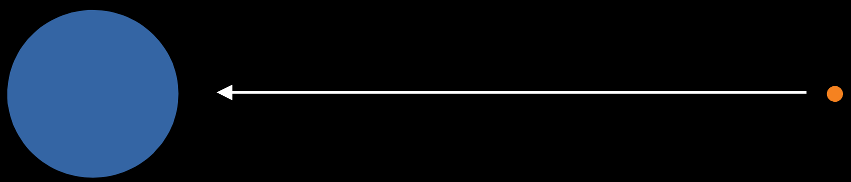
Accretion

Gravitational Energy



surface gravity: $g = \frac{GM}{r^2}$

grav. force: $F = mg$

work: $dE = Fdr$

$$E = m\int_R^{\infty} g\,dr = m\int_R^{\infty} \frac{GM}{r^2}\,dr$$

$$E = m\left[\frac{GM}{r}\right]_R^{\infty}$$

$$\boxed{E = \frac{GMm}{R}}$$

Examples: White dwarf

$M = 0.6\ M_{\odot}$

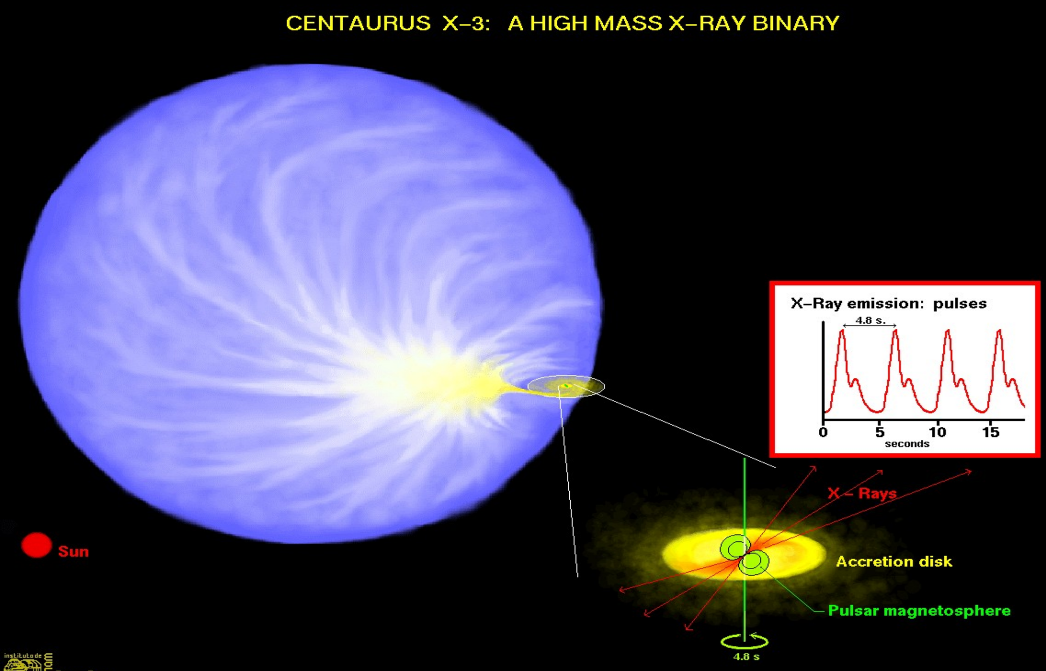
$R = 10\,000$ km

$E = GMm/R$

$m = 1$ g $\Rightarrow$

$E \approx 8 \times 10^{16}$ erg

Example: Neutron star



$M = 1.4\ M_{\odot}$

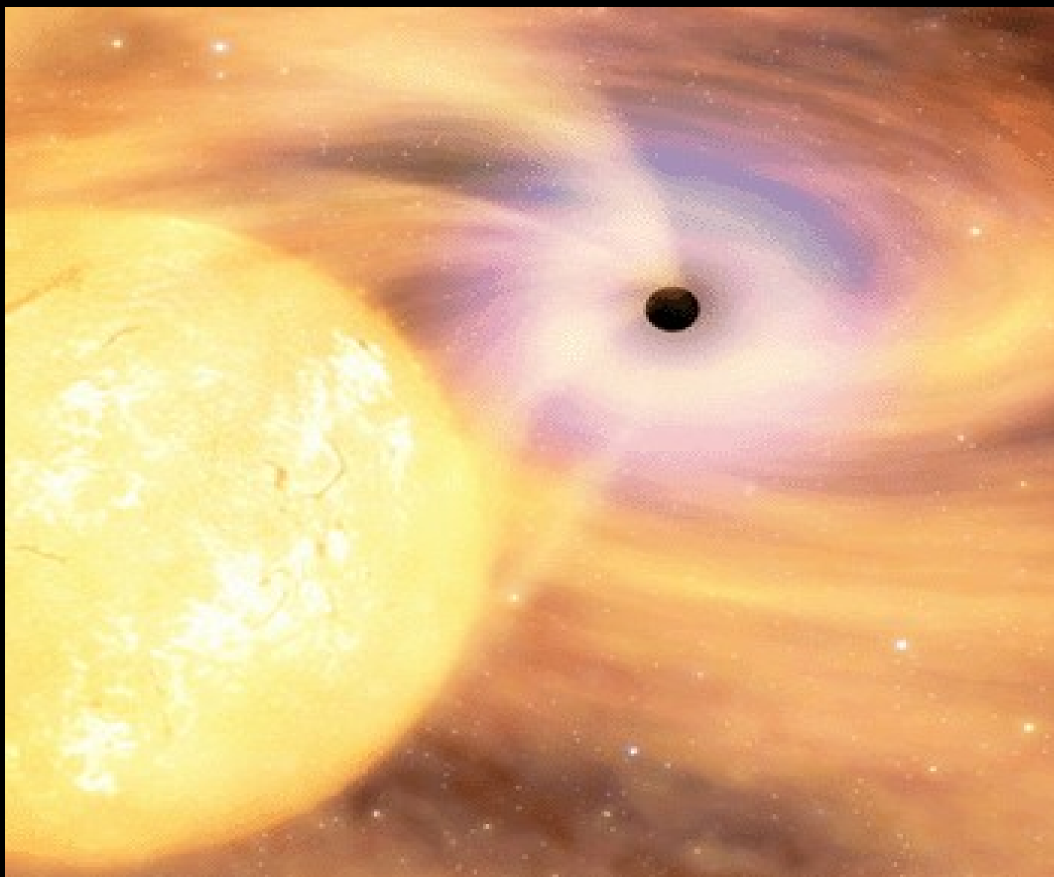
$R = 10$ km

$E = GMm/R$

$m = 1$ g $\Rightarrow$

$E \approx 2 \times 10^{20}$ erg

Example: Stellar black hole



$M = 6\ M_\odot$

$R \approx 2GM/c^2 \approx 18$ km

$E = GMm/R \approx 0.5\ mc^2$

$m = 1$ g $\Rightarrow$

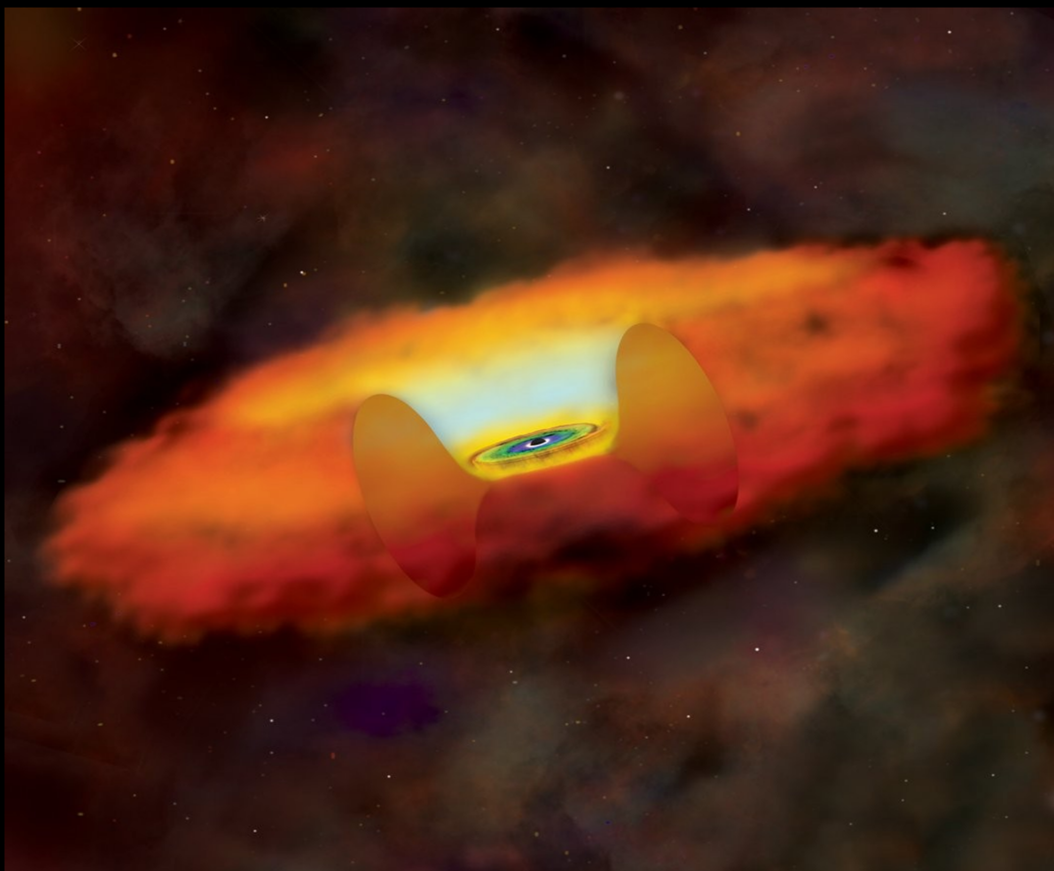
$E \approx 4 \times 10^{20}$ erg

$m = 1\ M_\odot \Rightarrow$

$E \approx 8 \times 10^{53}$ erg

⇒ If energy released in seconds/minutes: **GRB** luminosity (collapsar model)

Example: Active galactic nucleus (AGN)



$M = 10^{8}\,M_{\odot}$

$R = 2GM/c^{2}$

$E = GMm/R \approx 0.5\,mc^{2}$

$m = 1\text{ g} \Rightarrow$

$E \approx 4 \times 10^{20}$ erg

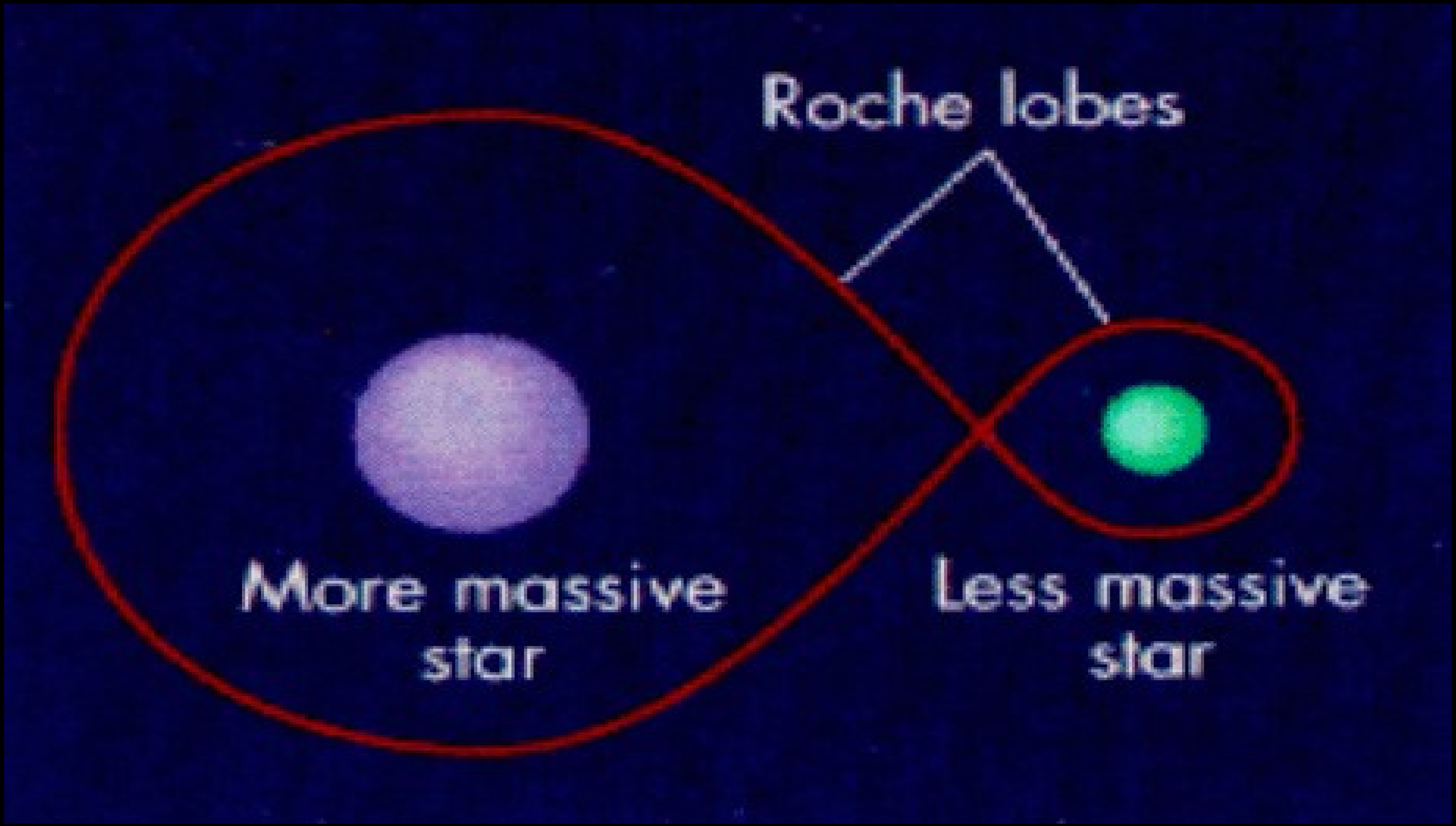
⇒ Are stellar BH as
bright as AGN?!

Accretion in binary systems

Compact star M , normal star M with $M_2 < M_1$

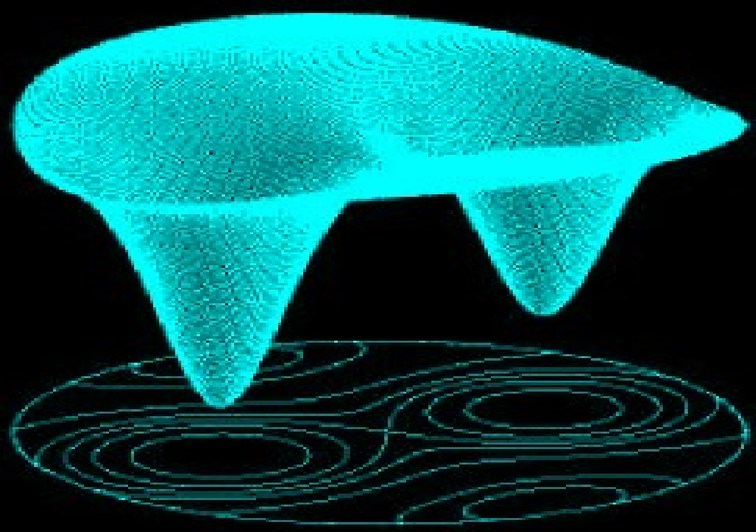
Normal star expanded or binary separation decreased => normal star feeds compact star

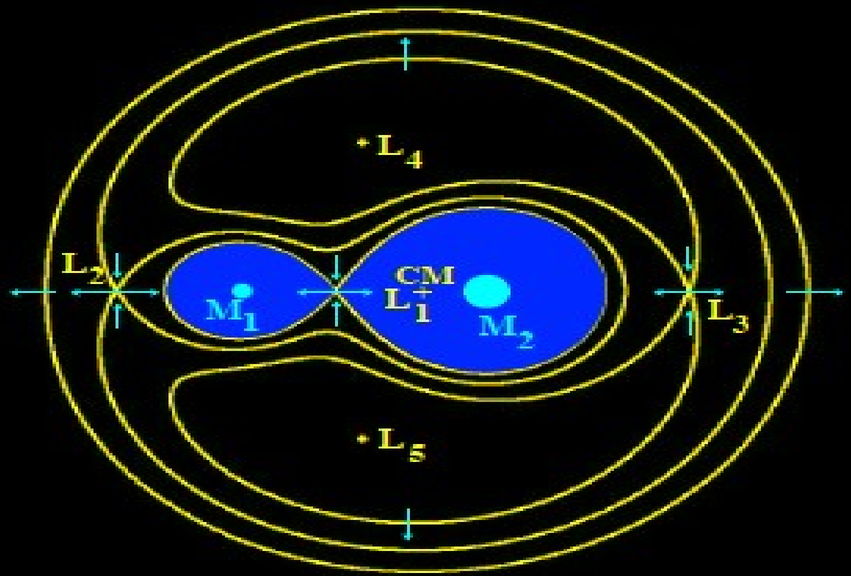
Roche Lobes



Roche lobes and Lagrangian points

Test particle in binary system: equipotential surface

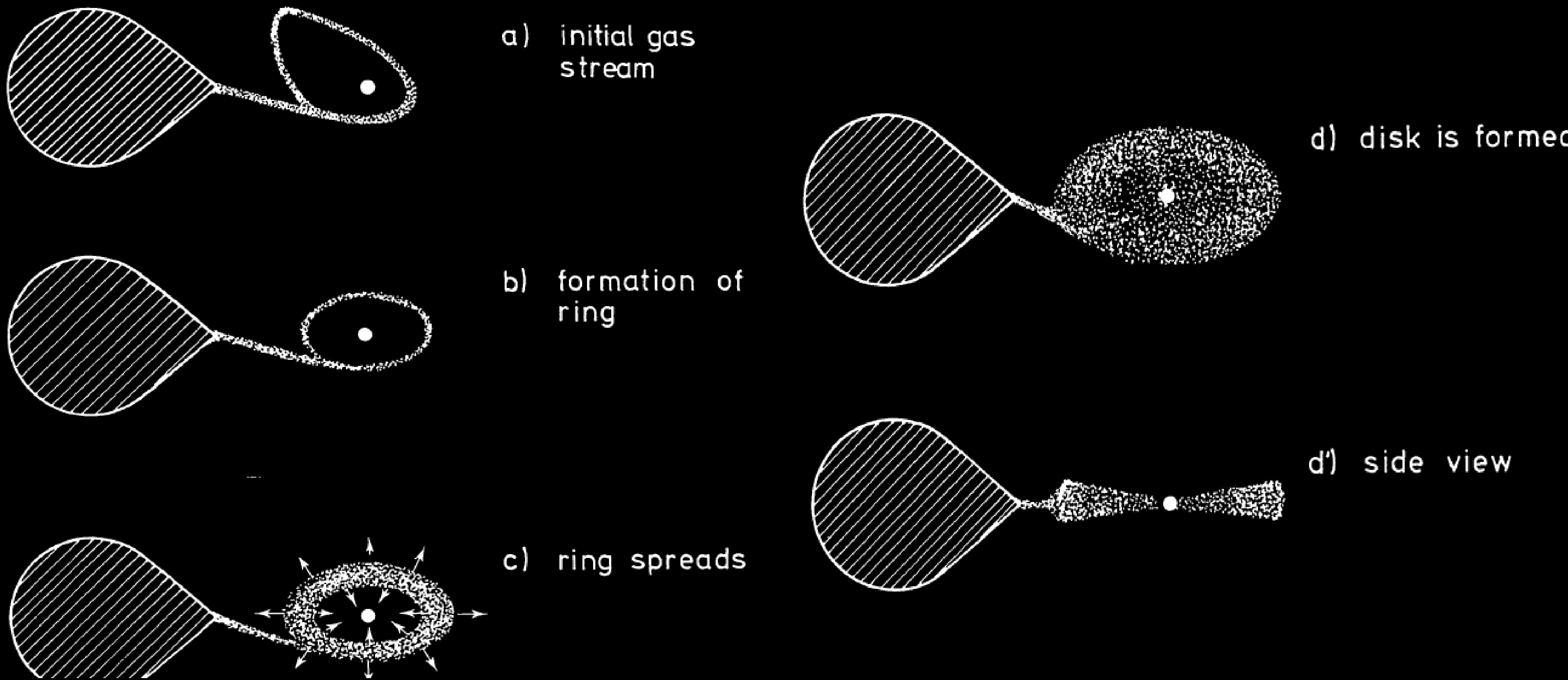




5 equilibrium points: Lagrangian points

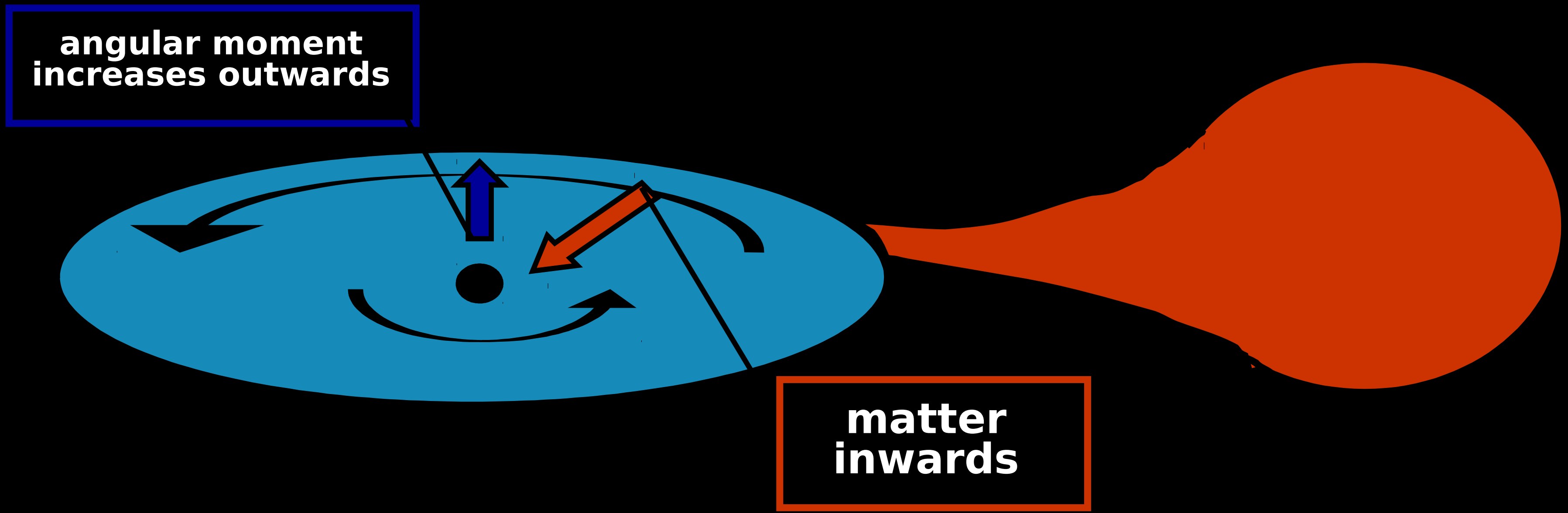
If a star fills its Roche lobe ⇒ mass transfer ⇒ accretion

Formation of an accretion disk



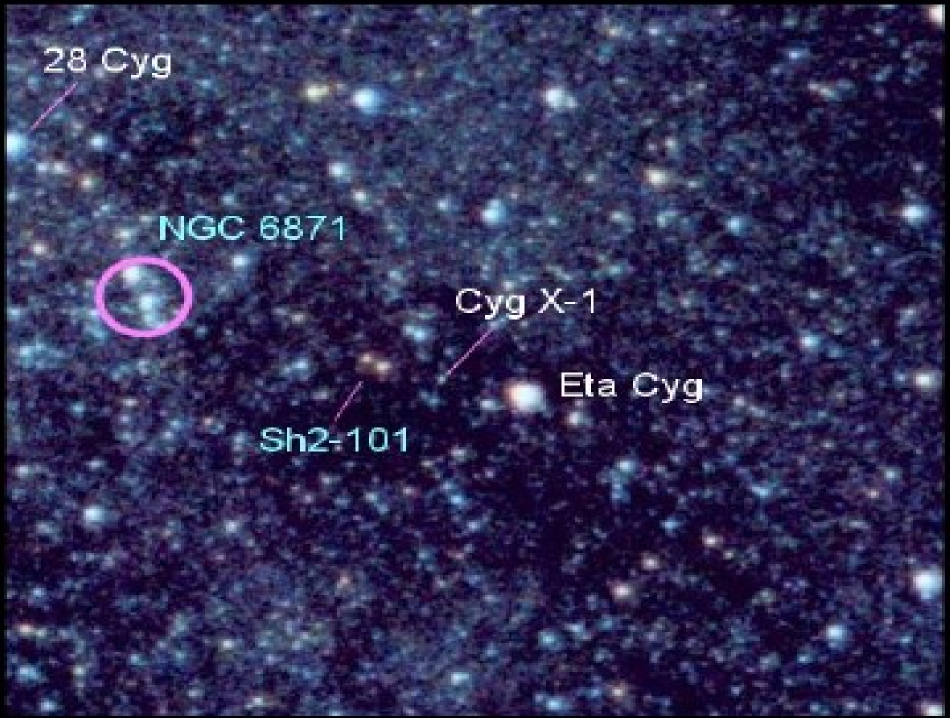
Accretion disk formation

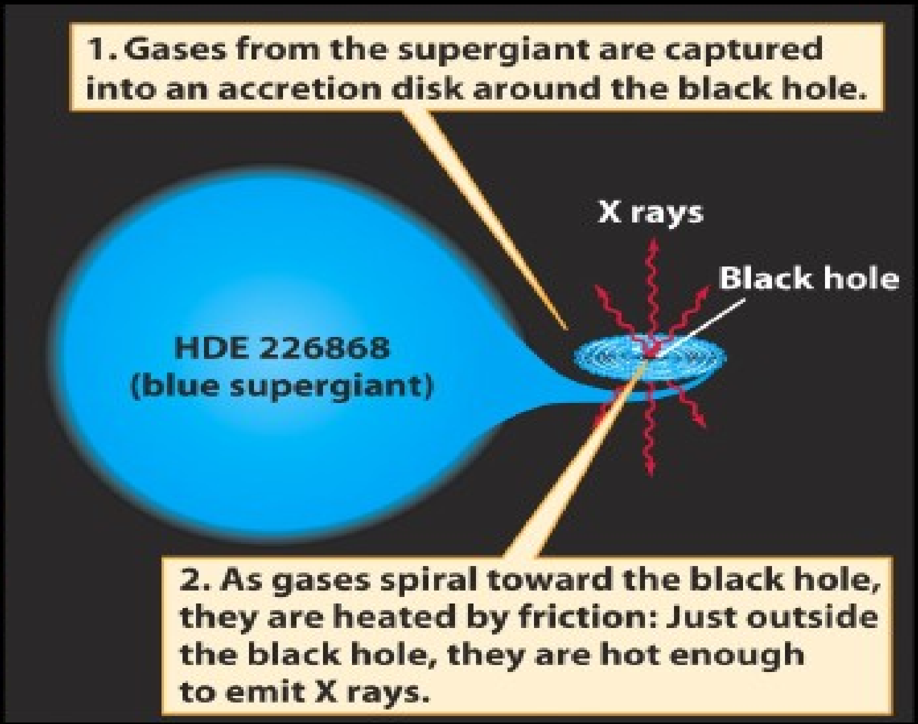
Matter circulates around the compact object:



Cyg X-1

Bright X-ray sources when in accreting binary systems





Cyg X-1: X-ray variability on <1 s timescale; $M \sim 15\ M_{\odot}$

Accretion disk

- Material transferred has high **angular momentum** so must lose it before accreting => disk forms
- Gas loses angular momentum through collisions, shocks, **viscosity** and magnetic fields: kinetic energy converted into **heat** and radiated.
- Matter sinks deeper into **gravity** of compact object

Accretion: gravitational power plant

potential energy: $\frac{GM_c m}{R_c}$

$\longrightarrow$ kinetic energy: $\frac{1}{2}mv^2$

$\longrightarrow$ thermal energy: $\frac{3}{2}kT$

$\longrightarrow$ radiation: $h\nu$

Accretion Disk Luminosity

For most accretion disks, total mass of gas in the disk is << *M* so we may **neglect self-gravity**

Hence the disk material is in circular **Keplerian orbits** with angular velocity

$$\boldsymbol{\Omega = (GM/R^3)/2 = v/R}$$

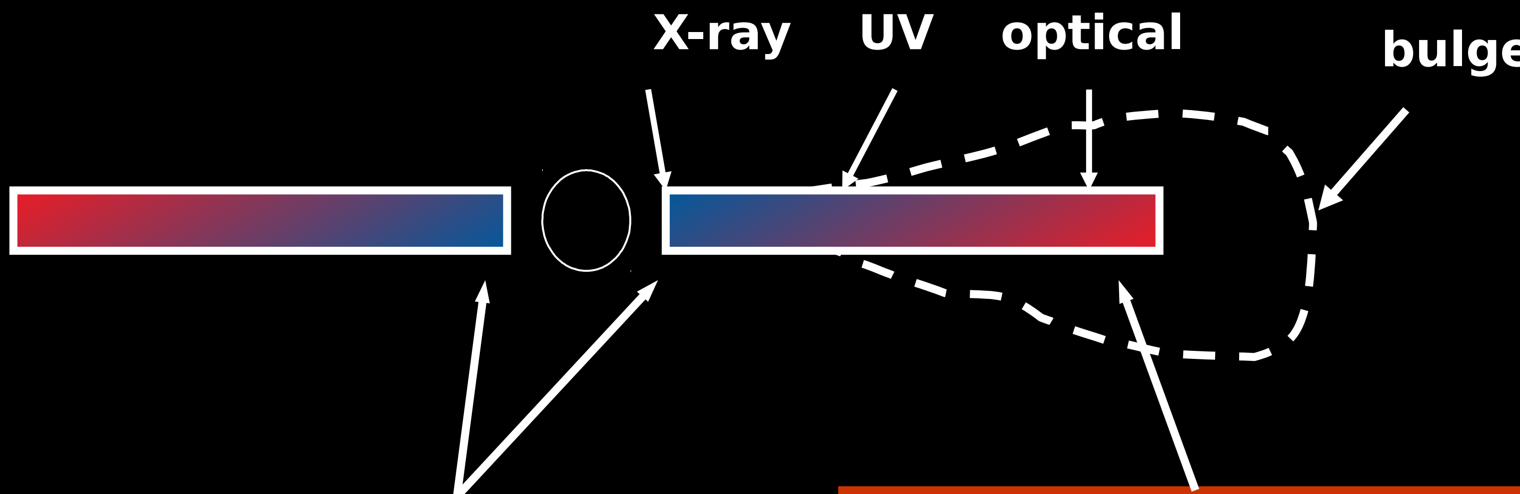
Energy of **particle** with mass *m* in the Kepler orbit of radius *R* just grazing the compact object is

$$\frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM}{R} = \frac{1}{2}E_{acc}$$

Gas particles start at large distances with negligible energy, thus

$$L_{disk} = \frac{GM\dot{M}}{2R} = \frac{1}{2}L_{acc}$$

Disk structure

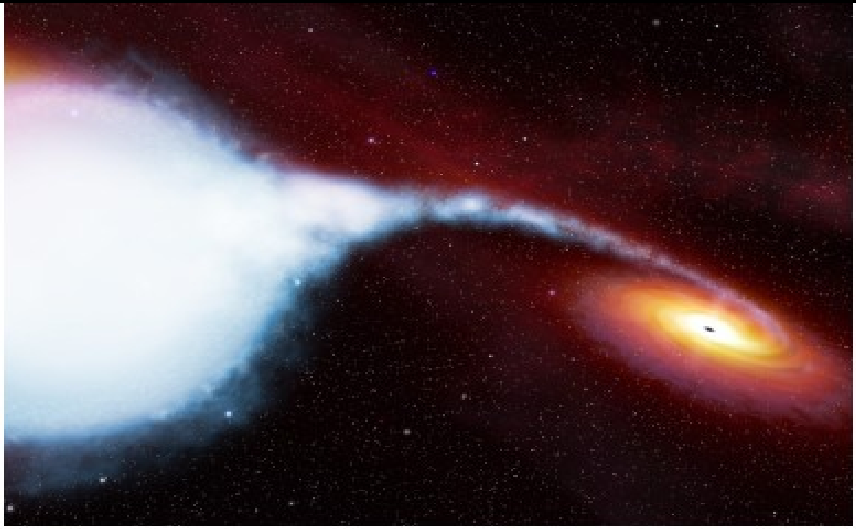


Hot, optically-thin inner region; emits bremsstrahlung

Outer regions are cool, optically-thick and emit blackbody radiation

The other half of the accretion luminosity is released very close to the star.

BH's accretion disks

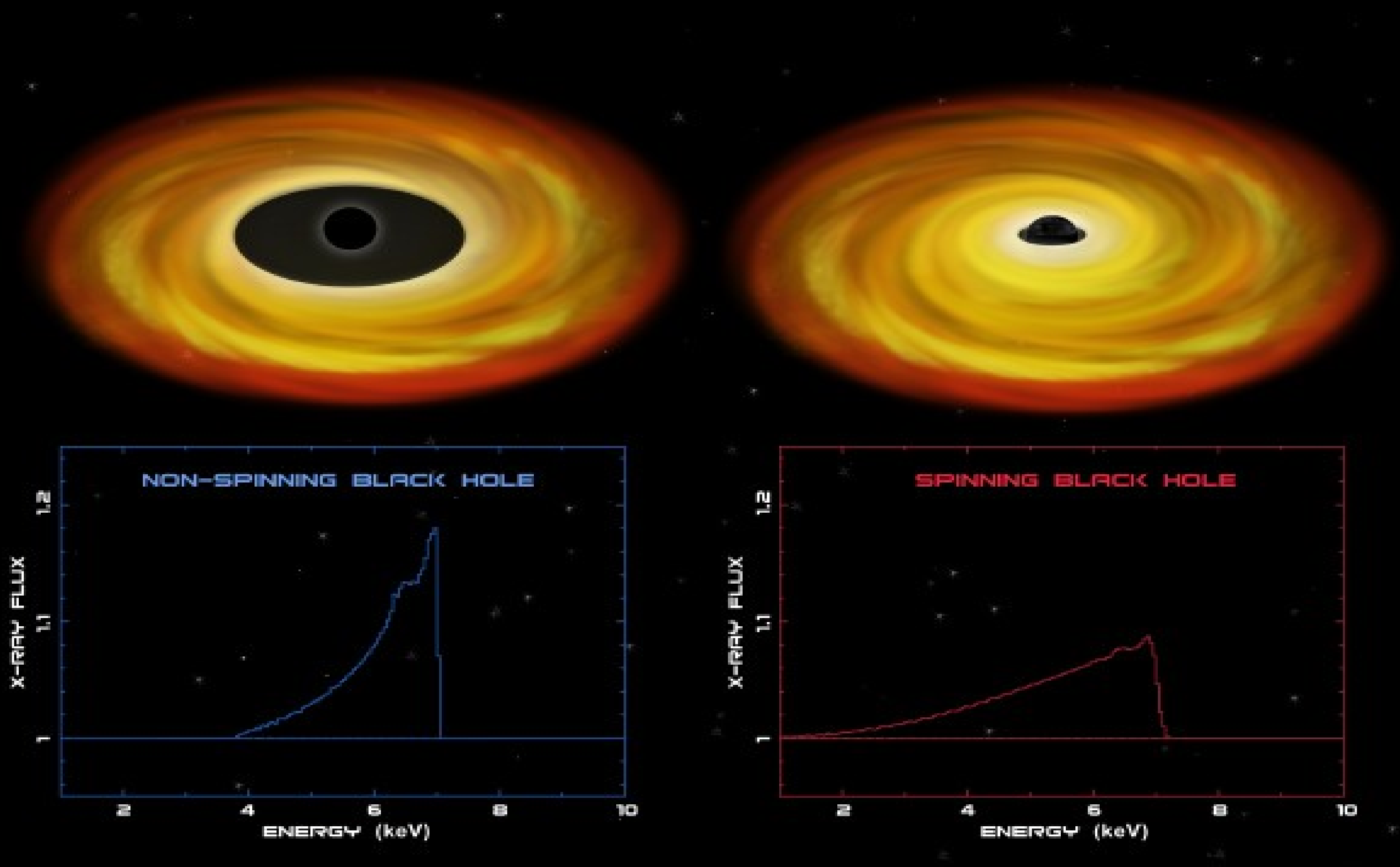


Gravitational energy at ISCO (R_{ISCO} =$3R_S$~ 100 km for a 10 $M_\odot$ BH):

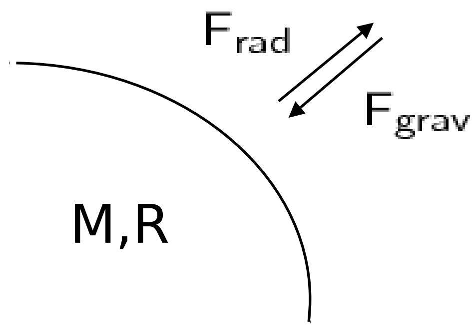
$$E_G \sim GmM/3R_s = GmMc^2/6GM = mc^2/6$$

Efficiency: $E_G/mc^2 \sim 1/6$ **~ 20%** » 0.7% (nuclear fusion)

Spinning vs non-spinning BHs



The Eddington luminosity



$$L = 4\pi R^2 F \implies F_{rad} = \frac{\sigma_T L}{4\pi R^2 c}$$

$$m_p = 1830 m_e \implies F_{grav} = \frac{GMm_p}{R^2}$$

$$F_{rad} = F_{grav} \implies \boxed{L_E = \frac{4\pi G M m_p c}{\sigma_T}}$$

Accretion rate: $\dot{M}$ (measured in [g/s] or [$M_\odot$/yr])

Accretion luminosity: $$L_{acc} = \frac{GM\dot{M}}{R} \ \text{[erg/s]}$$

Maximum accretion rate onto a **neutron star**:

$$L_{E,NS} \approx 1.8 \times 10^{38} \ \text{erg/s} \Rightarrow \dot{M}_{E,NS} = \frac{L_{E,NS} R}{GM} \approx 1.5 \times 10^{-8} \ \mathbf{M_o/yr}$$

Maximum accretion onto a **supermassive** (10^8) **black hole**:

$$L_{E,AGN} \approx 10^{46} \ \text{erg/s} \Rightarrow \dot{M}_{E,AGN} \approx 0.5 \ \mathbf{M_o/yr}$$

Characteristic temperatures

- Define temperature T_{rad} such that $h\nu \sim kT$
- Define 'effective' BB temp $T_{\boldsymbol{b}}$

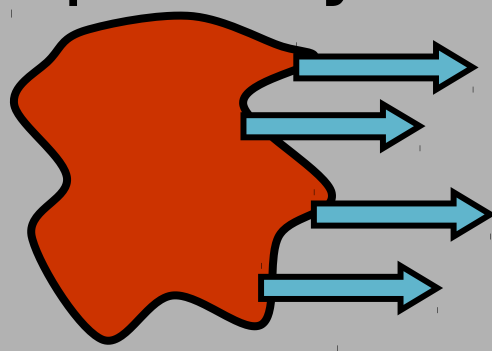
$$\boxed{T_b = (L_{acc} / 4\pi R^2 \sigma)^{1/4}}$$

- Thermal temperature, T_{th} such that:

$$G\frac{M(m_p + m_e)}{R} = 2 \times \frac{3}{2} kT_{th} \quad \Rightarrow \quad \boxed{T_{th} = \frac{GMm_p}{3kR}}$$

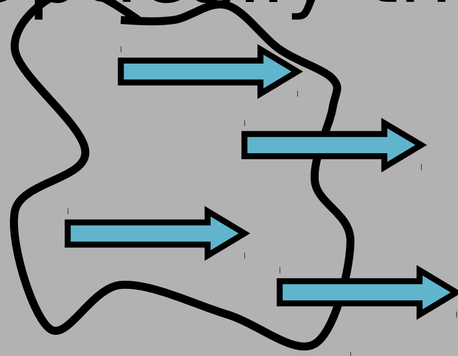
Accretion temperatures

- Optically-thick flow:



$$T_{rad} \sim T_b$$

- Optically-thin flow:



$$T_{rad} \sim T_{th}$$

Computing accretion temperatures

- In general, $$\boxed{T_b \le T_{rad} \le T_{th}}$$
- For a neutron star:

$$T_{th} = \frac{GMm_p}{3kR} \approx 7.5 \times 10^{11} K$$

$$T_b = (L_{acc} / 4\pi R^2 \sigma)^{1/4} \approx 2 \times 10^{7} K$$

assuming: $$L_{acc} \approx L_{Edd} = 1.3 \times 10^{38} \left(\frac{M}{M_{Sun}} \right) \text{erg/s}$$

Accreting NS and WD spectrum

- Thus expect photon energies in range:

$$\boxed{1\ \mathrm{keV} \le h\nu \le 100\ \mathrm{MeV}}$$

- Similarly for a stellar mass black hole

- For **white dwarf**, L ~ 10 erg/s, M ~ $M_{\odot}$, R = 5x10 cm,

$$\boxed{1\ \ \varepsilon\varsigma \le h\nu \le 100\ \mathrm{keV}}$$

=> optical, UV, X-ray sources

Accreting White Dwarfs in binary systems are called **Cataclismic Variables (CVs)**