



*Superfluidity in the crust  
of Neutron Stars:  
why should we care?*

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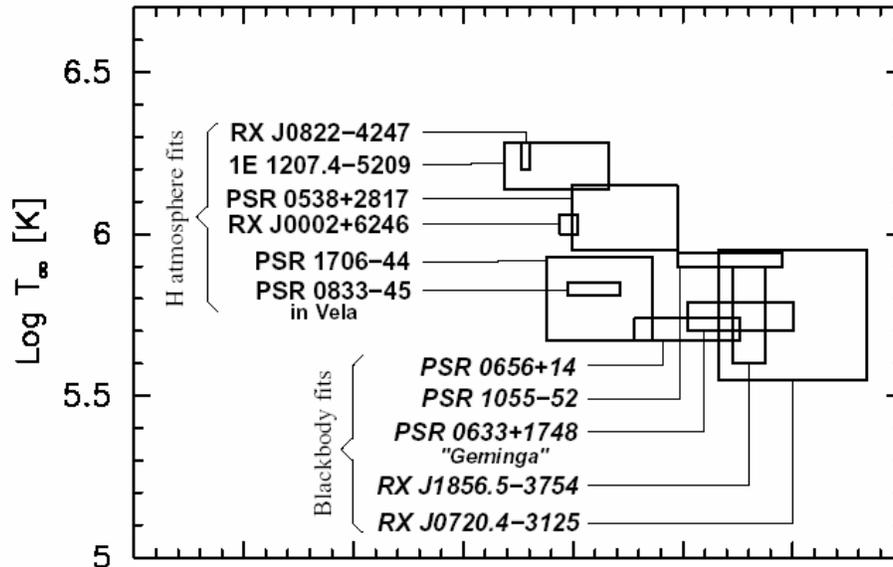
*Understanding superfluidity in the crust is crucial because:*

*1) Crust superfluidity directly determines how properties of the exotic inside are observed from the outside  
(cooling of Neutron Stars)*

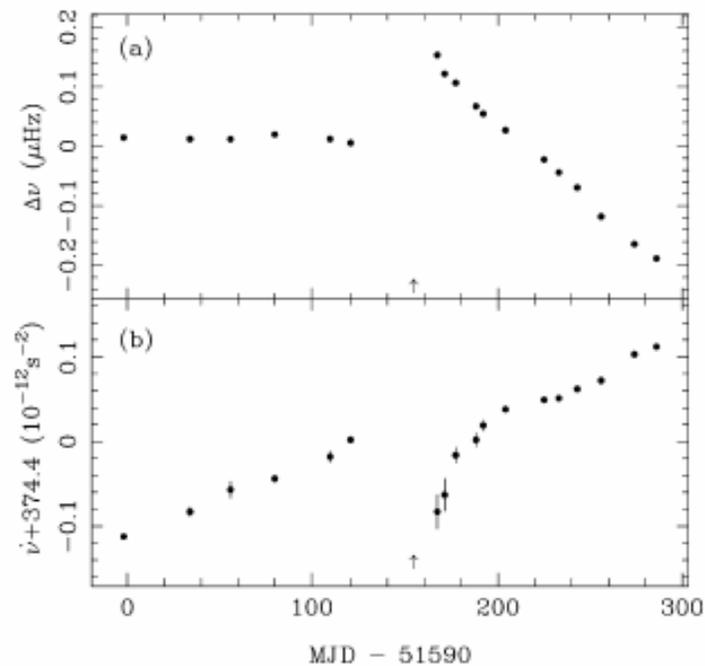
*2) Crust superfluidity directly affects additional observables, thus further constraining the structure of Neutron Stars  
(fast cooling & Pulsar glitches)*

# What can be observed

## ➤ Surface temperatures (isolated NS)

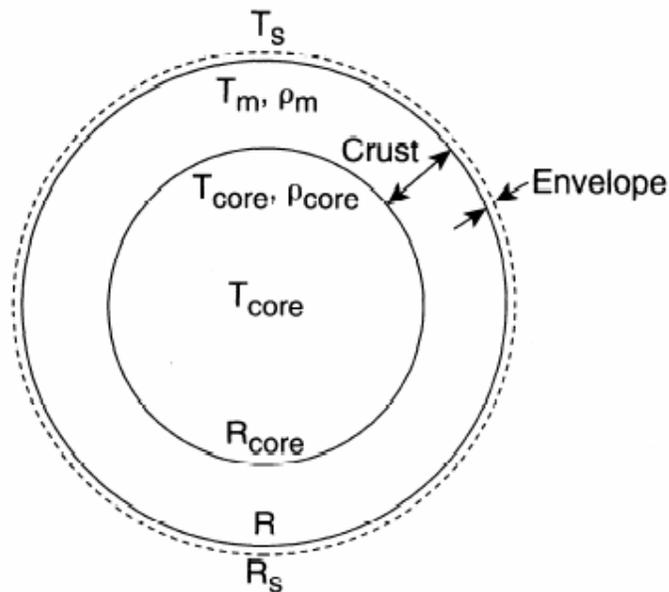


## ➤ Glitches in rotational frequency (Pulsars)



# Inside a neutron star

## ➤ Neutron star structure



**Crust** ( $\rho > \sim 10^6 \text{ g/cm}^3$ ) → **relativistic** electrons



**neutron-rich nuclei + sub-nuclear neutron matter**

## ➤ Crust properties

thermal → specific heat, emissivity

transport → thermal & electric conductivity, viscosity

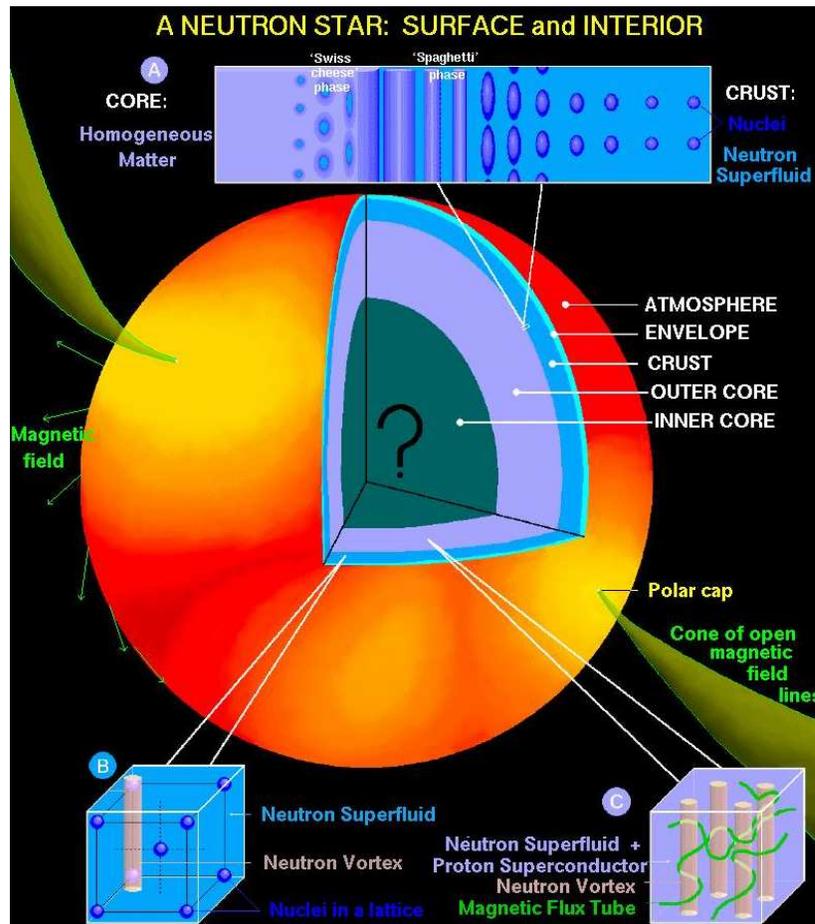
nuclear → equilibrium nuclides, pairing

mechanical → elasticity, superfluid motion



**nuclear core ⇒ crust ⇒ observable surface**

# Superfluidity in Neutron Stars



✦ **Inner crust** ( $\sim 10^{11} \text{ g/cm}^3 < \rho < \sim 10^{14} \text{ g/cm}^3$ )

gas of **unbound superfluid neutrons** ( $^1S_0$  pairing)  $\rightarrow n_G$

lattice of **neutron-rich nuclei** (Wigner-Seitz cells)  $\rightarrow R_N, R_{WS}$

$\rho_B$	$1.5 \times 10^{12}$	$9.6 \times 10^{12}$	$3.4 \times 10^{13}$	$7.8 \times 10^{13}$	$1.3 \times 10^{14}$
$n_G$	$4.8 \times 10^{-4}$	$4.7 \times 10^{-3}$	$1.8 \times 10^{-2}$	$4.4 \times 10^{-2}$	$7.4 \times 10^{-2}$
$R_{WS}$	44.0	35.5	27.0	19.4	13.77
$R_N$	6.0	6.7	7.3	6.7	5.2
$a$	0.77	0.83	0.94	1.12	1.25
$N$	280	1050	1750	1460	950
$Z$	40	50	50	40	32
$N_{bound}$	110	110	110	70	40

# Cooling scenarios

## ✦ Standard cooling of core

modified URCA process ( $\Delta t_9 \approx 1 \text{ yr}$ )

nucleon-nucleon bremsstrahlung ( $\Delta t_9 \approx 40 \text{ yr}$ )



low central density → low mass NS with stiff EOS

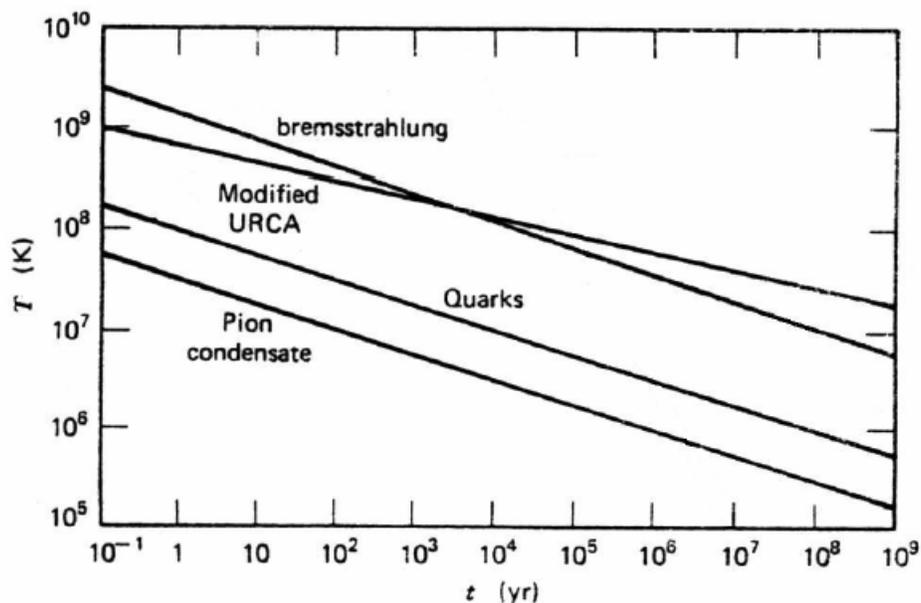
## ✦ Rapid cooling of core

direct URCA process ( $\Delta t_9 \approx 1 \text{ min}$ )

exotic  $\beta$ -decays (deconfined quarks, hyperons, meson condensates,  $\Delta t_9 \approx 1 \text{ hr}$ )

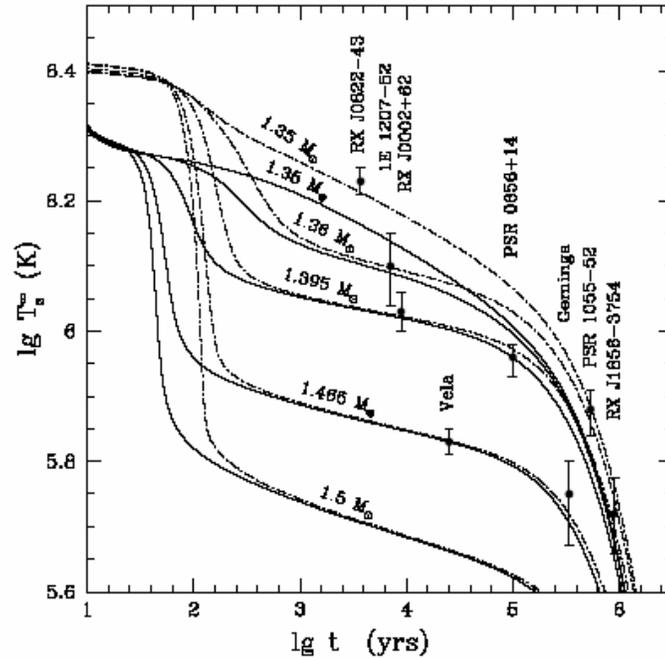


high central density → high mass NS or low mass NS with soft EOS

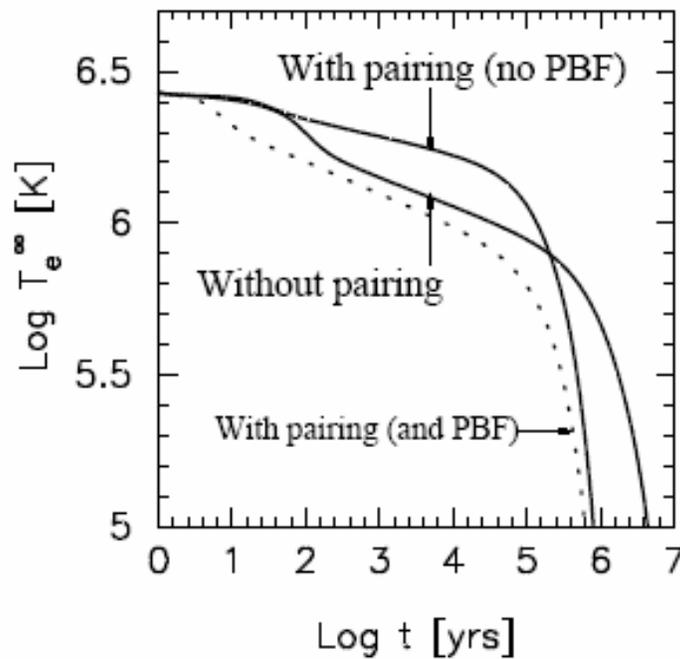


## Crust superfluidity and surface temperature

The *program*: cooling as a probe of NS structure



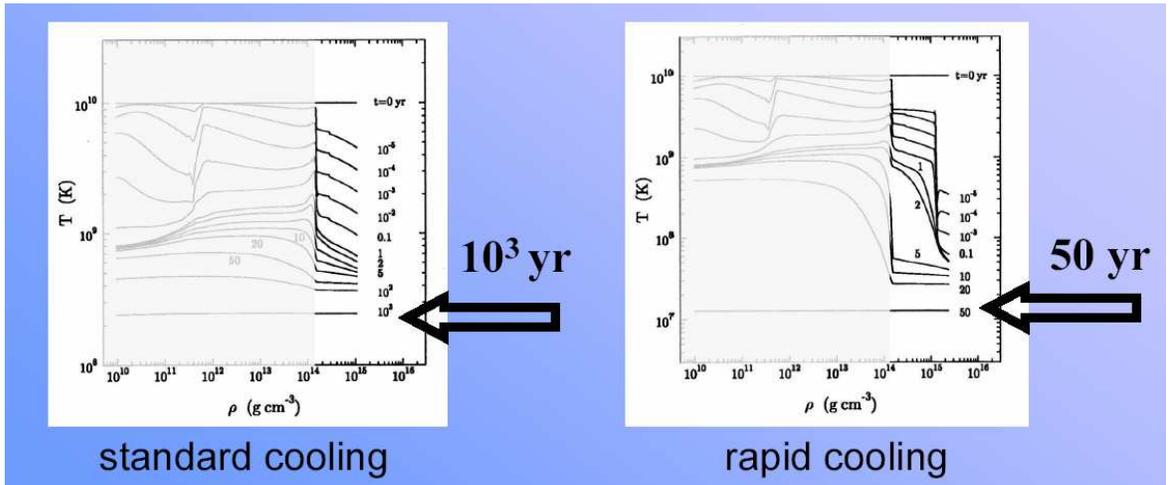
The *problem* to solve: superfluidity in the crust



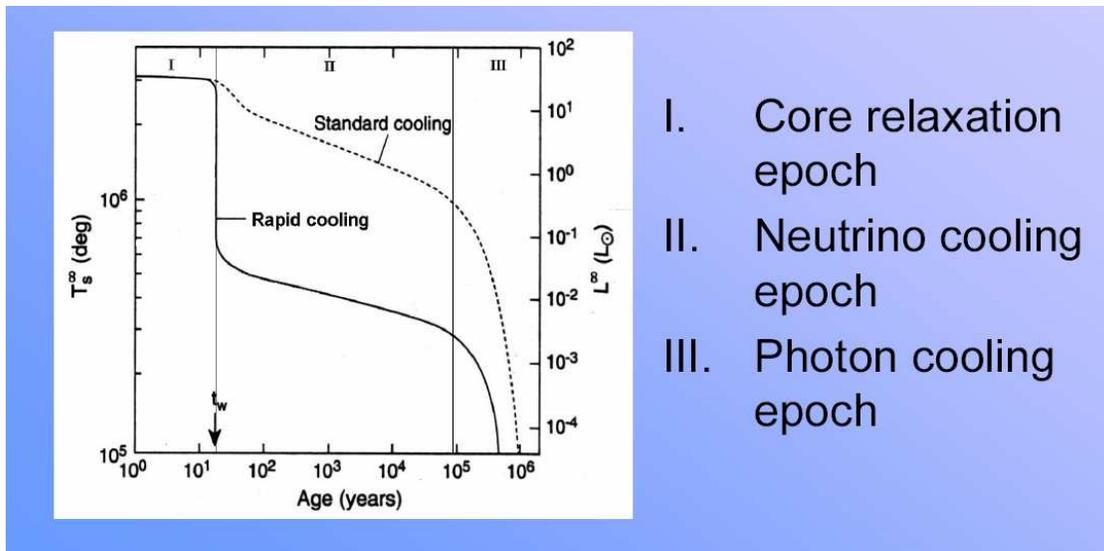
# Rapid cooling

## ✦ Temperature evolution

### Core temperature (internal properties)



### Surface temperature (observed)



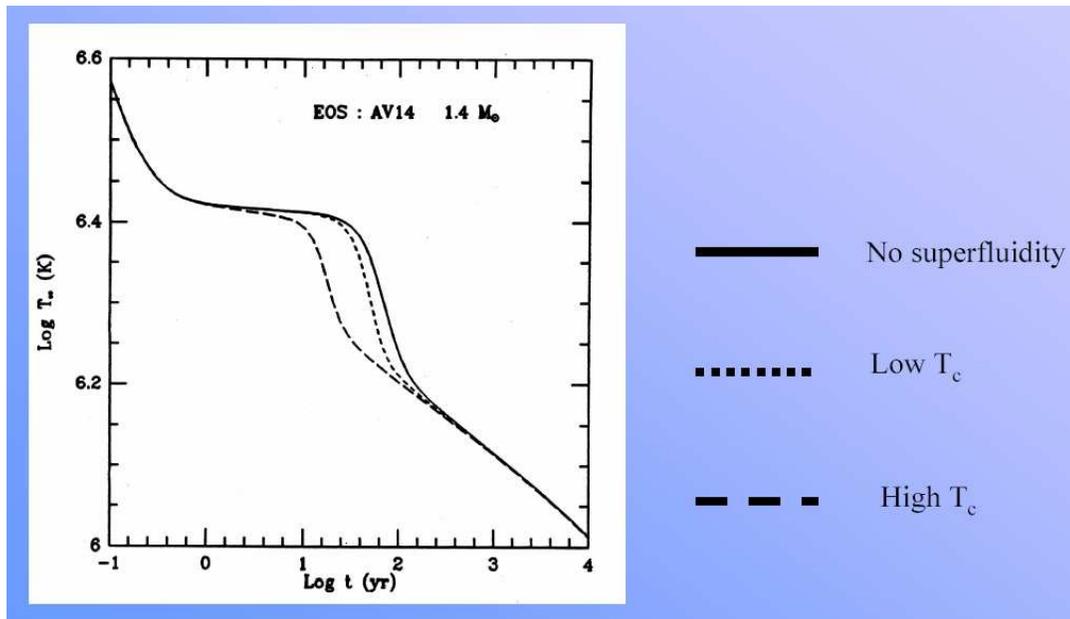
- I. Core relaxation epoch
- II. Neutrino cooling epoch
- III. Photon cooling epoch

crust properties  $\Rightarrow$  duration of epoch I (plateau)



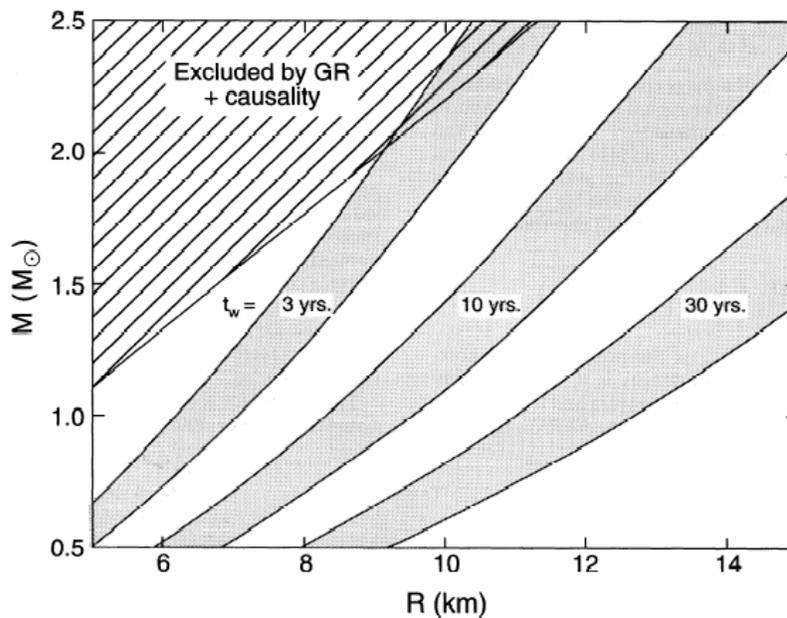
*if* rapid cooling  $\Rightarrow$  observe  $t_w$

## ✦ Crust superfluidity and rapid cooling



superfluidity  $\Rightarrow$  decrease of  $C_v \Rightarrow$  shorter  $t_w$

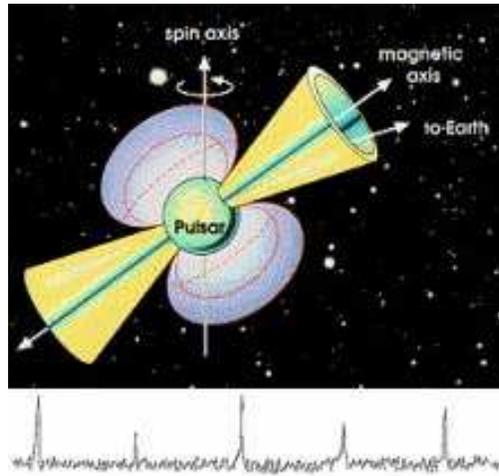
## ✦ Rapid cooling and EOS (LVPP 1994)



$t_w \Rightarrow M=M(R) \Rightarrow$  constraint on EOS

# Pulsar glitches

## ✦ Pulsar slow-down



emission of e.m. and gravitational waves  $\rightarrow \dot{E}_{\text{em}} > 0$

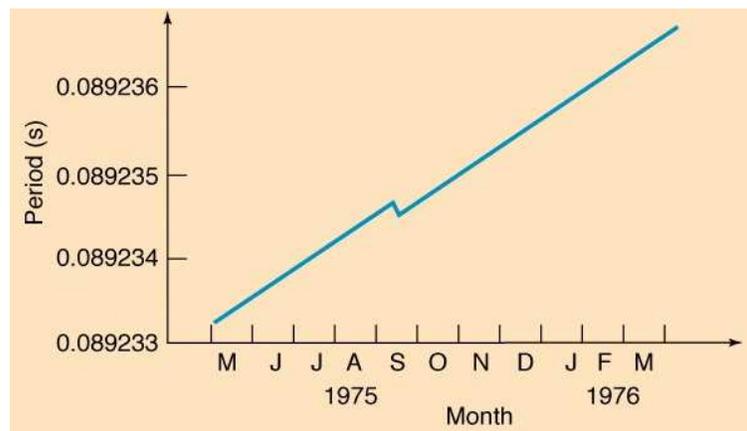
isolated neutron star  $\rightarrow \dot{E}_{\text{tot}} = \dot{E}_{\text{em}} + \dot{K}_{\text{rot}} = 0$

$\Downarrow$

decrease of rotational frequency  $\rightarrow \dot{\Omega} < 0$

## ✦ Rotational glitches

spin-ups  $\rightarrow \frac{\Delta\Omega}{\Omega} \approx 10^{-6}$  (Vela)  $\frac{\Delta\Omega}{\Omega} \approx 10^{-8}$  (Crab)

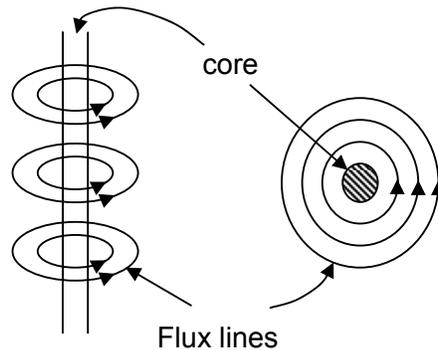


# Vortex theory for glitches

## Quantized vortex lines

superfluid  $\rightarrow$  irrotational flow  $\rightarrow \vec{\nabla} \times \vec{v}_s = 0$

classical vortex  $\rightarrow \vec{v}_s = \frac{C}{r} \hat{e}_\theta \rightarrow \vec{\nabla} \times \vec{v}_s = 2\pi C \delta^{(2)}(\vec{r}) \hat{e}_z$



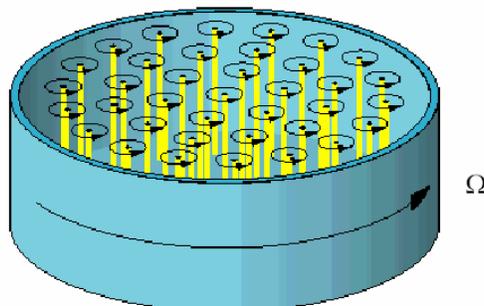
quantized vortex lines  $\rightarrow C = k \frac{\hbar}{2m_N} \quad (k=1,2,\dots)$

quantized vorticity  $\rightarrow \oint \vec{v}_s \cdot d\vec{l} = k \frac{\hbar}{2m_N} \quad (k=1,2,\dots)$

## Rotating vessel: Feynman-Onsager formula

$\vec{\nabla} \times (\vec{\Omega} \times \vec{r}) = 2\vec{\Omega} \neq 0 \rightarrow$  no rigid rotations  $\vec{\Omega}$

array of parallel vortices (if  $\Omega \gg \Omega_{cr}$ )  $\rightarrow \langle \vec{v}_s \rangle = \sum_i \vec{v}_{si}$



$$E_{\Omega} \text{ minimized by } \langle \vec{v}_s \rangle = \vec{\Omega} \times \vec{r}$$

⇓

$$\text{uniform density of vortices} \rightarrow n_v = \frac{N_v}{\pi R^2} = \frac{4m_N}{h} \Omega$$

⇓ ⇓

superfluid angular momentum quantized in vortices

### ✦ Vortex pinning and Magnus force

if superfluid vortices are pinned  $\rightarrow \dot{n}_v = 0 \rightarrow \dot{\Omega}_s = 0$

but slow-down of normal component  $\rightarrow \dot{\Omega}_n < 0$

⇓

rotational lag of components  $\rightarrow \Delta \Omega = \Omega_s - \Omega_n > 0$

outward drag force on vortex  $\rightarrow f_{\text{mag}} \propto \Delta \Omega$

### ✦ Vortex un-pinning and glitches

since  $\Delta \dot{\Omega} > 0 \rightarrow f_{\text{mag}}$  increases with time

pinning energy  $\rightarrow$  maximum pinning force  $f_{\text{pin}}$

when  $f_{\text{mag}} \geq f_{\text{pin}} \rightarrow$  unpinning of many vortices

⇓

transfer of angular momentum to the star surface

⇓ ⇓

normal component spin-up  $\Rightarrow$  pulsar glitch

## ✦ Vortex energy in the inner crust

inner crust (nuclei + neutron gas)  $\rightarrow n_N \gg n_G$

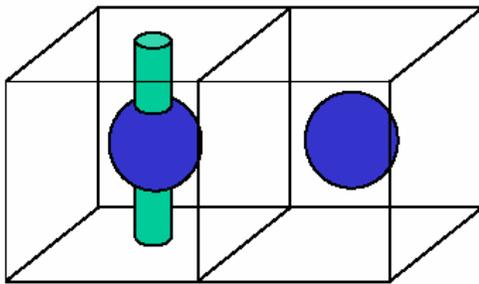
vortex energy density is density-dependent  $\rightarrow \varepsilon_s = \varepsilon_s(n)$



presence and position of nuclei modify  $\varepsilon_s(n)$

## ✦ Vortex-nucleus interaction and pinning

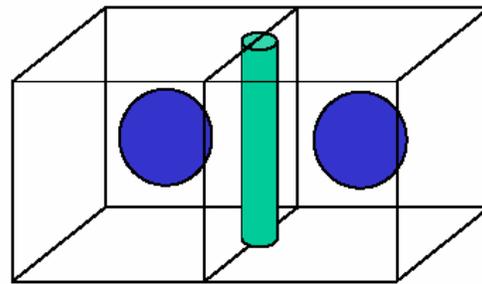
vortex-nucleus configurations in nuclear lattice



Nuclear pinning



$E_{NP}$



Interstitial pinning



$E_{IP}$

pinning energy  $\rightarrow \Delta E_{pin} = E_{NP} - E_{IP}$



$\Delta E_{pin} < 0 \Rightarrow$  nuclear pinning

$\Delta E_{pin} > 0 \Rightarrow$  interstitial pinning (*superweak*)

$\infty$

*Glitches as evidence of macroscopic superfluidity!*