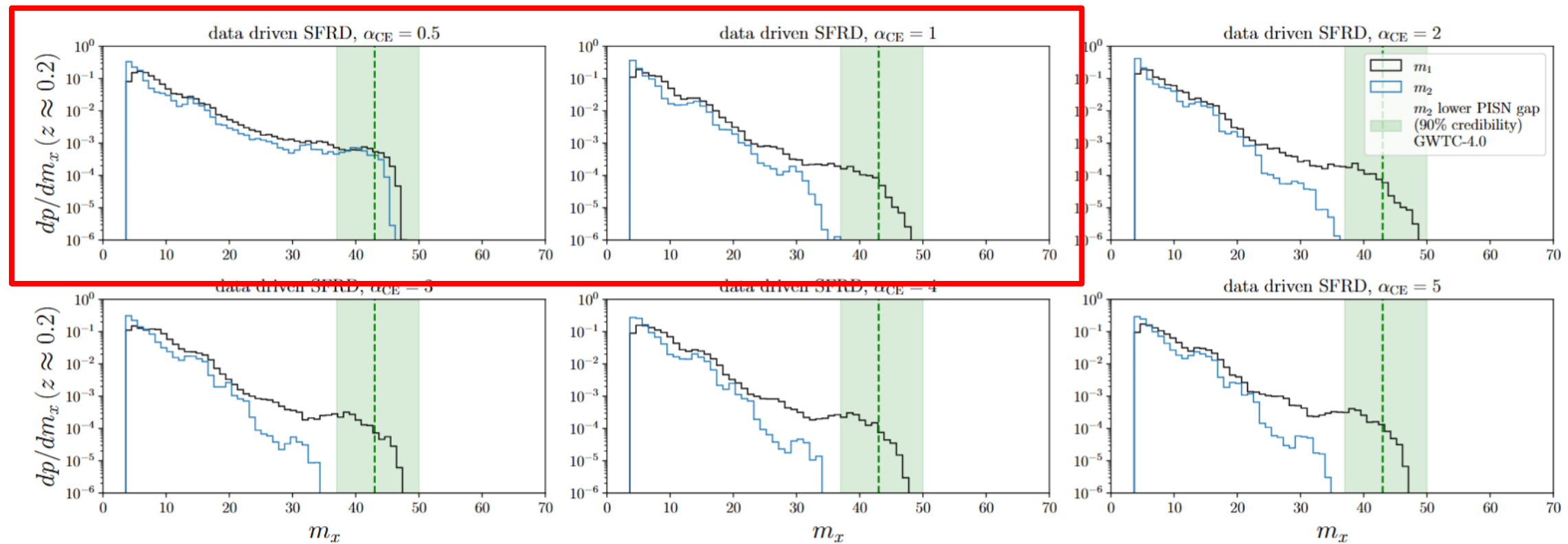


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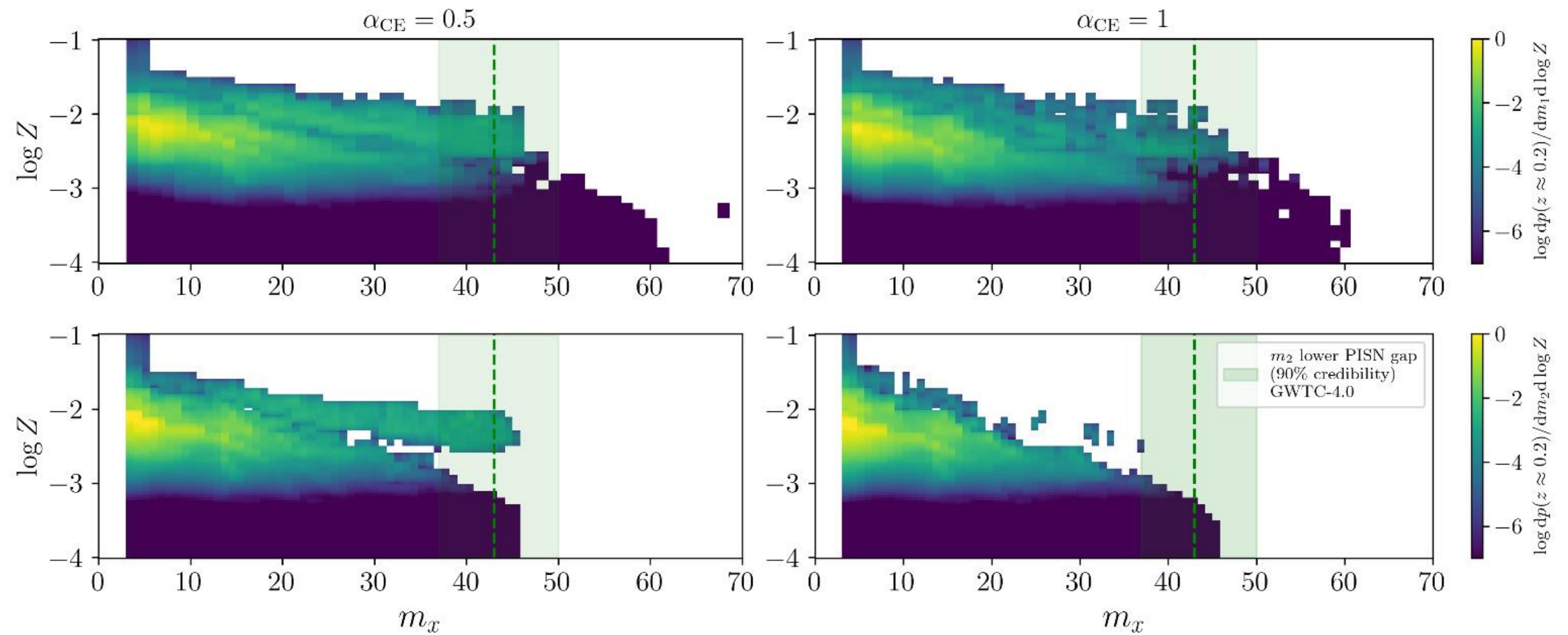
2. A data driven cosmic SFR seems to produce a different PISN cut for the same common envelope efficiency -> **Cosmological dependence**

3. In our model we find a transition around  $\alpha_{CE} = 1$

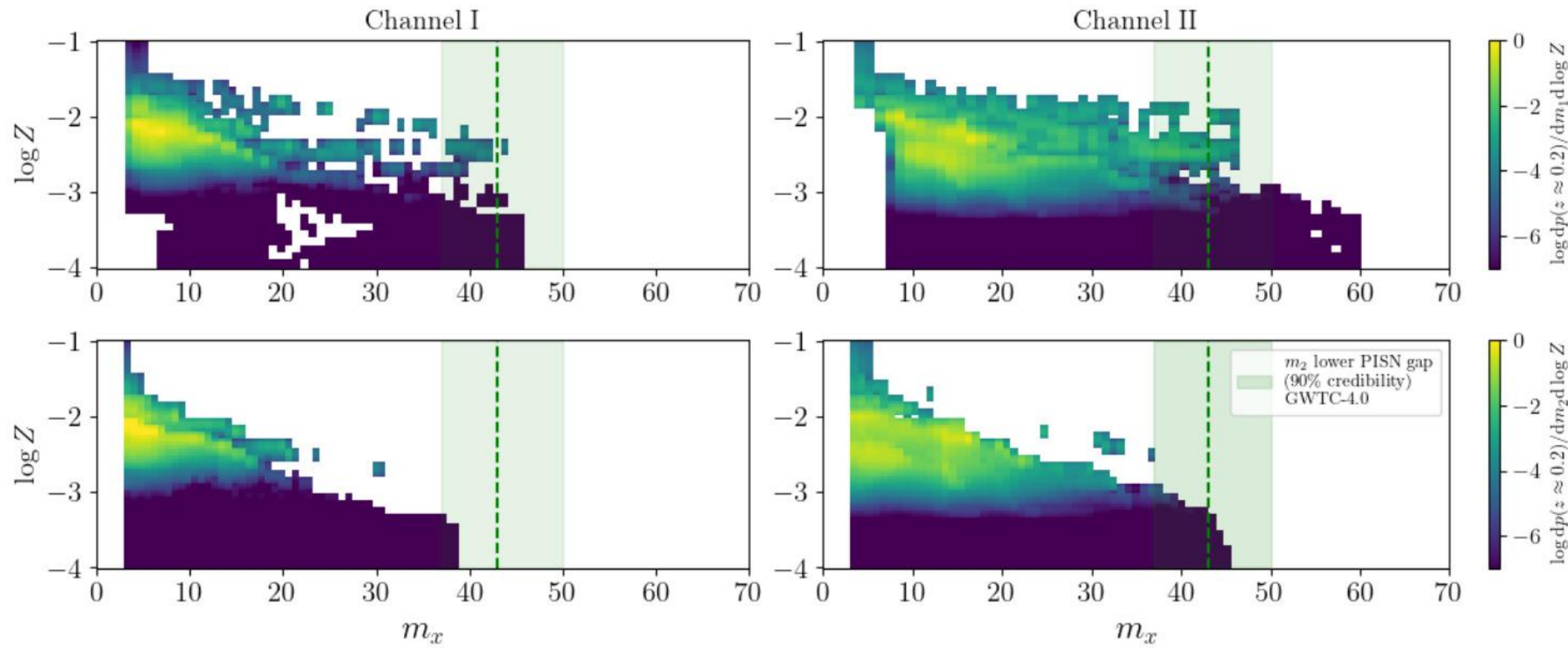


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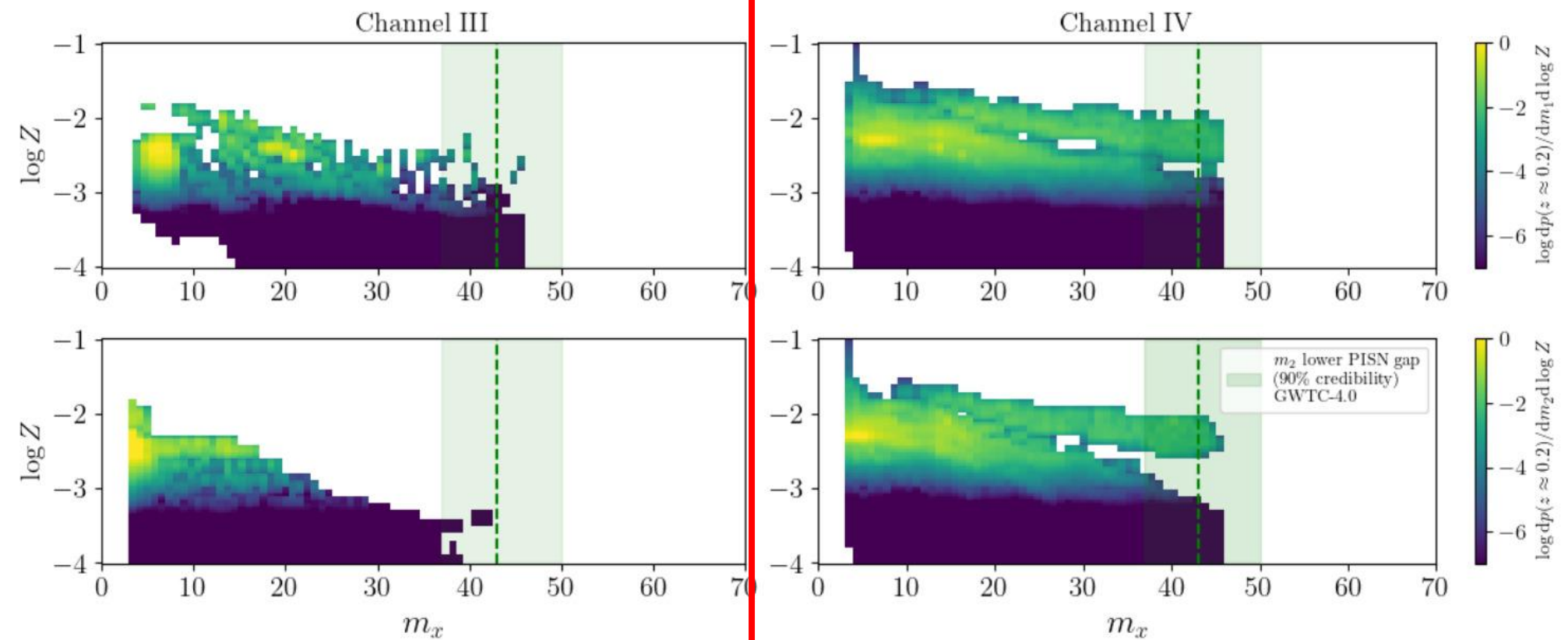


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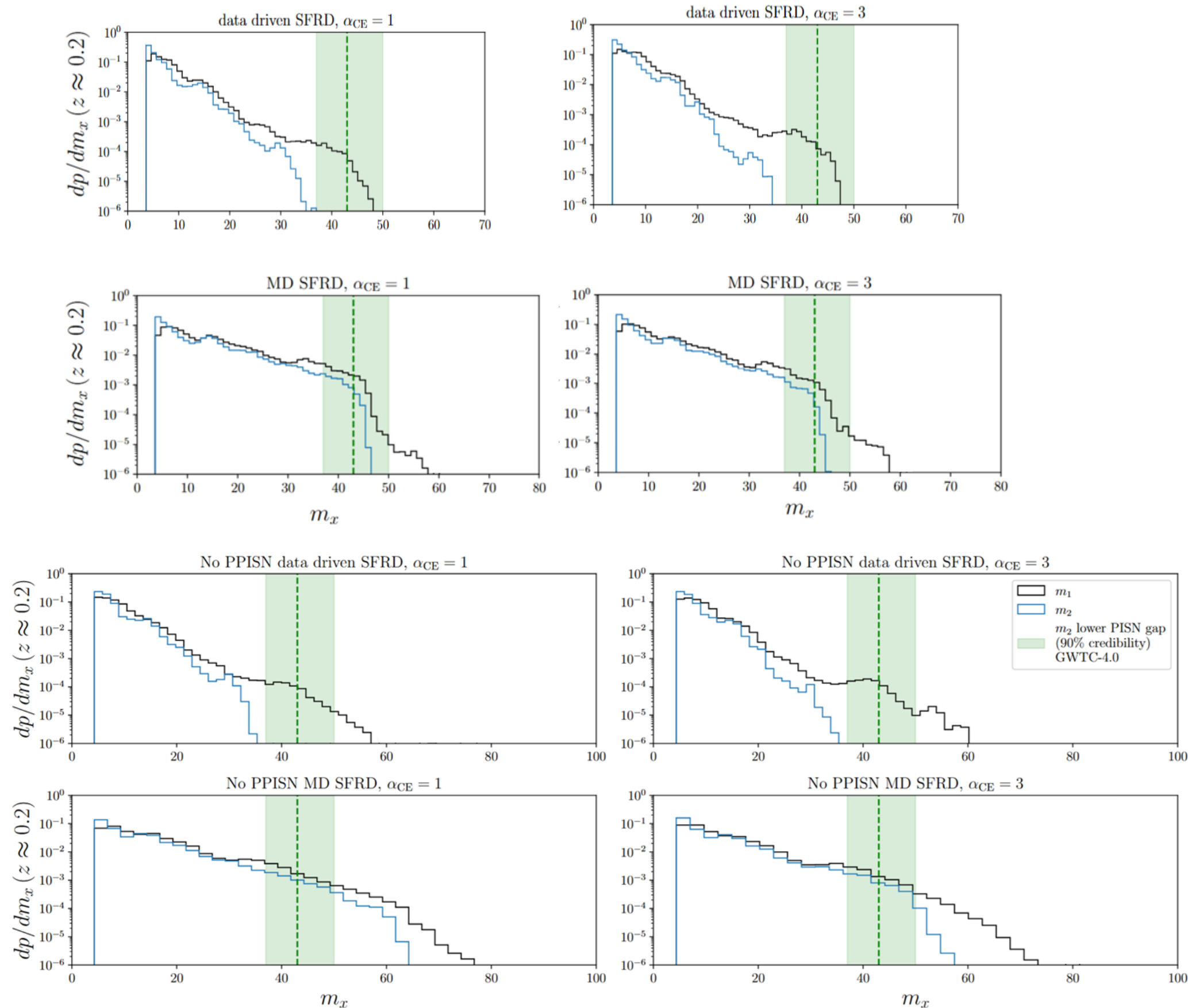
F.S. Broekgaarden, et al., MNRAS 516 (2022) 5737

**Interpretation:** The data driven SFR gives a lot of weight to higher metallicities, and this feature gets picked up, differently from MD.

These systems survive because they can get really close by a double combination of low CE efficiency and double envelope ejection (double naked core system)



# Comparing extremes: PISN vs. No PISN



4. Similarly to what was found by Olejak A., arXiv 2026, binary stellar evolution (**different PISN prescriptions**) influence the position of the PISN gap for  $m_1$ .

However, it seems to be slightly insensitive to the position of the PISN cut in  $m_2$ , therefore **Cosmological model could be dominant over the  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$  rate.**

# Stellar mergers in Young Star Clusters

**Collaborator:** Ugo Niccolò Di Carlo

**Aim:** run N-body simulations for a young stellar cluster to investigate IMF upper limits, initial conditions and formation pathways of the most massive R136 stars

**Methods:**

used PeTar latest version [github.com/lwang-astro/PeTar](https://github.com/lwang-astro/PeTar) a hybrid Nbody code: force of distant particles computed w/ Barnes-Hut tree algorithm, Hermite integrator for closer particles, SDAR for close distance multiples

tested different initial conditions (SC of  $M=10^5 M_{\odot}$ ,  $R = 0.8$  pc):

**2.1)** 5 Kroupa mass upper limits (20, 50, 80, 100, 150)  $M_{\text{sun}}$

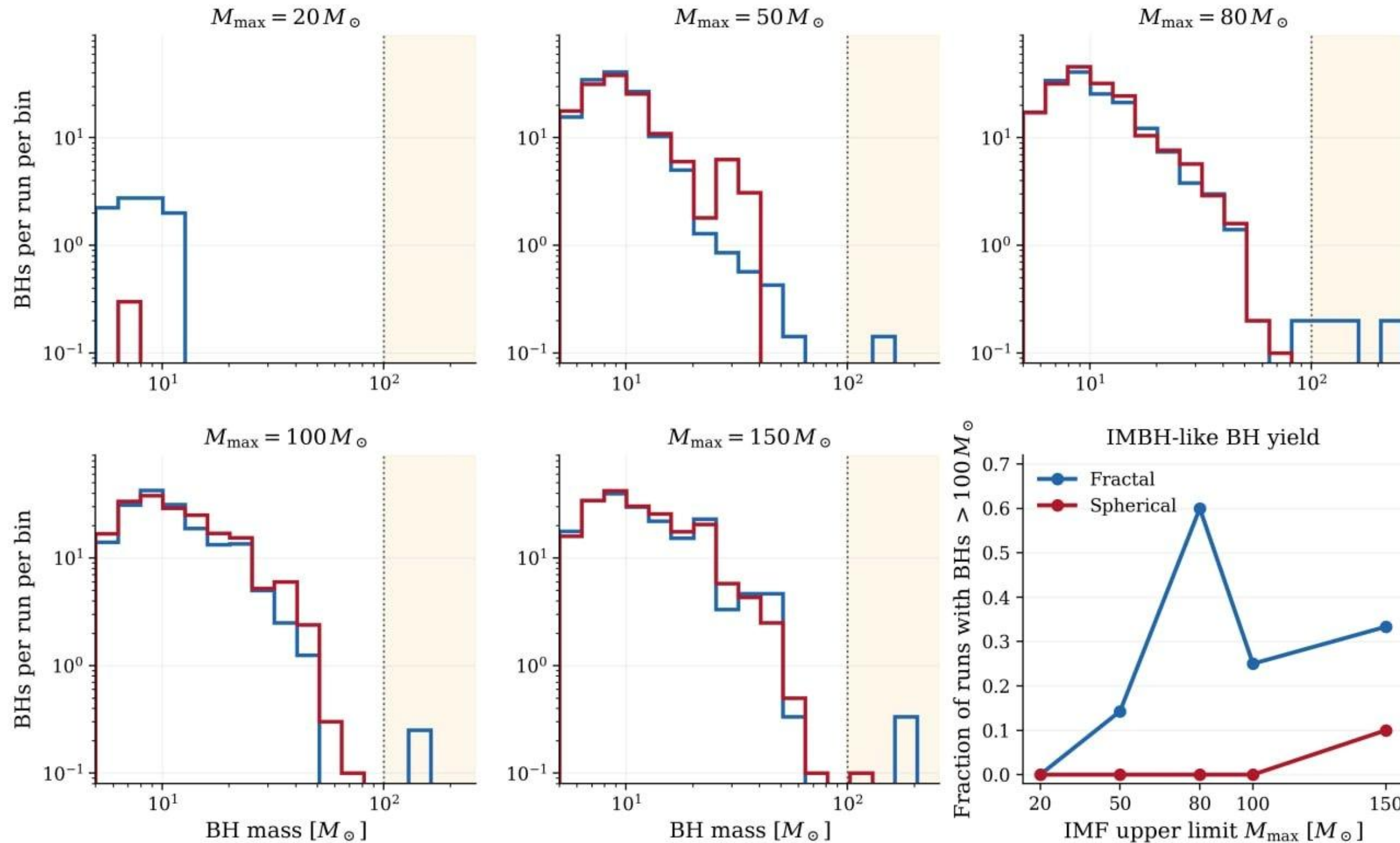
**2.2)** 10 realizations for 2 different density distributions (*Spherical* and *Fractal* w/ fractal dimension 1.6) integrated up to 10 Myrs (actually 5 fractal realizations reach 10 Myrs, but 10 reach up to 2 Myrs)

**Results:** (in preparation)

# BH mass spectra

Black-hole Mass Spectra At 10 Myr

— Fractal — Spherical > 100  $M_{\odot}$  region



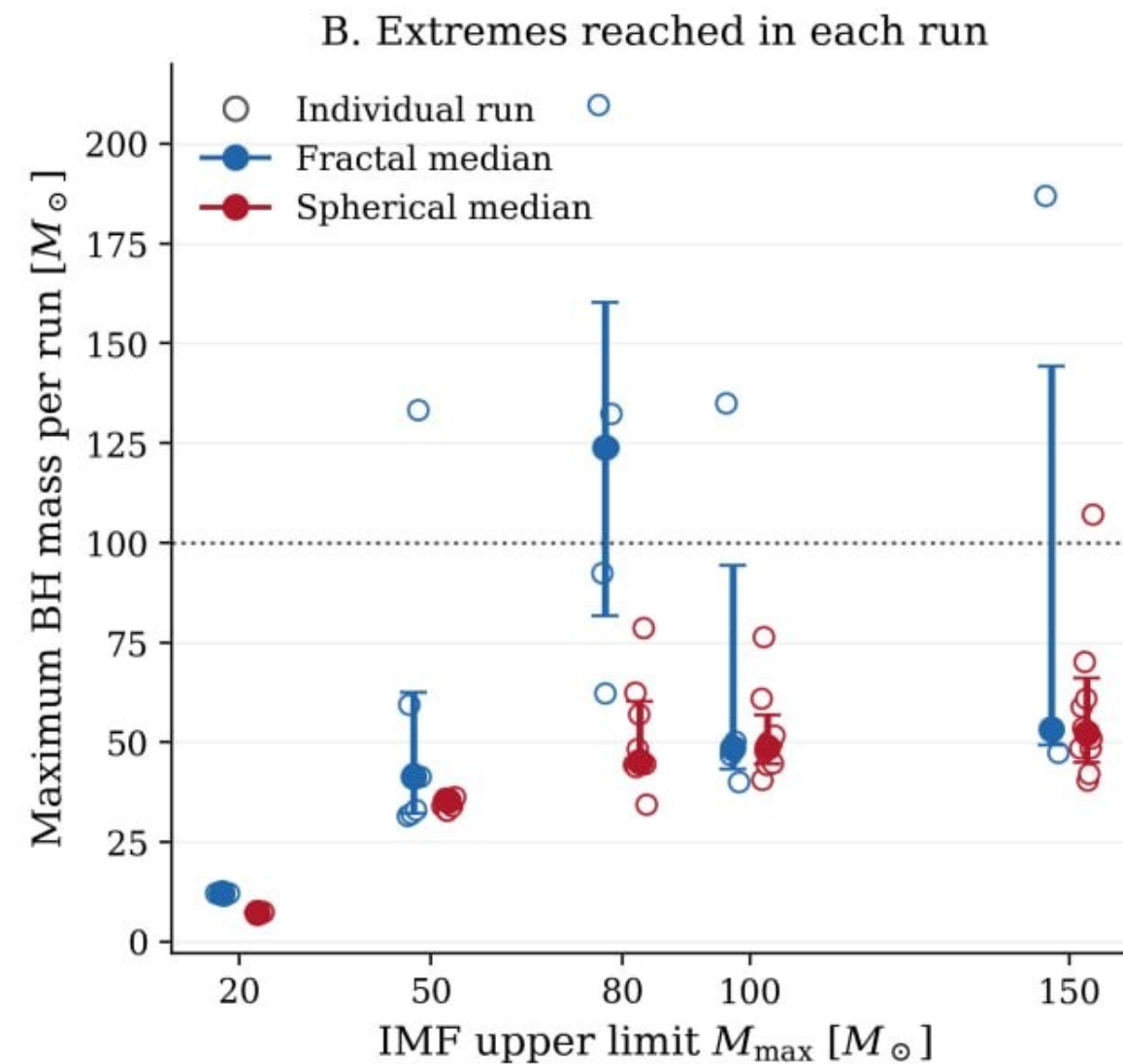
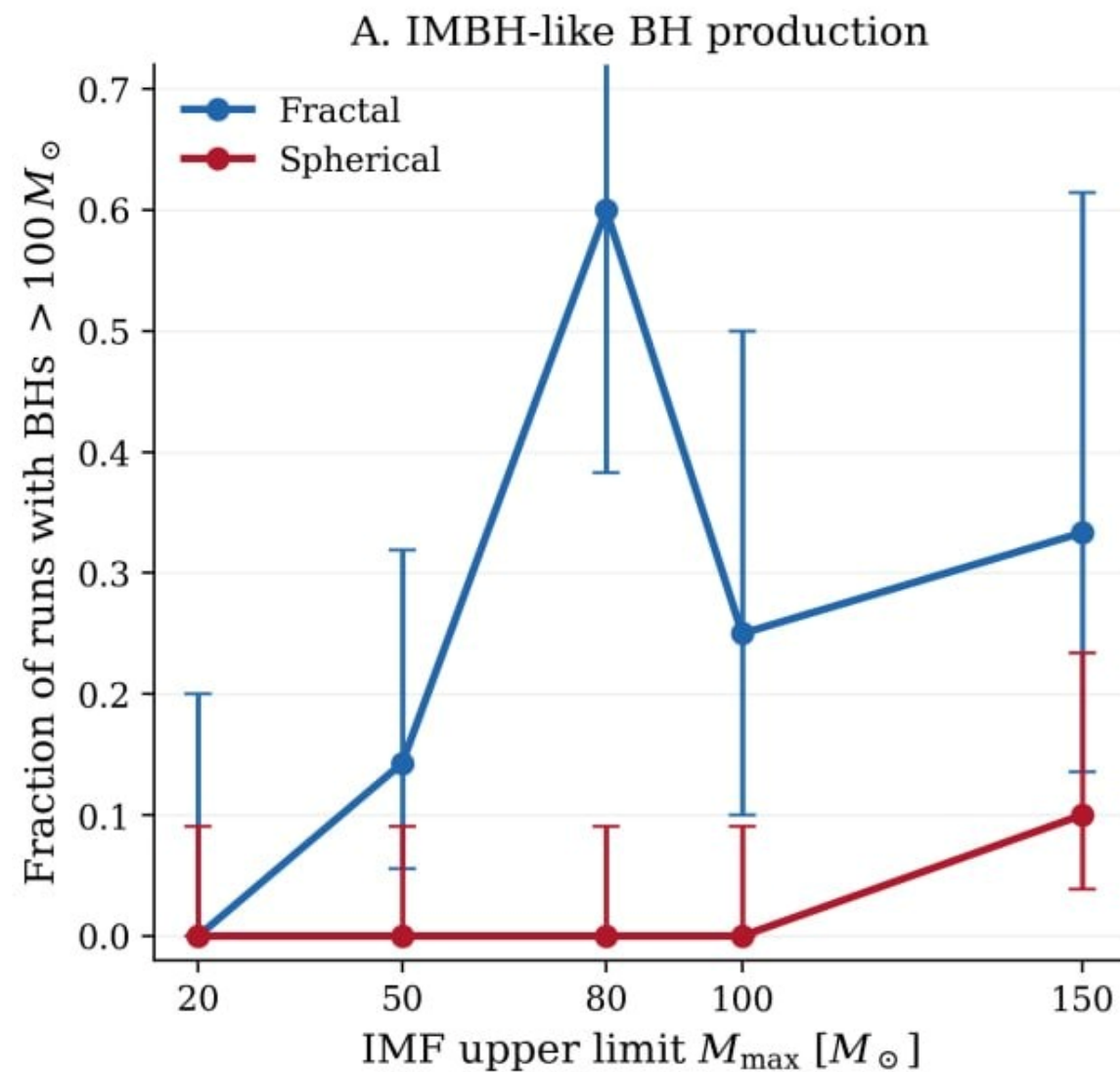
- a) Simulations up to 10 Myr for a total of 50 (S) + 28 (F)
- b) **Heavier BHs formed with Fractal initial condition**
- c) **w/ Fractal IMBH formed from  $50 M_{\odot}$  IMF limit, w/ Spherical only at  $150 M_{\odot}$**

Next steps:

- 1) check formation pathways of IMBHs
- 2) increase statistics: run more simulations and for longer

# Initial conditions and IMBH formation

Fractal Versus Spherical Formation Of IMBH-like BHs



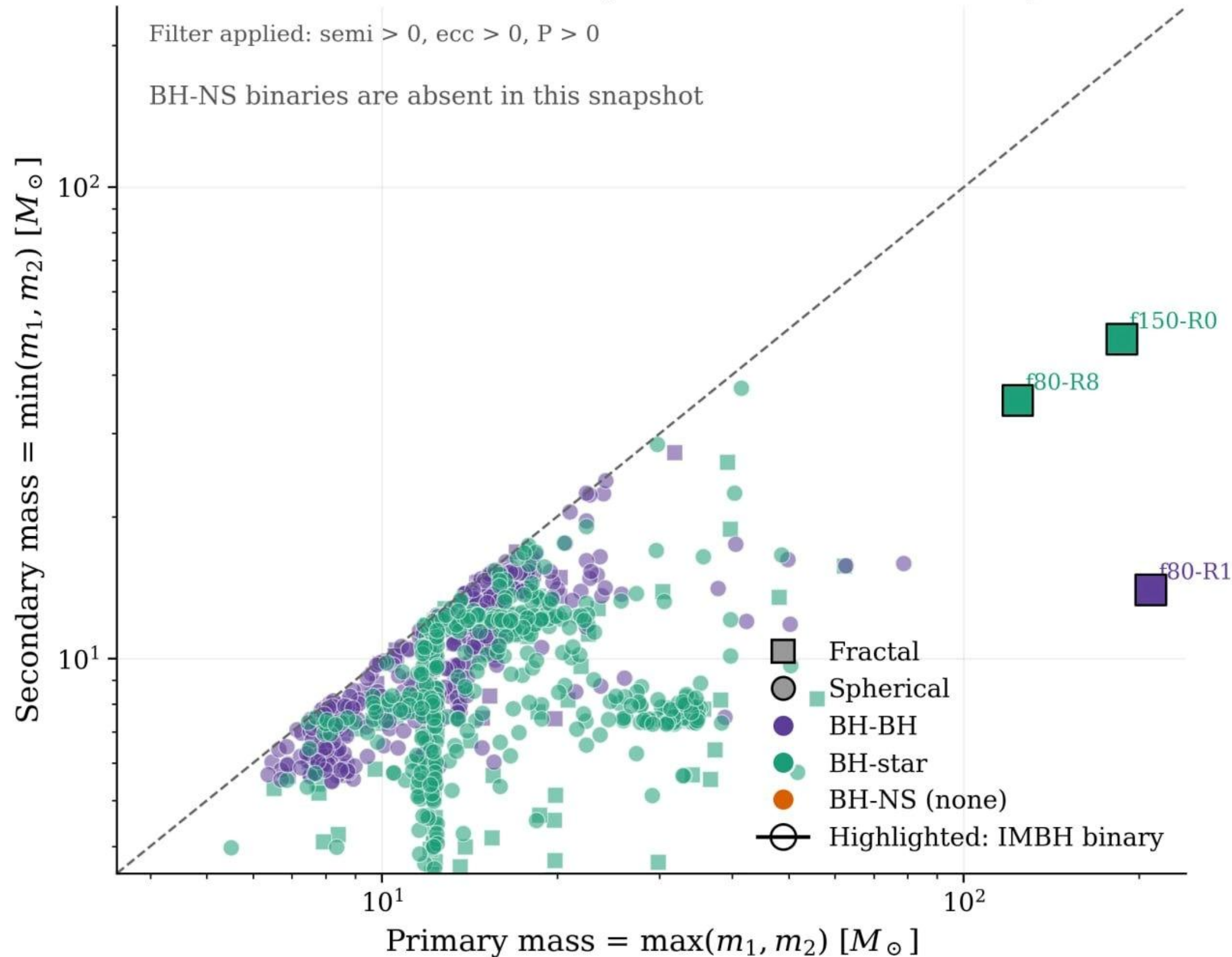
IMF limit of  $80 M_{\odot}$  and Fractal gave the maximum yield of IMBH in each run

Generally more IMBH for Fractal IC and more massive BH masses formed

Even with more simulations (error bars) the same picture would hold

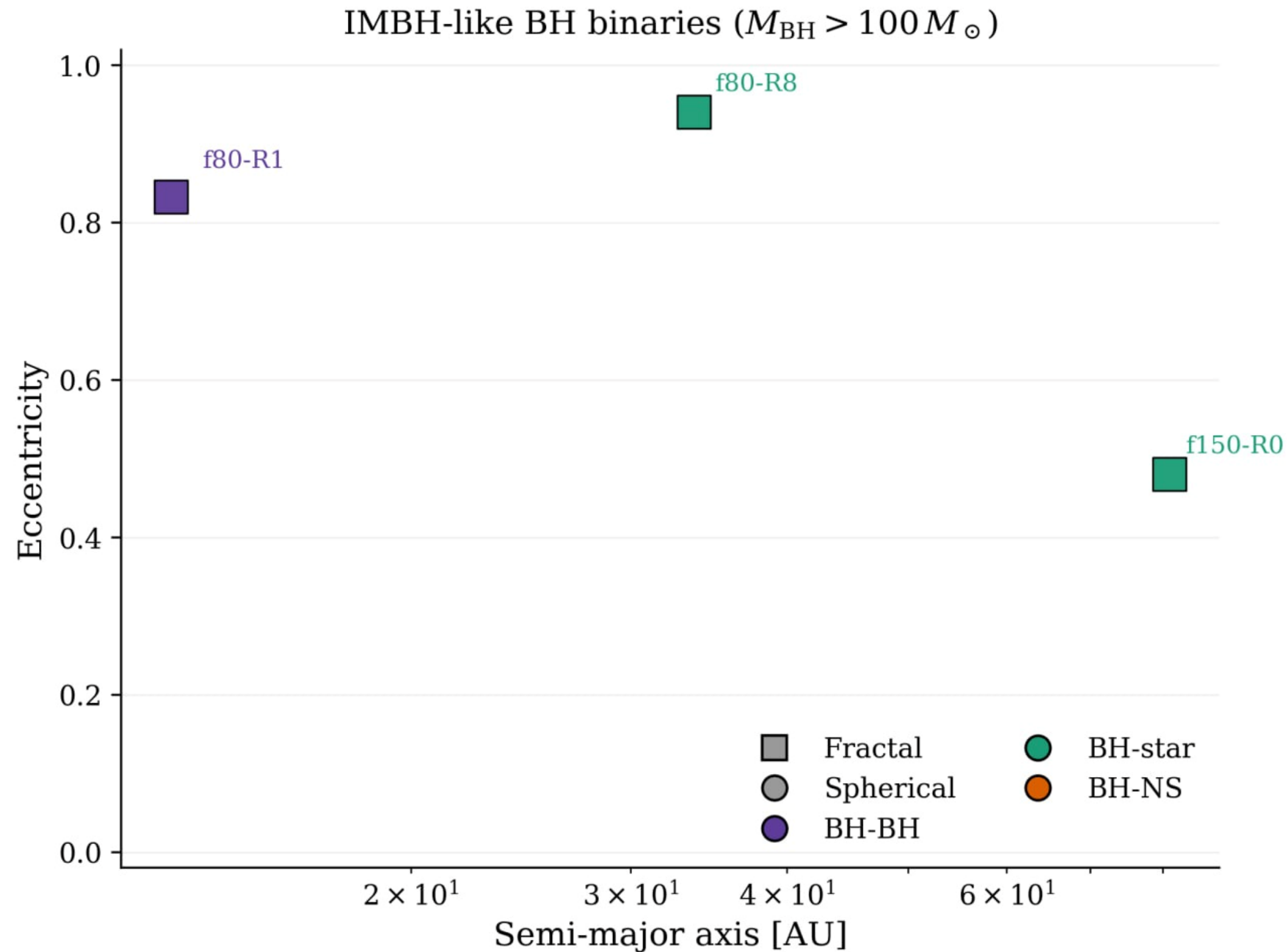
# IMBH in binaries

All binaries containing at least one BH at 10 Myr



At 10 Myr we have only 3  
IMBH binary candidates, 1  
in a BBH  
No NSBH binaries

# IMBH binaries



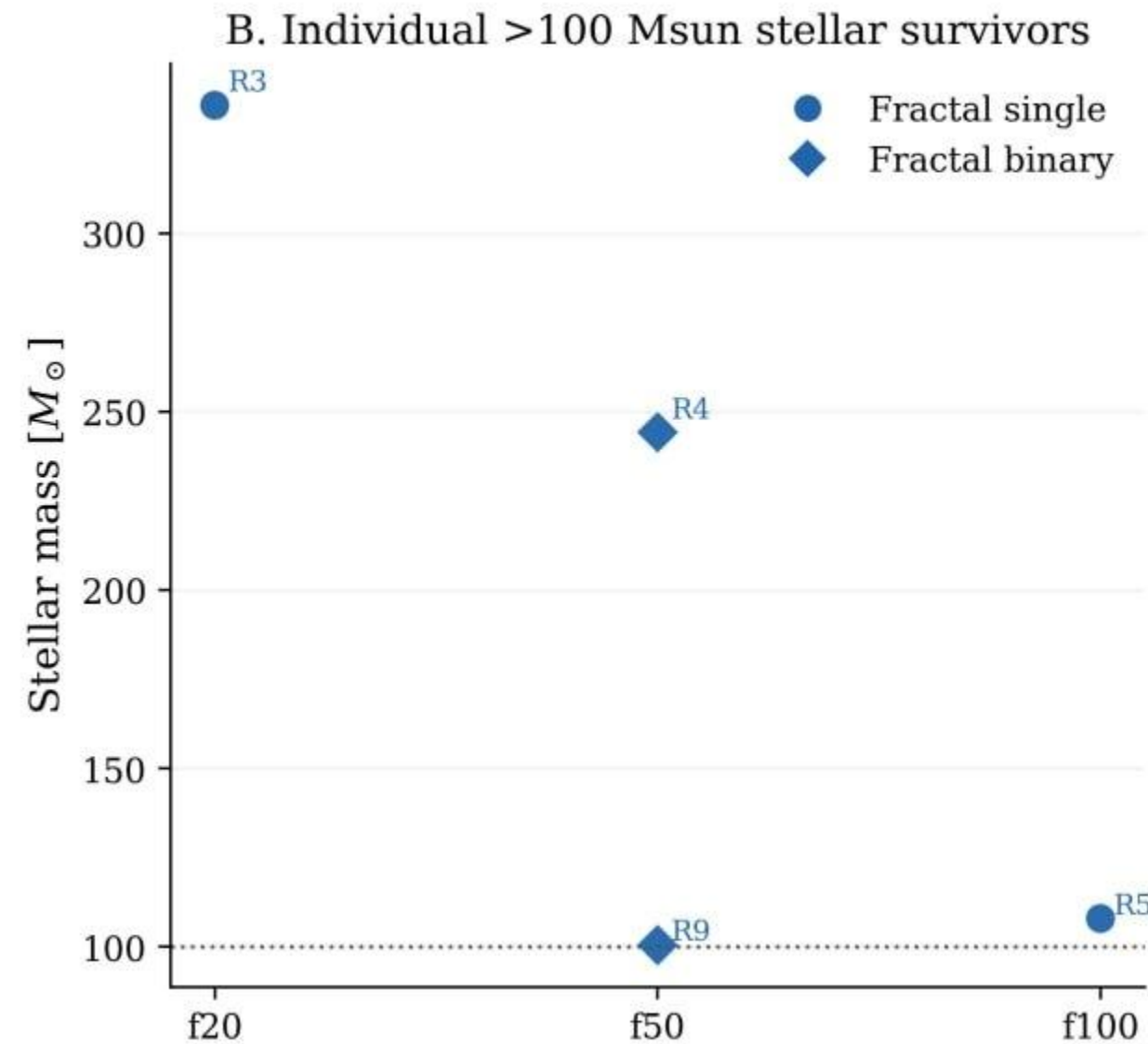
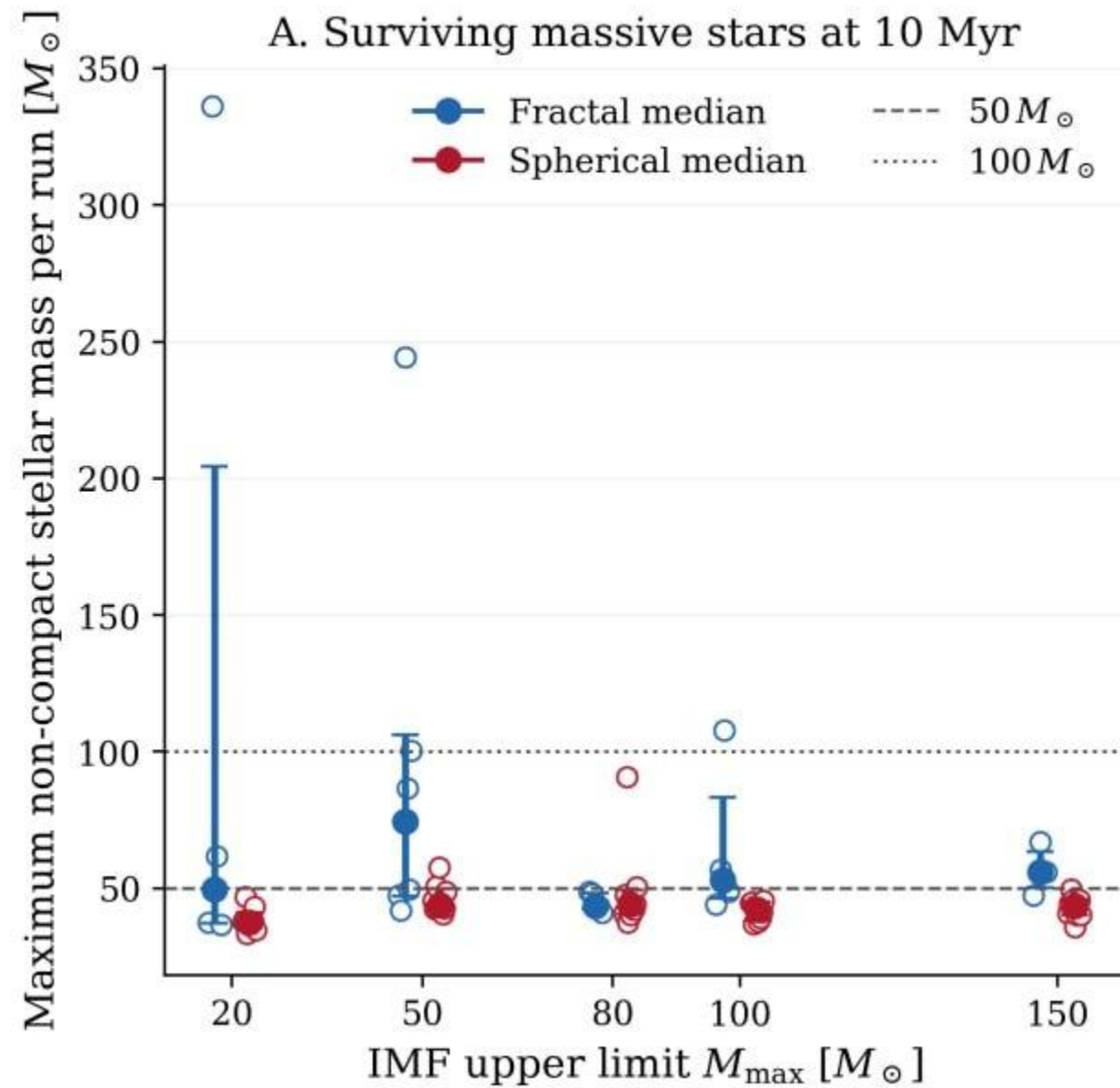
We have one hard binary of stellar BH - IMBH (F, 80 IMF): **GW inspiral time of  $10^5$  Gyr...**

It is located in the core of the star cluster and we expect **repeated hardenings** following a **merger in 100-200 Myr**

Two eccentric BH-star binaries: MS - BH and CHeB - BH

# BH binary status

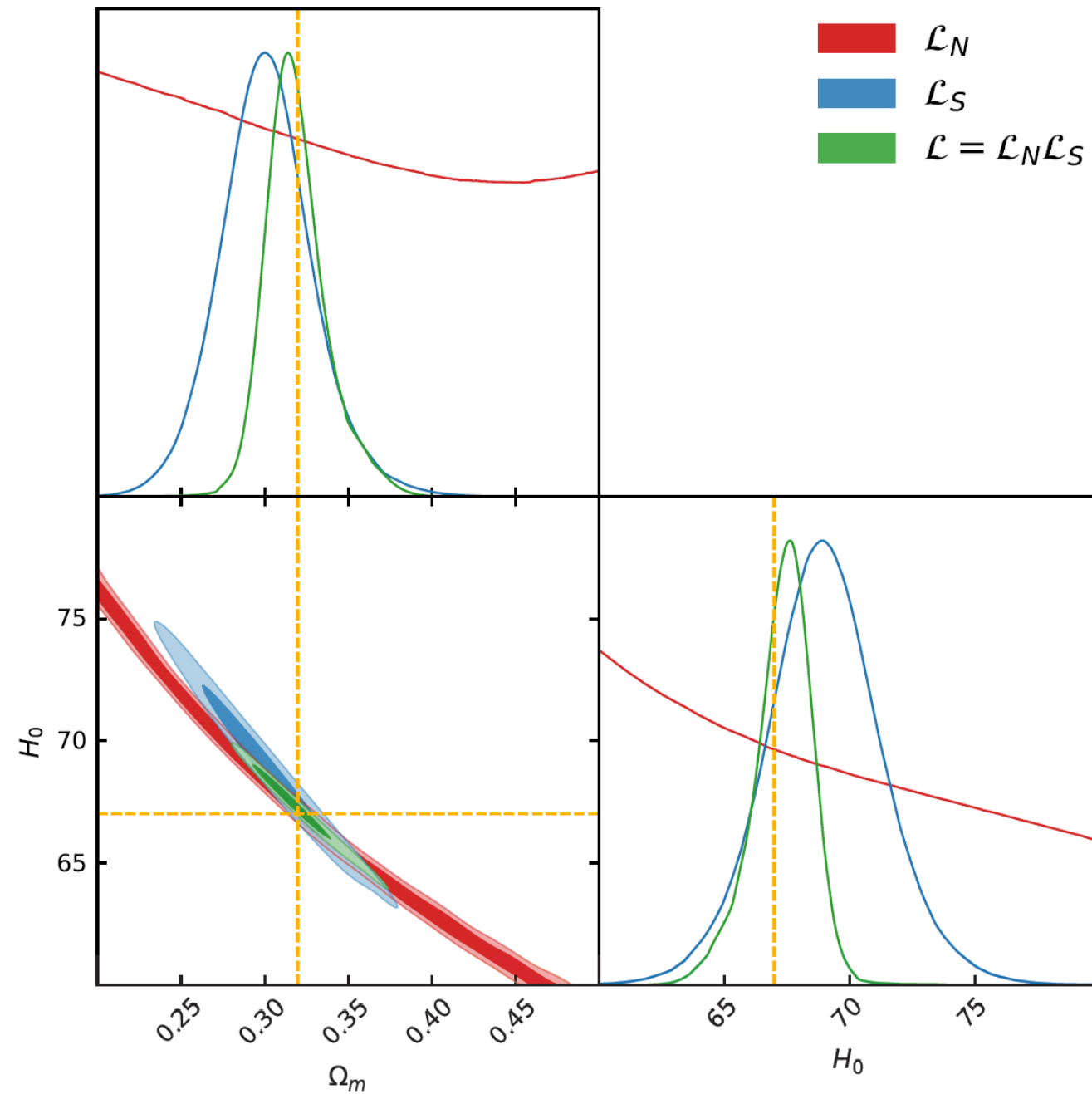
## Massive Stellar Survivors At 10 Myr



Massive stars w/  $M > 100 M_{\odot}$  are found only w/ Fractal IC at different IMF limits. Stars in Spherical IC are mostly below  $50 M_{\odot}$

Understanding the formation and evolution of massive progenitors can give insights on R136 most massive stars.

# Simulation based population inference

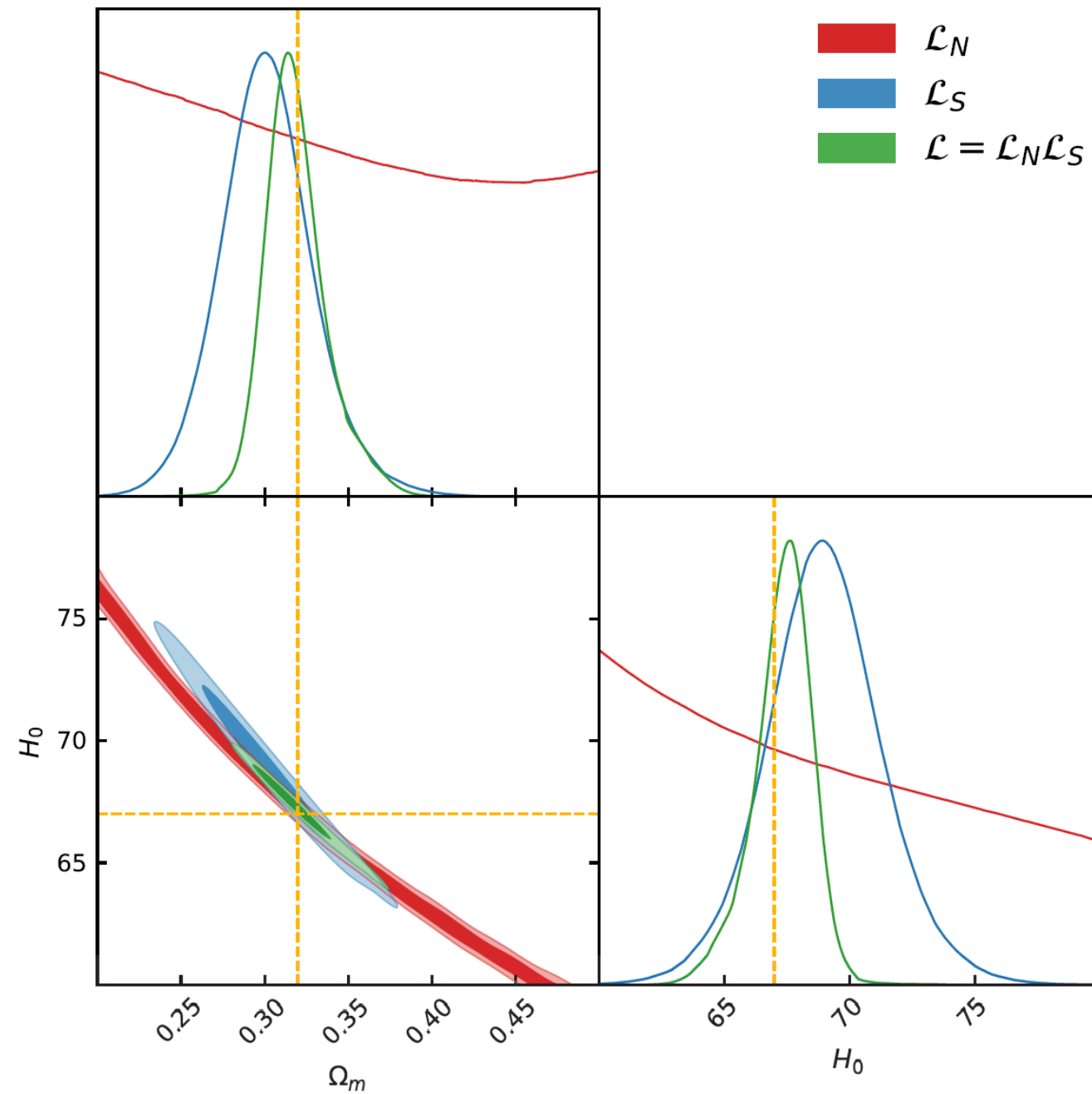


$$\mathcal{L}(\{\vec{x}_i\}, N_{\text{obs}} | \Lambda) \propto e^{-N_{\text{det}}} N_{\text{det}}^{N_{\text{obs}}} \prod_{i=1}^{N_{\text{obs}}} \frac{\int d\vec{\theta} p(\vec{x}_i | \vec{\theta}) p(\vec{\theta} | \Lambda)}{\int d\vec{\theta} p(\text{det} | \vec{\theta}) p(\vec{\theta} | \Lambda)}$$

Computing the hierarchical likelihood for the next generation is computationally expensive due to large data-parameter space:  $O(10^5 \times 10 \times 10)$

**New methods are required: e.g. SBI** (B.K. Miller, et al., Adv.NIPS 7, 2021)

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**Simulator:** a code that produces data realizations  $\vec{x}_j$  given a set of hyper-parameters  $\Lambda_j$ , for  $j=1, \dots, R$

- 1) Extract  $R$   $\Lambda$ s from a prior  $\pi(\Lambda)$  and construct the set  $\{(\vec{x}_1, \Lambda_1), \dots, (\vec{x}_R, \Lambda_R)\}$  **these will behave as samples from  $p(\vec{x}, \Lambda)$**
- 2) **Reshuffle randomly the samples** in the set to obtain samples from marginals  $p(\vec{x})$  and  $p(\Lambda)$
- 3) **Train a neural network** that computes the **marginal posterior-to-prior ratio** (Swyft: [github.com/undark-lab/swyft](https://github.com/undark-lab/swyft))

$$r(\vec{x}, \Lambda) = \frac{p(\vec{x}, \Lambda)}{p(\vec{x})p(\Lambda)}$$

- 4) **Reweight  $r$  by  $p(\Lambda)$**  and get posterior samples

# Simulation based population inference

