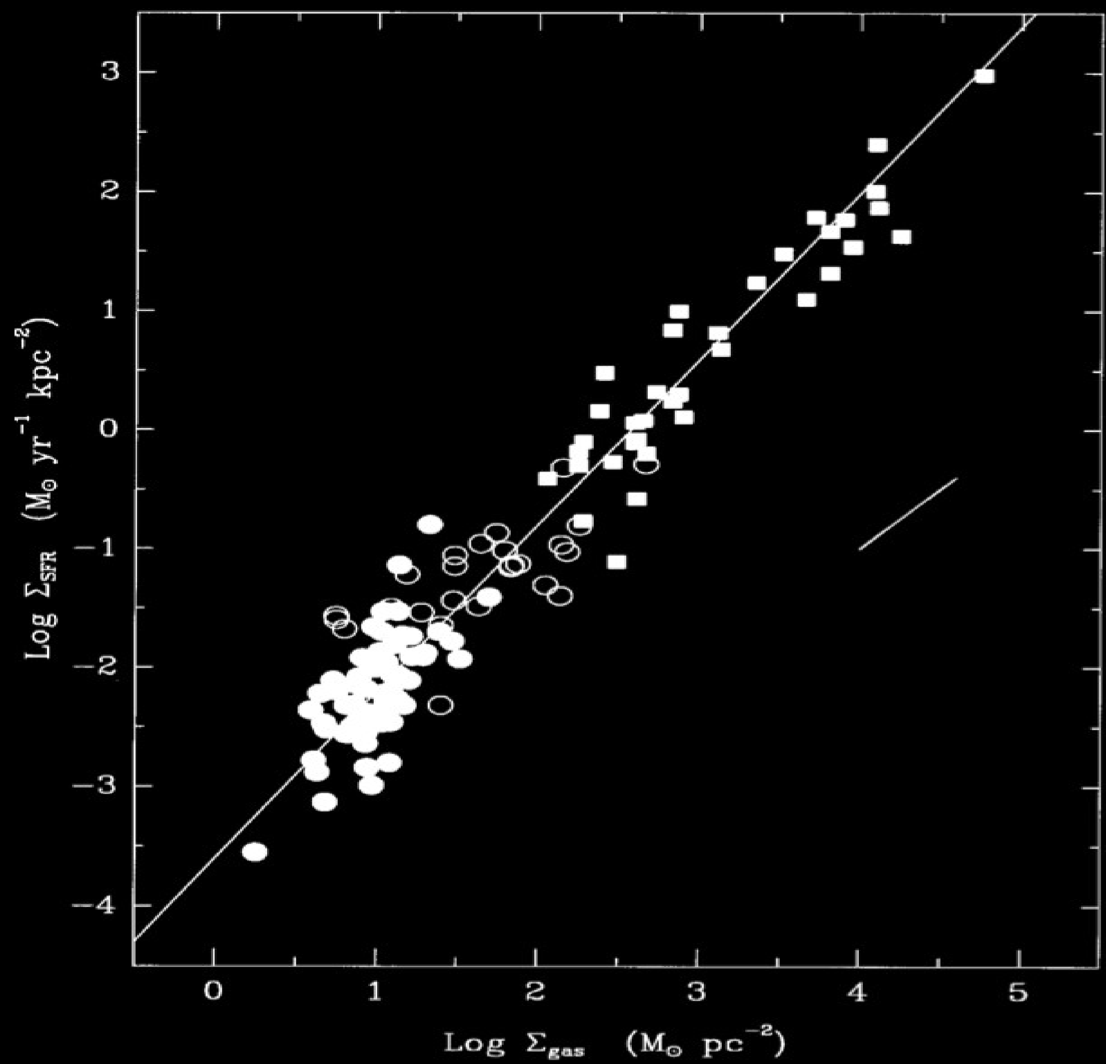
Stars

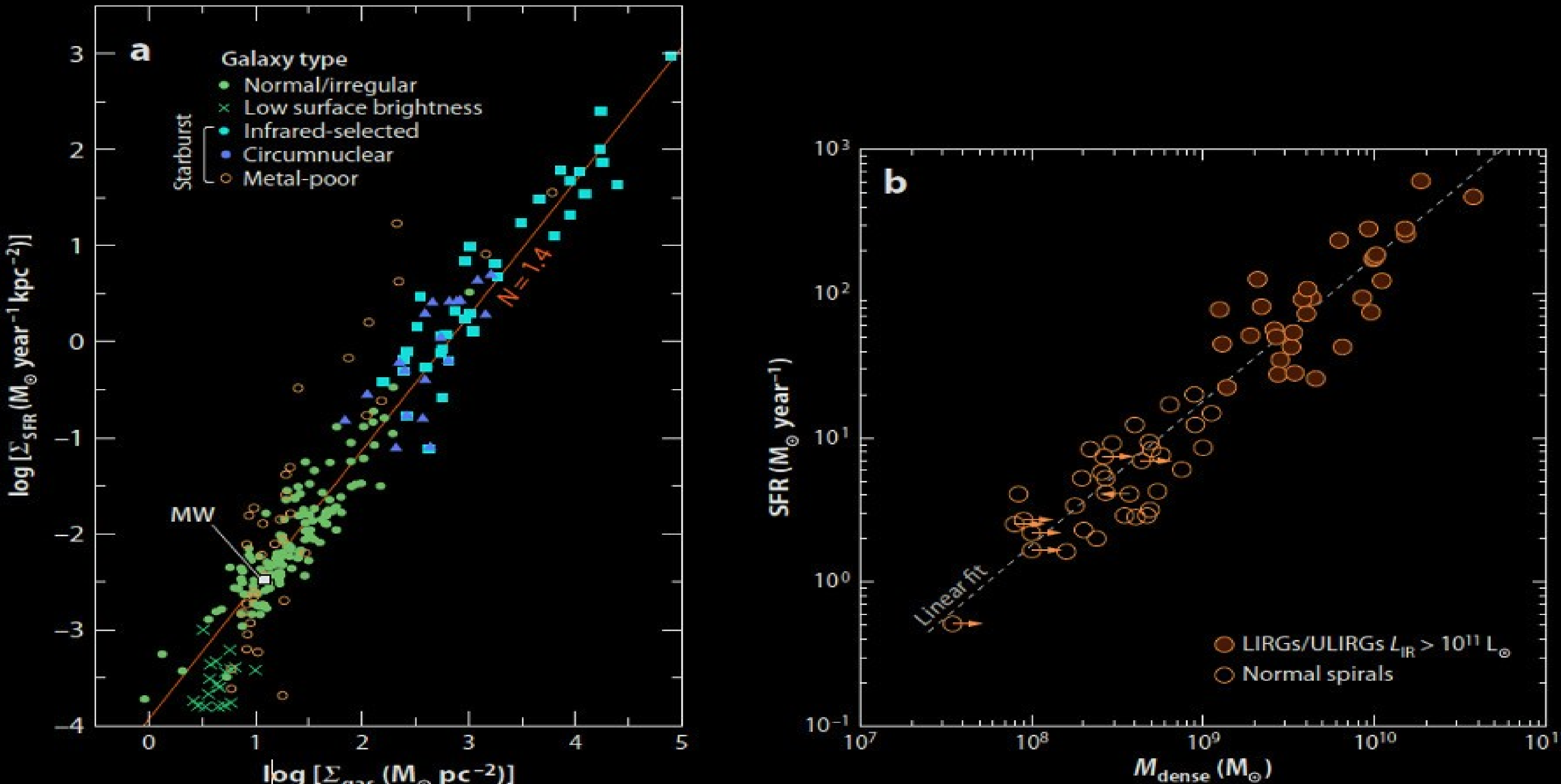
- *Star formation*
- *Stellar equations*
- *Stellar evolution*
- *Final stages of stellar evolution*

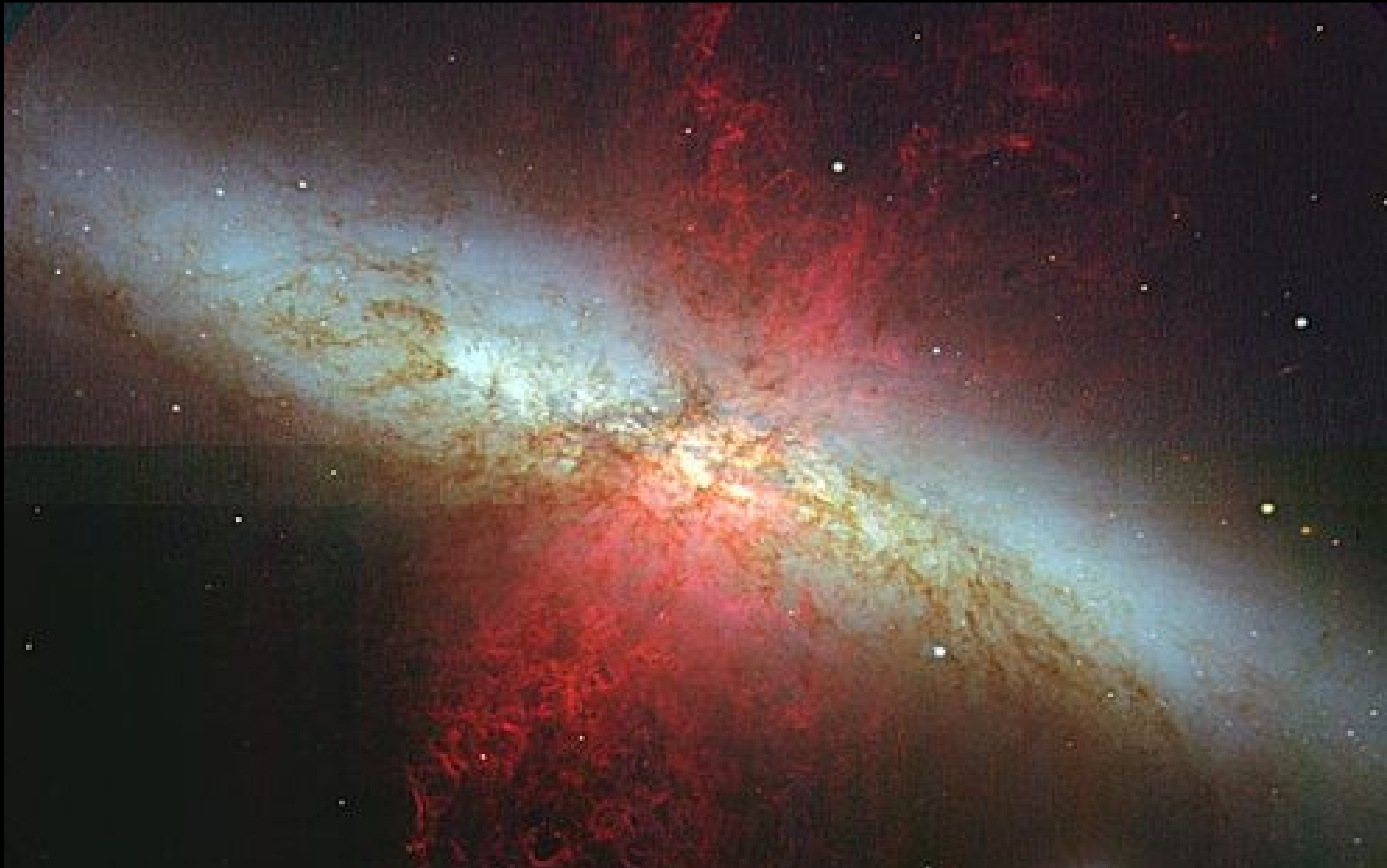
Star formation

Schmidt-Kennicut law



Schmidt-Kennicut law





Jeans Theory

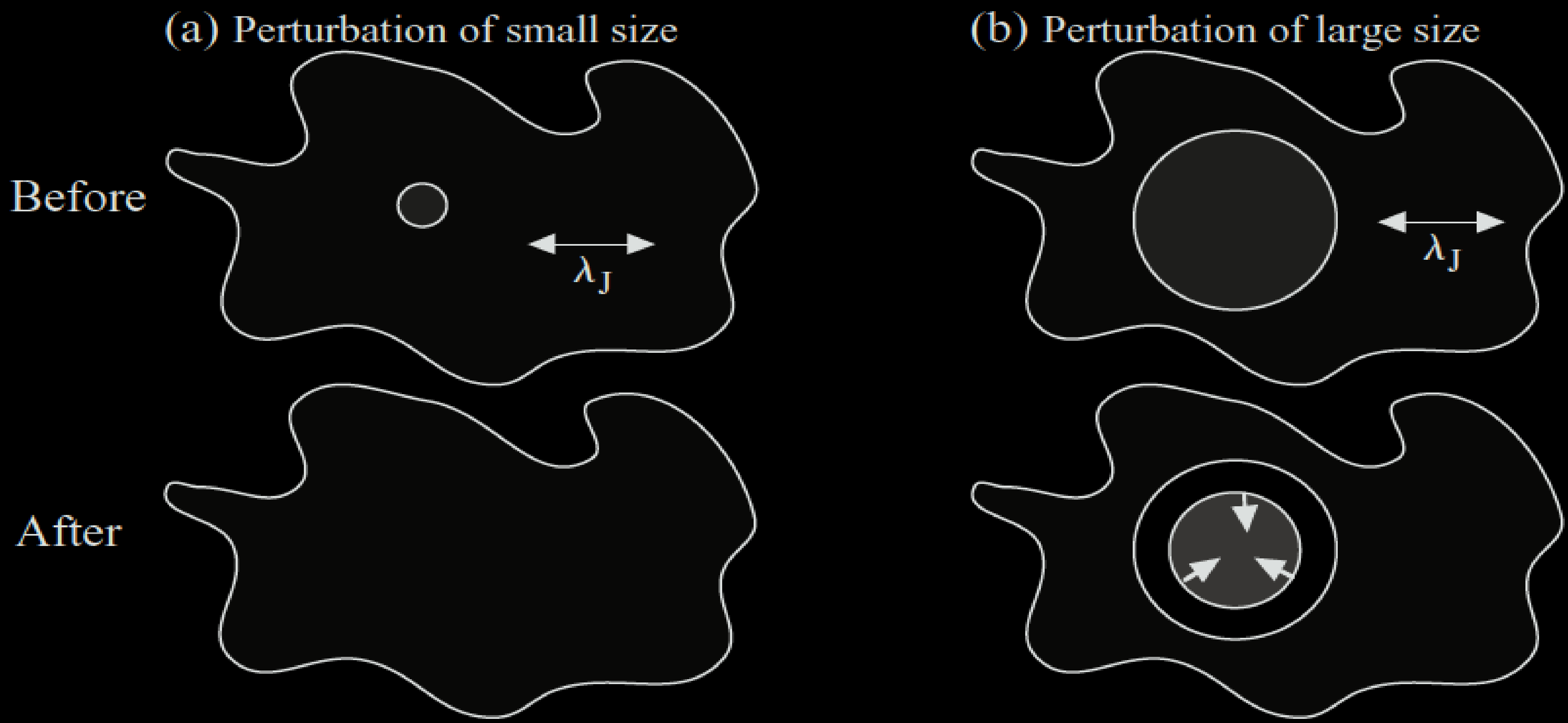
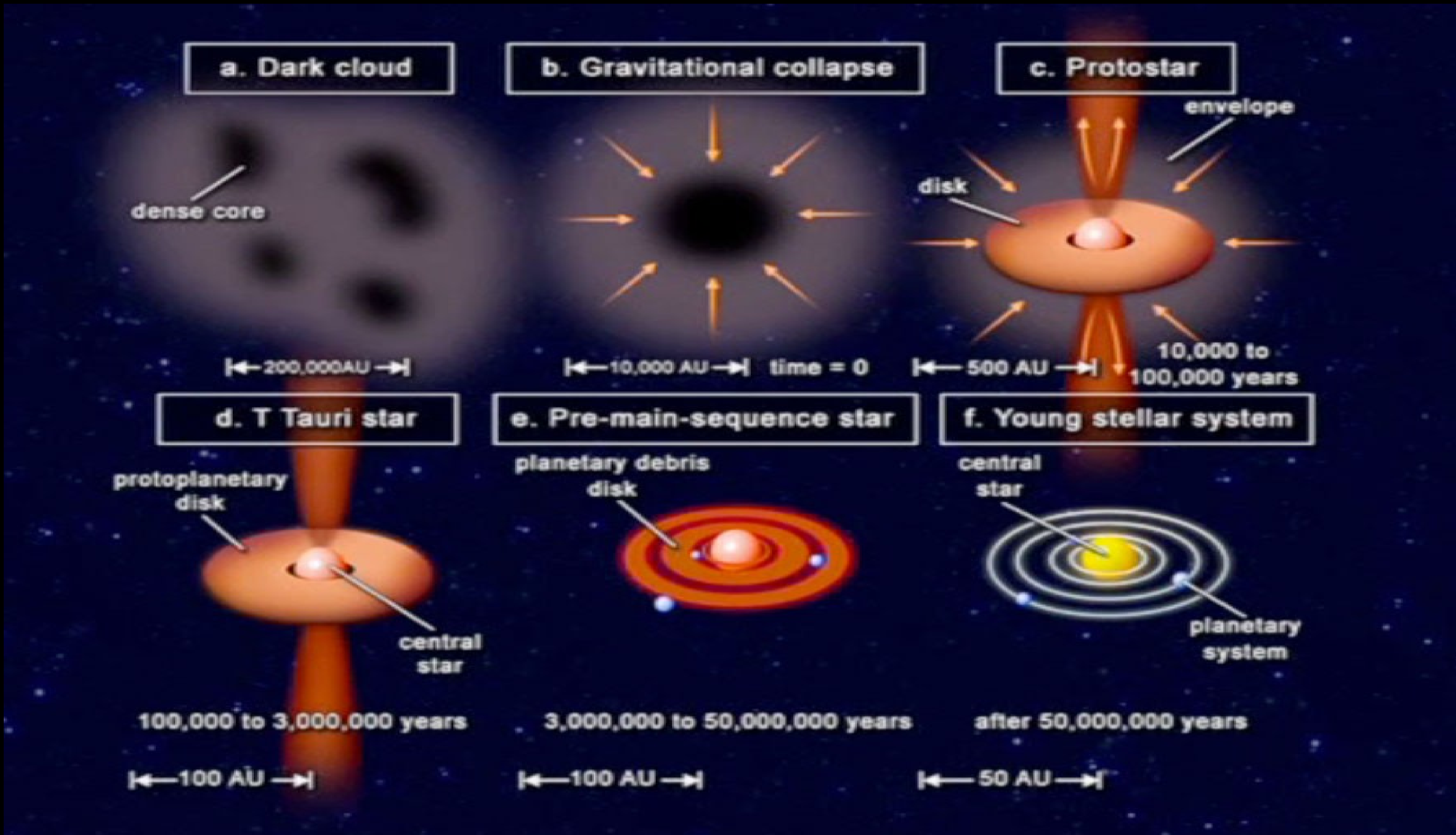
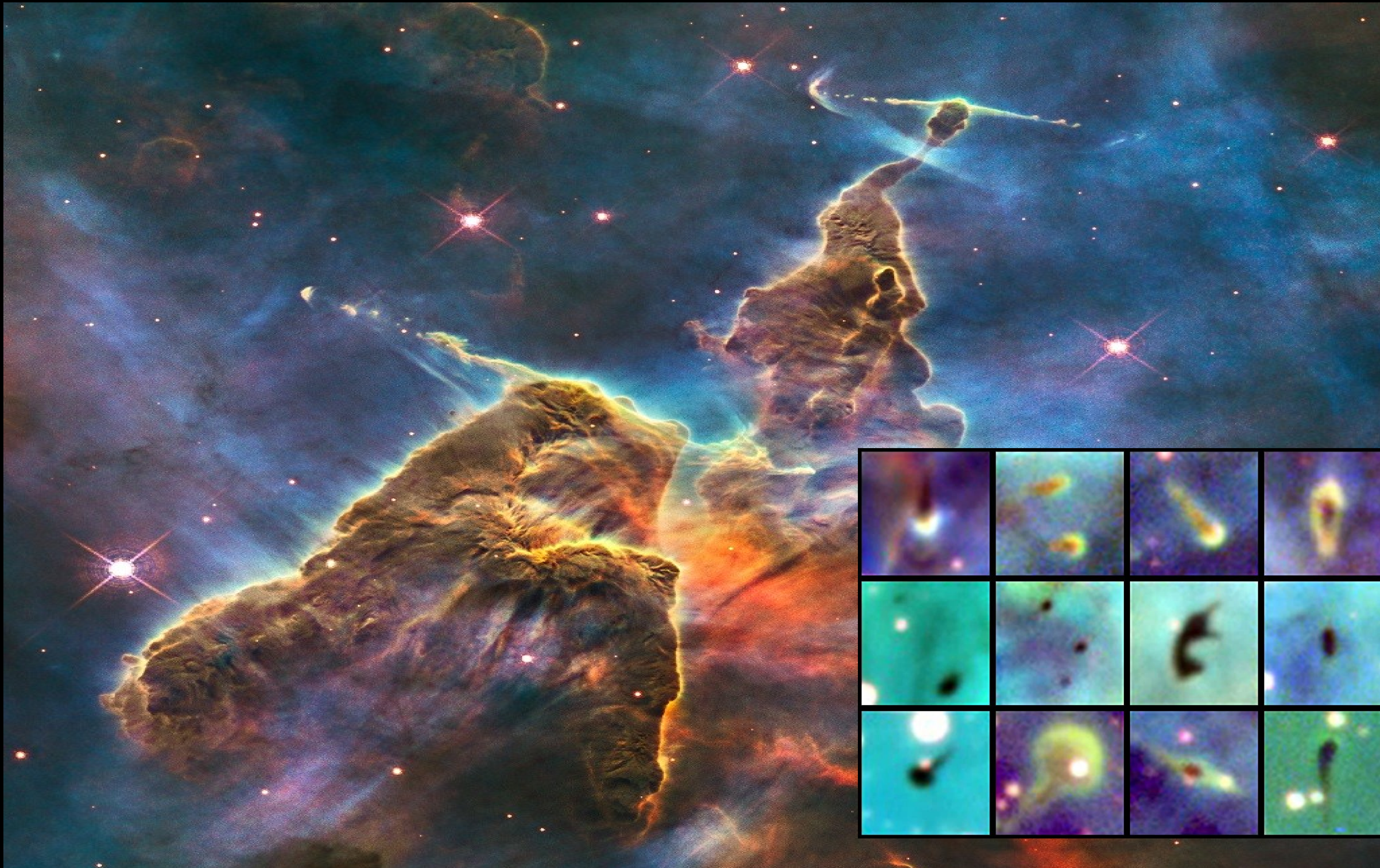


Table 12.4 The Jeans criterion and the contents of giant molecular clouds.

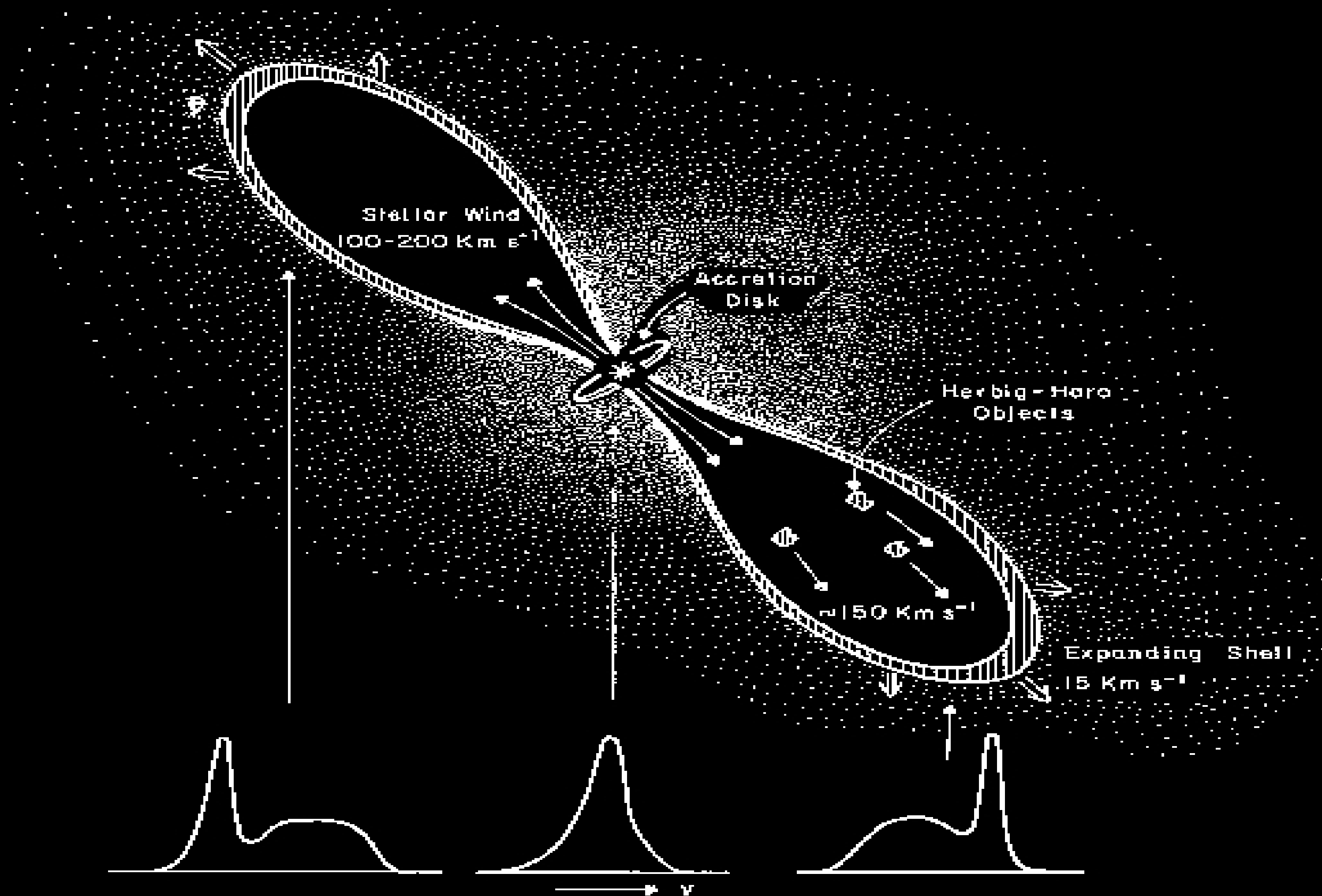
	GMC	Clump	Dense core
Size	50 pc	10 pc	0.1 pc
Mass	$10^5\ M_\odot$	$30–10^3\ M_\odot$	$3–10\ M_\odot$
Number density	$10^8\ \mathrm{m^{-3}}$	$5 \times 10^8\ \mathrm{m^{-3}}$	$5 \times 10^{10}\ \mathrm{m^{-3}}$
Temperature	15 K	10 K	10 K
Jeans length	4 pc	1.5 pc	0.15 pc
Jeans mass	$600\ M_\odot$	$100\ M_\odot$	$30\ M_\odot$

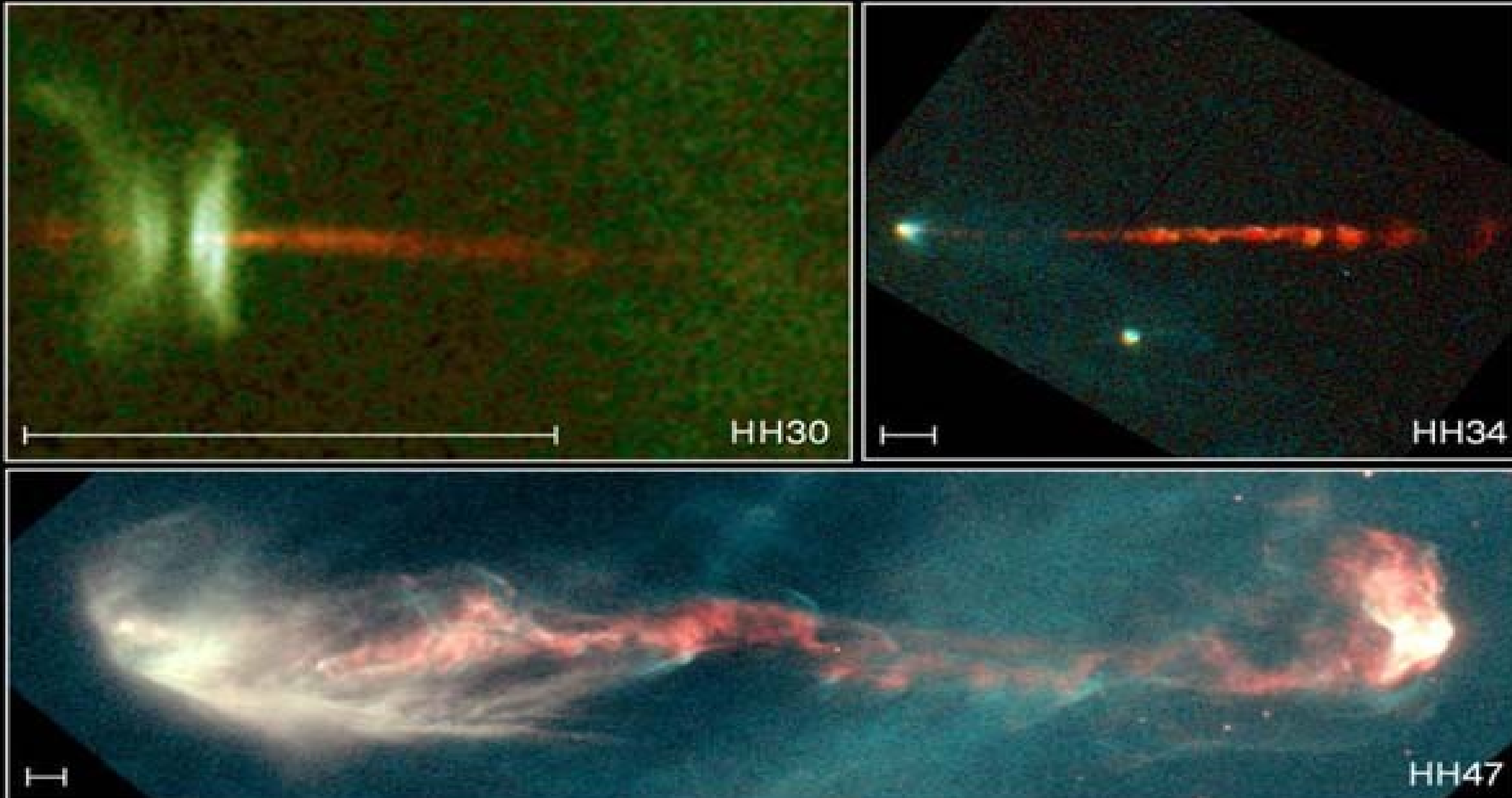

a. Dark cloud
dense core
200,000AU
b. Gravitational collapse
10,000 AU
time = 0
c. Protostar
envelope
disk
500 AU
10,000 to
100,000 years
d. T Tauri star
protoplanetary
disk
central
star
100,000 to 3,000,000 years
100 AU
e. Pre-main-sequence star
planetary debris
disk
3,000,000 to 50,000,000 years
100 AU
f. Young stellar system
central
star
planetary
system
after 50,000,000 years
50 AU


HST–Star Formation in the Orion Nebula
Red = Nitrogen
Green = Hydrogen
Blue = Oxygen
Proplyds
Trapezium



Bipolar outflows



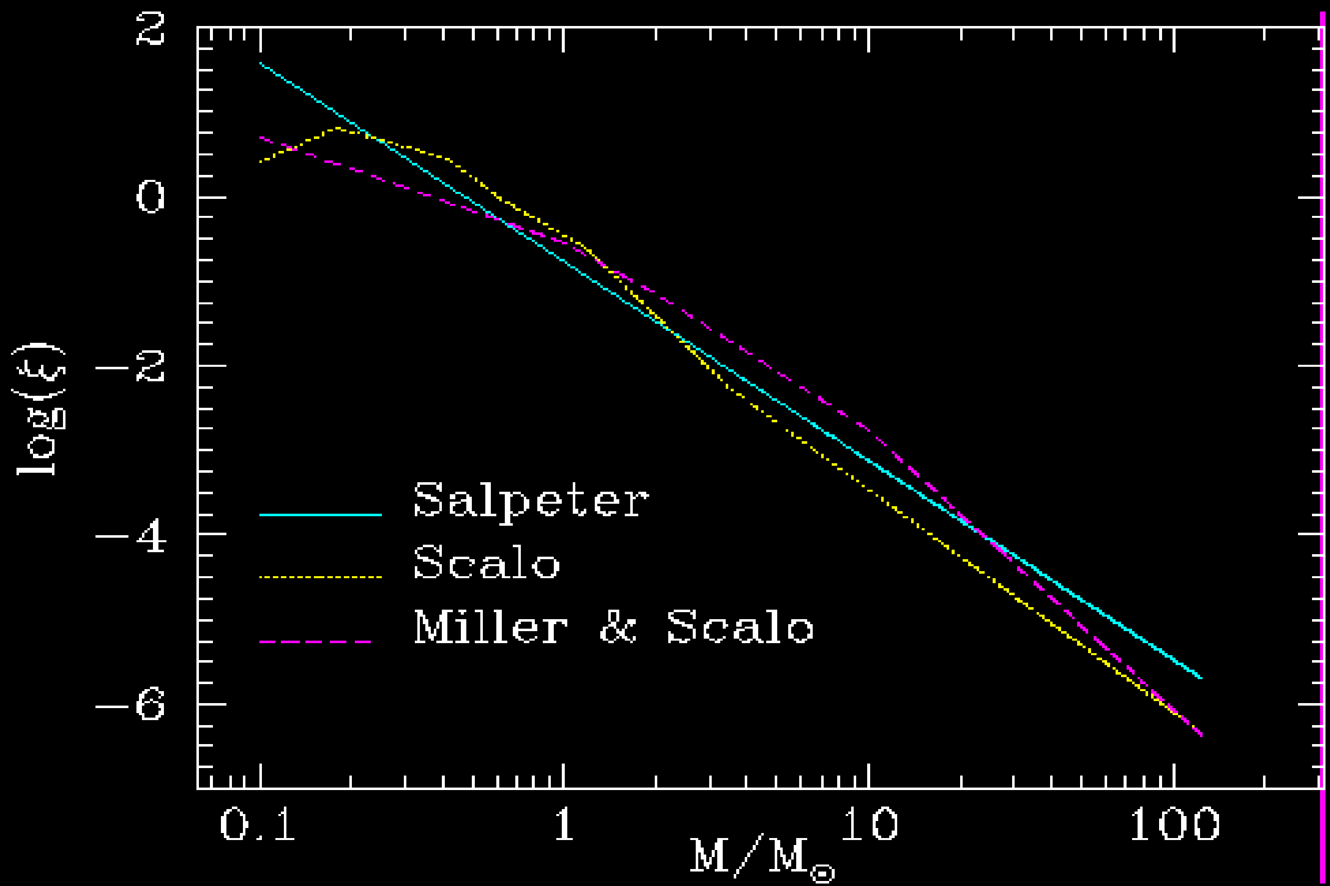


Jets from Young Stars HST · WFPC2

PRC95-24a · ST Scl OPO · June 6, 1995

C. Burrows (ST Scl), J. Hester (AZ State U.), J. Morse (ST Scl), NASA

Initial Mass Function



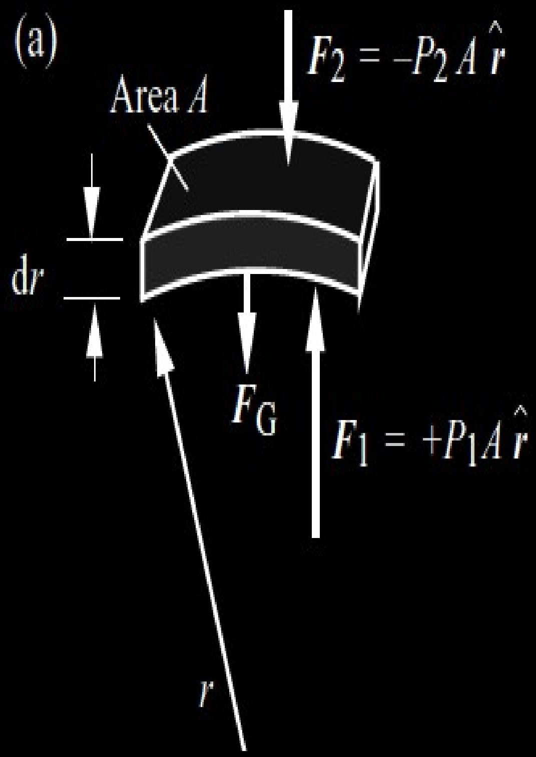
Hydrostatic equilibrium

balance between gravity and gas pressure

$$F_g = \frac{GM(r)}{r^2}\rho(r)drdA$$

$$F_p = [P(r) - P(r+dr)]\,dA = -\frac{dP}{dr}drdA$$

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$



Stellar Equations

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$

1) Hydrostatic equilibrium

Stellar Equations

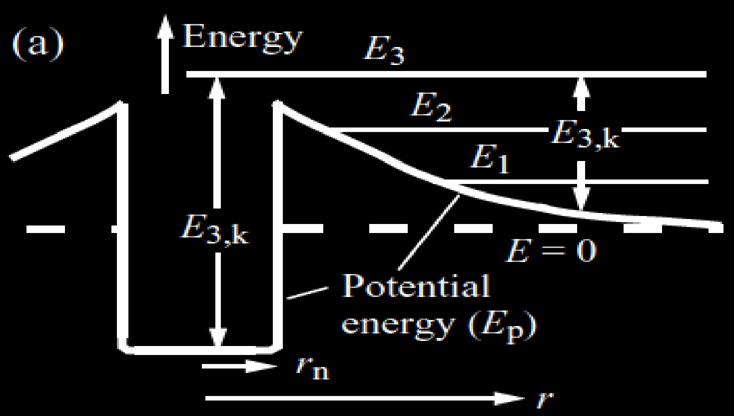
$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$

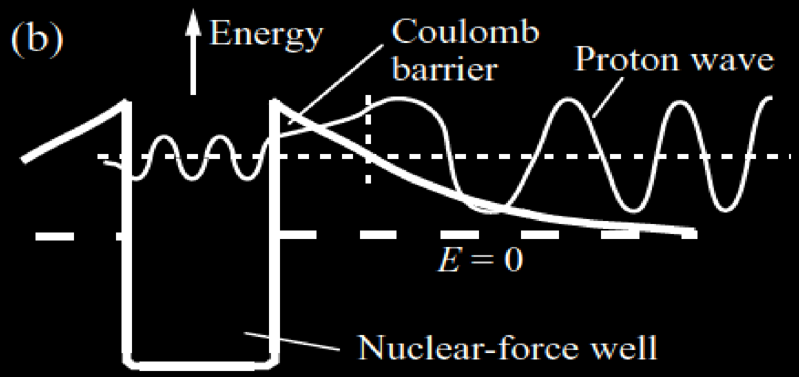
1) Hydrostatic equilibrium

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

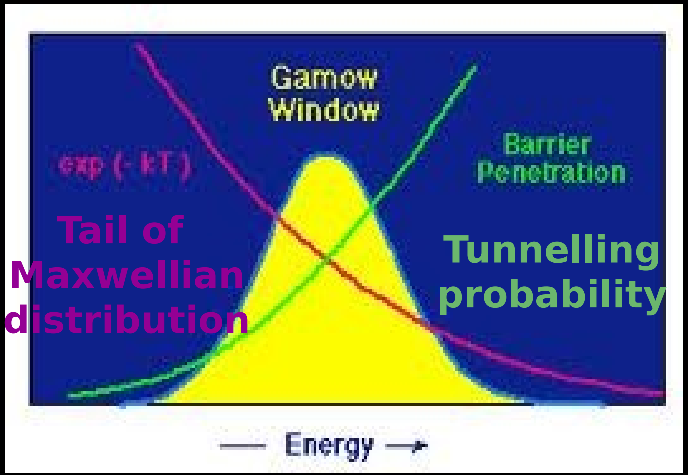
2) Conservation of mass

Nuclear reactions



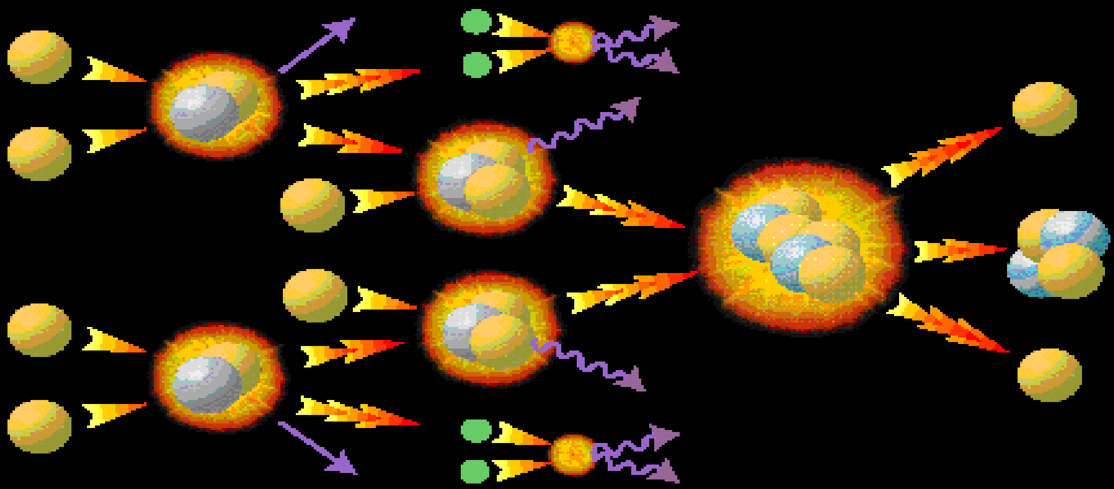


- T > 10^{10} K would be required to surmount **Coulomb barrier**
- Quantum effects (**tunnelling**) allow nuclear reactions at much lower temperatures (low, and strongly T-dependent, efficiency)



Proton-proton (pp) chain

Most of the nuclear energy from stars is produced by the fusion of four hydrogen atoms into a helium nucleus: the pp chain



$$ {}^1_1H + {}^1_1H \rightarrow {}^2_1D + e^+ + \nu_e $$

$$ {}^2_1D + {}^1_1H \rightarrow {}^3_2He + \gamma $$

$$ {}^3_2He + {}^3_2He \rightarrow {}^4_2He + {}^1_1H + {}^1_1H $$

$$ 6\,{}^1H^+ \longrightarrow {}^4He^{++} + 2\,{}^1H^+ + 2e^+ + 2\nu + 2\gamma $$

pp Chain

The energy released by the pp chain is simply the mass decrement between the initial and final nuclei

$$6\,{}^{1}H^{+} \longrightarrow {}^{4}He^{++} + 2\,{}^{1}H^{+} + 2e^{+} + 2\nu + 2\gamma$$

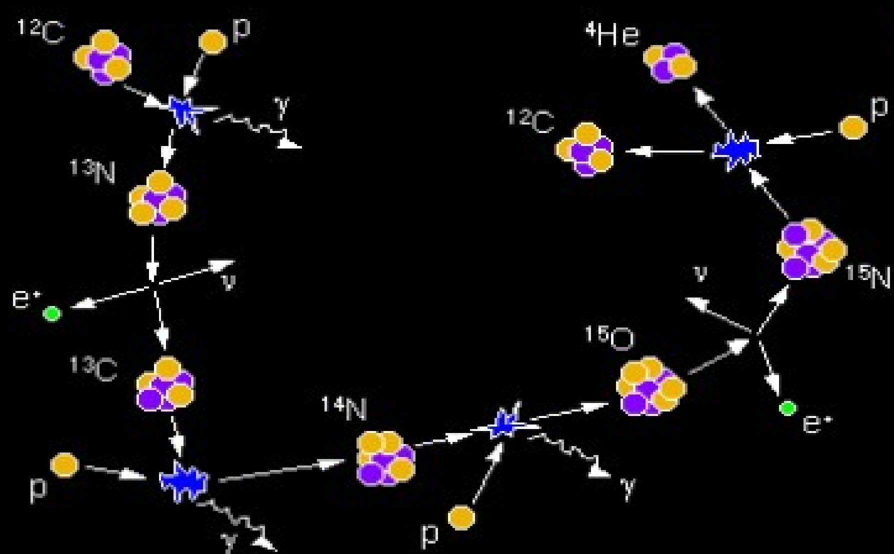
Energy released

Mass difference between initial and final nuclei

$$\begin{aligned}\Delta E &= \Delta m c^2 \\ &= (M_{6H} - M_{2H} - M_{He})c^2 \\ &\sim 26\ \mathrm{MeV}\end{aligned}$$

CNO Chain

The CNO cycle commences once the stellar core temperature reaches 1.4×10^7 K and is the primary source of energy in stars of mass **M > 1.5 M$_\odot$**



Hydrogen in

$$^{12}C + {}^{1}H \longrightarrow {}^{13}N + \gamma$$

$$^{13}N \longrightarrow {}^{13}C + e^{+} + \nu$$

$$^{13}C + {}^{1}H \longrightarrow {}^{14}N + \gamma$$

$$^{14}N + {}^{1}H \longrightarrow {}^{15}O + \gamma$$

$$^{15}O \longrightarrow {}^{15}N + e^{+} + \nu$$

$$^{15}N + {}^{1}H \longrightarrow {}^{12}C + {}^{4}He$$

Helium out

C is only a **catalyst** for the CNO reaction
How much energy is released?

Nuclear reatcions

Many nuclear reactions can occur in stars, with relative efficiencies depending on temperature, density and abundances of chemical elements

⇒ different reactions are dominant in different stages of **stellar evolution**

Nuclear Fuel	***Process***	***Threshold Temperature***	***Products***
H	p-p chain	~ 4 x 10^6 K	He
H	CNO cycle	15 x 10^6 K	He
He	3α	100 x 10^6 K	C, O
C	C + C	600 x 10^6 K	O, Ne, Na, Mg
O	O + O	1000 x 10^6 K	Mg, S, P, Si
Si	Disintegration	3000 x 10^6 K	Co, Fe, Ni

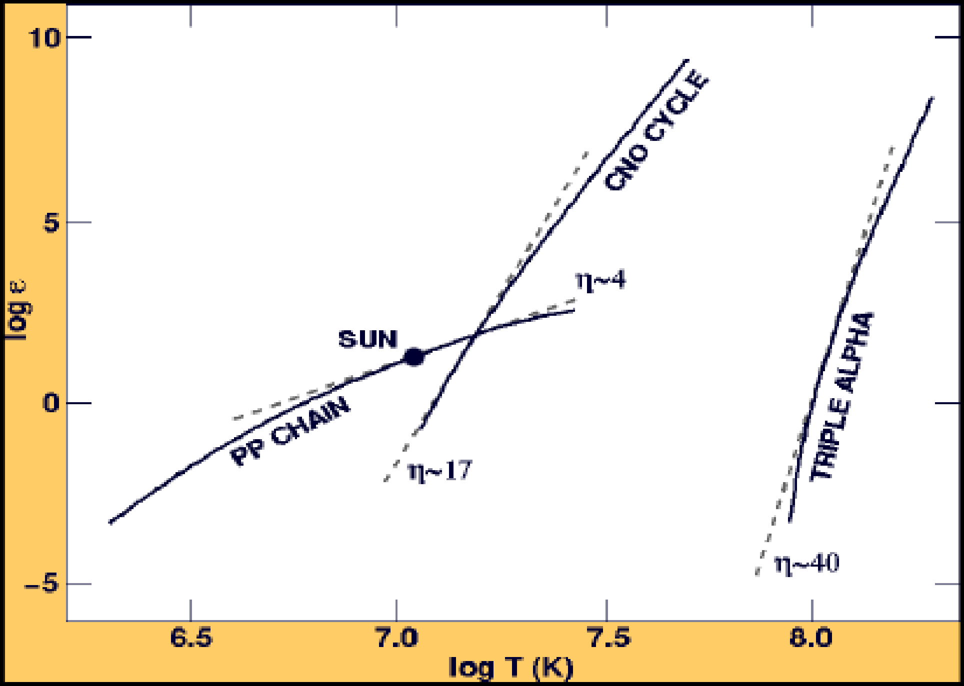
Nuclear reactions

The energy generation rate **ε (energy/mass)** is proportional to the number of interactions per second and strongly depends on temperature:

$$\varepsilon_{PP} \propto \rho X_H^2 T^{4.6}$$

$$\varepsilon_{CNO} \propto \rho X_H X_{CNO} T^{16.7}$$

$$\varepsilon_{3\alpha} \propto \rho^2 T^{40}$$



Stellar Equations

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$

1) *Hydrostatic equilibrium*

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

2) *Conservation of mass*

$$\frac{dL}{dr} = 4\pi r^2 \rho(r)\epsilon$$

3) *Energy generation*

Opacity: $\kappa_\nu=\alpha_\nu/\rho$

- Opacity in a star is a function of composition and temperature.

- Determined by the details of how photons interact with particles (atoms, ions, free electrons).

- If the opacity varies slowly with λ it determines the star continuous spectrum (continuum). A rapid variation of opacity with λ produces dark absorption lines in the spectrum.

Optically thin cloud: $\tau << 1$

- Chances are small that a photon will interact with particle

- Can effectively see right through a cloud

- In the optically thin regime, the amount of extinction (absorption plus scattering) is linearly related to the amount of material: double the amount of gas, double the extinction

→ if we can measure the amount of light absorbed (or emitted) by the gas, we can calculate exactly how much gas there is

Optically thick cloud: $\tau >> 1$

- Certain that a photon will interact many times with particles before it finally escapes from the cloud
- Any photon entering the cloud will have its direction changed many times by collisions, which means that its "output" direction has nothing to do with its "input" direction.

→ ***Cloud is opaque***

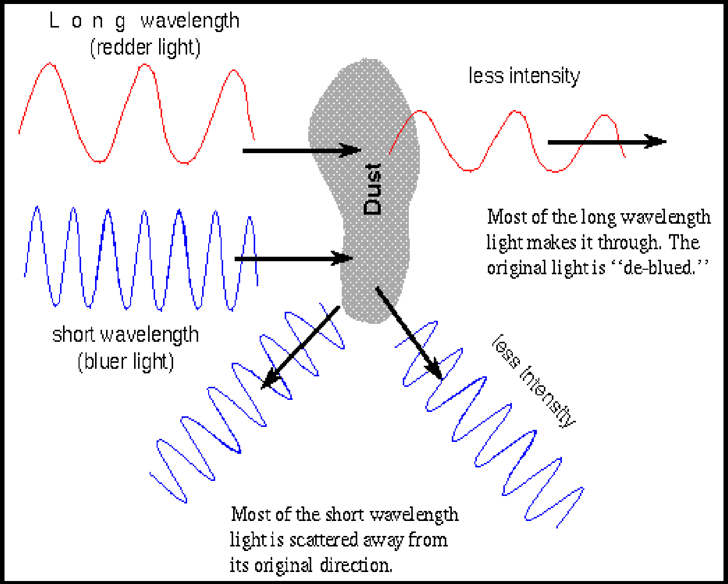
- You can't see through an optically thick medium; you can only see light emitted by the very outermost layers.

→ ***you can't observe interior of a star, but only the surface (photosphere)***

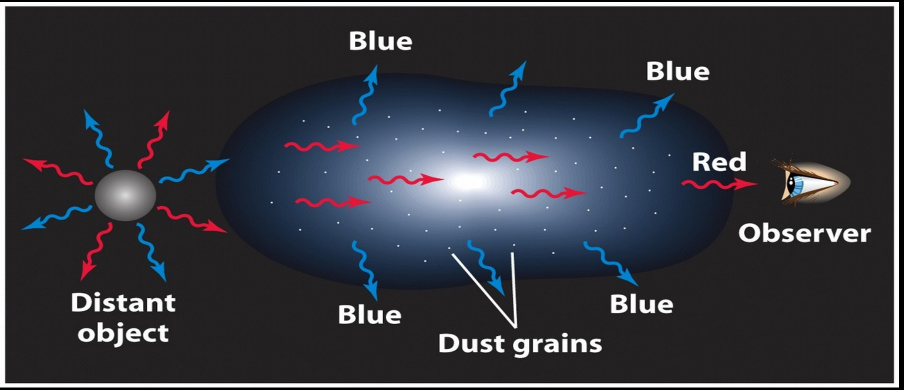
- The spectrum of the radiation emitted by optically thick material is a **blackbody**

- **Bound-Bound absorption:** Small, except at those discrete wavelengths capable of producing a transition (*absorption lines*)
- **Bound-Free absorption:** *Photoionisation*. Occurs when photon has sufficient energy to ionize atom. The freed e^- can have any energy, thus this is a source of continuum opacity
- **Free-Free absorption:** *Bremsstrahlung*. A free electron absorbs a photon, causing its speed to increase. It is a source of continuum opacity and important at high temperatures (it needs free e^-).
- **Electron scattering:** *Thomson scattering*. A photon is scattered, but not absorbed by a free electron.
- **Dust extinction:** Only important for very cool stellar atmospheres and cold interstellar medium

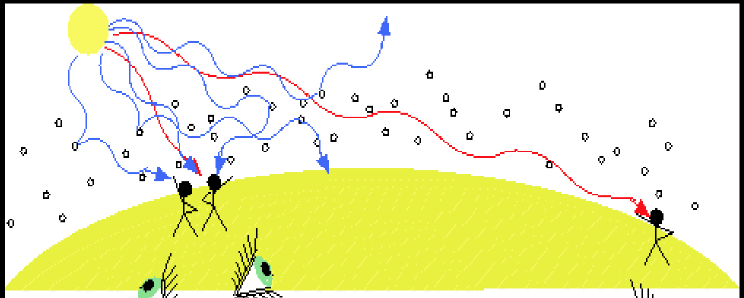
Dust and light



Dust **extinction** and **reddening** in astronomical optical/UV observations



Why is the sky **blue** (and **red** at sunset)?



Stellar Equations

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$

1) Hydrostatic equilibrium

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

2) Conservation of mass

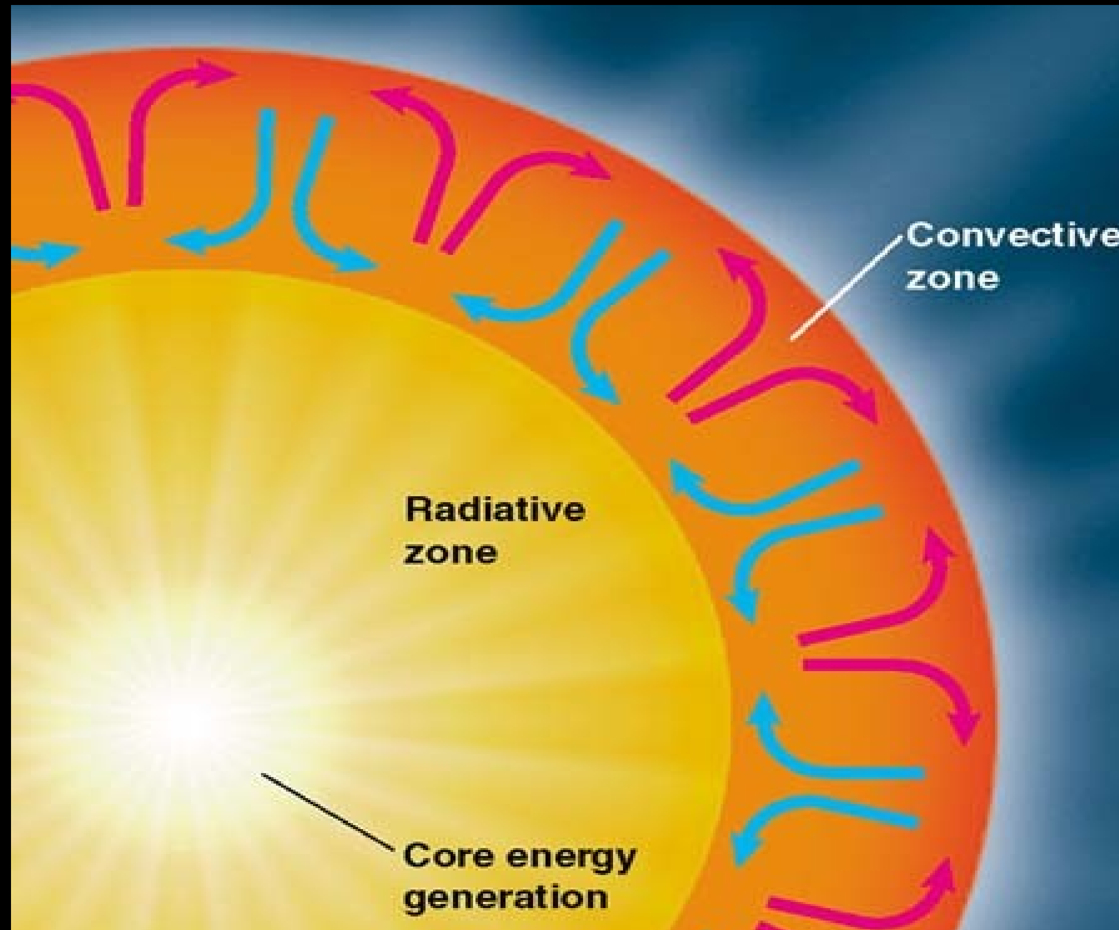
$$\frac{dL}{dr} = 4\pi r^2 \rho(r)\epsilon$$

3) Energy generation

$$\frac{dT}{dr} = -\frac{3k\rho}{16\pi ac\, r^2 T^3}\, L(r)$$

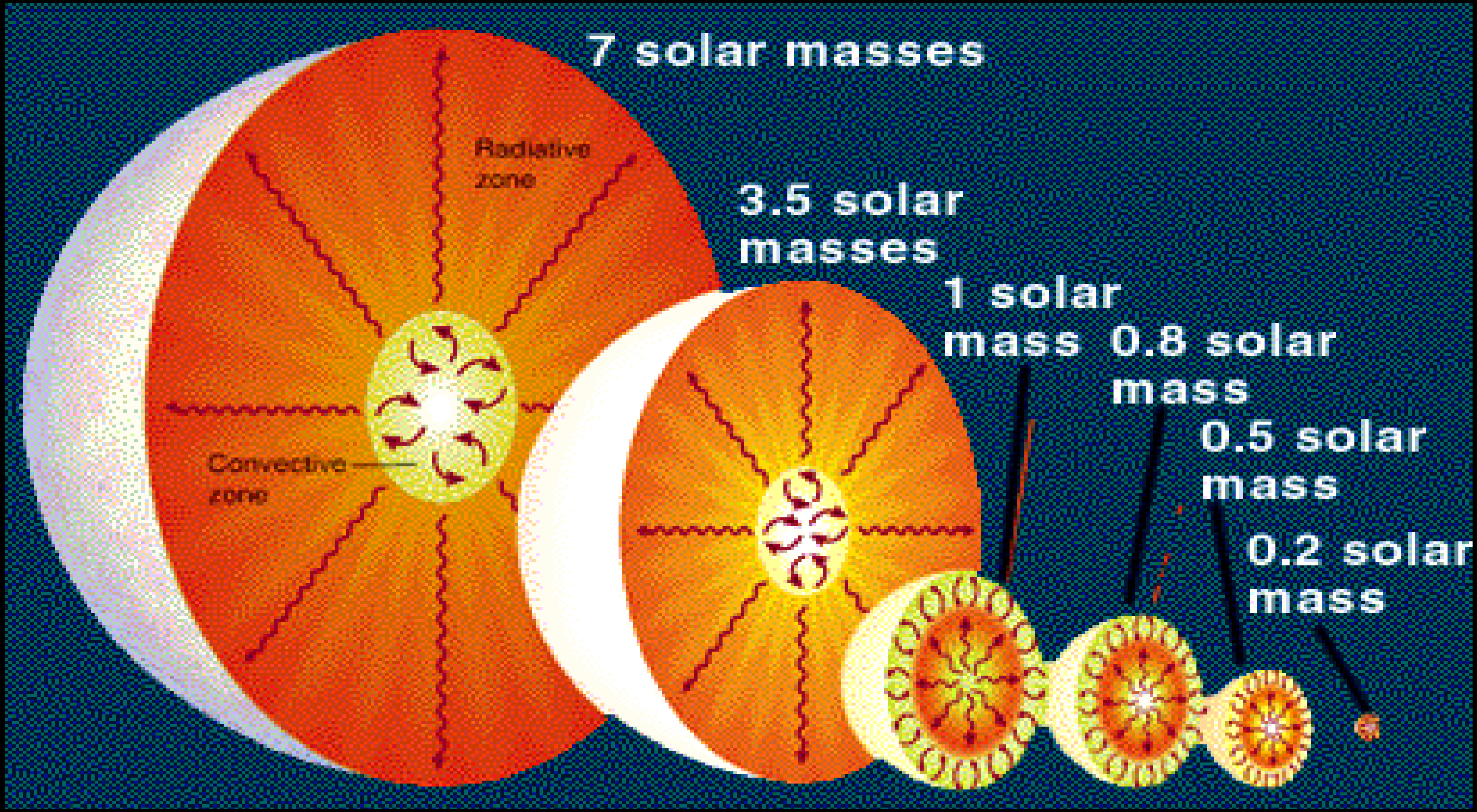
4) Energy transport

Energy Transport in the Sun



In the sun, energy is transported via radiation in the central regions, but by convection in the outer

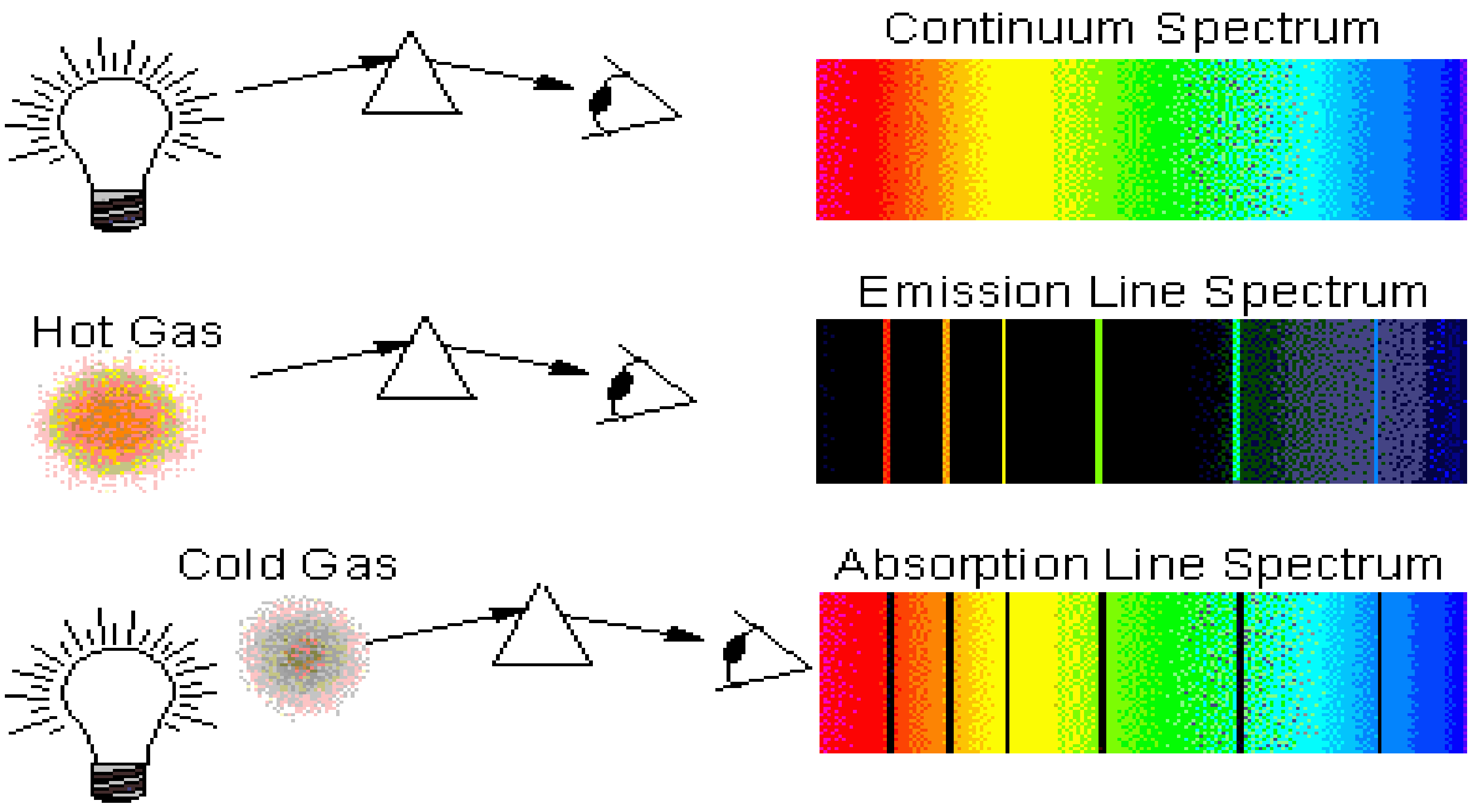
Energy Transport inside Stars



The structure and evolution of stars is accurately modeled with only a few well understood laws of physics ⇒ stellar models.

Spectra of stellar photospheres

Stellar spectra

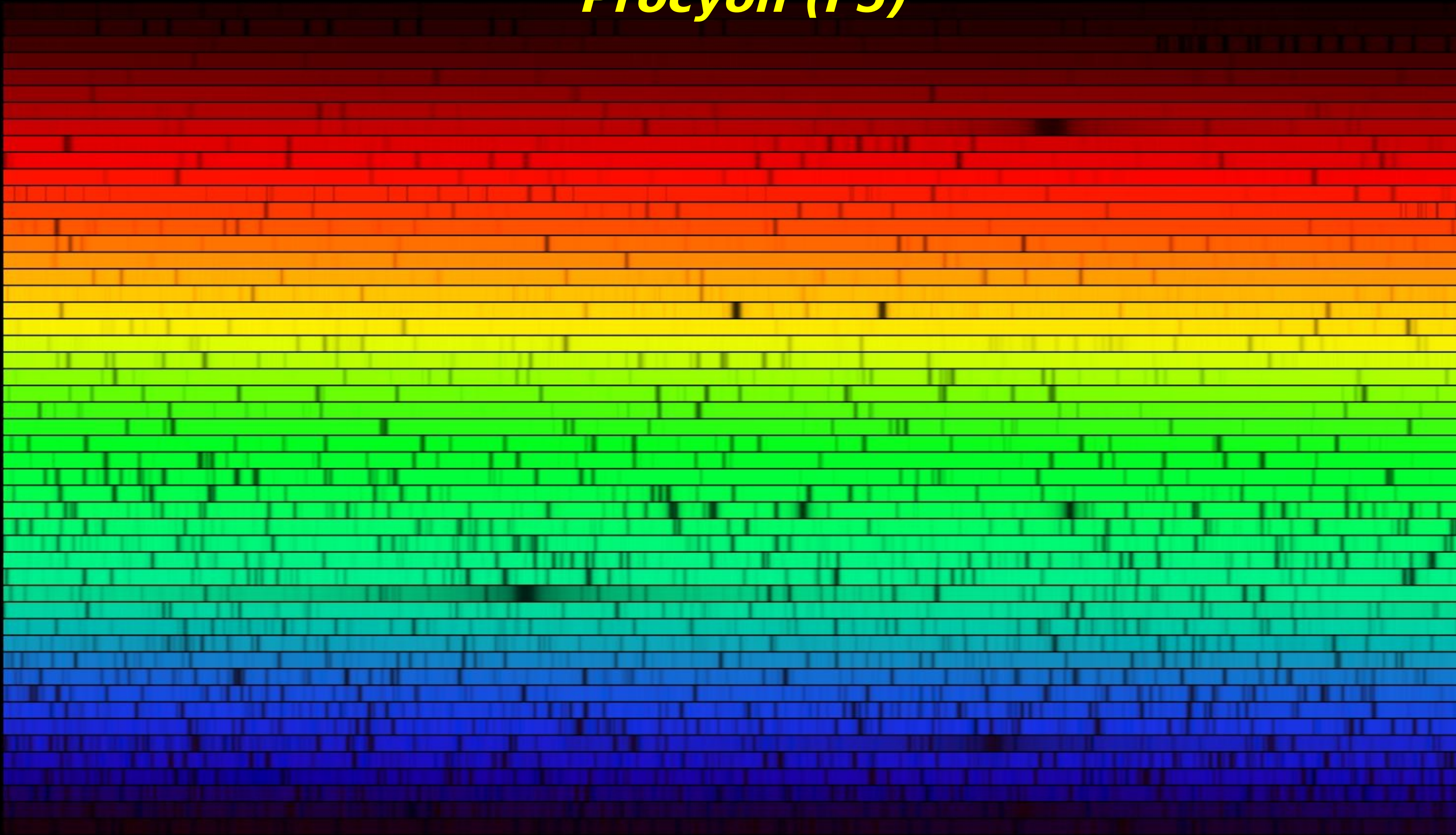


- ***Stellar spectra?***

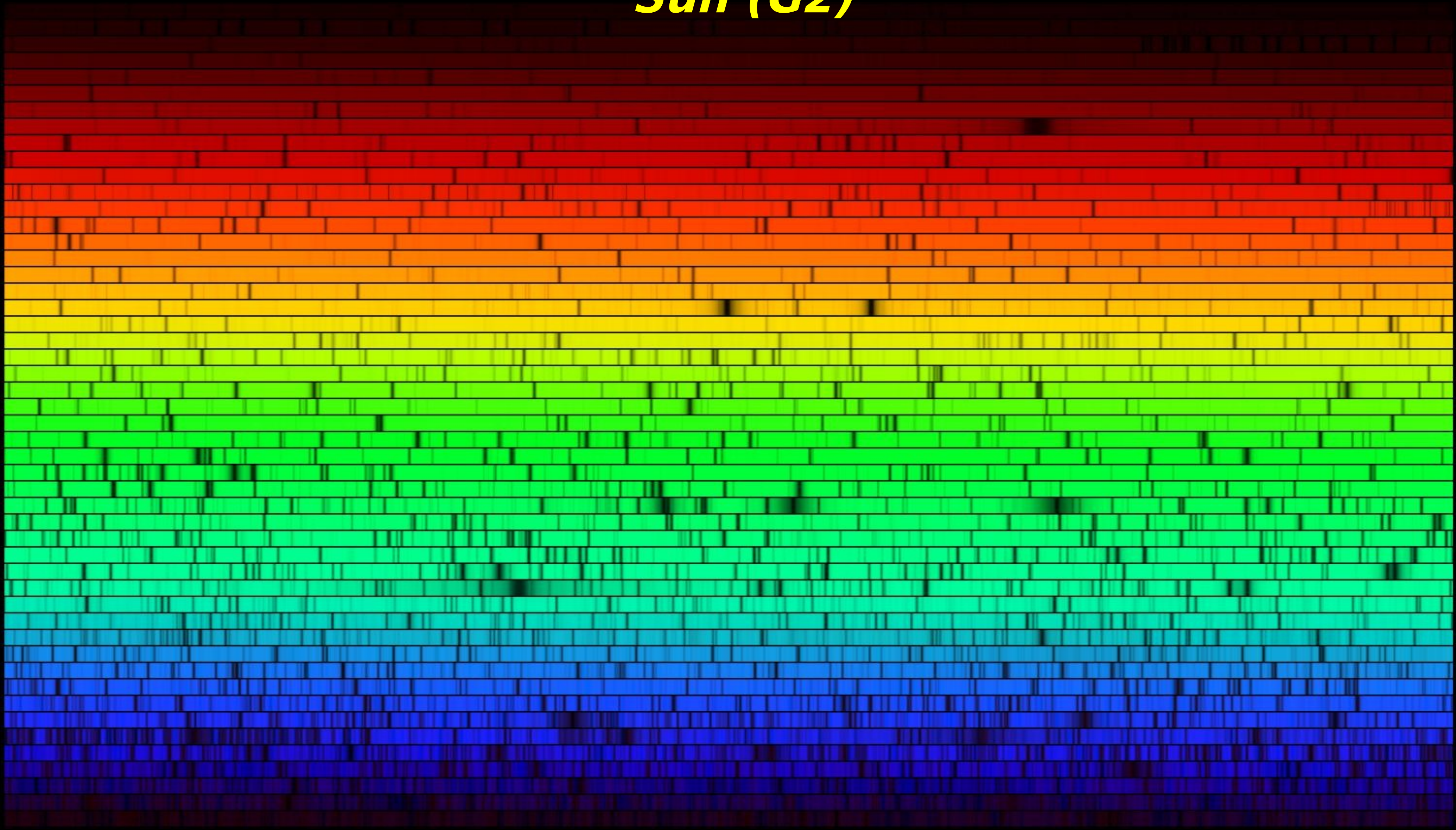
Based on their absorption lines (T indicators)
⇒ spectral types: from warm to cool

“**O**h **B**e **A**
Fine **G**irl
Kiss **M**e”

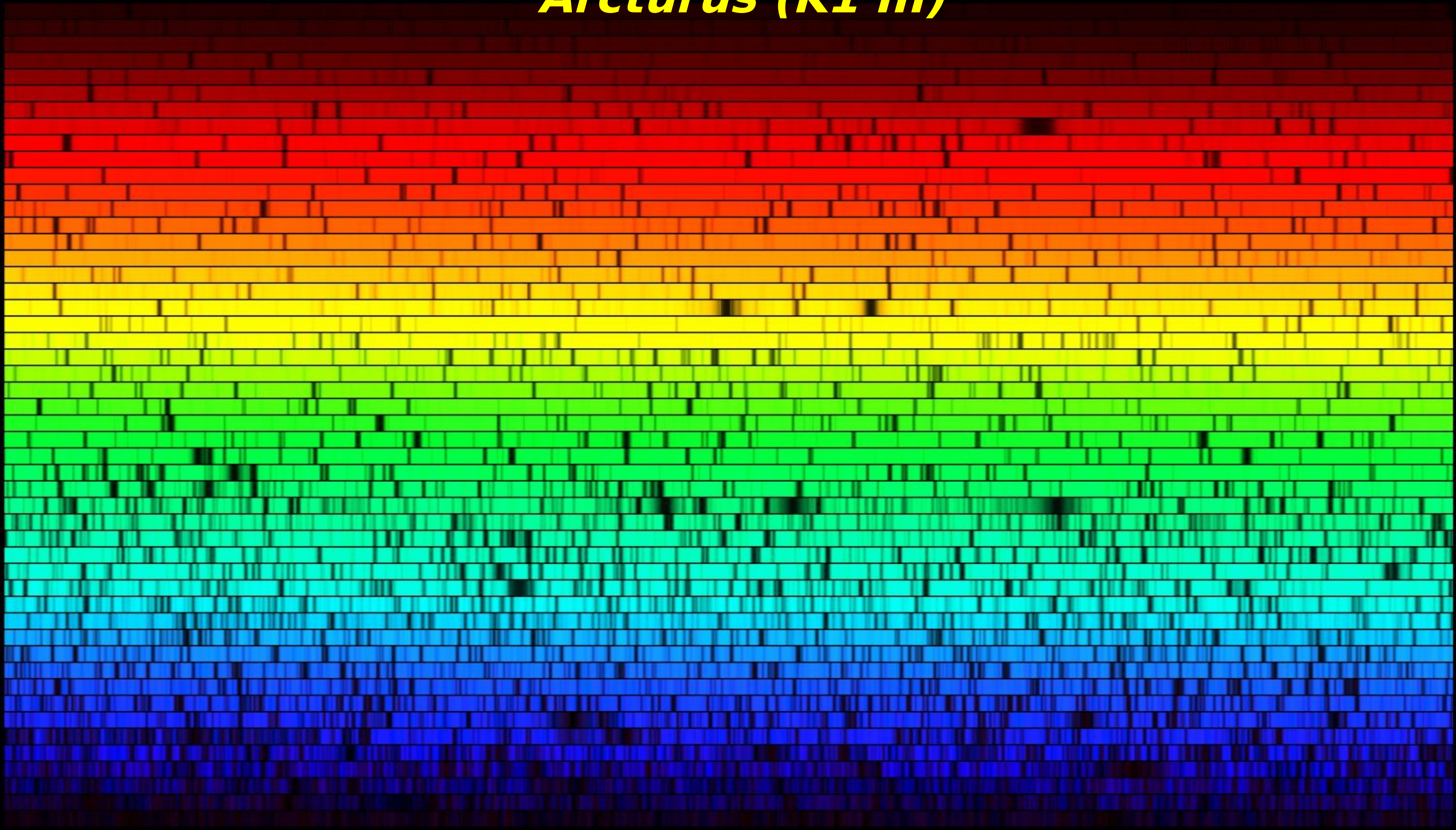
Procyon (F5)

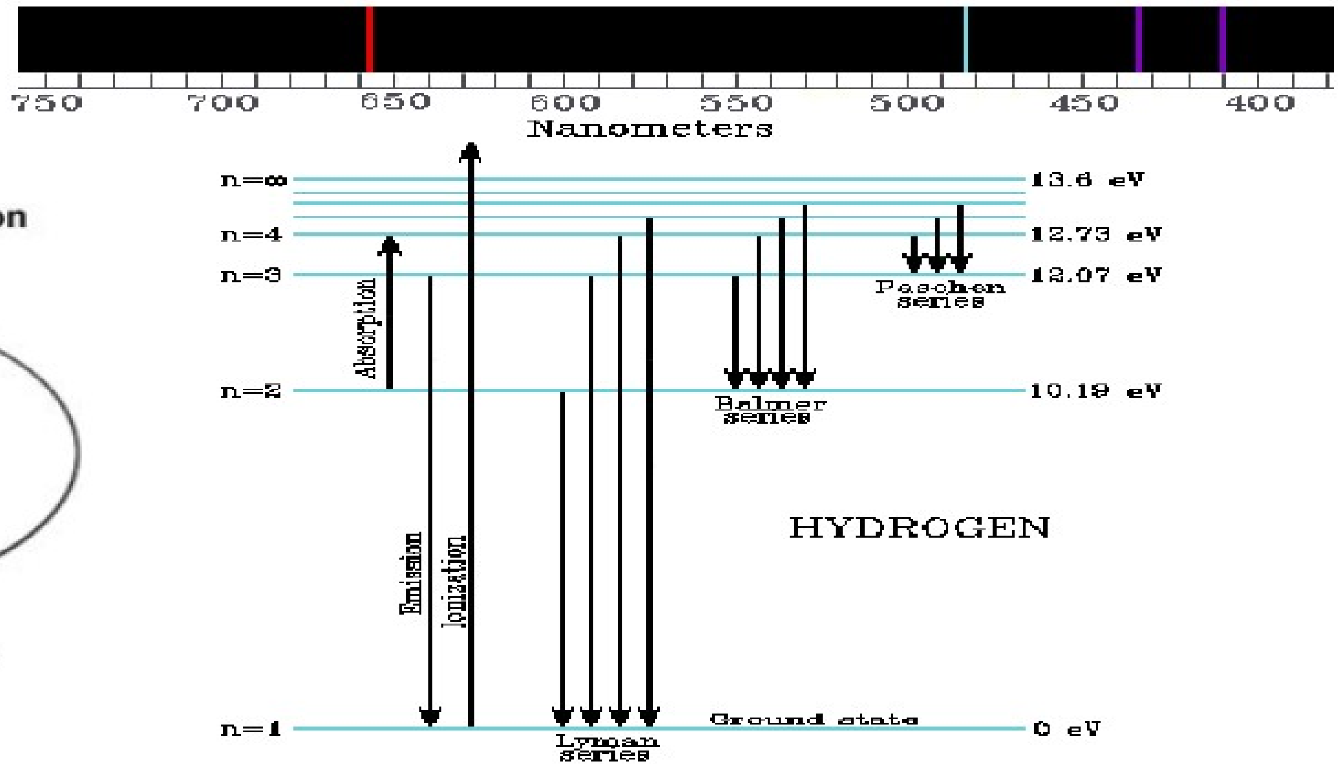


Sun (G2)

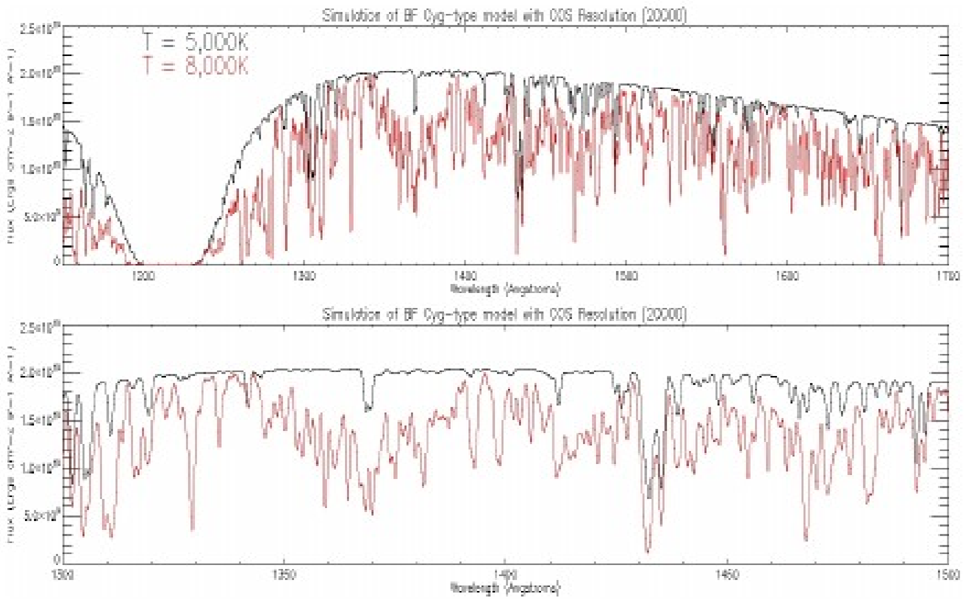


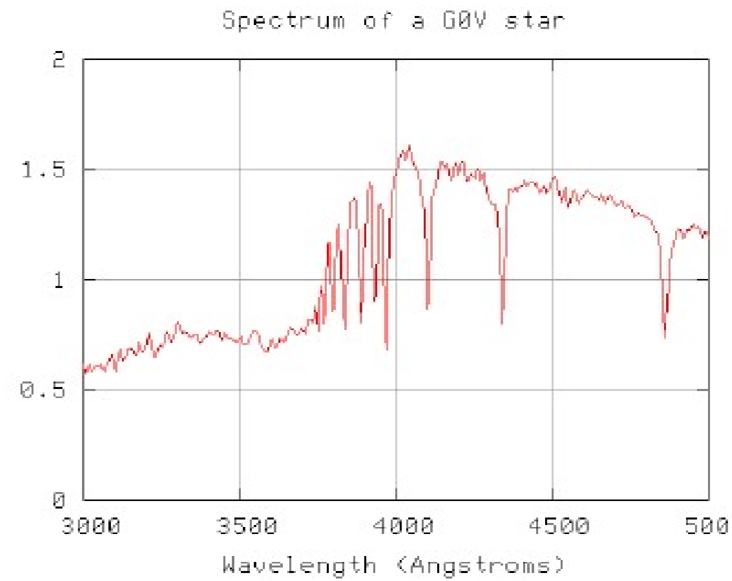
Arcturus (K1 III)





Structure of the H atom → produces spectral features

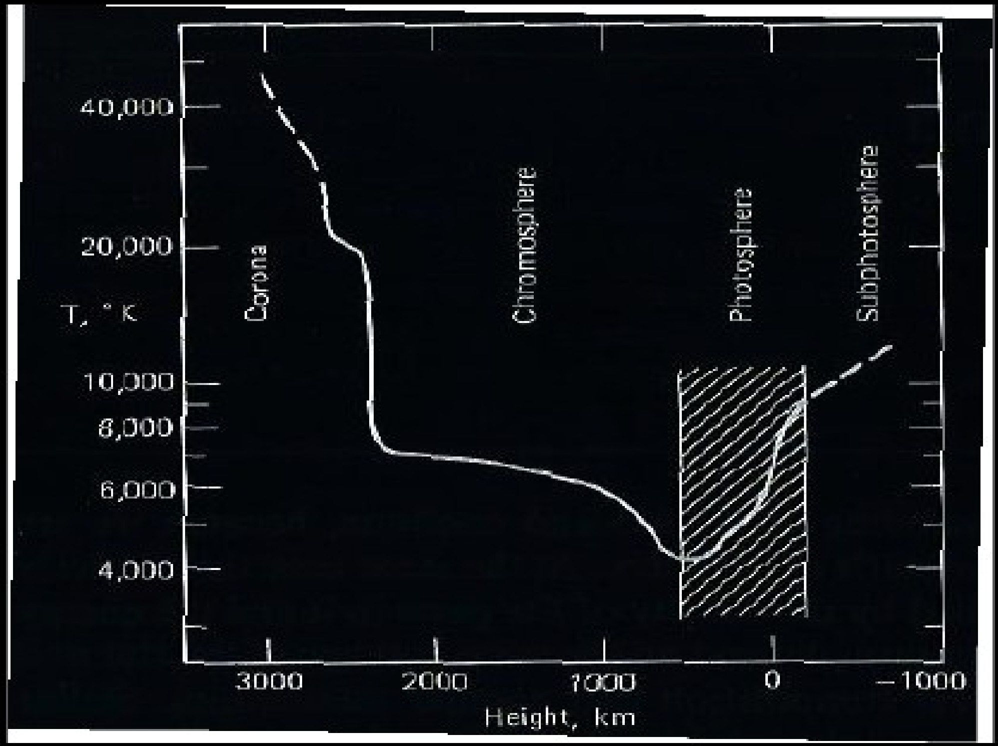




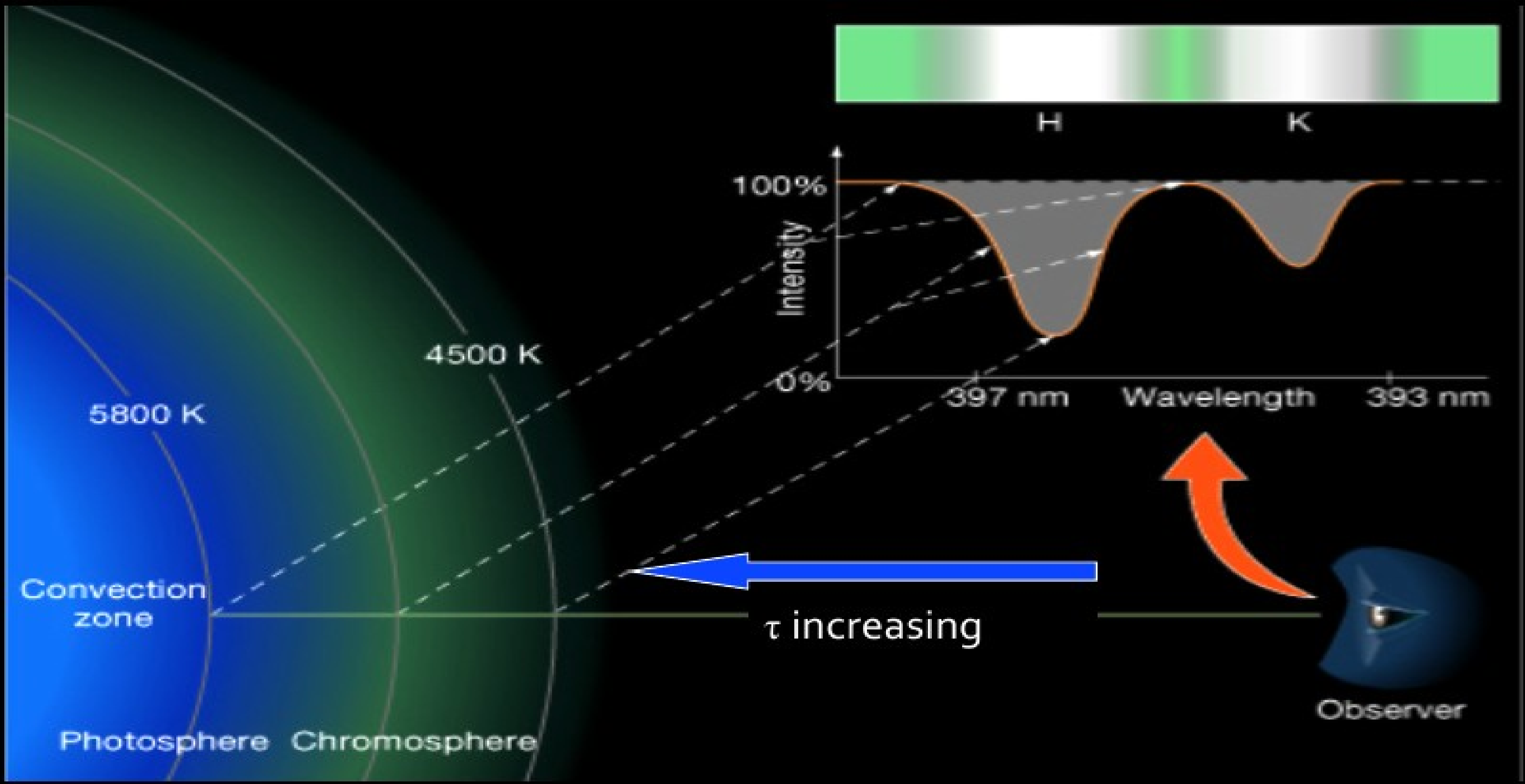
Modelled opacity in the UV due to gas at 5,000K (black) and 8,000K (red). The opacities are due to lines, mostly HI, FeII, SiII, NI, OI, MgII

Balmer series bound-bound transitions (note the Balmer edge → continuous, so bound-free)

- The lower the optical depth, the deeper into the star we see
- For weak lines (lower optical depth) the deeper the line formation region
- For strong lines (higher optical depth), the shallower the line formation region

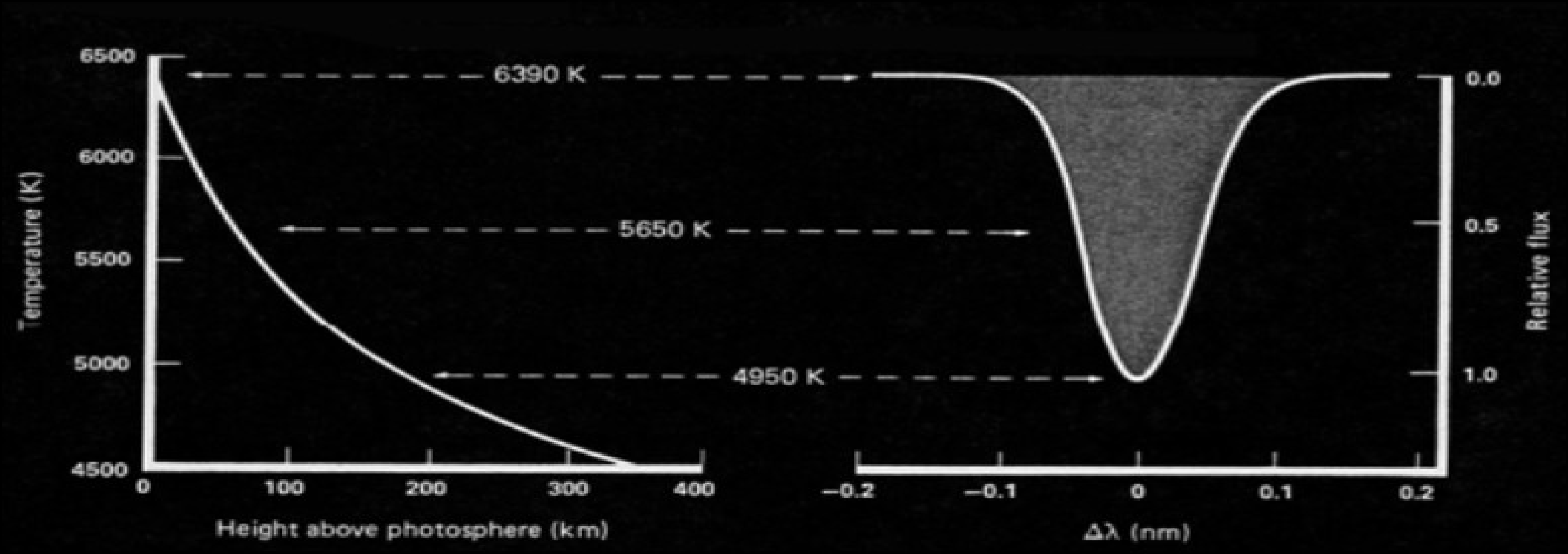


Temperature structure of solar atmosphere

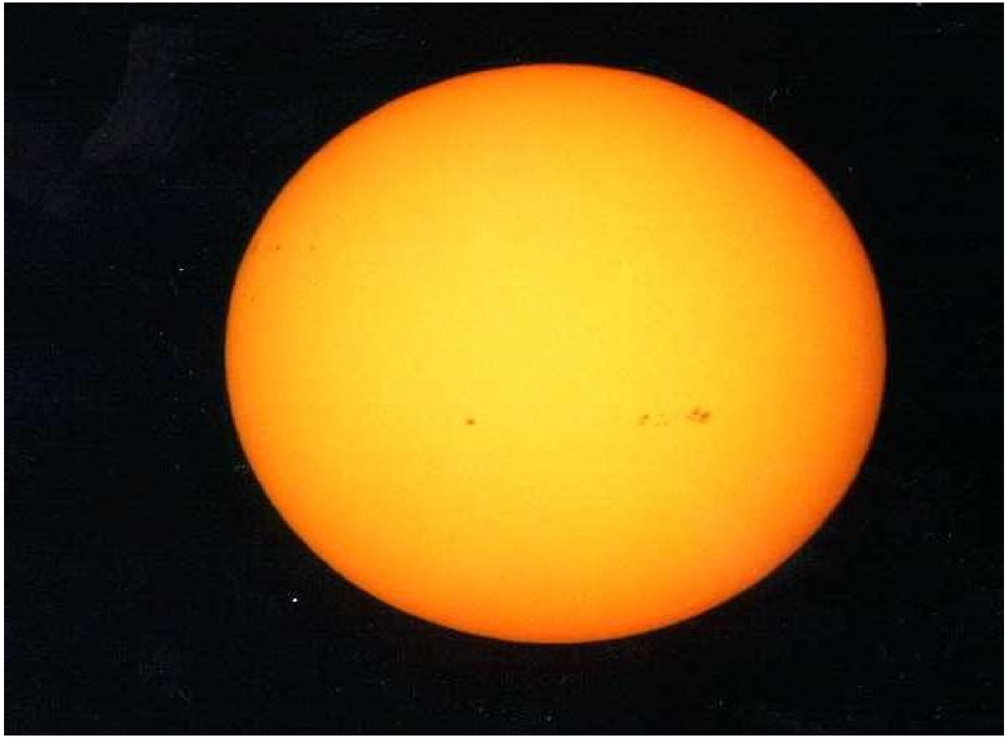


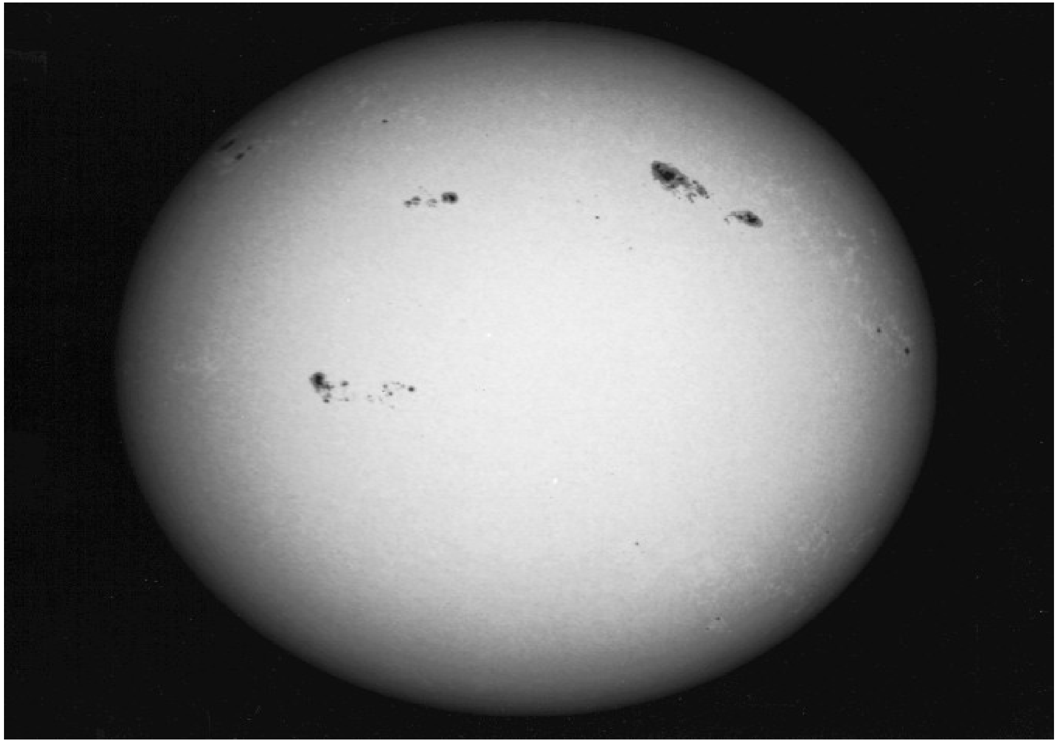
Formation of absorption lines on the Sun

- Formation of absorption features can also be understood in terms of the temperature of the local source function decreasing towards the line centre



Limb Darkening





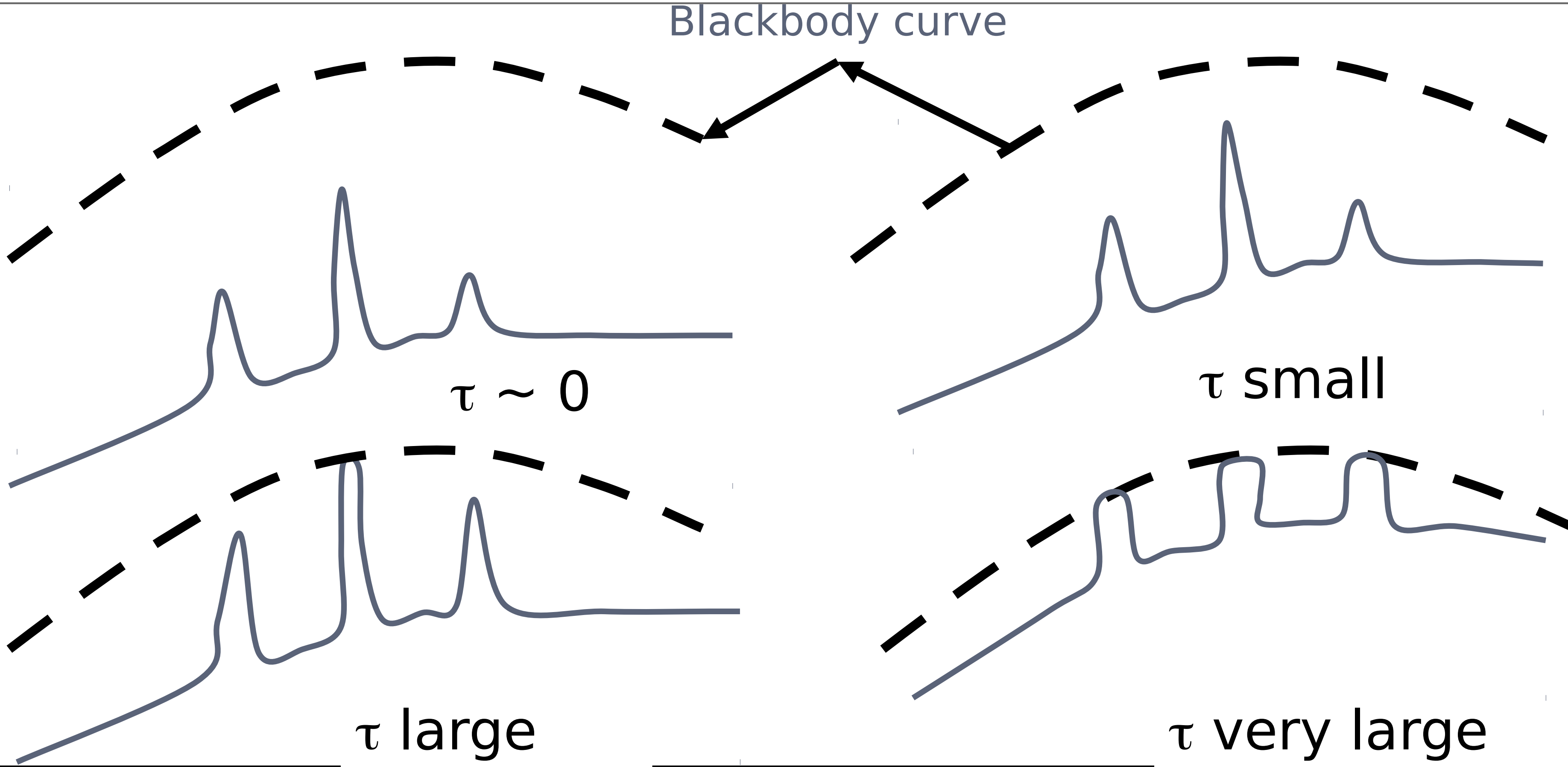
The Sun → redder at the edges, also dimmer at the edges…

Thermalisation

- Consider a uniform slab of gas of thickness L and temperature T that radiates like a blackbody, with an absorption coefficient α_ν which is small everywhere except at a strong line of frequency ν_0

- Compare the emitted intensity in the line relative to the neighbouring continuum for different limiting optical thicknesses of the slab

Approach to thermalisation



At a given temperature, BB has the largest luminosity

⇒ maximum efficiency

Emission or absorption?

Spherical BB with T_c surrounded by shell with T_s.
Emission or absorption at ν_0 if $\alpha_{\nu 1} << \alpha_{\nu 0}$?

T_c > T_s $\Rightarrow B_\nu(T_c) > B_\nu(T_s)$

$$\boxed{I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + S_\nu (1 - e^{-\tau_\nu})}$$

Case A:

$\alpha_{\nu 1}$ small $\Rightarrow I_{\nu 1} \approx B_{\nu 1}(T_c)$

$I_{\nu 0}(0) > S_{\nu 0}(T_s) = B_{\nu 0}(T_s)$

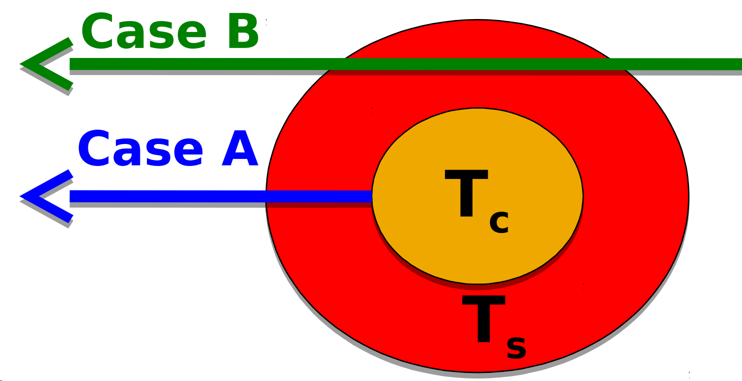
$\Rightarrow I_{\nu 0} = S_{\nu 0} + (I_{\nu 0}(0) - S_{\nu 0})\, e^{-\tau_{\nu 0}} > S_{\nu 0}$

$dI_{\nu 0}/d\tau_{\nu 0} = S_{\nu 0} - I_{\nu 0} < 0 \Rightarrow$ absorption

Case B:

$I_\nu(0)=0$

$I_{\nu 0} < S_{\nu 0} = B_{\nu 0}(T_s) \Rightarrow dI_{\nu 0}/d\tau_{\nu 0} = S_{\nu 0} - I_{\nu 0} > 0 \Rightarrow$ emission

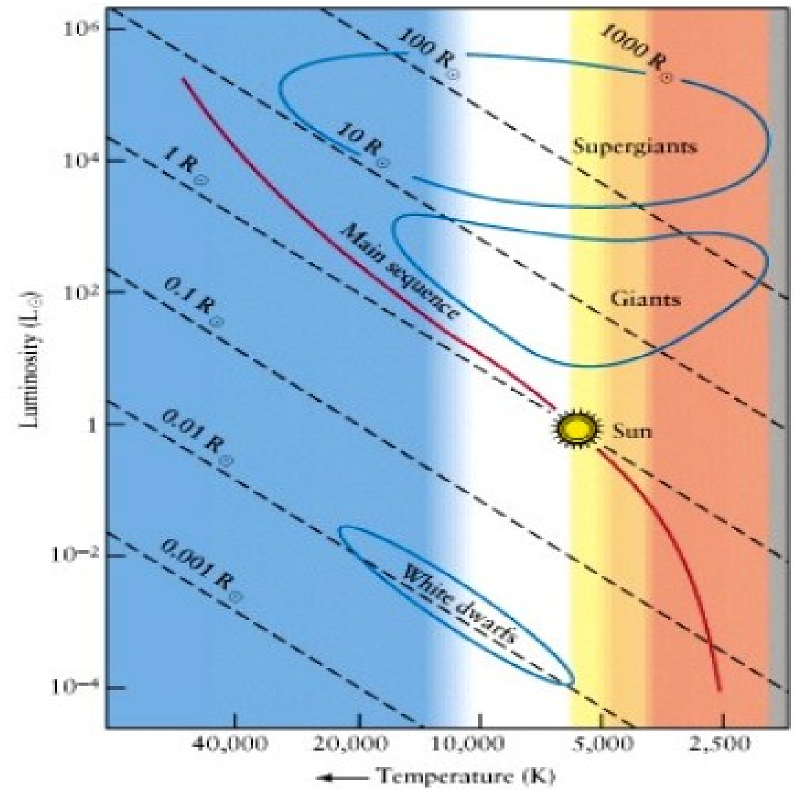


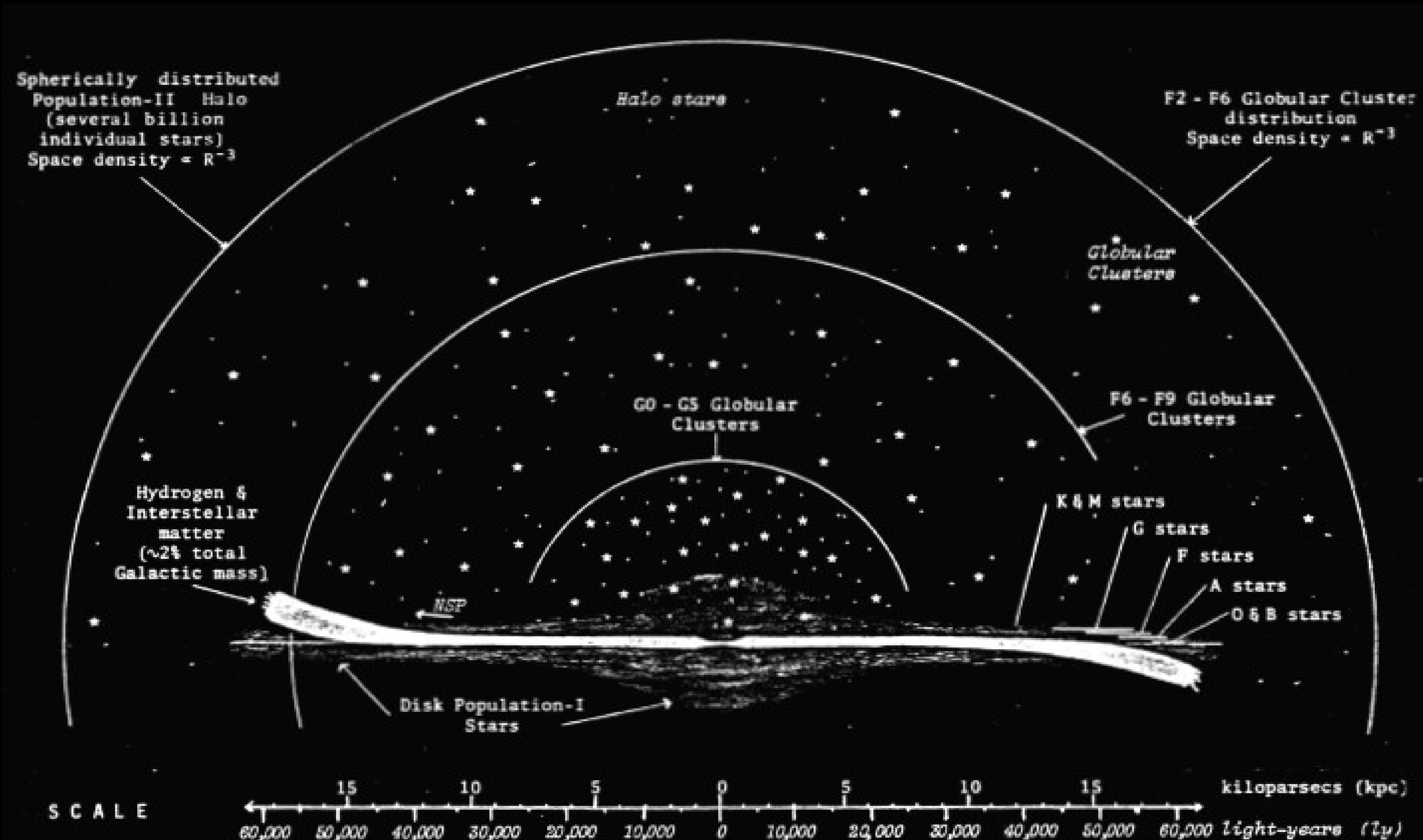
$L \propto T^4$

Why the Main Sequence is not a straight line?

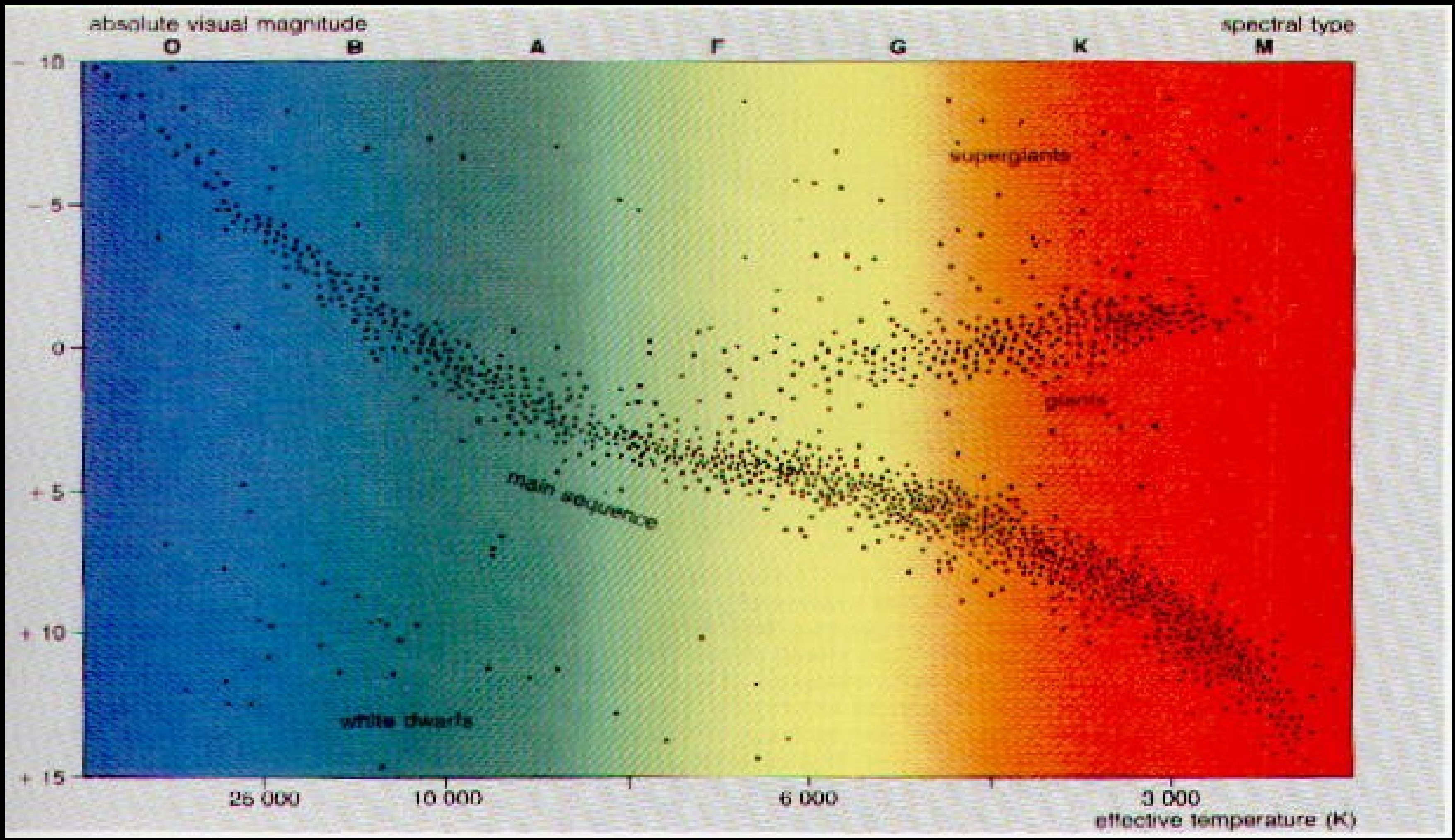
$$L = 4\pi R^2 \sigma T^4$$

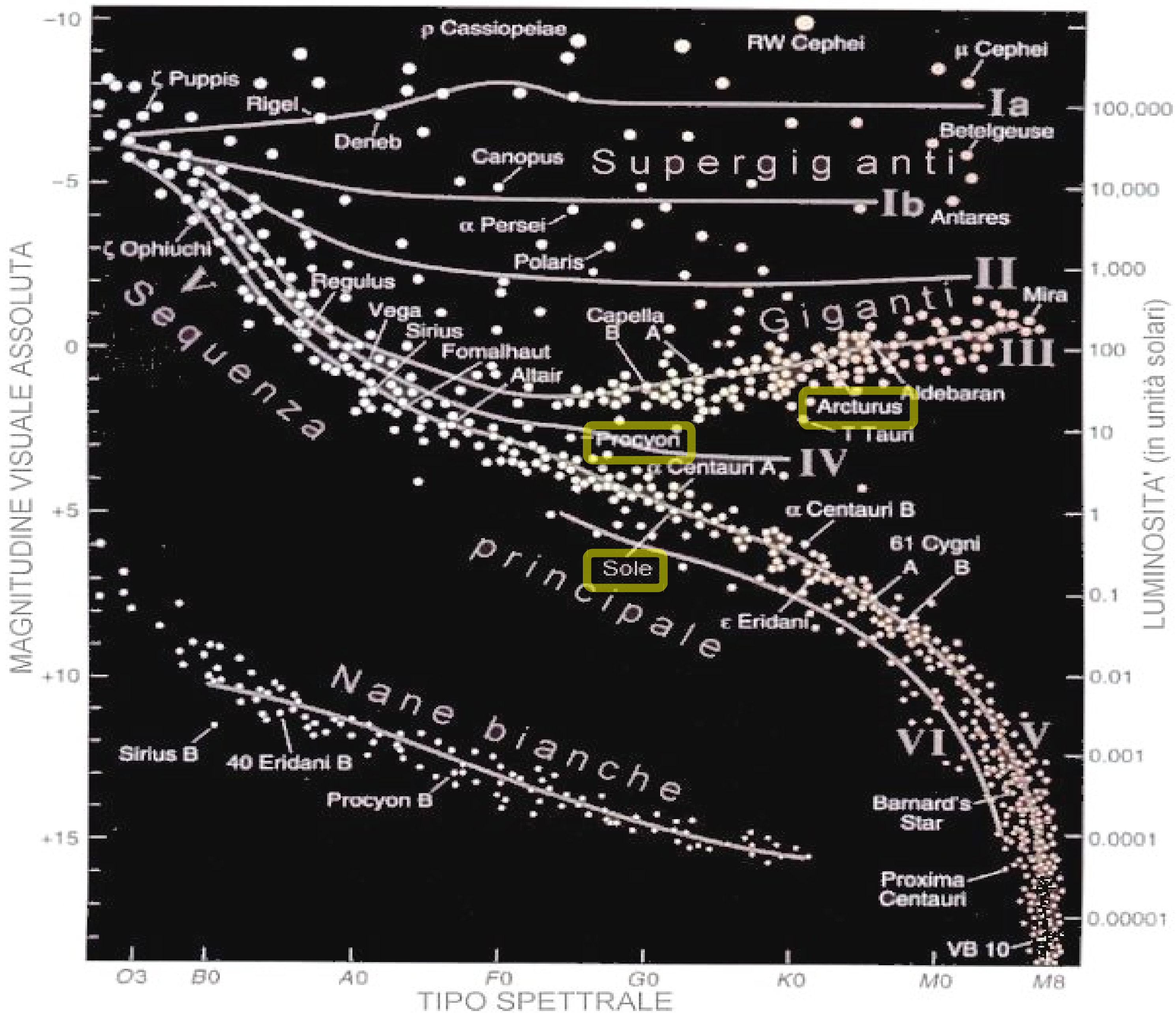
defines lines of **constant radius**




Spherically distributed Population-II Halo (several billion individual stars) Space density ∝ R⁻³
Halo stars
F2 - F6 Globular Cluste: distribution Space density ∝ R⁻³
Globular Clusters
F6 - F9 Globular Clusters
G0 - G5 Globular Clusters
Hydrogen & Interstellar matter (~2% total Galactic mass)
NSP
K & M stars
G stars
F stars
A stars
O & B stars
Disk Population-I Stars
SCALE
15 10 5 0 5 10 15 kiloparsecs (kpc)
60,000 50,000 40,000 30,000 20,000 10,000 0 10,000 20,000 30,000 40,000 50,000 60,000 light-years (ly)

Hertzsprung-Russel Diagram





ρ Cassiopeiae
RW Cephei
μ Cephei
ζ Puppis
Rigel
Ia
Deneb
Canopus
Supergiganti
Betelgeuse
Ib
Antares
α Persei
ζ Ophiuchi
Polaris
II
Regulus
V
Mira
Vega
Sirius
Capella
B
A
Giganti
III
Fomalhaut
Altair
Sequenza
Aldebaran
Arcturus
Procyon
γ Tauri
α Centauri A
IV
α Centauri B
principale
Sole
61 Cygni
A
B
ε Eridani
Nane bianche
VI
V
Sirius B
40 Eridani B
Procyon B
Barnard's Star
Proxima Centauri
VB 10
−10
−5
0
+5
+10
+15
O3
B0
A0
F0
G0
K0
M0
M8
TIPO SPETTRALE
MAGNITUDINE VISUALE ASSOLUTA
100,000
10,000
1,000
100
10
1
0.1
0.01
0.001
0.0001
0.00001
LUMINOSITA' (in unità solari)

Hydrostatic Thermostat

Nuclear fusion reactions are **temperature** sensitive:

- Higher Core Temperature = More Fusion

BUT

- More fusion makes the core hotter
- Hotter core leads to even more fusion

Why don't stars **explode** like

Hydrogen Bombs?

If the reactions run **too fast**:

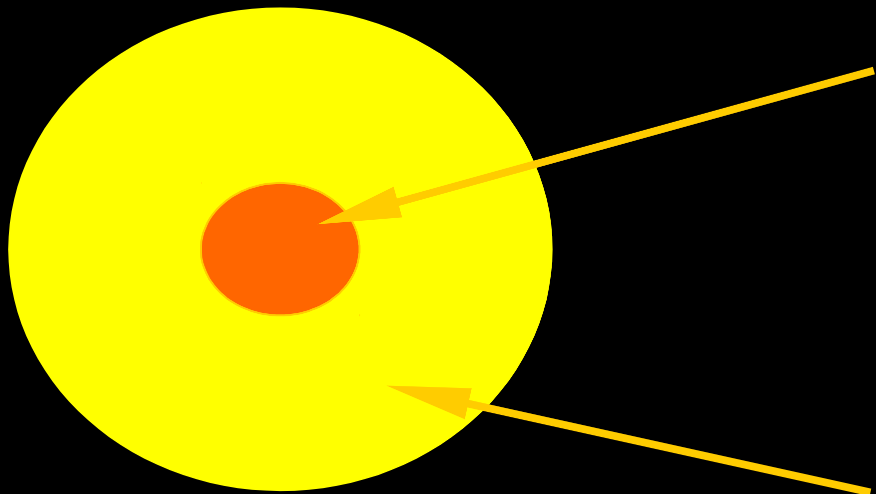
- The core heats up ⇒ higher Pressure (P)
- Higher P ⇒ expansion
- Expansion cools core, slowing the rate of fusion

If the reactions run **too slow**:

- The core cools ⇒ lower P
- Lower P ⇒ contraction
- Contraction heats core, increasing the fusion rate

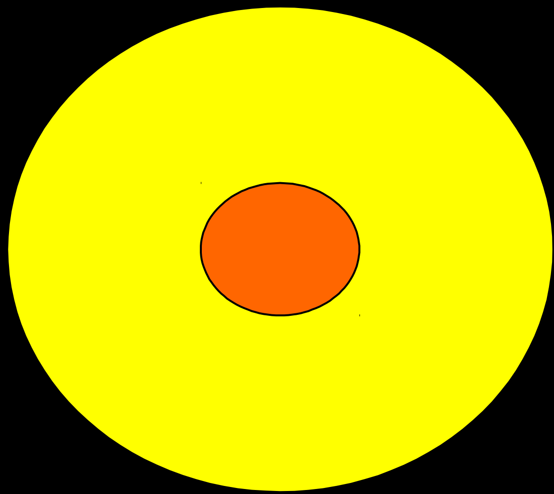
Result is like a **thermostat**

Sun’s Structure

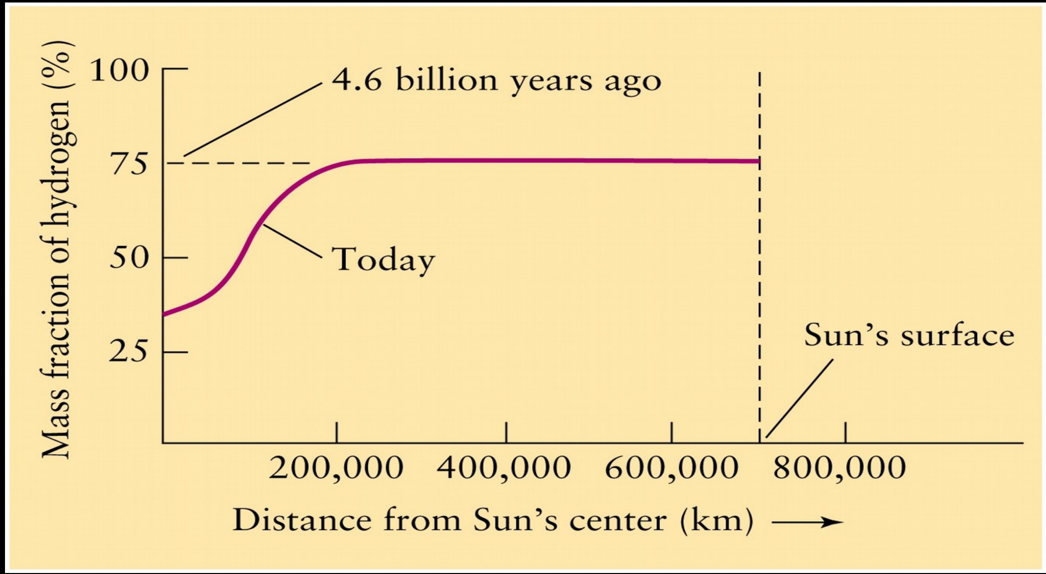


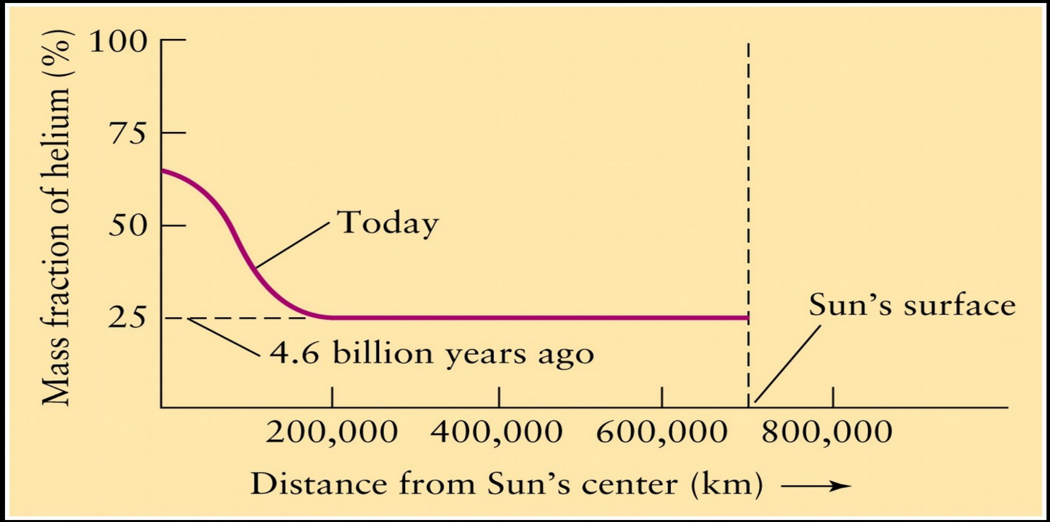
- Core
  - Where nuclear fusion occurs
- Envelope
  - Supplies gravity to keep core hot and dense

Main Sequence Evolution



- Core starts with same fraction of hydrogen as whole star
- Fusion changes H → He
- Core gradually shrinks and Sun gets hotter and more luminous

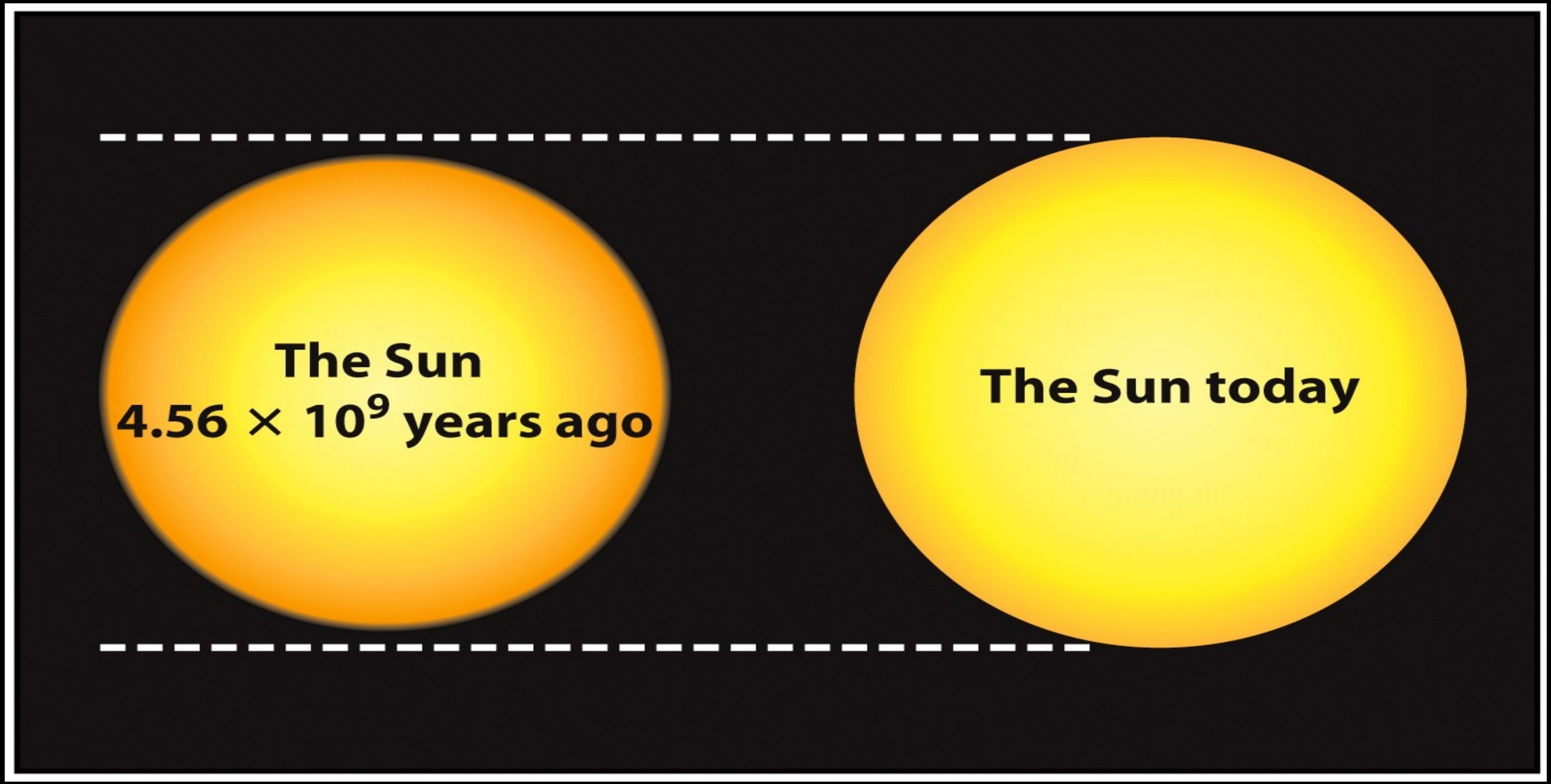




Since reaching the Main Sequence, H has been depleted in the core, while He has been built up there

We do not see this on the surface!

Gradual change in size of Sun



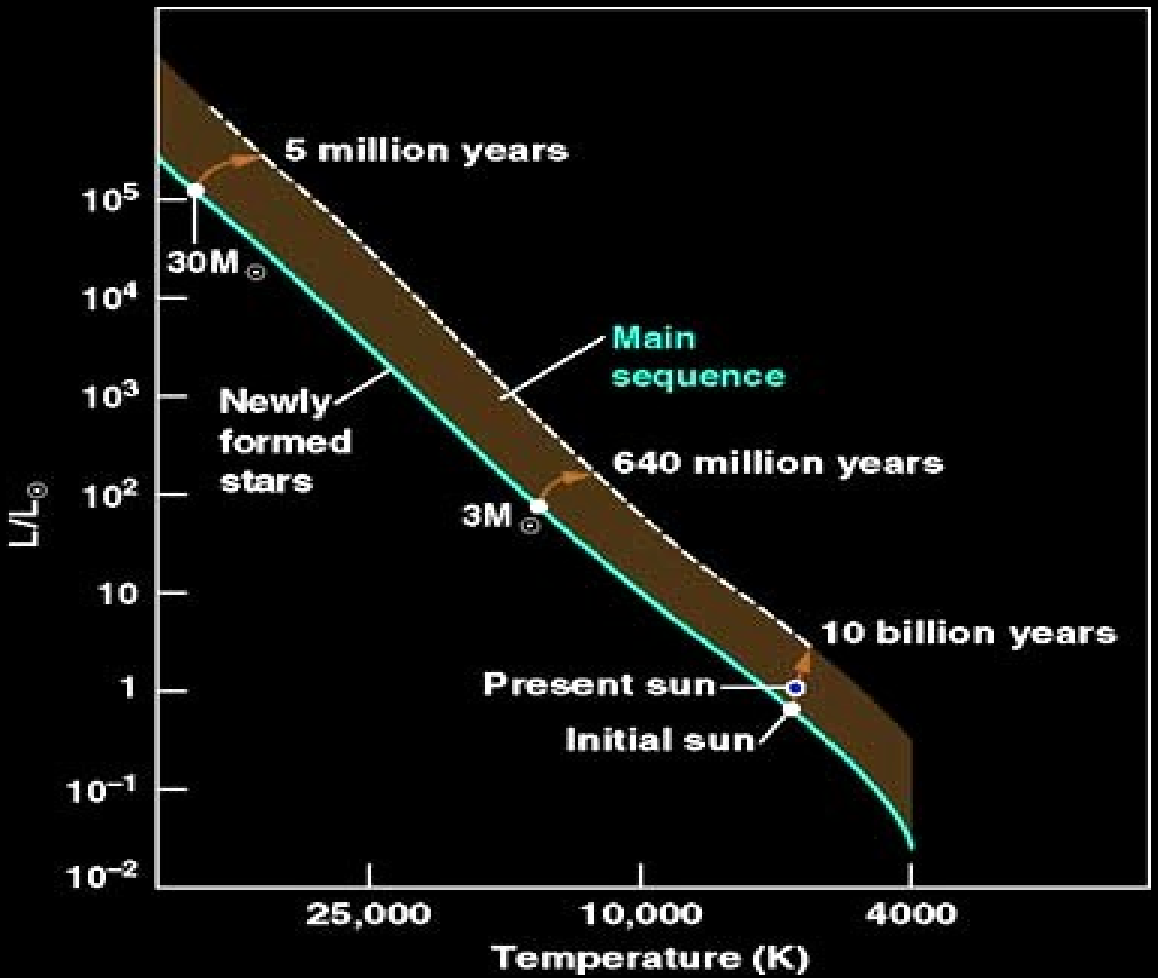
Now 6% larger, 5% hotter ⇒ 40% brighter

Main Sequence Evolution

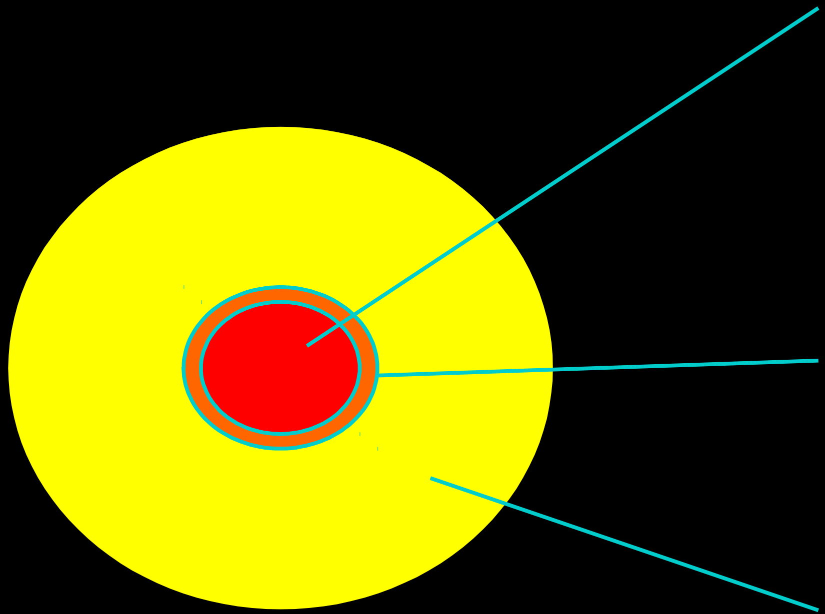
When stars initiate H burning in their cores, they are located on the ***zero-age main sequence*** (ZAMS).

As they age, they evolve slowly away from the ZAMS.

Most stars, regardless of their mass, spend roughly 90% of their total lifetimes as main sequence stars.

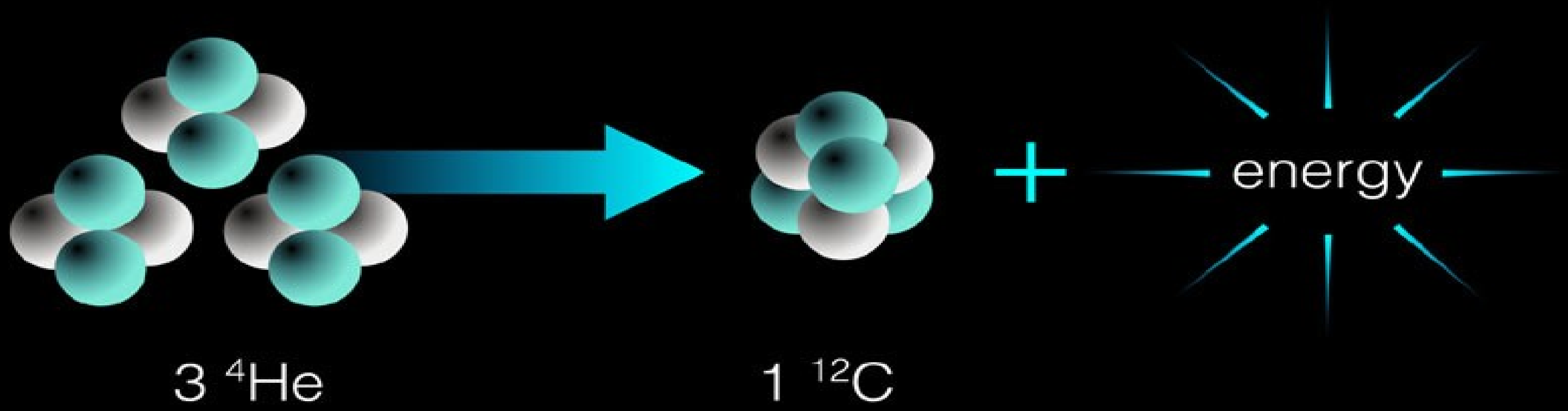


Red Giant Phase



- He core
  - No nuclear fusion
  - Gravitational contraction produces energy
- H layer
  - Nuclear fusion
- Envelope
  - Expands because of increased energy production
  - Cools because of increased surface area

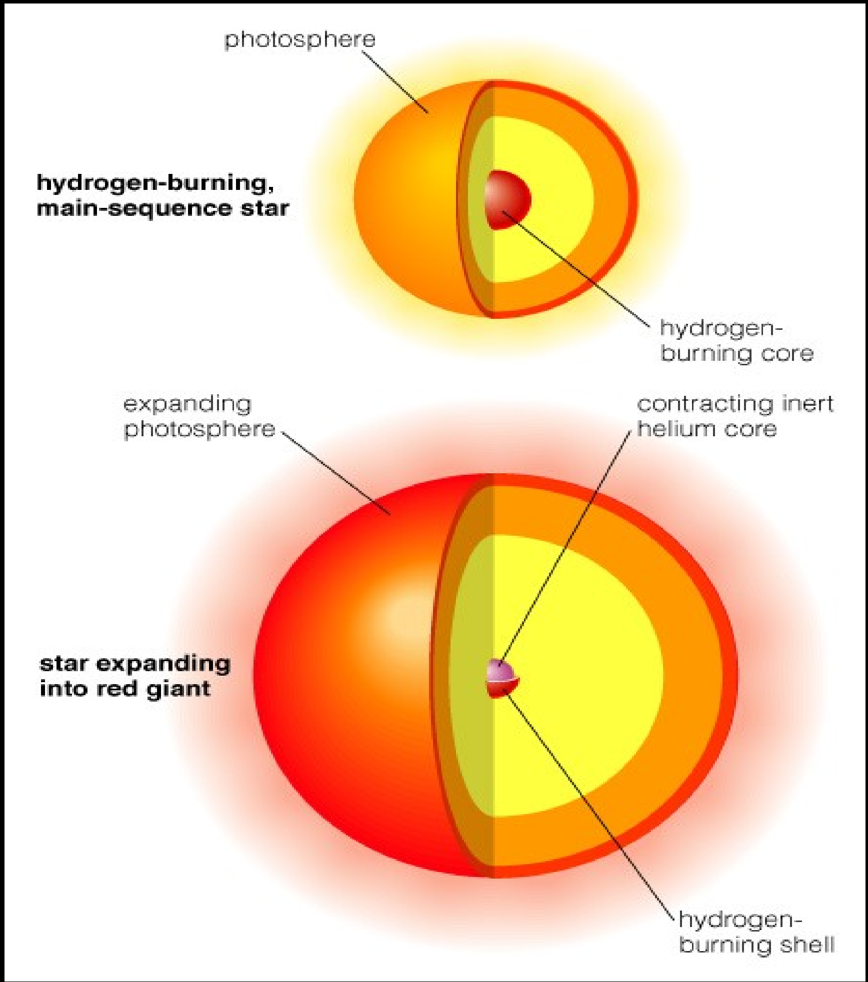
Helium fusion



Helium fusion does not begin right away because it requires higher temperatures than hydrogen fusion—larger charge leads to greater repulsion

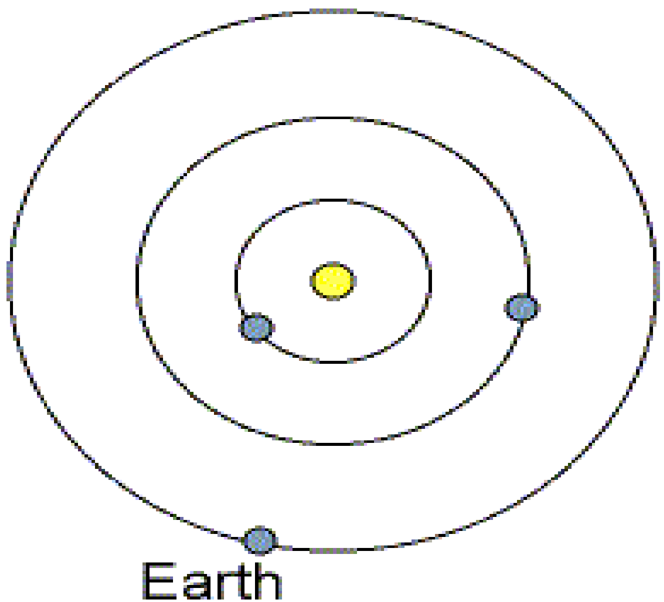
Fusion of two helium nuclei doesn't work, so helium fusion must combine three He nuclei to make carbon

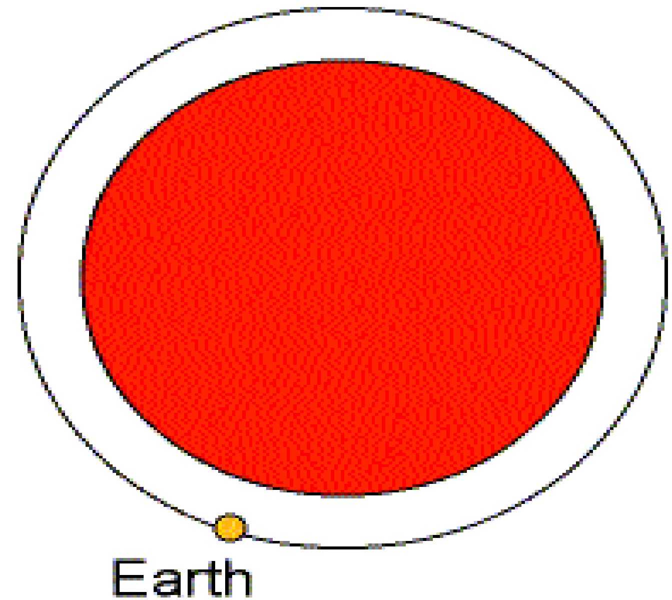
Broken Thermostat



- As the core contracts, H begins fusing to He in a shell around the **degenerate** core
- Luminosity increases because the core thermostat is broken (no nuclear reactions) ⇒ the increasing fusion rate in the shell does not stop the core from contracting

Sun’s Red Giant Phase

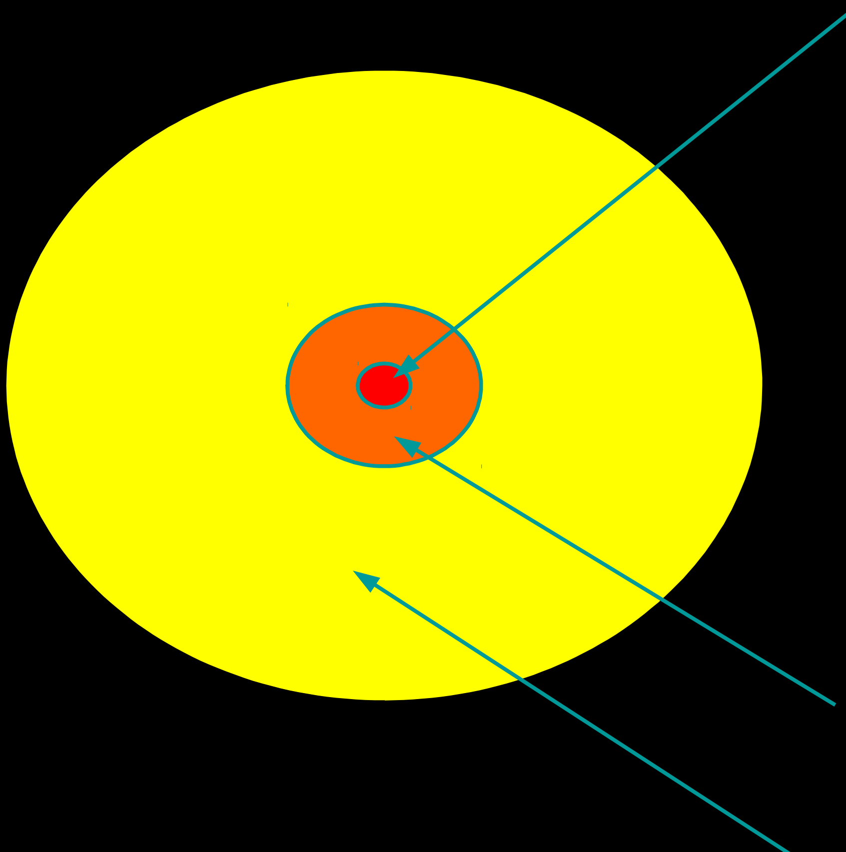




Now: hot core + warm surface; small size.

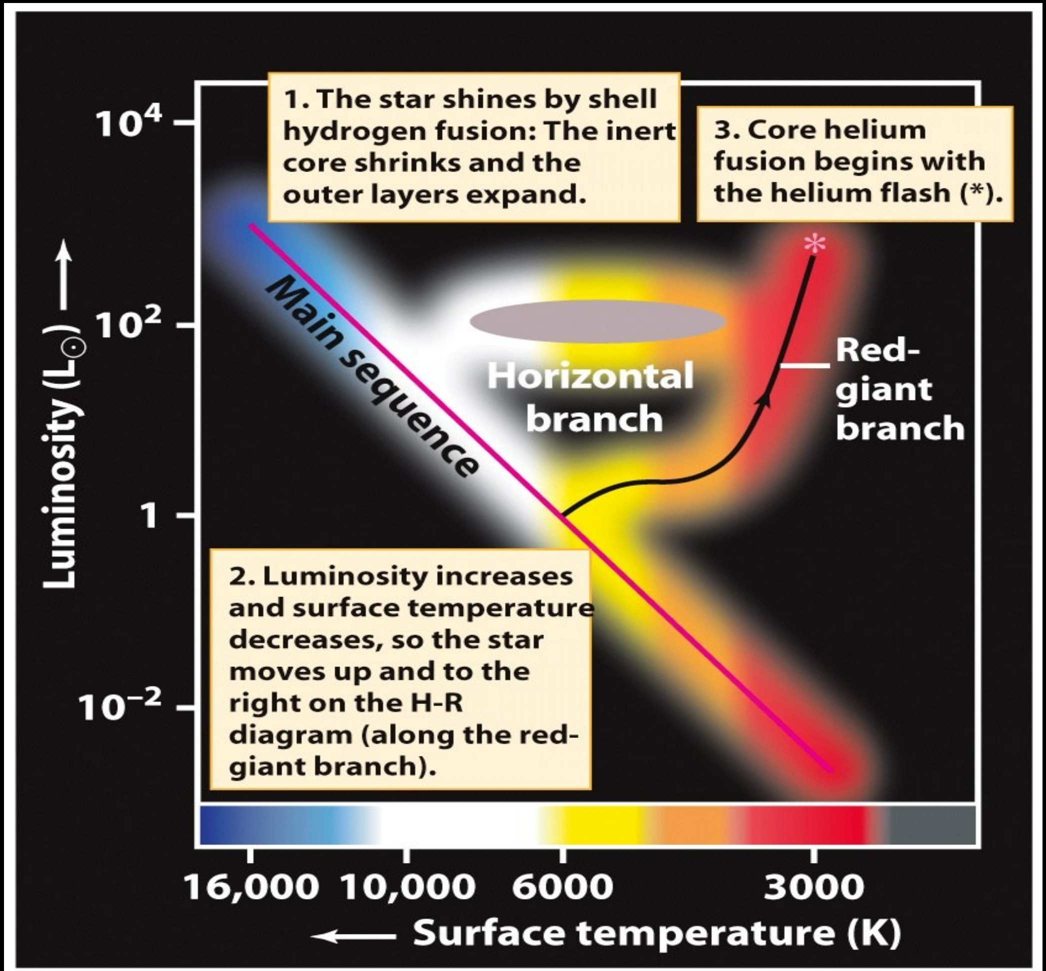
Future: very hot core + cool surface. Large size

Helium Flash



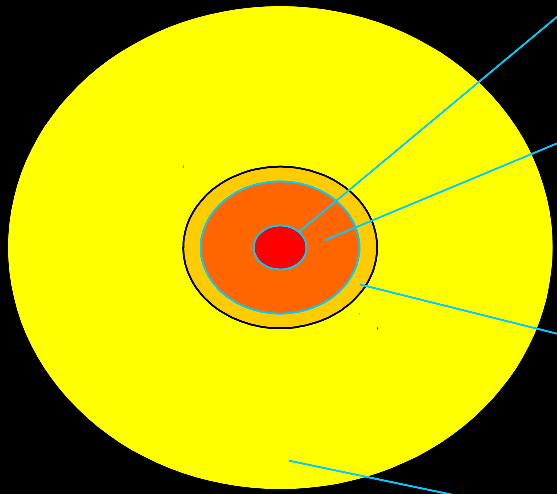
- He core
  - Eventually the core gets hot enough to fuse Helium into Carbon.
  - This causes the temperature to increase rapidly to 300 million K and there's a sudden flash when a large part of the Helium gets burned all at once.
  - We don't see this flash because it's buried inside the star.
- H layer
- Envelope

Movement on HR diagram



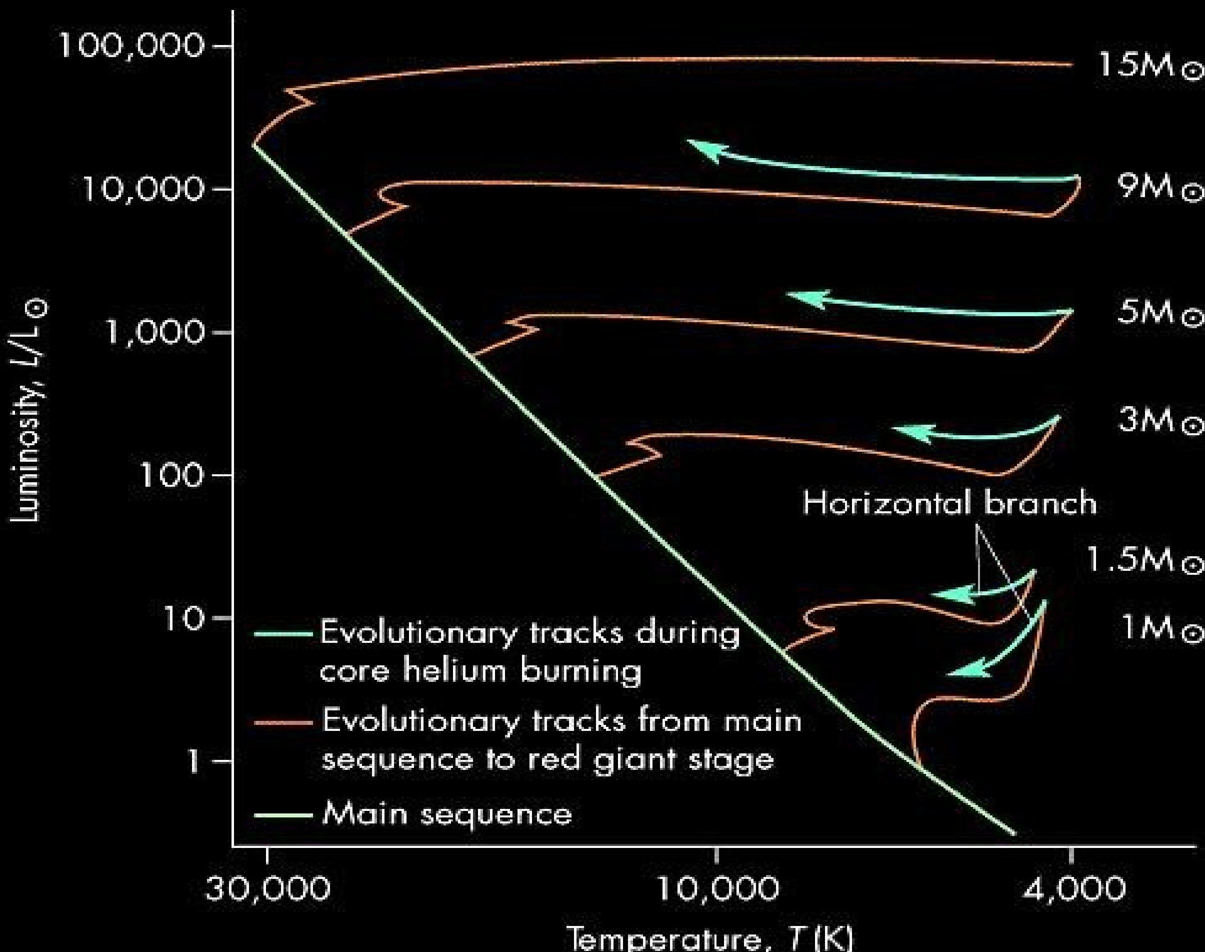
Before the helium flash: A red-giant star

Red Giant after Helium Ignition



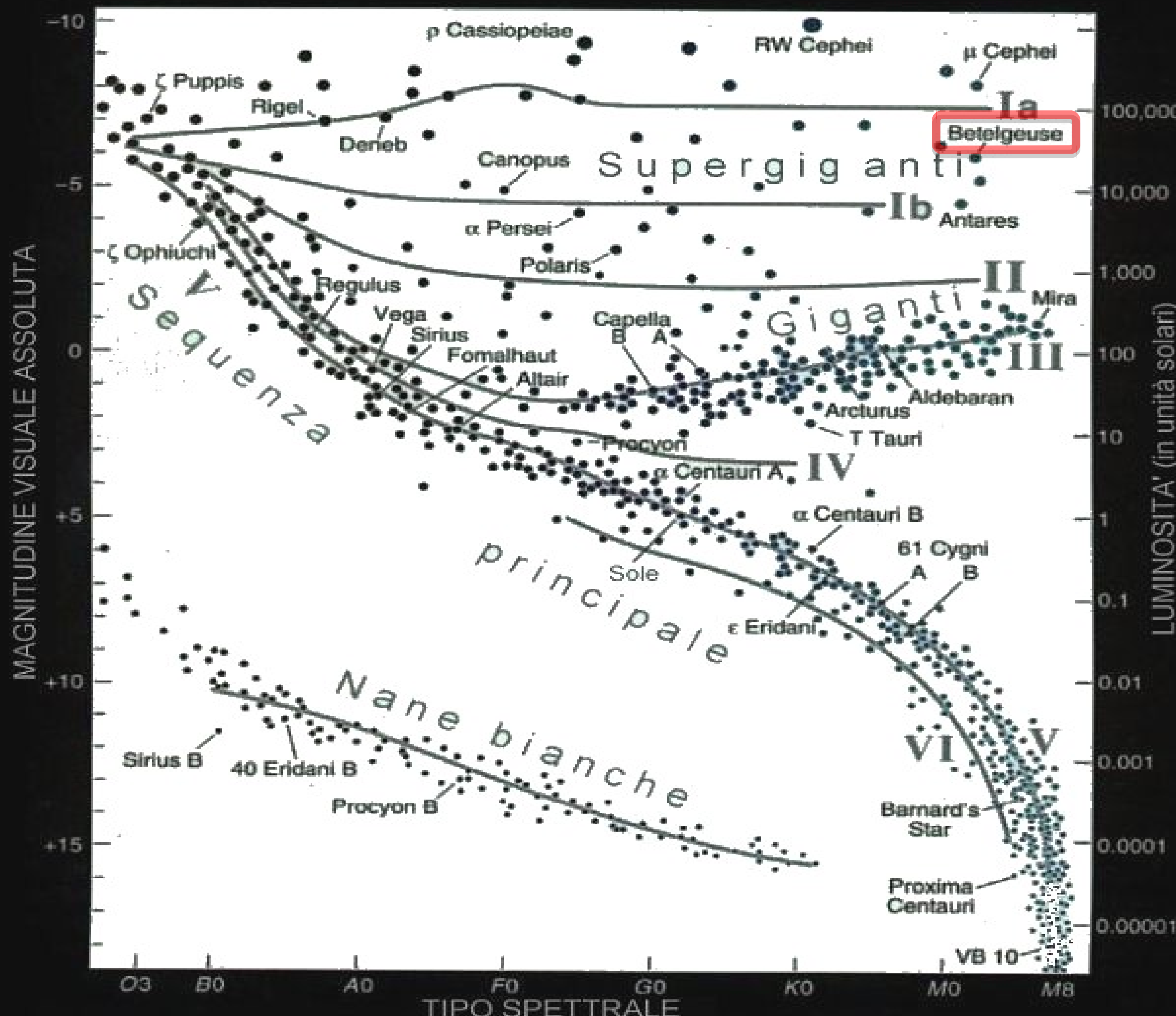
- He burning core
  - Fusion burns He into C, O
- He rich core
  - No fusion
- H burning shell
  - Fusion burns H into He
- Envelope

Sun moves onto Horizontal Branch



Sun burns He into Carbon and Oxygen in the core

Sun becomes hotter and smaller (L~constant): **Horizontal Branch**



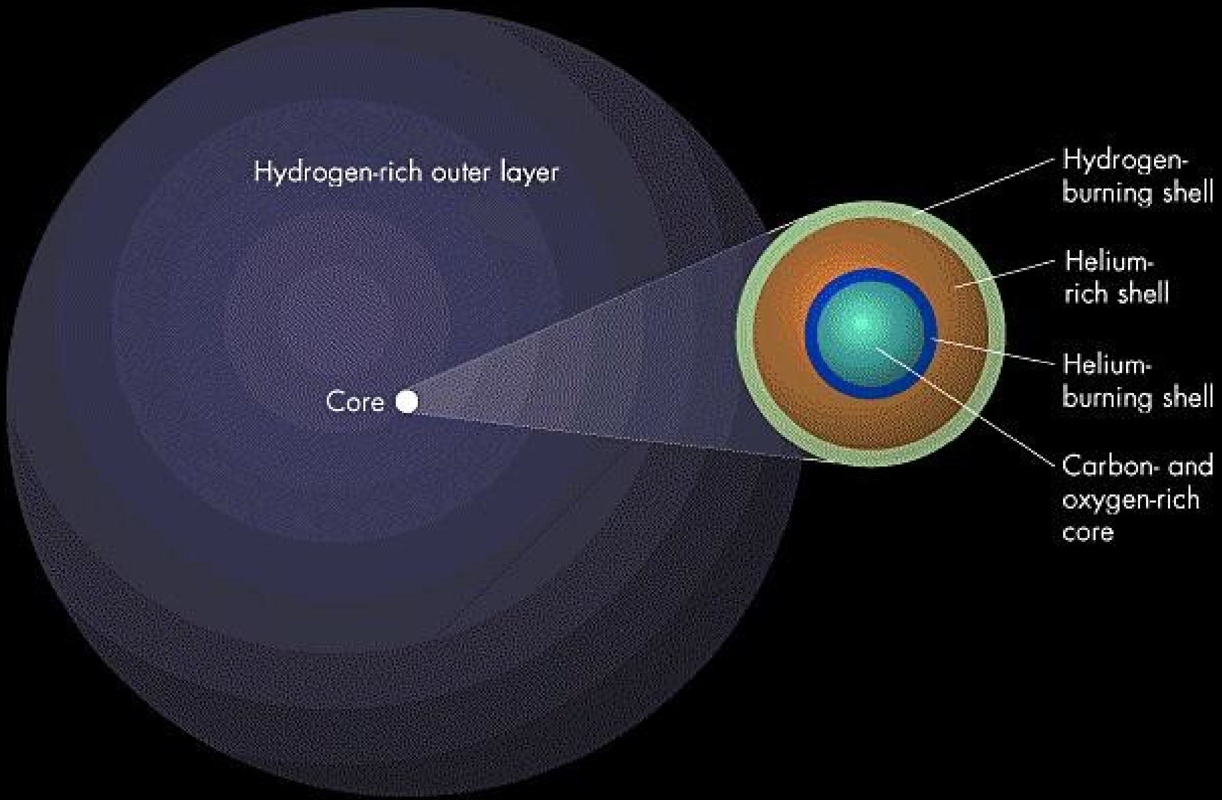
ρ Cassiopeiae
RW Cephei
μ Cephei
ζ Puppis
Rigel
Deneb
Canopus
Ia
Betelgeuse
Supergiganti
Ib
Antares
α Persei
ζ Ophiuchi
Polaris
II
Regulus
Mira
Vega
Sirius
Capella
B A
Giganti
III
Fomalhaut
Altair
Aldebaran
Arcturus
T Tauri
Procyon
IV
α Centauri A
α Centauri B
61 Cygni
A B
Sole
ε Eridani
V
Sequenza principale
Nane bianche
VI
V
Sirius B
40 Eridani B
Procyon B
Barnard's Star
Proxima Centauri
VB 10
MAGNITUDINE VISUALE ASSOLUTA
−10
−5
0
+5
+10
+15
LUMINOSITA' (in unità solari)
100,000
10,000
1,000
100
10
1
0.1
0.01
0.001
0.0001
0.00001
O3
B0
A0
F0
G0
K0
M0
M8
TIPO SPETTRALE

Helium burning in the core stops

H burning is continuous

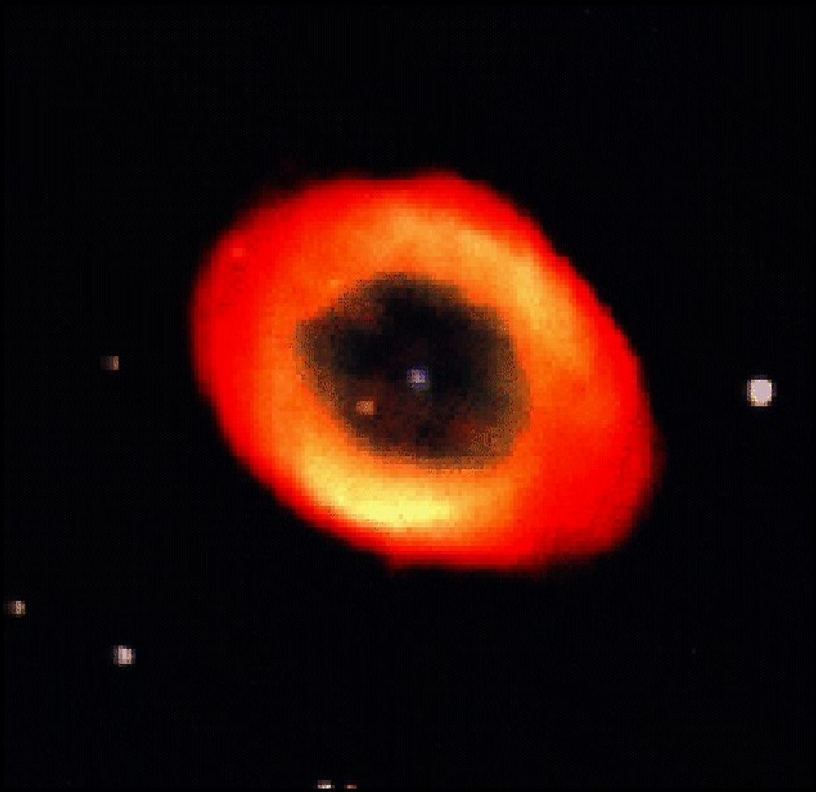
He burning happens in "thermal pulses"

Core is degenerate

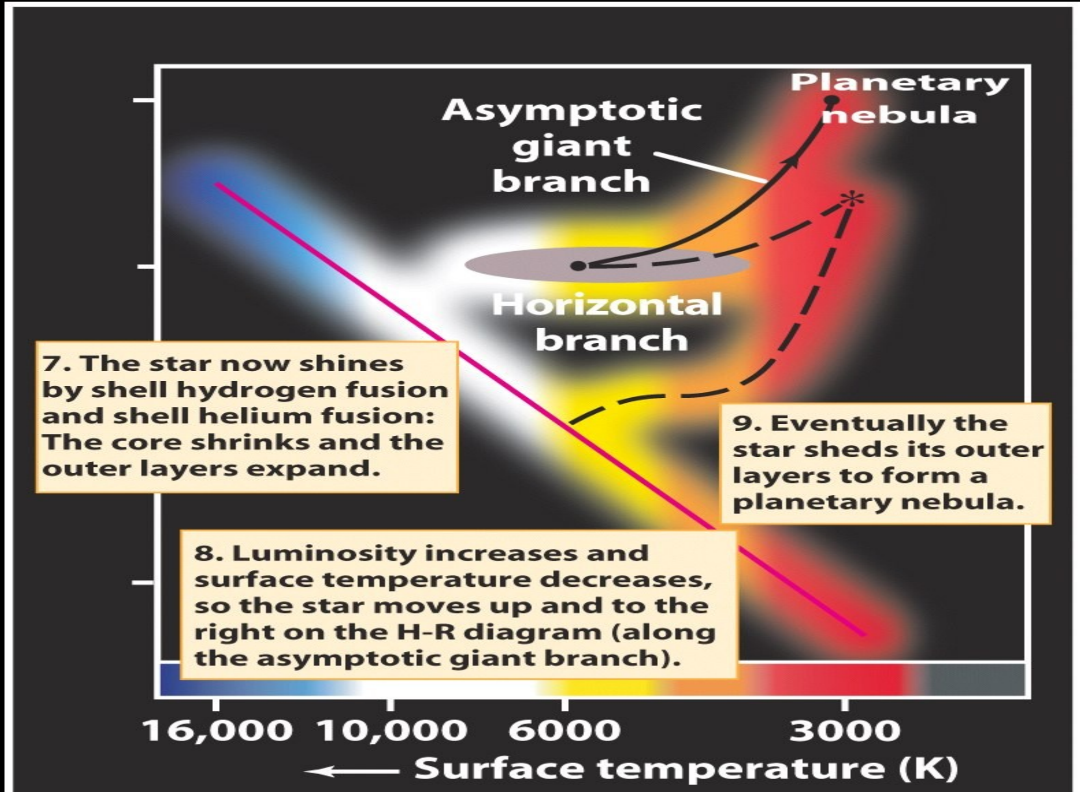


Sun loses mass via winds

- Creates a “planetary nebula”
- Leaves behind core of carbon and oxygen surrounded by thin shell of hydrogen
- Hydrogen continues to burn

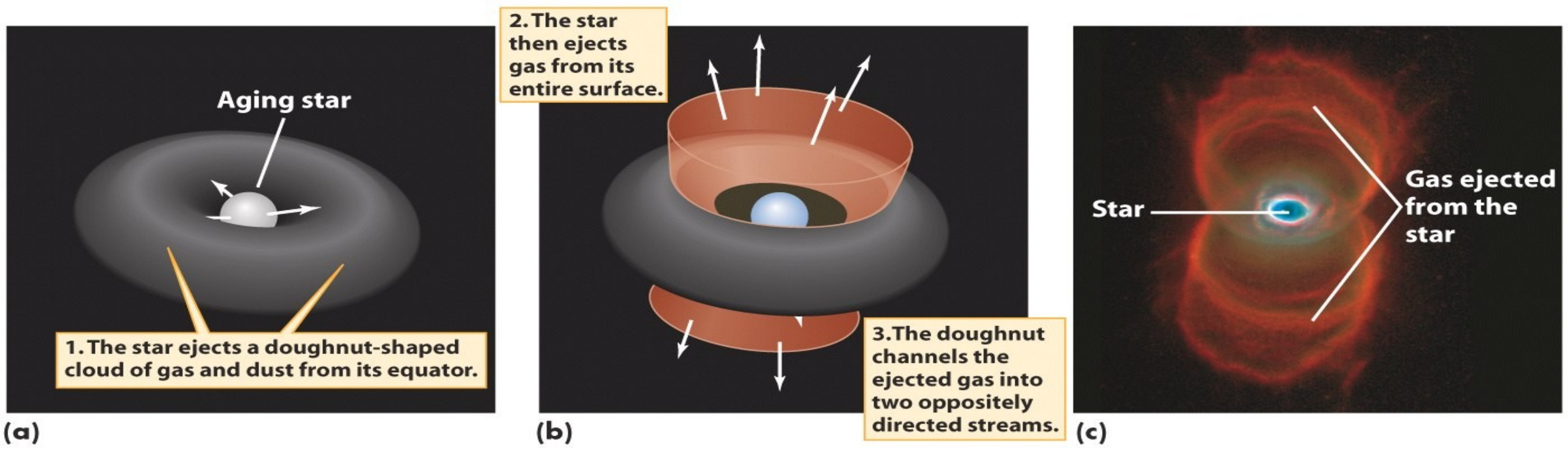


Sun moves onto Asymptotic Giant Branch (AGB)



After core helium fusion ends: An AGB star

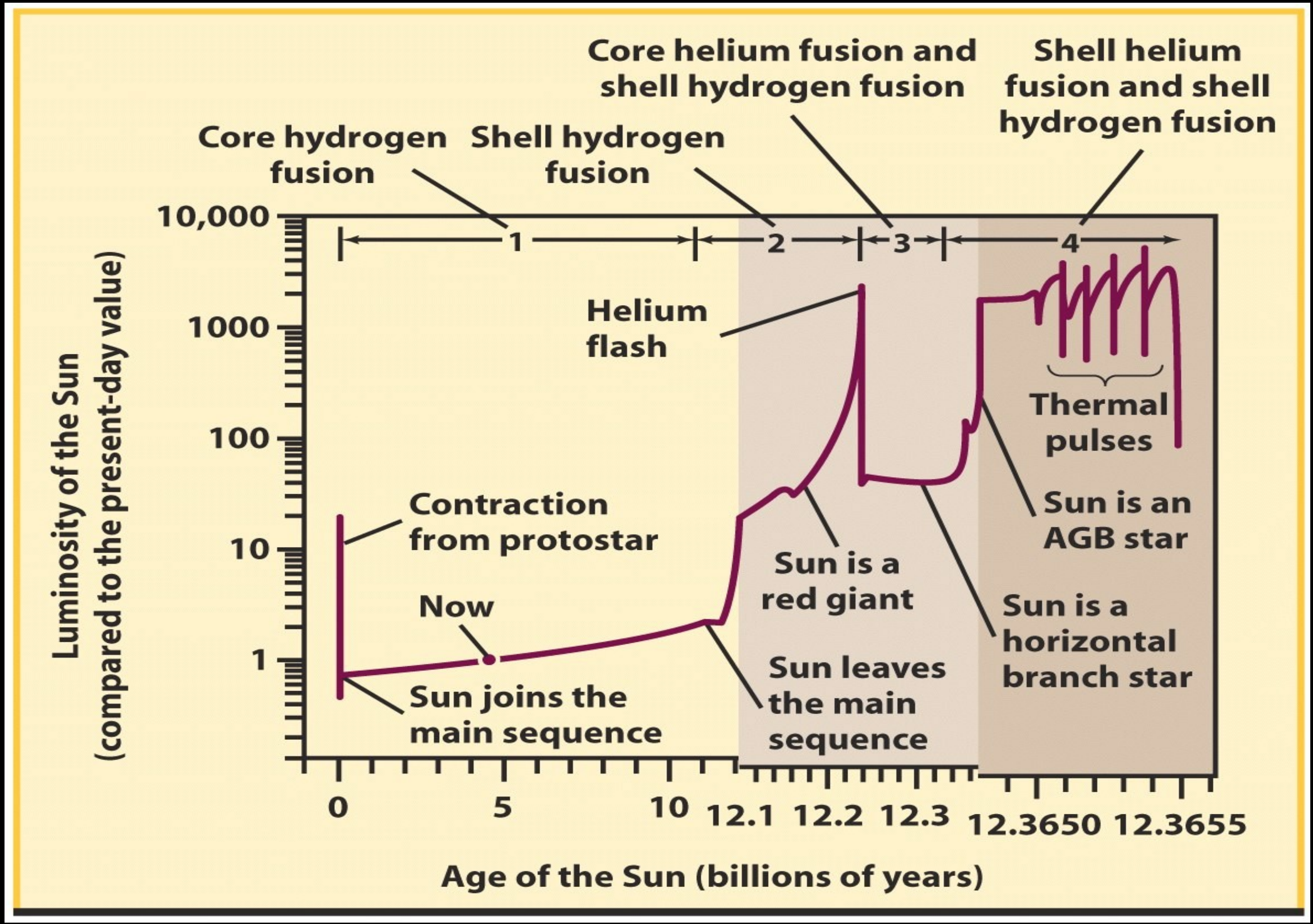
Bipolar planetary nebulae



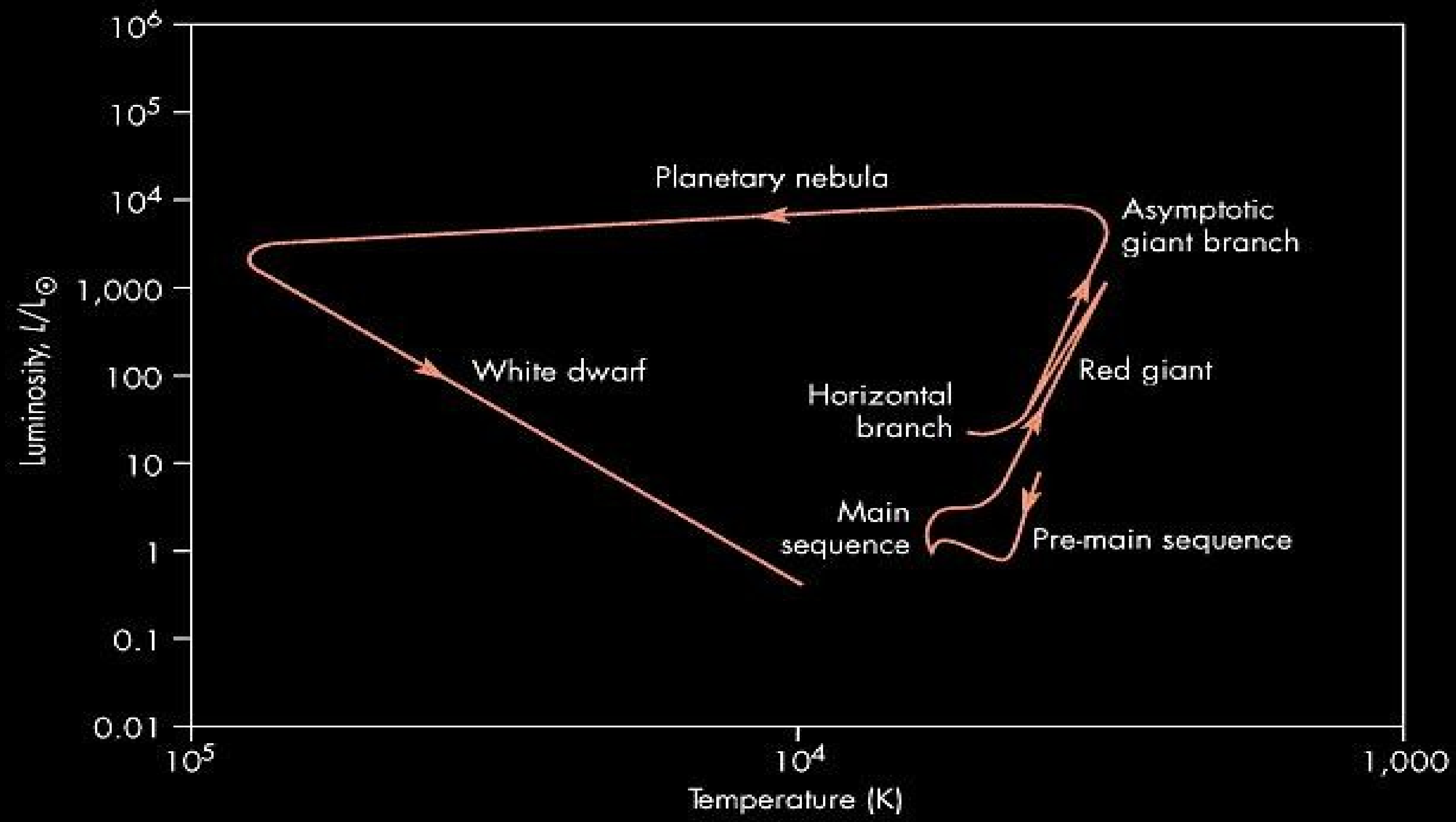
White dwarf

- Star burns up rest of hydrogen
- Nothing remains but degenerate core of Oxygen and Carbon
- “White dwarf” cools but does not contract because core is degenerate
- No energy from fusion, no energy from gravitational contraction
- White dwarf slowly fades away…

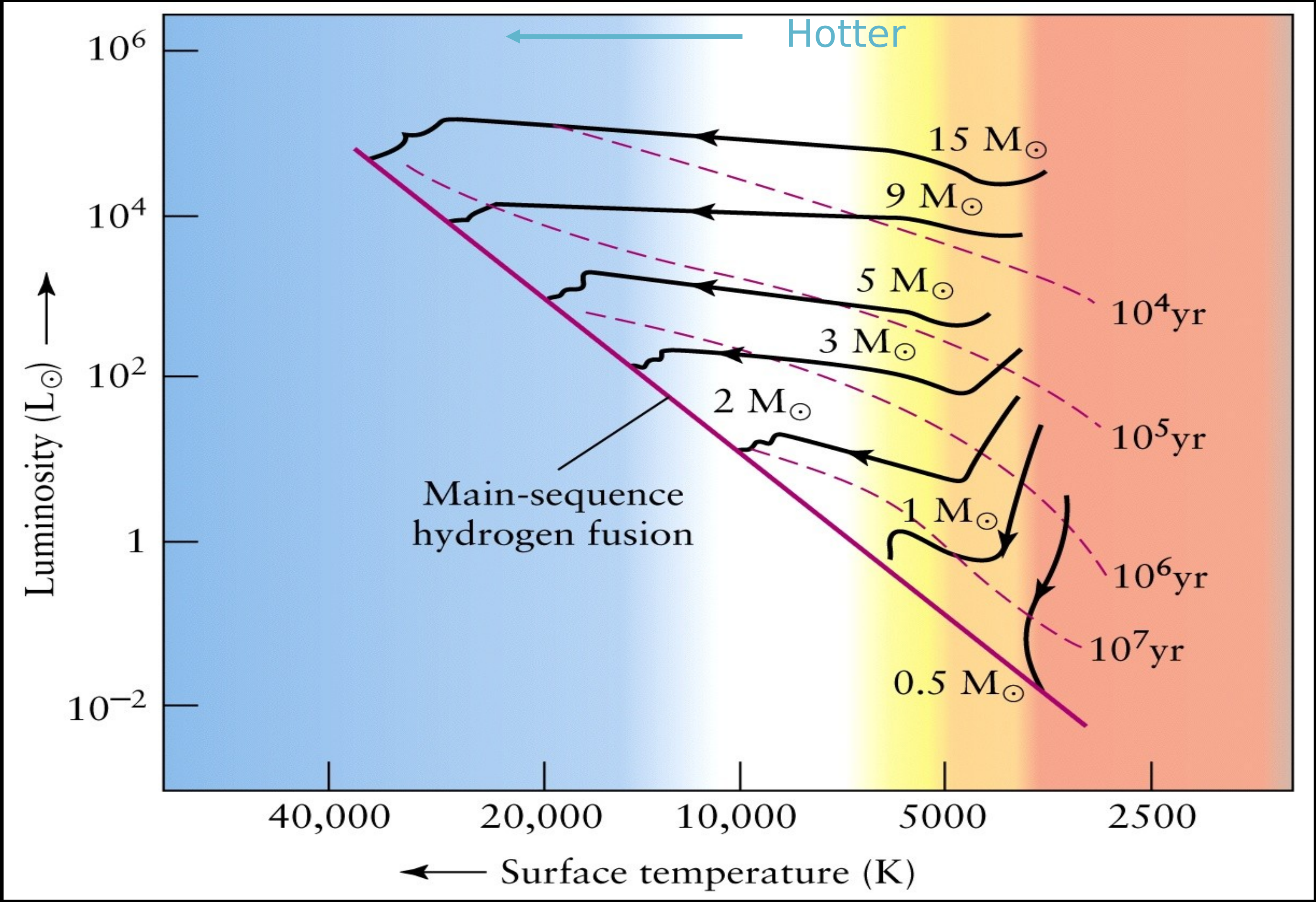
Time line for Sun’s evolution



Evolution on HR diagram



Higher mass protostars contract faster



Higher mass stars spend less time on the main sequence

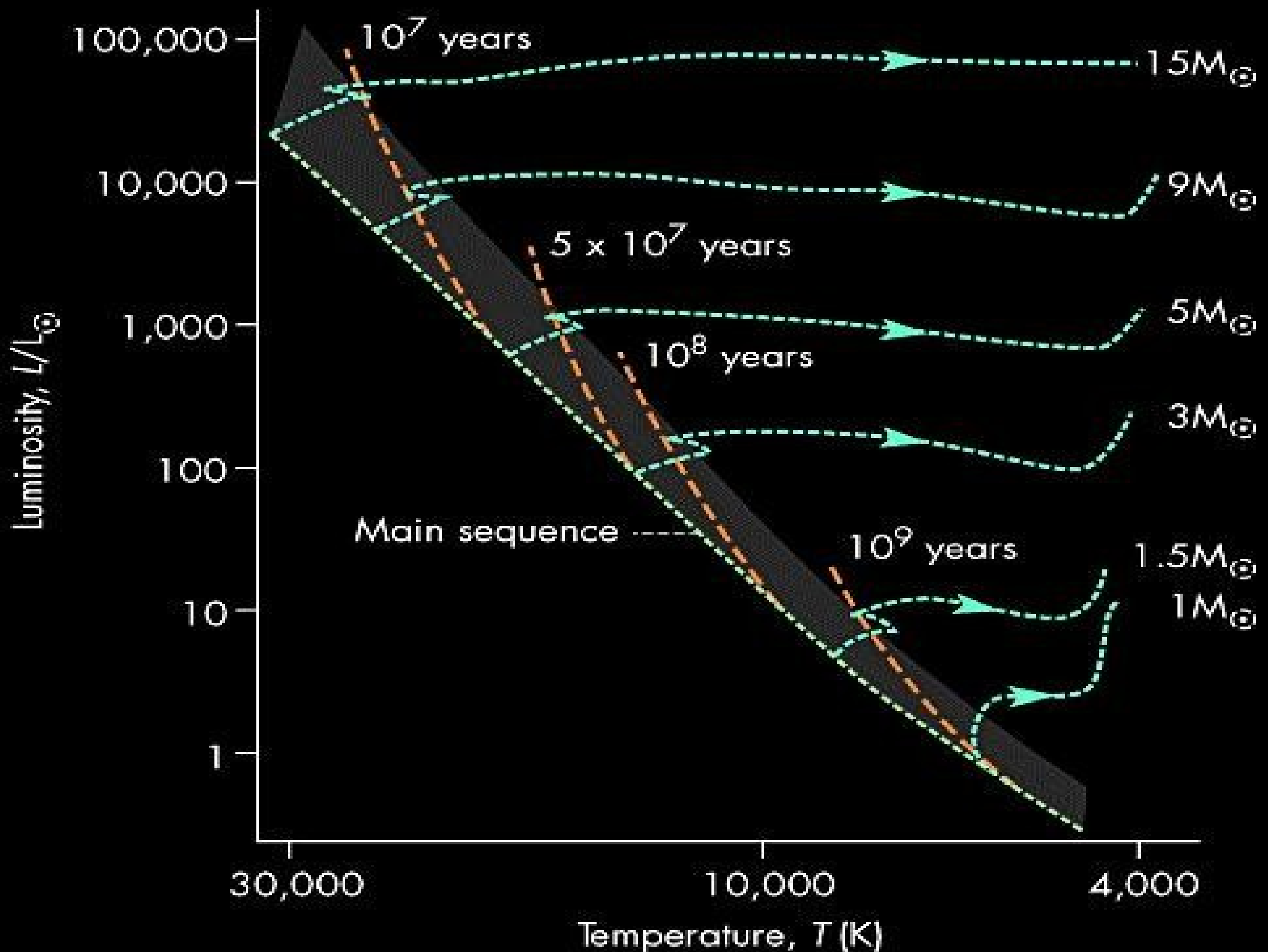


Table 21-1 Main-Sequence Lifetimes

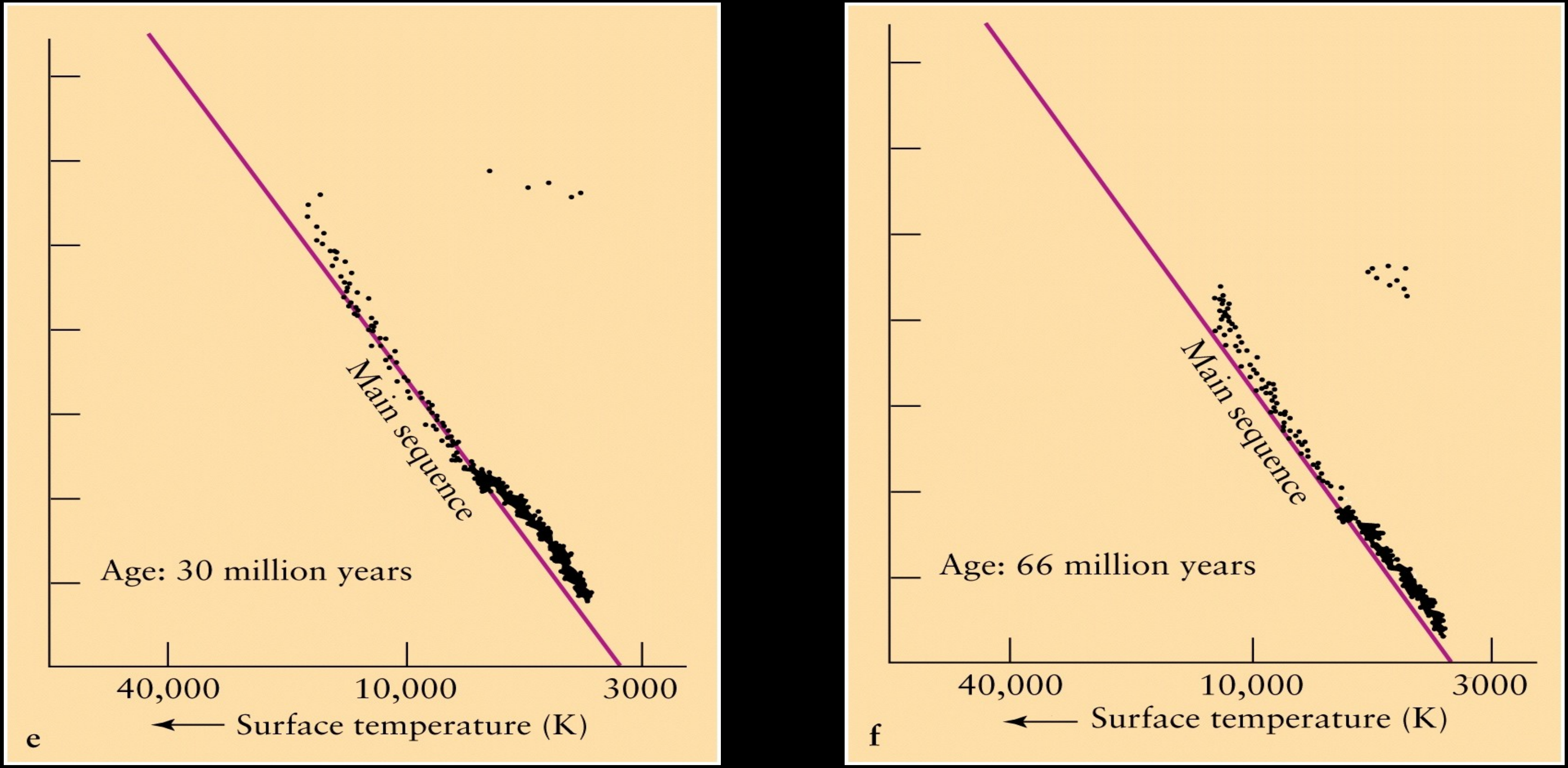
Mass ($M_\odot$)	Surface temperature (K)	Spectral class	Luminosity ($L_\odot$)	Main-sequence lifetime (10^6 years)
25	35,000	O	80,000	3
15	30,000	B	10,000	15
3	11,000	A	60	500
1.5	7000	F	5	3000
1.0	6000	G	1	10,000
0.75	5000	K	0.5	15,000
0.50	4000	M	0.03	200,000

Determining the age of a star cluster

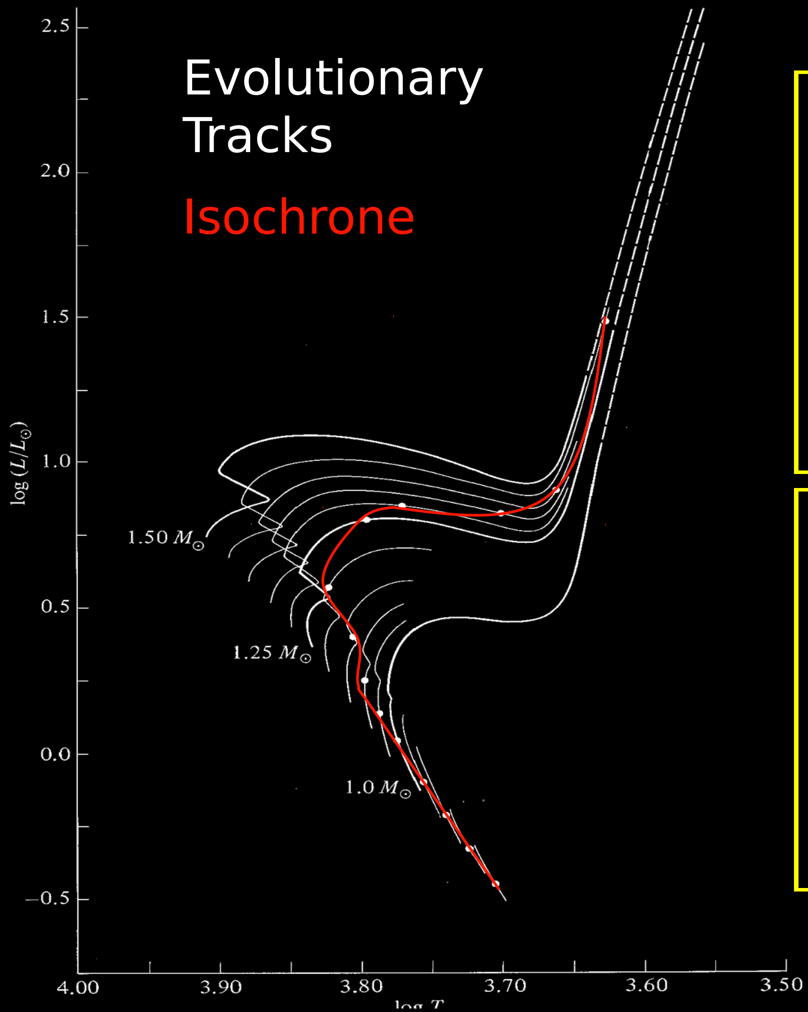
- Imagine we have a cluster of stars that were all formed at the same time, but have a variety of different masses

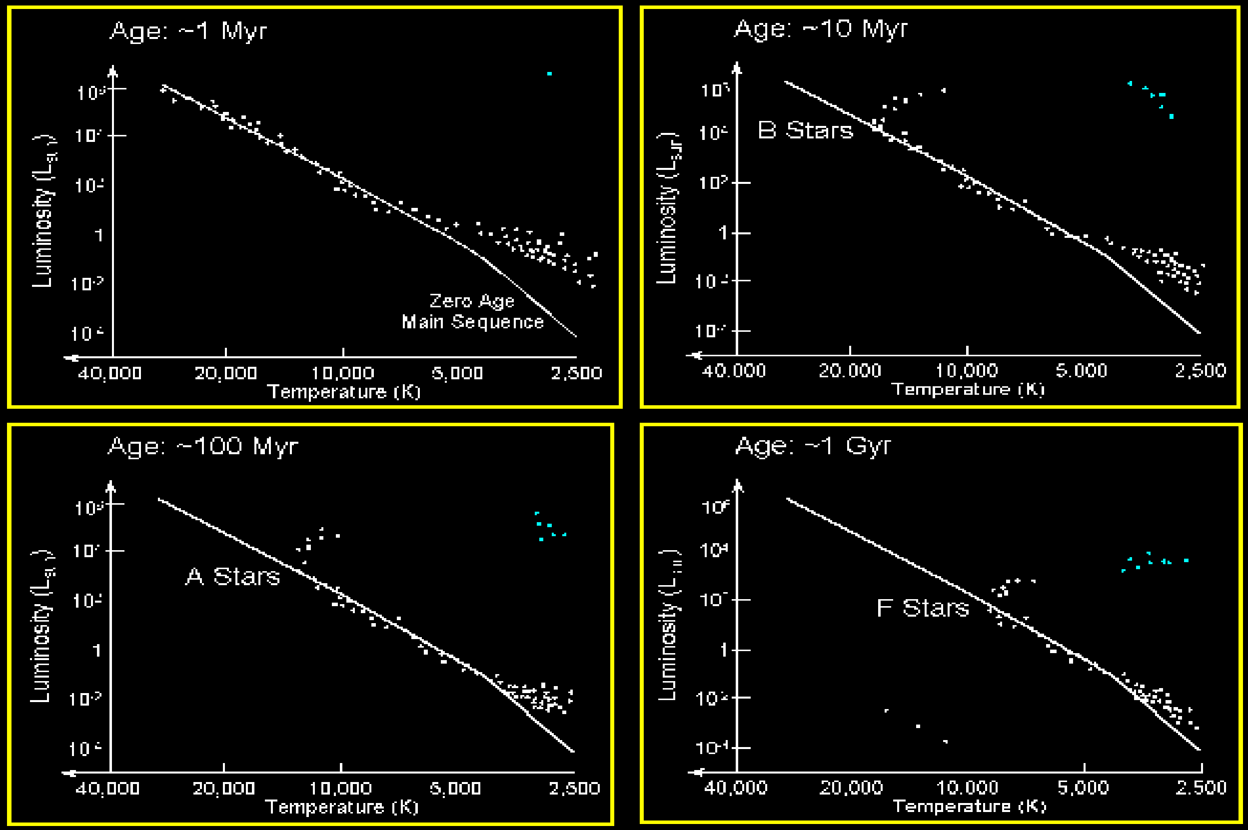
- Using what we know about stellar evolution is there a way to determine the age of the star cluster?

For a group of stars formed at the same approximate time, the more luminous ones evolve faster.

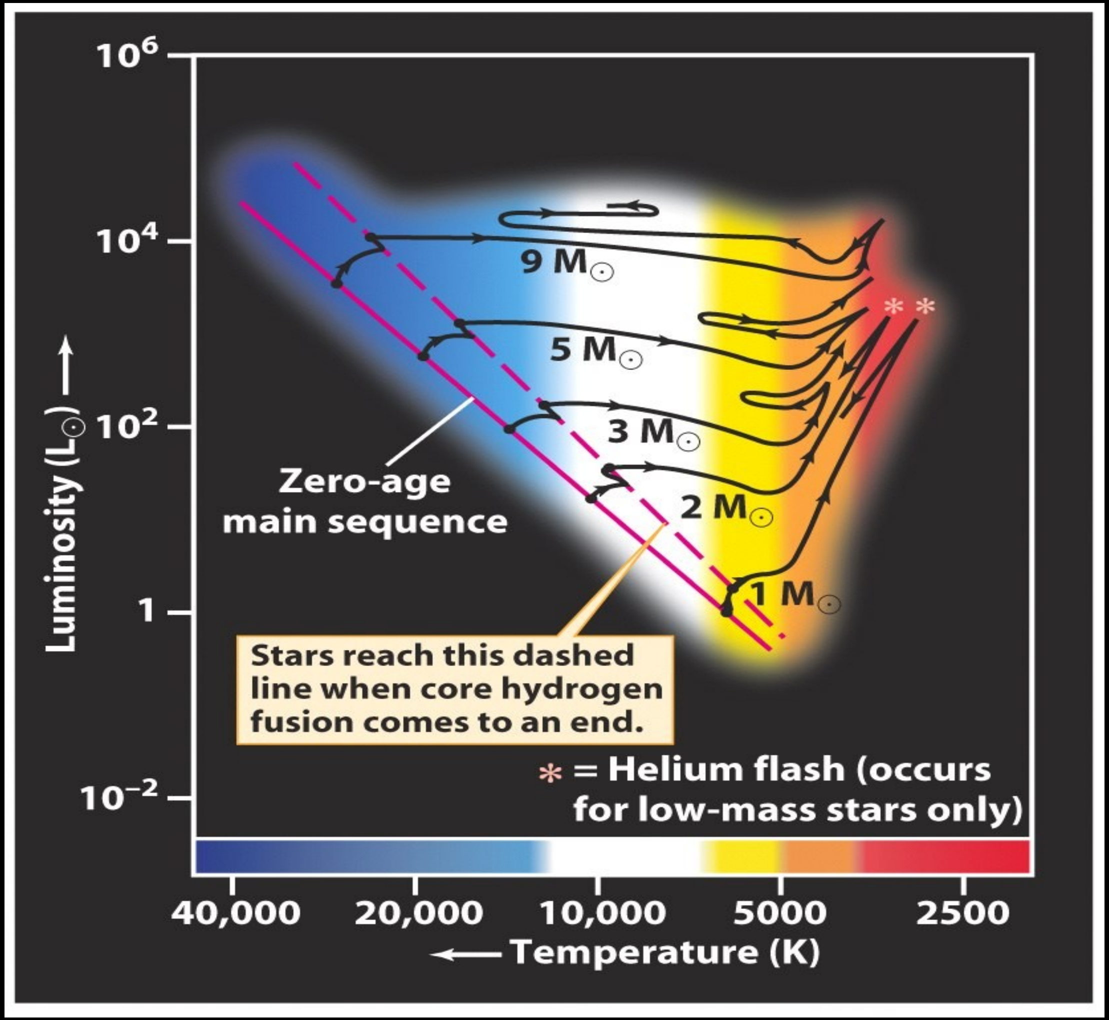


Cluster age and turn-off point

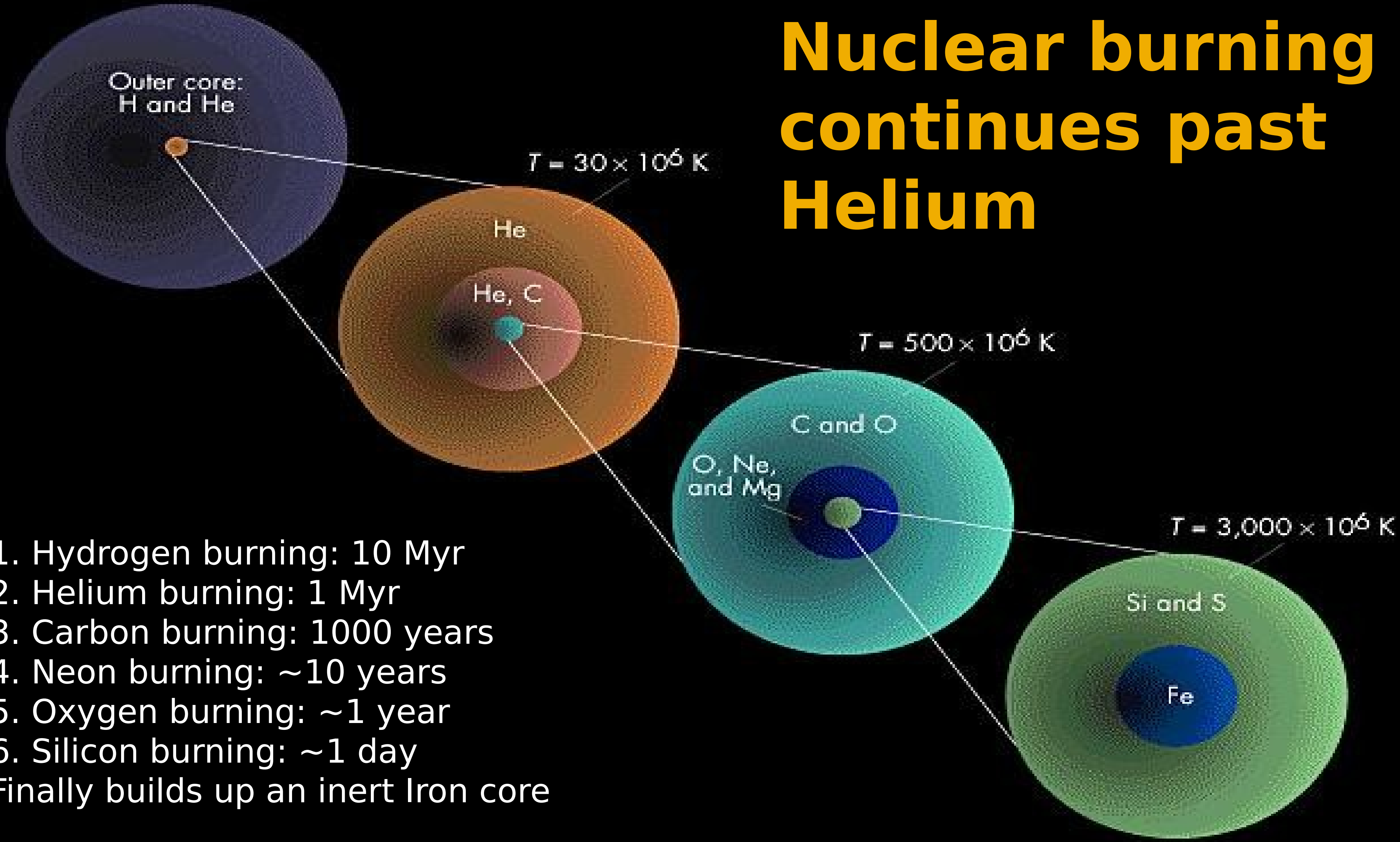




Higher mass stars do not have helium flash



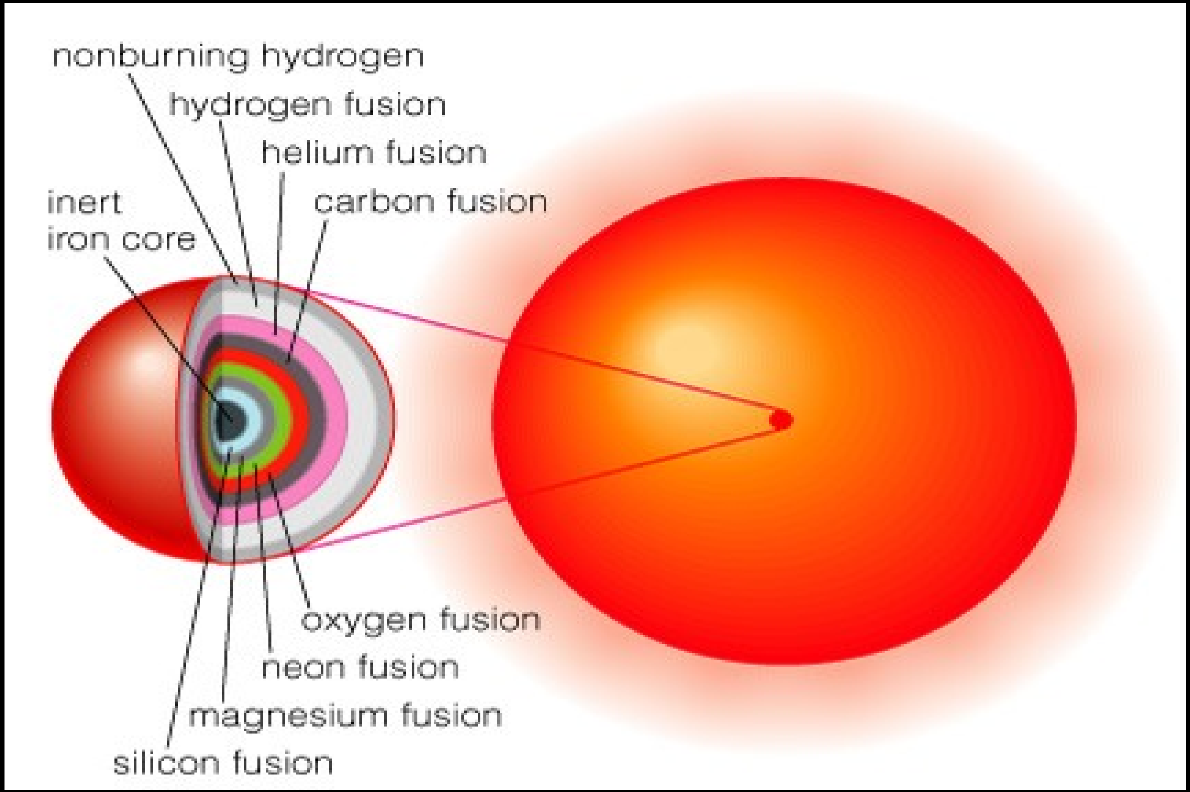
Nuclear burning continues past Helium



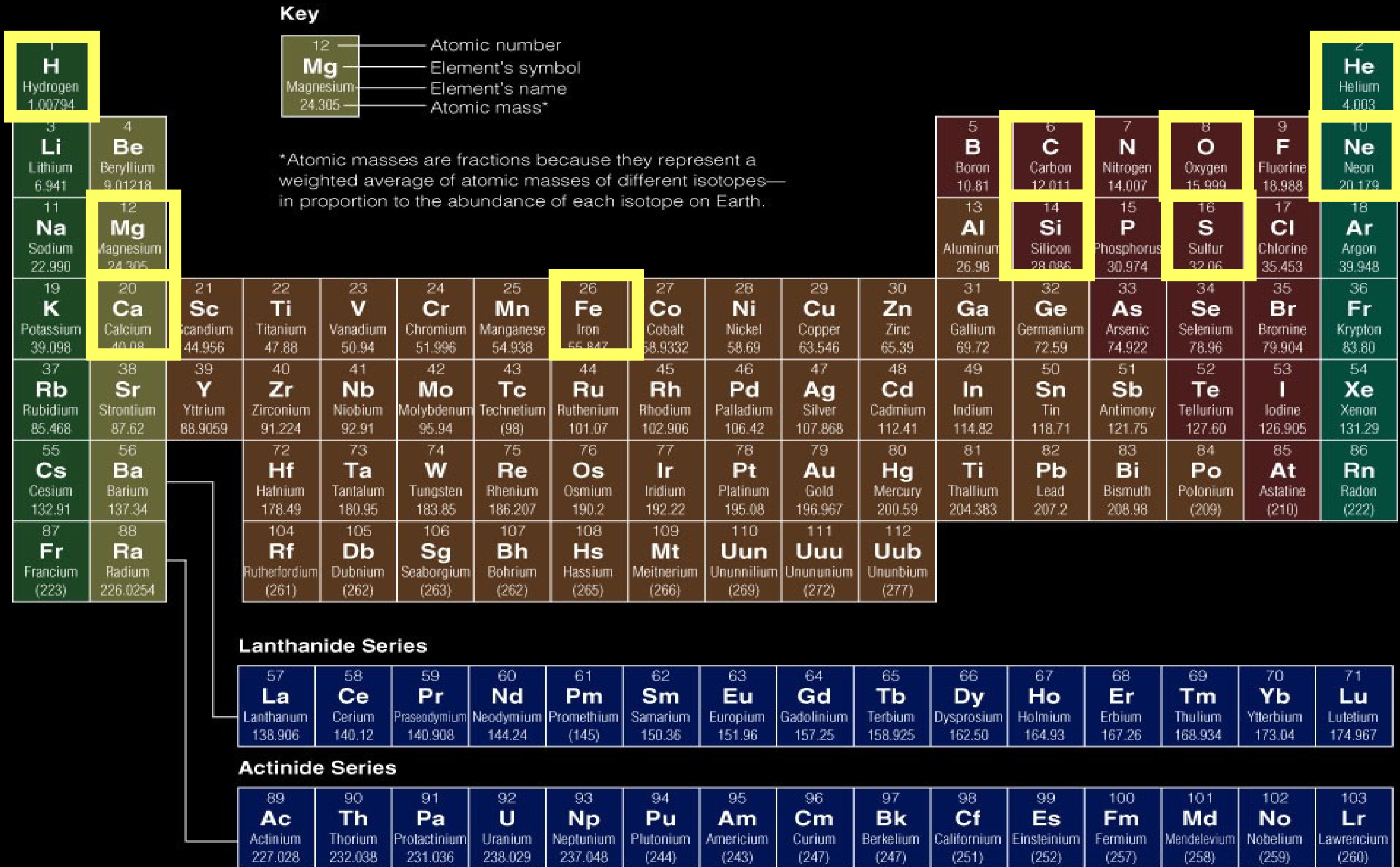
1. Hydrogen burning: 10 Myr
2. Helium burning: 1 Myr
3. Carbon burning: 1000 years
4. Neon burning: ~10 years
5. Oxygen burning: ~1 year
6. Silicon burning: ~1 day

Finally builds up an inert Iron core

Multiple Shell Burning

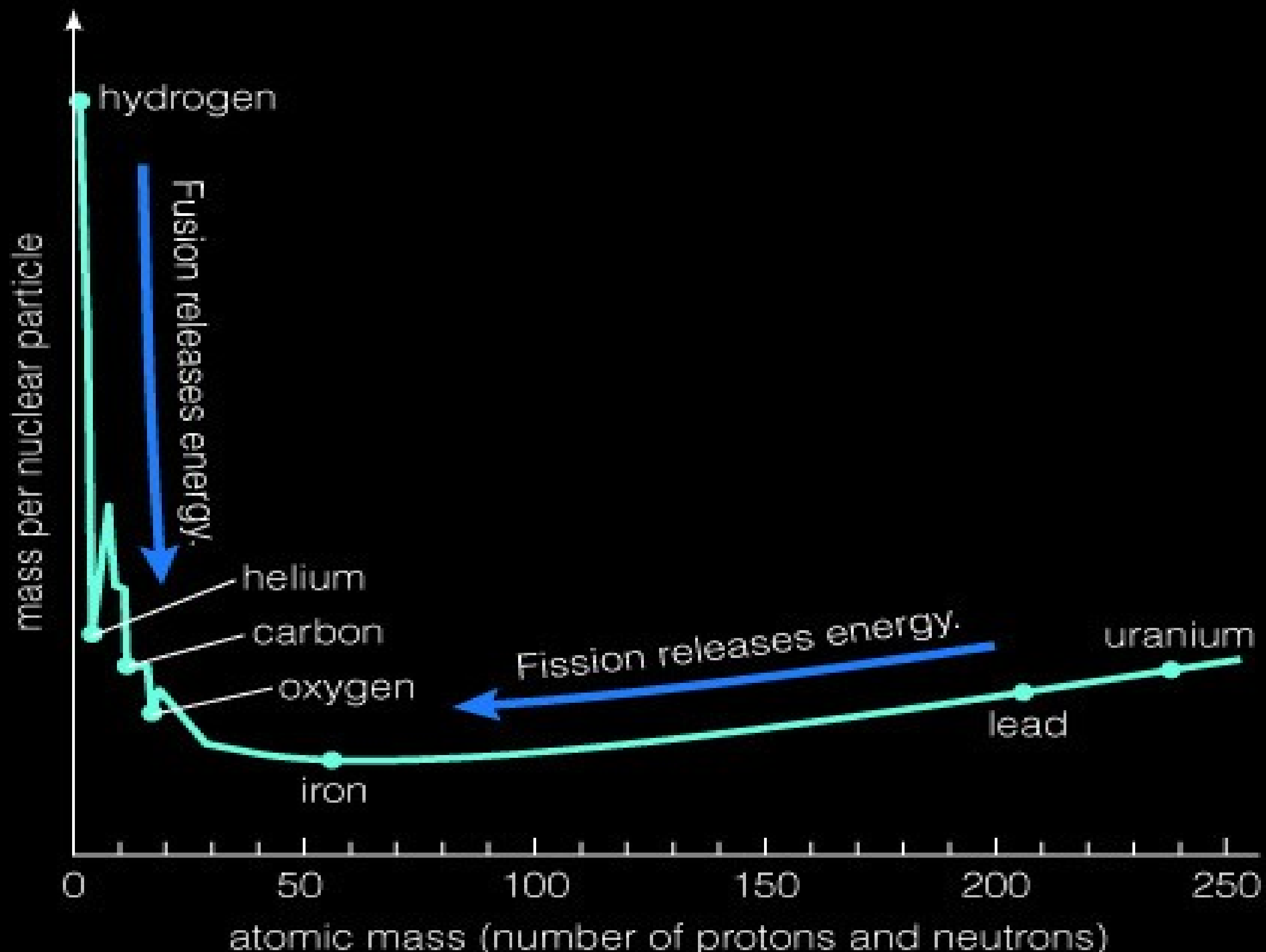


- Advanced nuclear burning proceeds in a series of nested shells



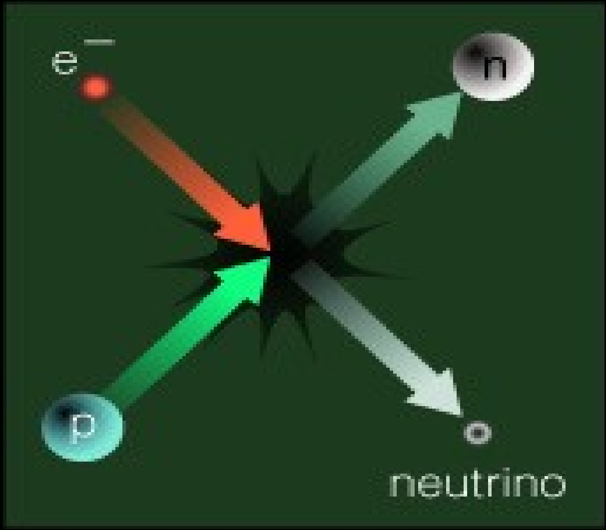
Advanced reactions in stars make elements like Si, S, Ca, Fe

Why does fusion stop at Iron?

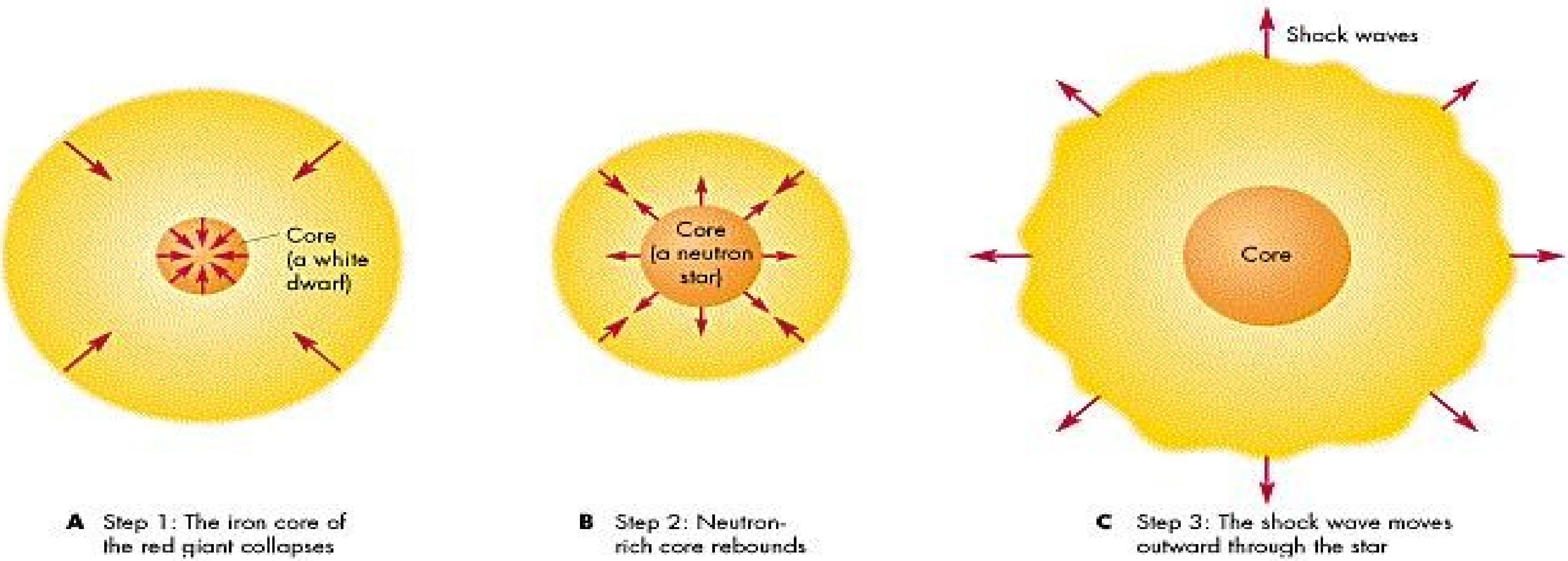


Core collapse

- Iron core is degenerate and grows until it is too heavy to support itself
- Core collapses and iron nuclei are converted into neutrons with the emission of neutrinos
- Core collapse stops, neutron star is formed
- Rest of the star bounces off the new neutron star (also pushed outwards by the neutrinos)



Supernova explosion



Neutron capture and beta decay

- Interaction between nuclei and free neutrons (**neutron capture**)
- Neutrons capture by heavy nuclei **is not limited by the Coulomb barrier**, so could proceed at relatively **low temperatures**.
- If enough neutrons available, chain of reactions:

$$I(A, Z) + n \rightarrow I_1(A+1, Z)$$
$$I_1(A+1, Z) + n \rightarrow I_2(A+2, Z)$$
$$I_2(A+2, Z) + n \rightarrow I_3(A+3, Z) \quad \ldots\text{etc}$$

- If a radioactive isotope is formed it will undergo **β-decay**, creating a new element:

$$I_N(A+N, Z) \rightarrow J(A+N, Z+1) + e^- + \bar{\nu}_e$$

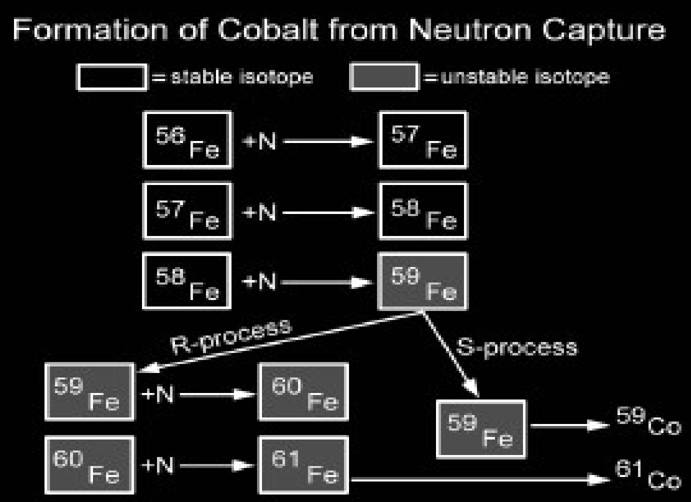
- If new element is stable, it will resume **neutron capture**, otherwise may undergo series of **β-decays**

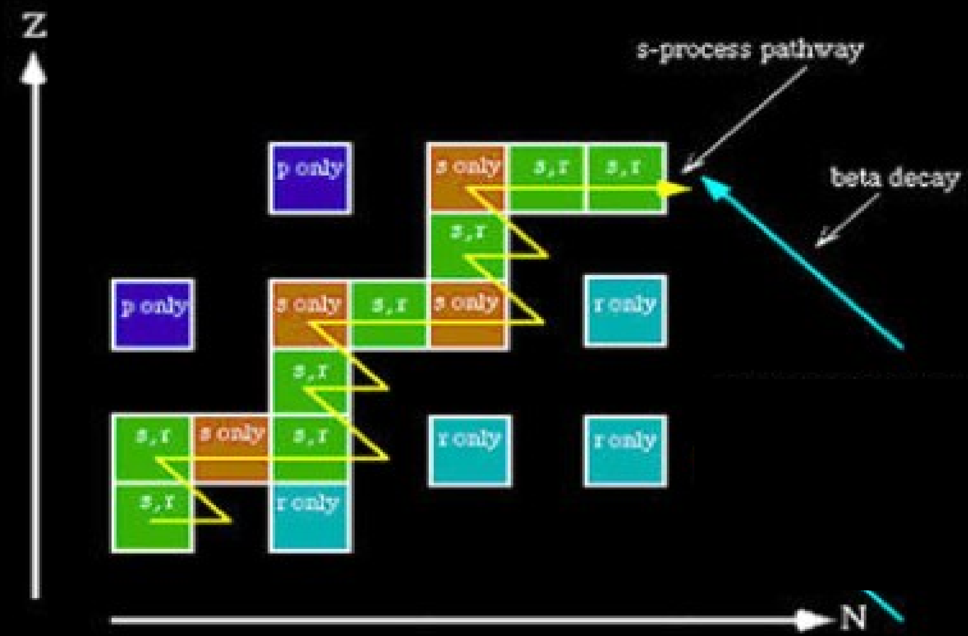
$$J(A+N, Z+1) \rightarrow K(A+N, Z+2) + e^- + \bar{\nu}_e$$
$$K(A+N, Z+2) \rightarrow L(A+N, Z+3) + e^- + \bar{\nu}_e$$

s-process and r-process

Stable nuclei may undergo only neutron captures, unstable ones may undergo both, with the outcome depending on the timescales for the two processes.

Timescales: neutron capture reactions may proceed more ***slowly*** or more ***rapidly*** (if many neutrons are available) than the competing β-decays: ***s-process*** or ***r-process***.





Nucleosystesis from NS merging