

# Cosmology

# Cosmology

- What is the Universe made of?
- How big is it?
- How old is it?
- How did it form?
- What will happen to it?

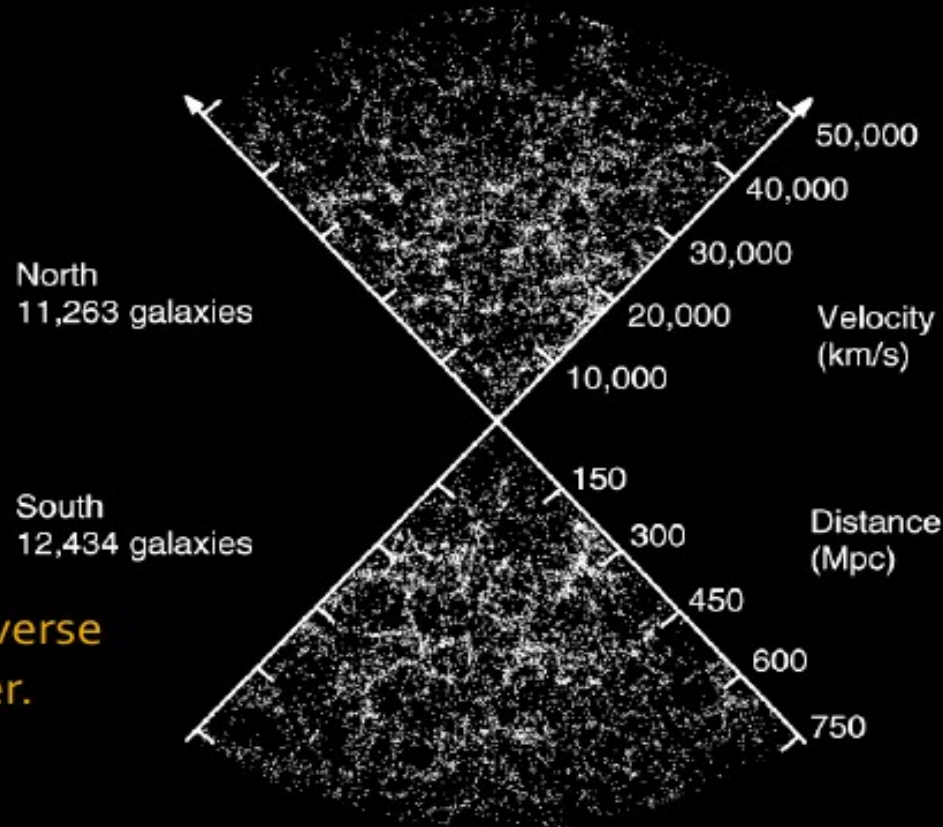


# The cosmological principle

**The Cosmological Principle** : the universe is homogeneous and isotropic on sufficiently large scales

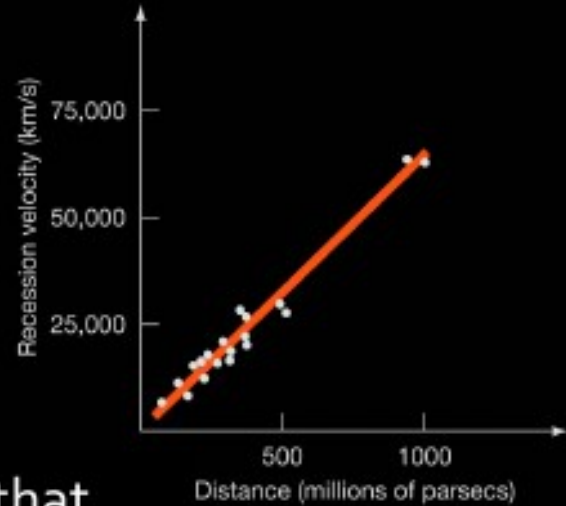
The universe looks pretty much like this everywhere: “walls” and “voids” are present but no larger structures are seen....

It follows that the Universe has no “edge” or center.



# Hubble Law

$$\text{recession velocity} = H_o \times \text{distance}$$



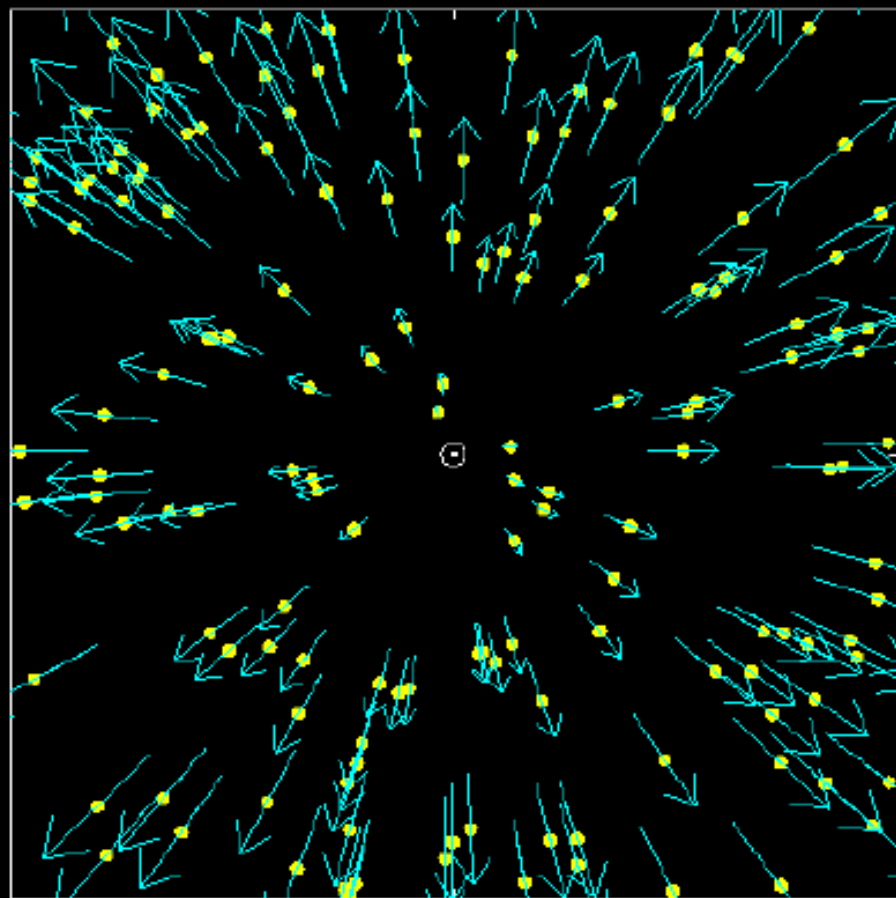
- The cosmological principle does not imply that the Universe is *constant* at all times (this was once thought to be the case - **Steady State Universe**).
- Universal **expansion** points to a beginning of the Universe and implies that the Universe is changing over time

# Hubble Law

In a homogenous Universe, what does the farther away the faster they move away mean?

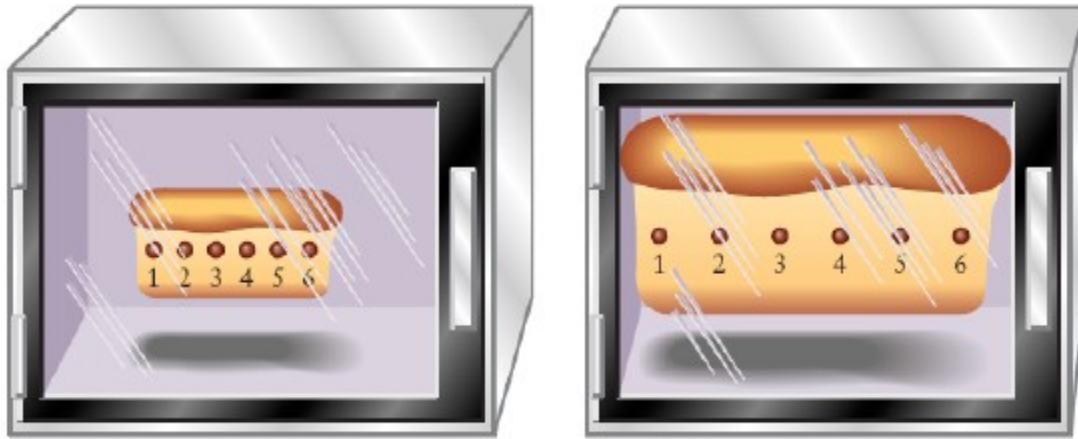
Are we at the **center of the Universe**?!

*GALAXY MOTION: ARTIST'S CONCEPTION*

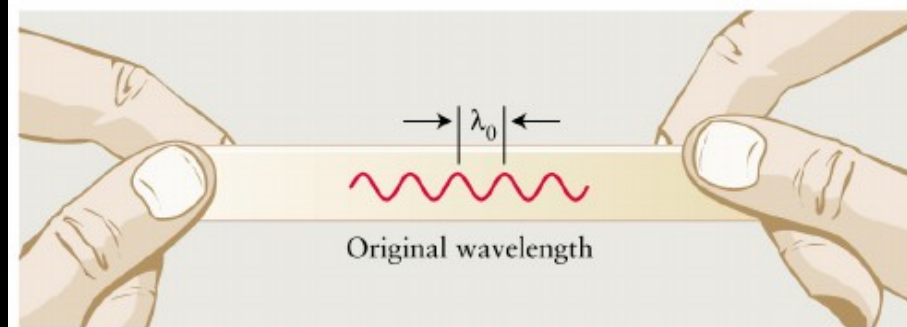


*⊙ = YOU ARE HERE*

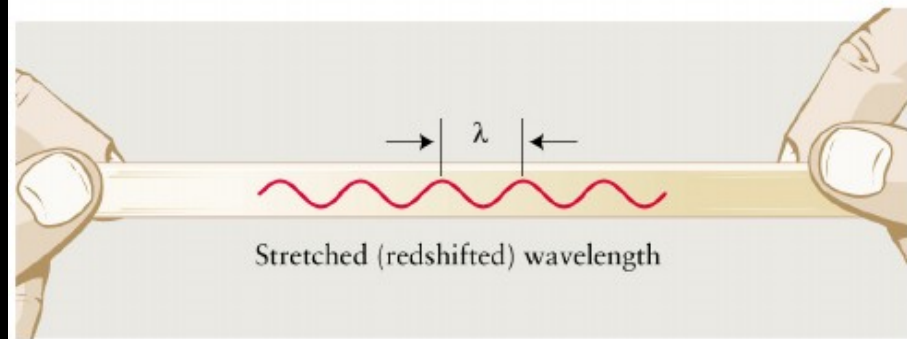
# Expansion and redshift



Just as all the chocolate chips move apart as the cake rises, all the galaxies recede from each other as the universe expands.



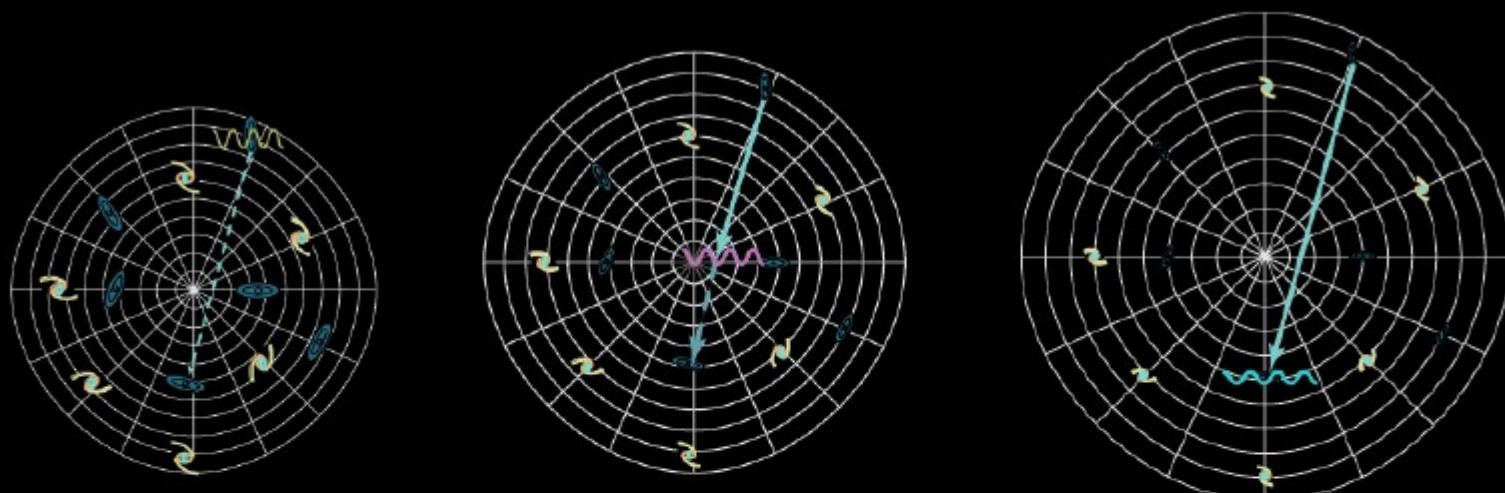
a A wave drawn on a rubber band ...



b ... increases in wavelength as the rubber band is stretched.



As Universe expands,  $\lambda$  expands proportionally to the scale factor  $\mathbf{R(t)} = r(t)/r(t_0)$ .



If radiation is emitted at wavelength  $\lambda_1$  at epoch  $t_1$ , and detected at wavelength  $\lambda_2$  at epoch  $t_2$ , then

$$\frac{\lambda_2}{\lambda_1} = \frac{R(t_2)}{R(t_1)}$$

If we let  $t_1$  be some arbitrary time, and  $t_2$  be the reference epoch  $t_0$  (for which  $R = 1$ ), this becomes

$$\frac{\lambda_0}{\lambda} = \frac{1}{R(t)}$$

We define the *redshift*,  $z$ , to be

$$z = \frac{\Delta\lambda}{\lambda}$$

where  $\Delta\lambda = \lambda_0 - \lambda$ , so  $\lambda_0/\lambda = 1 + z$ . This gives

$$1 + z = \frac{1}{R(t)}$$

Remember, since the radiation is emitted before the reference epoch,  $R(t) < 1$ , so  $z > 0$ .

Since redshift arises due to expanding wavelength of all photons traveling through an expanding Universe, it is called a **cosmological redshift**.

As a consequence, we don't normally convert a  $z$  to a **distance**, since we need to assume a particular model for how  $R(t)$  has evolved.

We often need to study events (objects) spread out in space and time so we must compute distances between 2 events in **4D spacetime**

Space-time interval in **spherical coordinates**

$$(ds)^2 = (cdt)^2 - R^2(ct) \left[ \frac{(dr)^2}{(1-\kappa r^2)} + r^2 [(d\theta)^2 + \sin^2\theta (d\phi)^2] \right]$$

*Robertson-Walker metric*

where  $R$  is the scale factor of the Universe and  $r, \theta, \phi$  are the usual spherical coordinates of objects (like galaxies). These are the *comoving coordinates* of a point in space. If the expansion of the universe is homogeneous and isotropic, comoving coordinates are constant with time.

The **photons** from distant galaxies follow *null geodesics* (where  **$ds=0$** ).



# Big Bang

- In the 1940s, based on Hubble's Law, George **Gamow** proposed that the universe began in a colossal explosion
- In the 1950s, the term **BIG BANG** was coined by an unconvinced Sir Fred Hoyle
- In the 1990s, there was an international competition to **rename** the BIG BANG with a more appropriate name, but no new name was selected

# Big Bang

- If the universe is expanding, it must have been **smaller in the past**
- If it was smaller in the past, then something must have made it **begin** to expand
- This “event” is called the **BIG BANG**
- The **age of the universe** is simply the separation distance of the most distant galaxies divided by their recessional velocities

# The age of the universe

Hubble:  $v = H D$   
 $D$ : distance between 2  
particles (galaxies)

If **constant** speed,  
distance = speed x time

$$D = v t$$

Put it all together:

$$v = H D = H (v t) = H t v$$

$$\text{so: } H t = 1$$

$$t = 1/H$$

“expansion age” of Universe

# The age of the universe

Other methods to date the Universe:

- **Radioactivity** in Rocks
  - uranium decays to lead
  - decay is "clock": tells time since uranium made in star
  - age > 10 billion yrs
- **Globular clusters**
  - oldest stars
  - age about 13 billion years

Best estimate (Planck, from CMB):

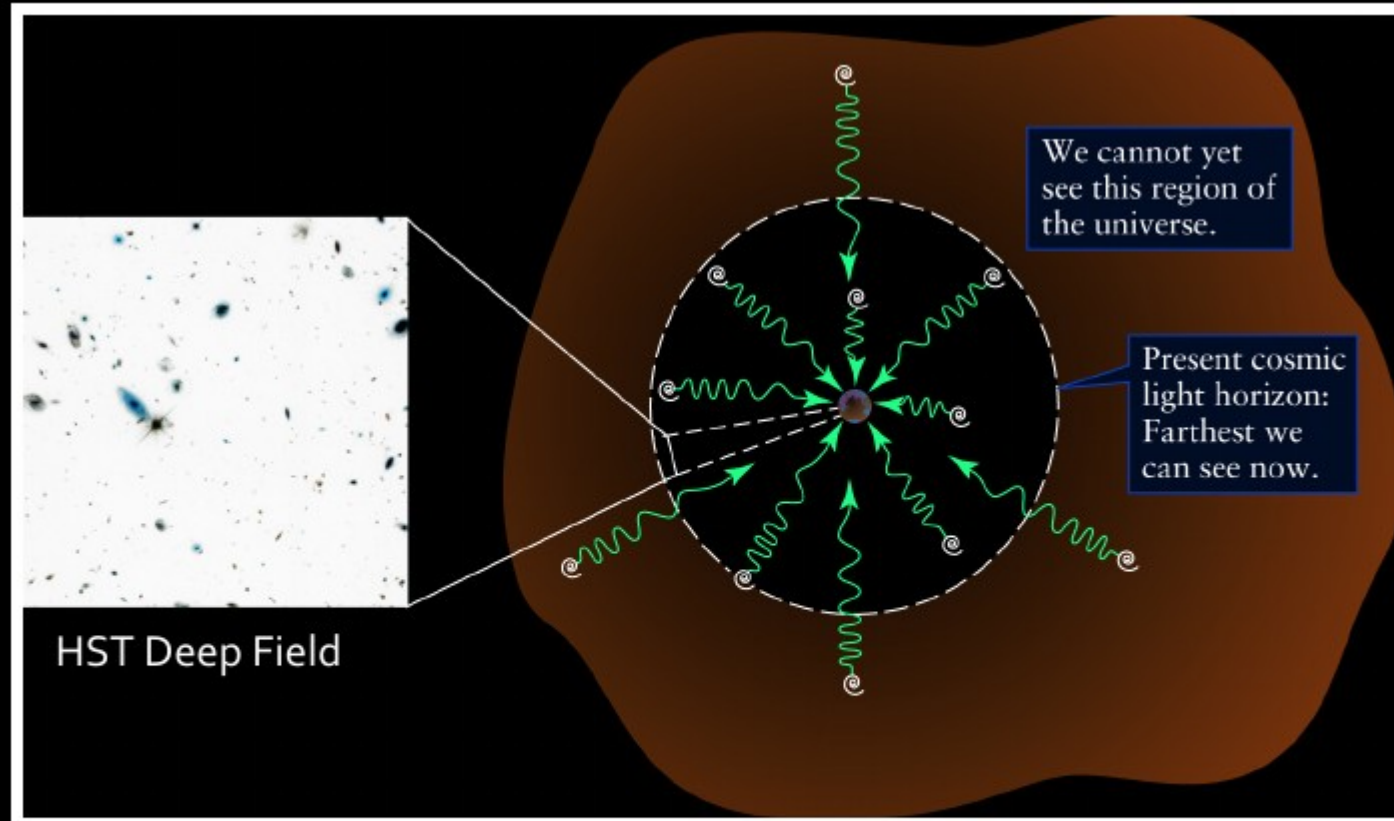
- Age  $t = \mathbf{13.82 \text{ billion years}}$



Globular Cluster NGC 6093



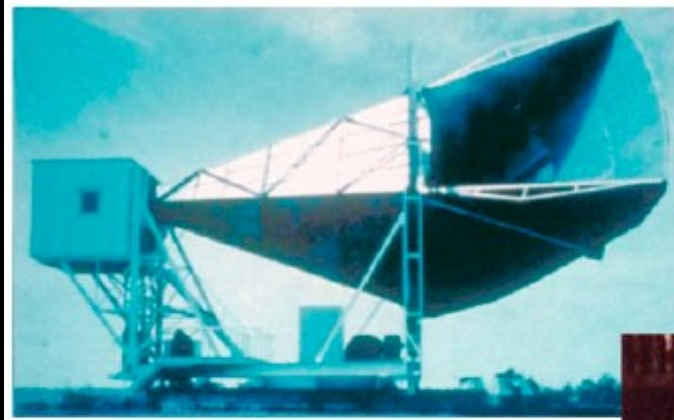
# The observable universe



The cosmic light **horizon** today is about 13.8 billion light-years away in all directions. The **farthest galaxies** we can see are as they were within a few hundred million years after the Big Bang.

# CBM

- If the early Universe was so hot, we should be able to see the **blackbody** radiation, **redshifted**
- It is redshifted down to the **microwaves**. Called the Cosmic Microwave Background (**CMB**).
- First detected by Robert **Wilson** and Arno **Penzias** in 1965 (1978 Nobel prize).



Microwave Receiver



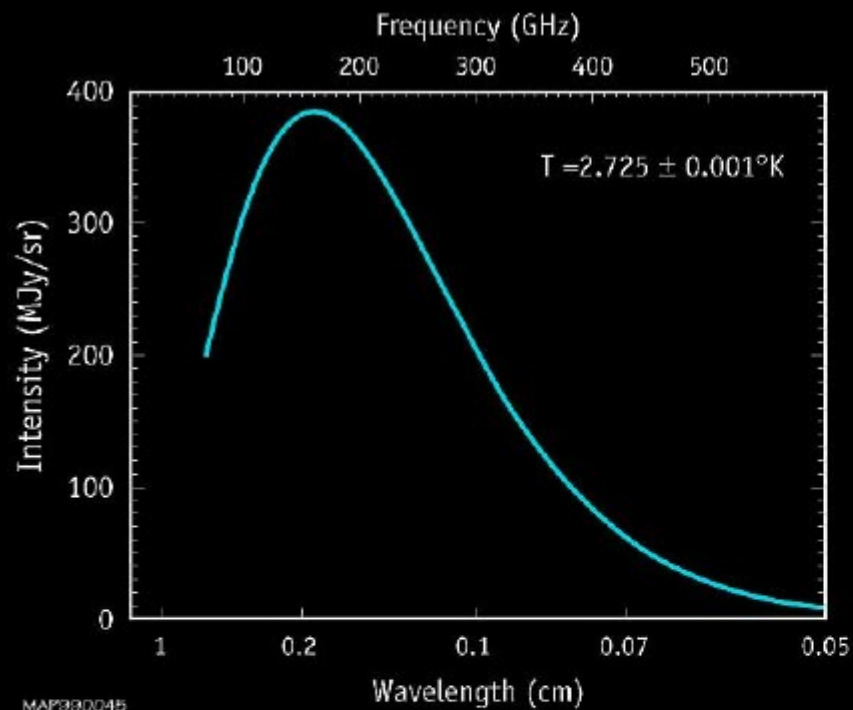
MAP990045

Robert Wilson



Arno Penzias





Cosmic  
Background  
Explorer  
(COBE) satellite  
(launched in  
1989)

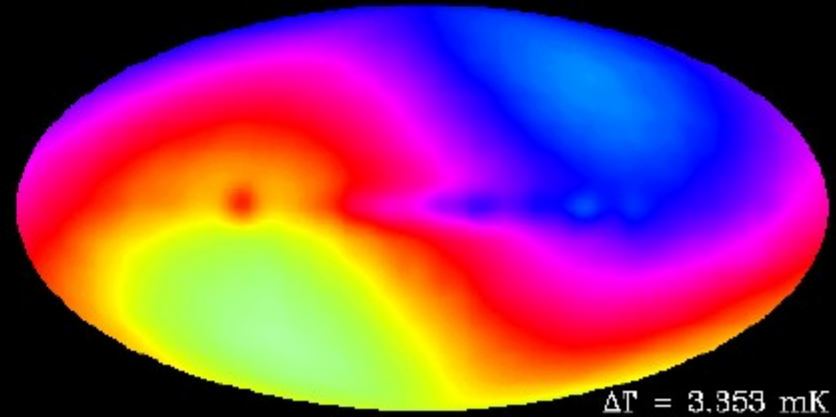
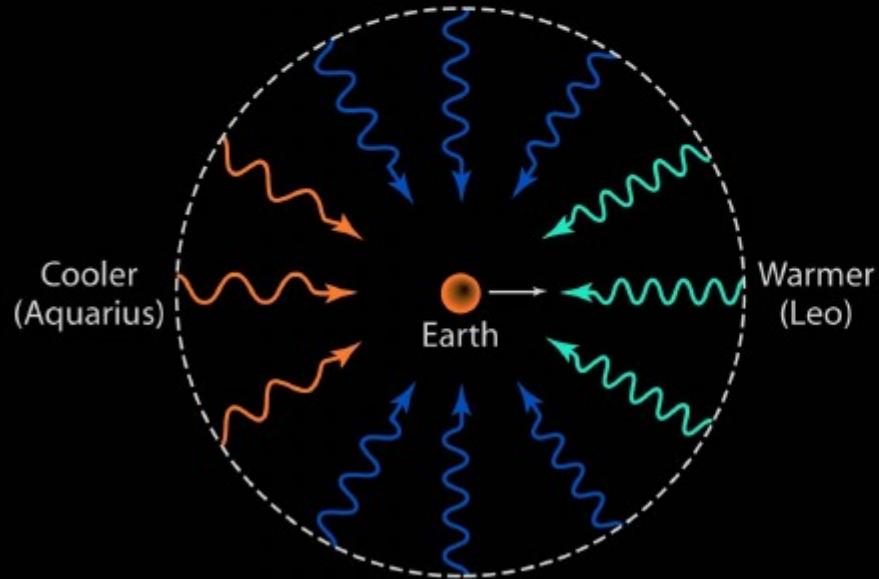


Is the spectrum perfectly Planckian  
and the temperature identical in  
every direction?

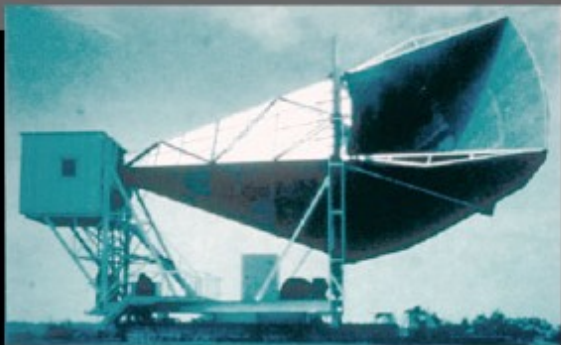




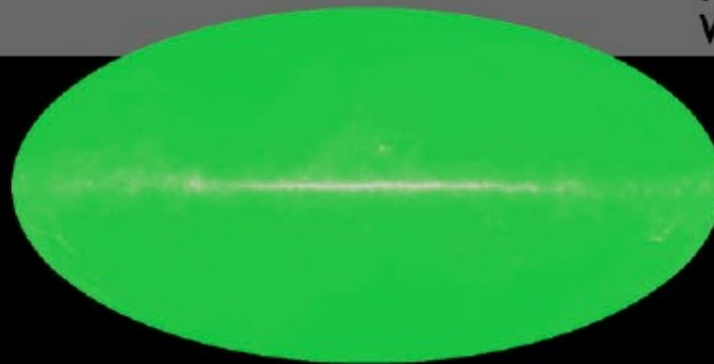
- Small scale variation, due to our movement with respect to the background.
- We are moving at  $\sim 600$  km/s



1965



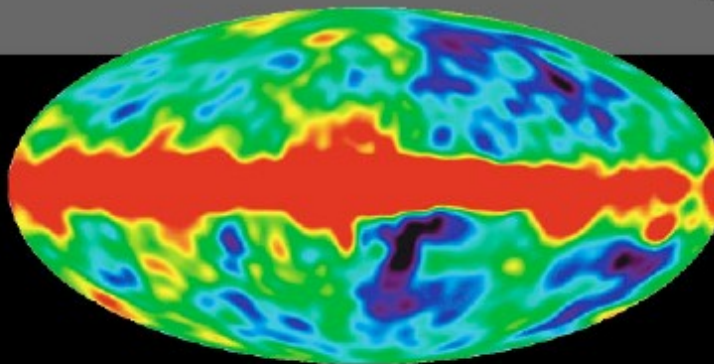
Penzias and  
Wilson



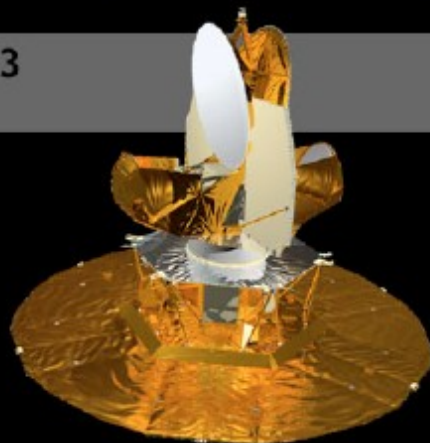
1992



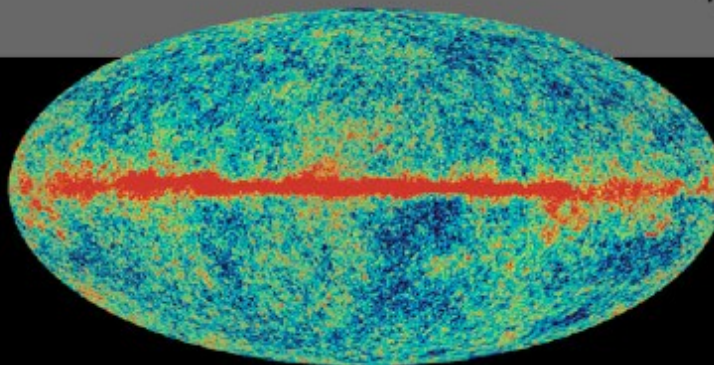
COBE

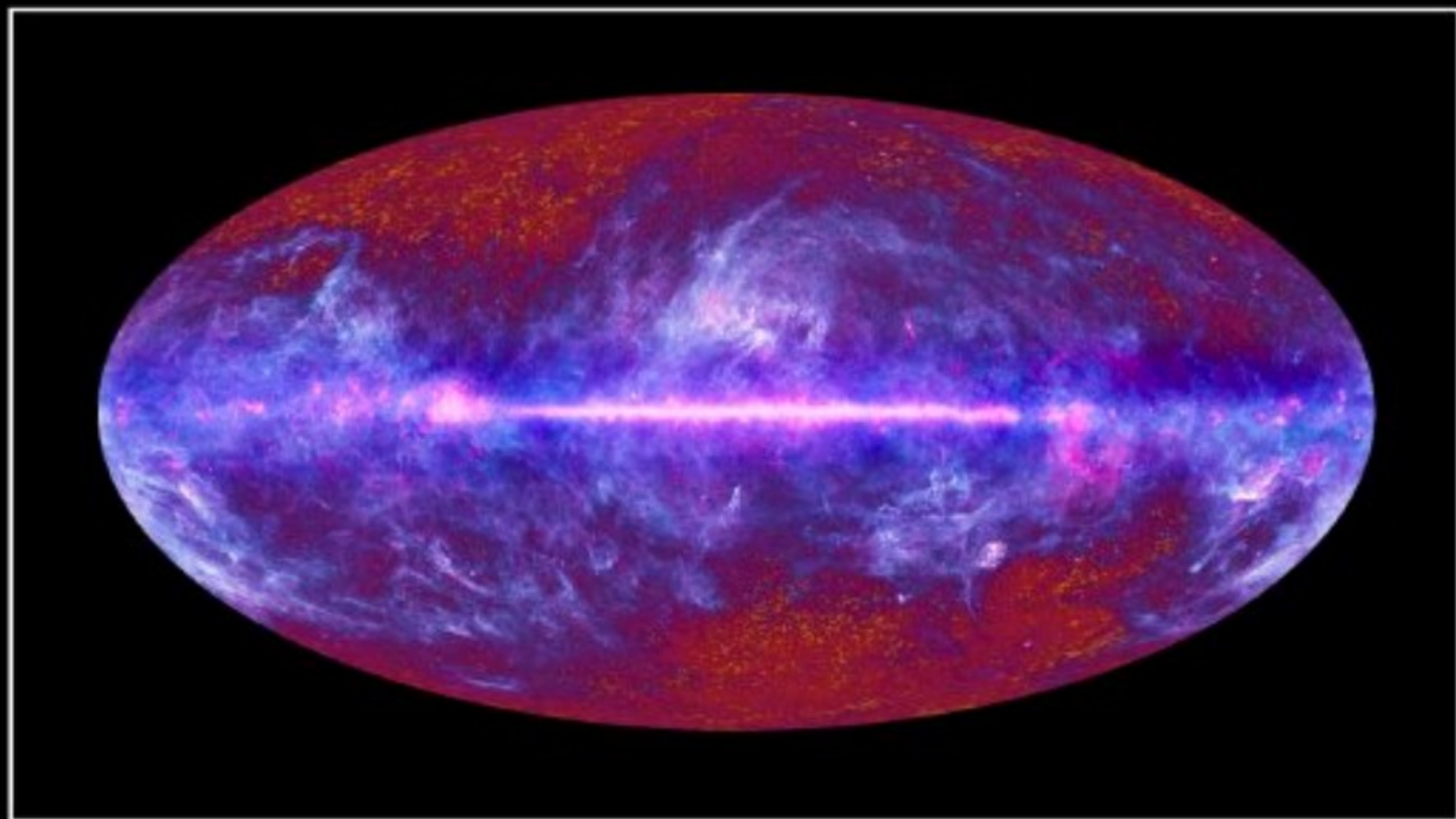


2003

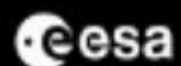


WMAP



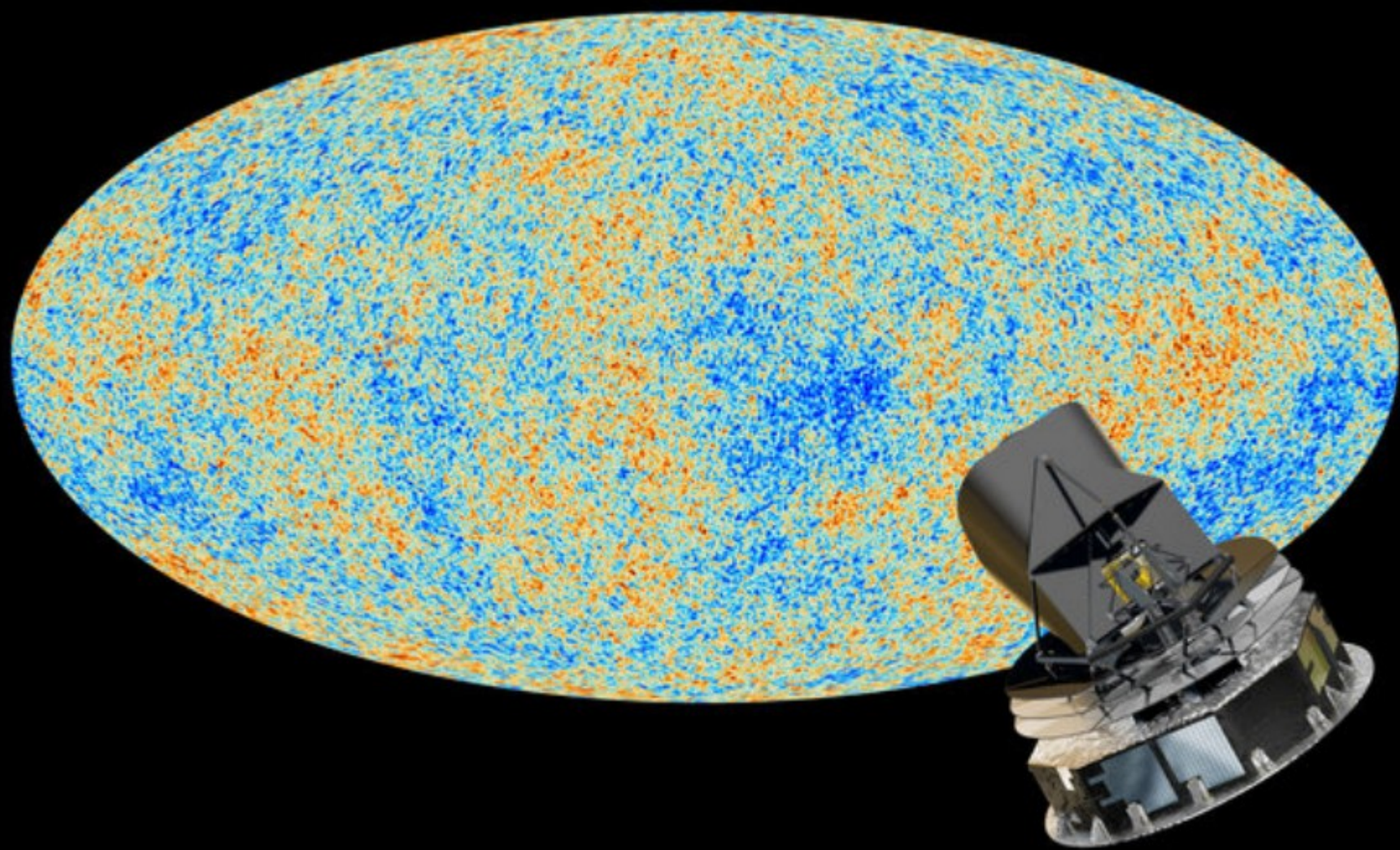


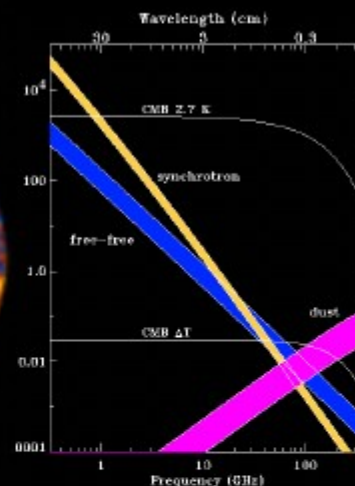
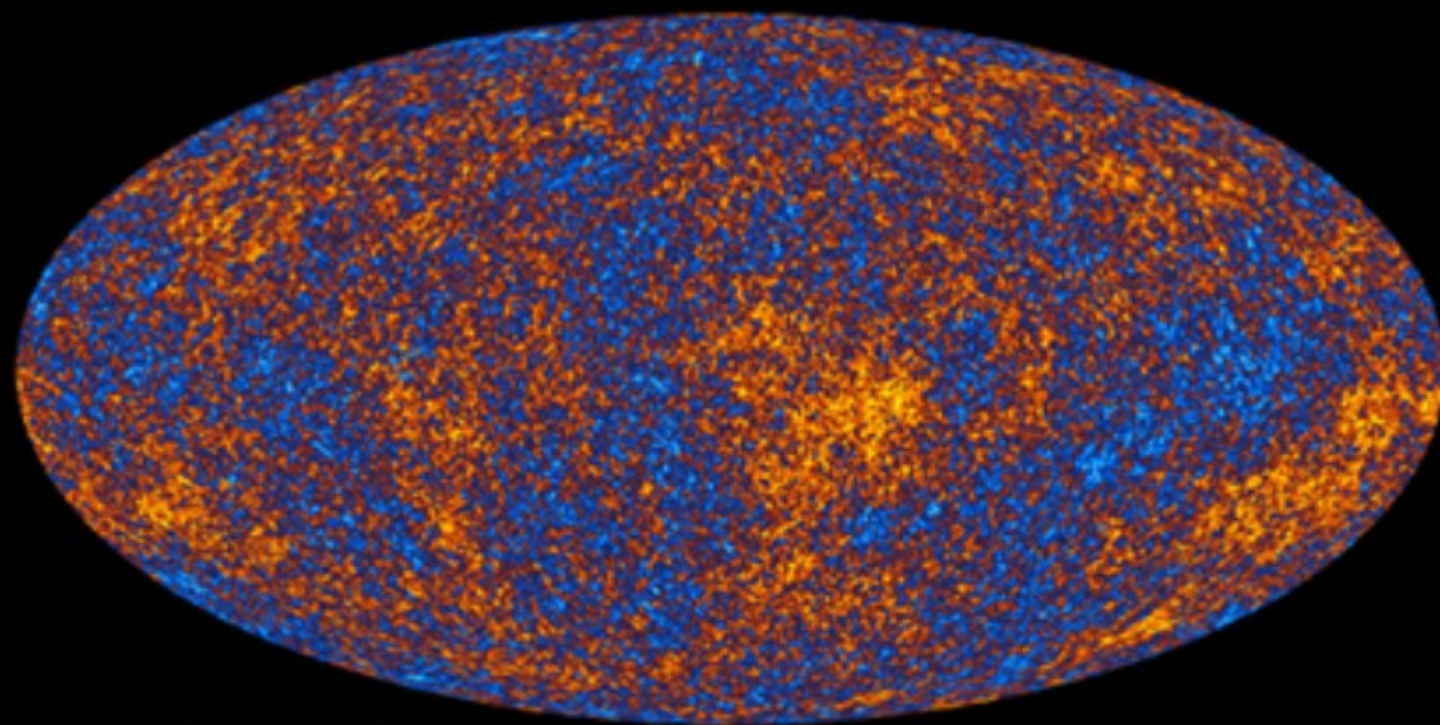
The Planck one-year all-sky survey



(c) ESA, HFI and LFI consortia, July 2010



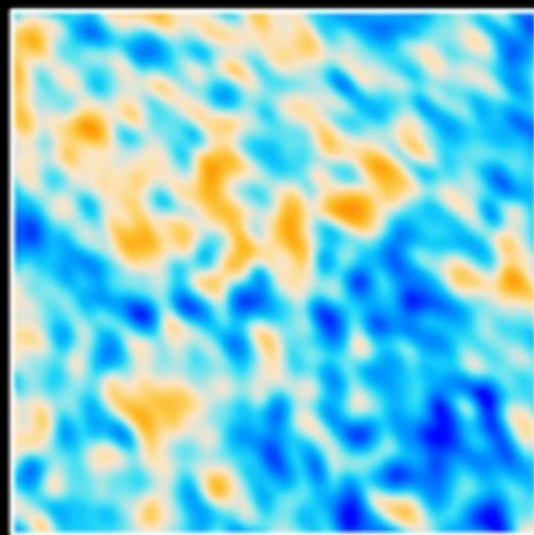
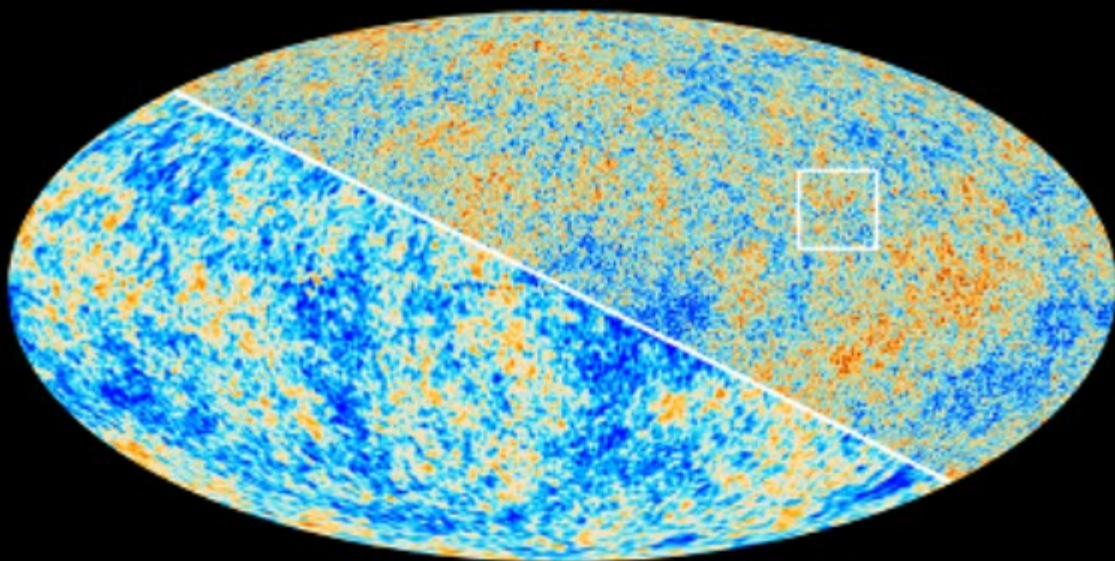




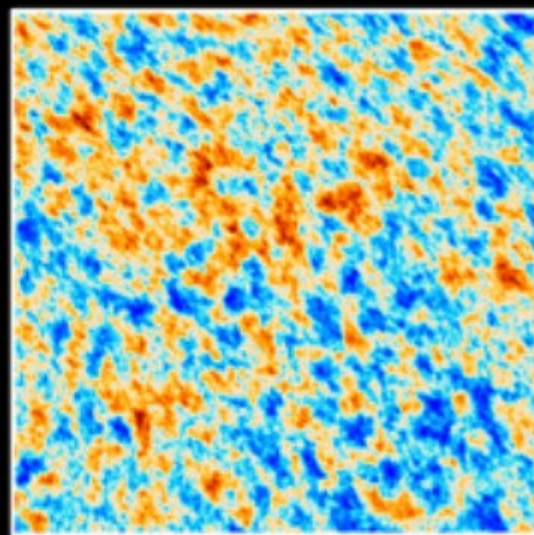
These small ( $< 1/100,000$ ) perturbations are the **fluctuations** that caused the **large scale structures** we see today. Discovered by COBE (2006 Nobel Prize to George **Smoot** and John **Mather**)



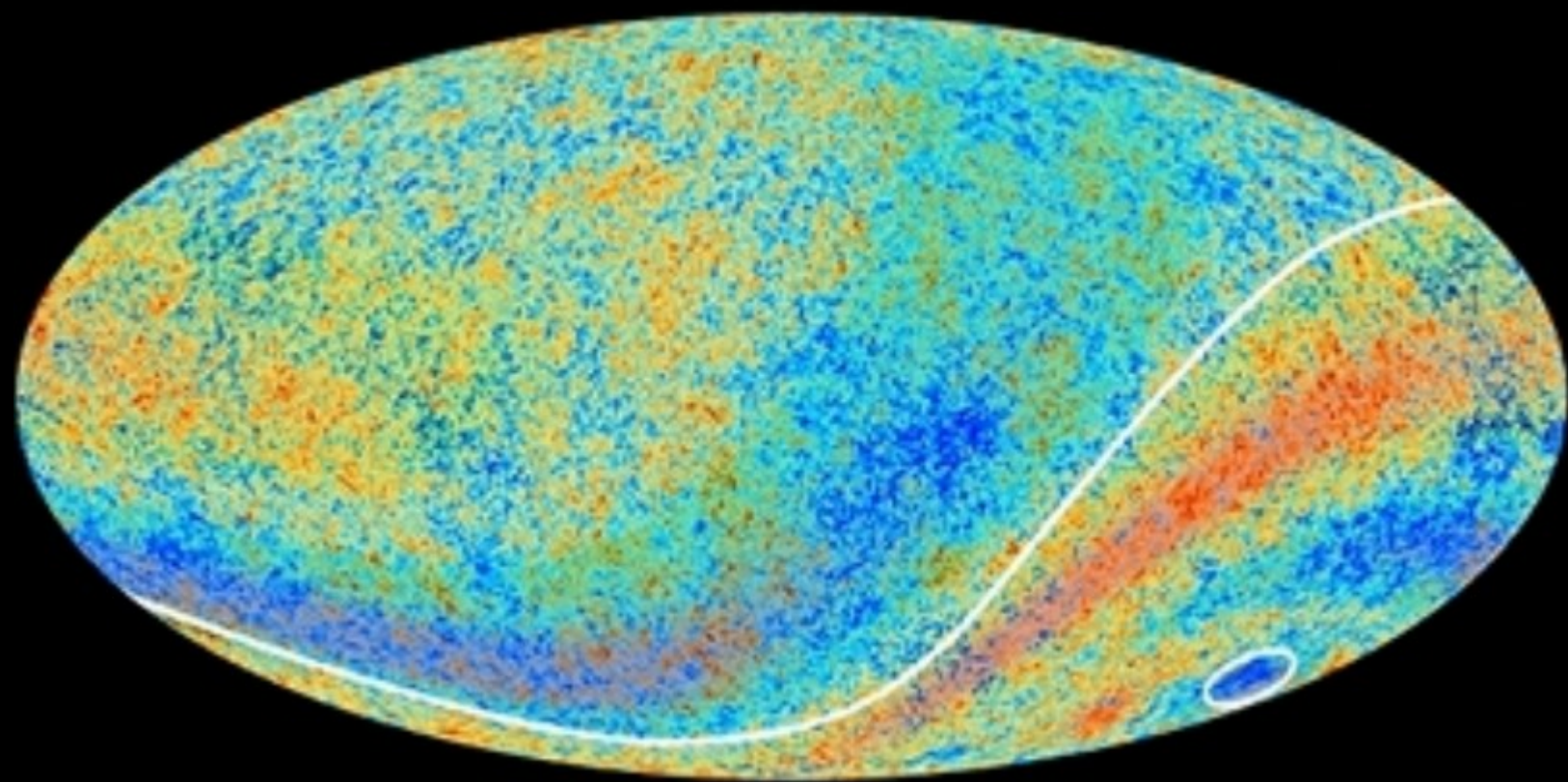
*The Cosmic Microwave Background as seen by Planck and WMAP*



*WMAP*



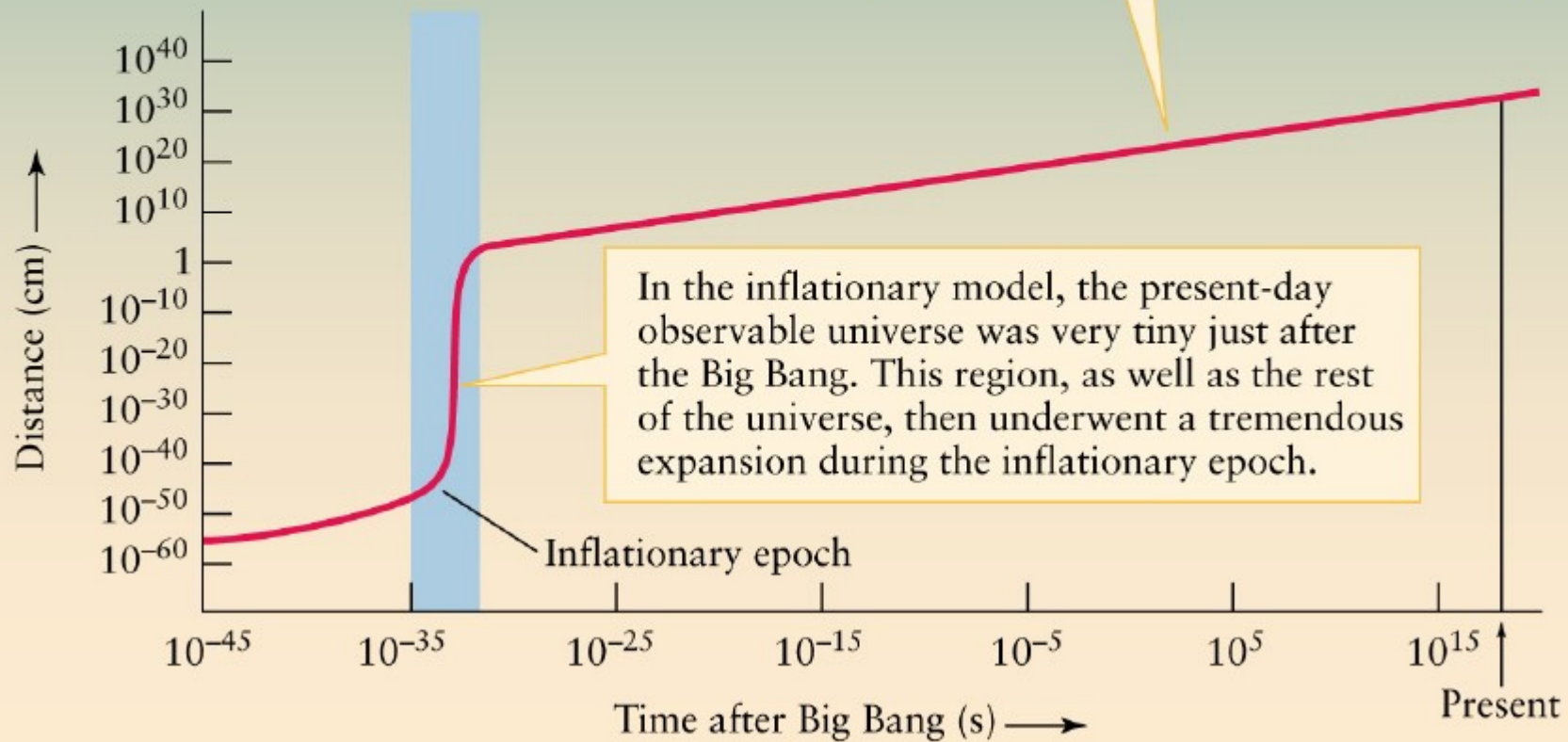
*Planck*



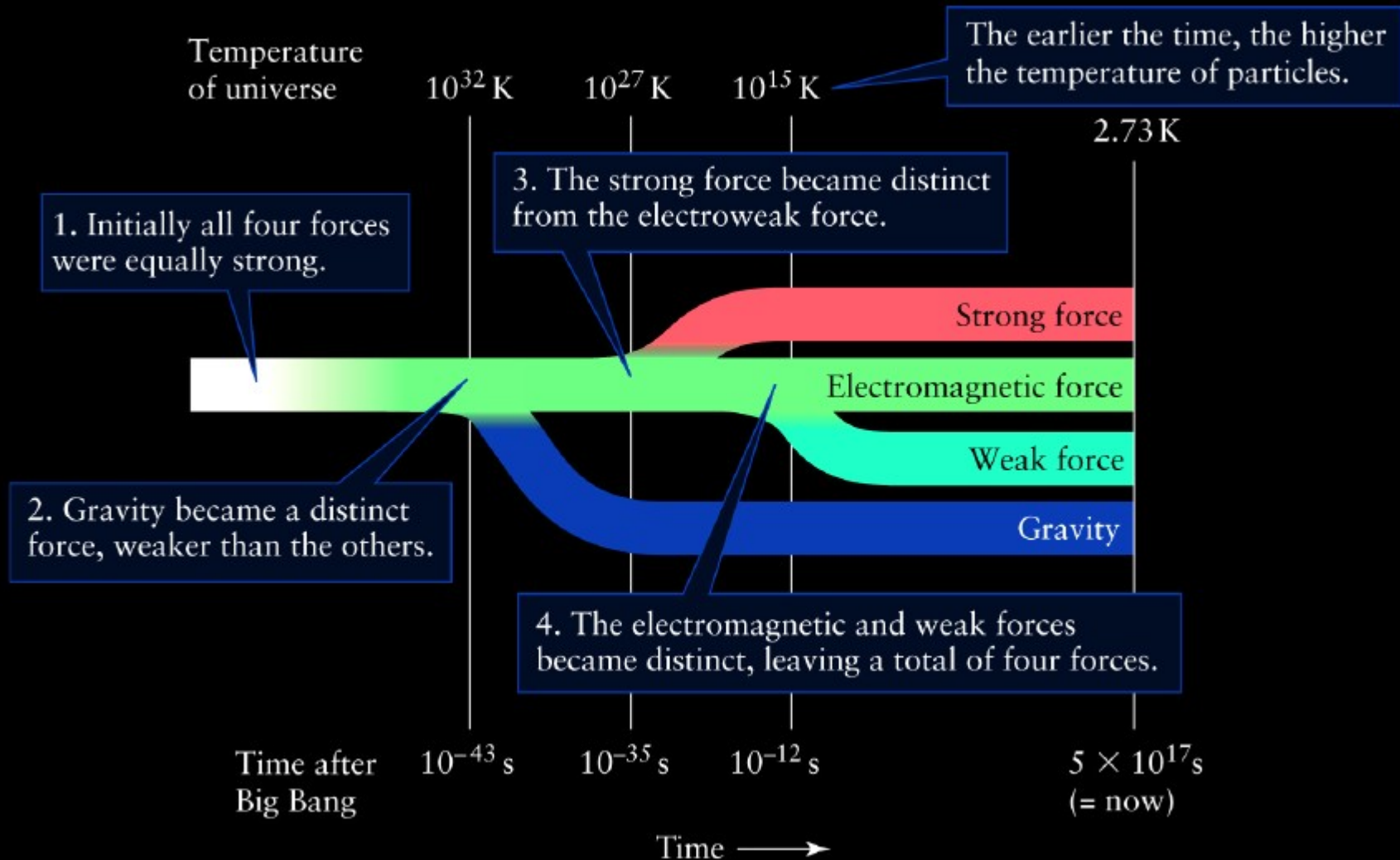


# Inflation

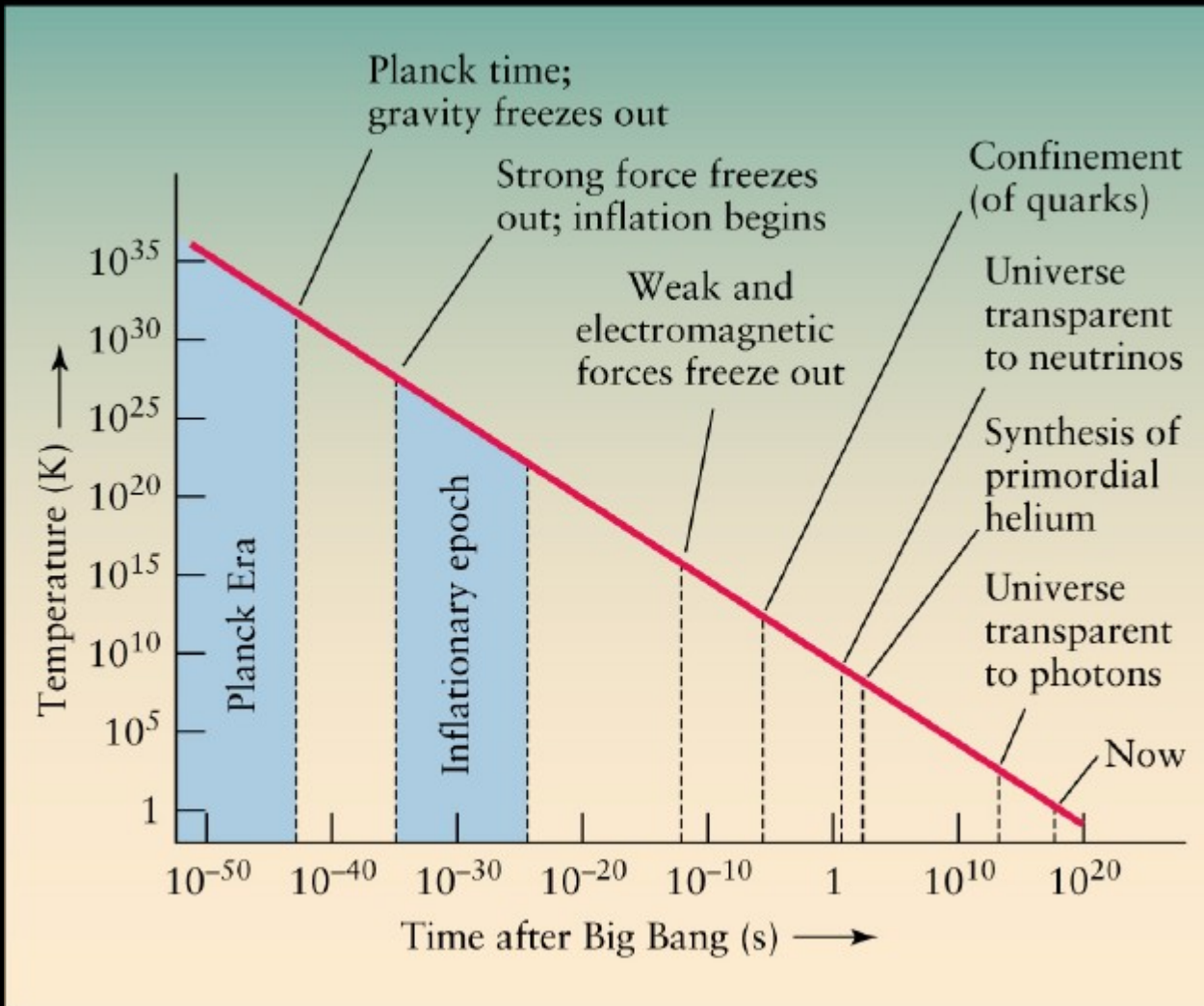
- PROBLEM: If the universe is at least 26 billion light years across (13 billion in each direction), how could both sides have exactly the **same temperature** if they couldn't "communicate"?
- SOLUTION: During the first fraction of second, the universe underwent a rapid expansion, called the **inflationary epoch**, in which it became many times larger than its original size.



# Unification of the four forces



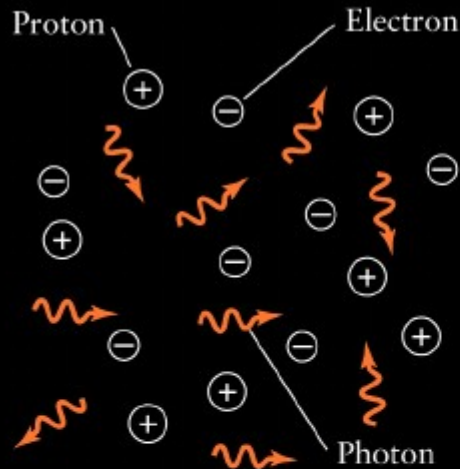
# The history of the universe



# Cosmological Nucleosynthesis

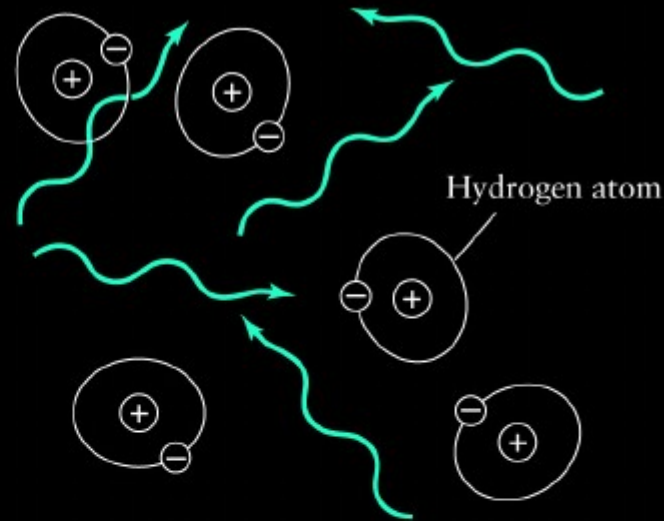
Periodic Table of the Elements

# Recombination era



a Before recombination:

- Temperatures were so high that electrons and protons could not combine to form hydrogen atoms.
- The universe was opaque: Photons underwent frequent collisions with electrons.
- Matter and radiation were at the same temperature.



b After recombination:

- Temperatures became low enough for hydrogen atoms to form.
- The universe became transparent: Collisions between photons and atoms became infrequent.
- Matter and radiation were no longer at the same temperature.

*When (temperature, redshift) did recombination occur?  $T \sim 3000$  K;  $z \sim 1000$*

# Friedmann equations

*General relativistic approach:* Insert metric into Einstein equation to obtain differential equation for  $R(t)$ :

Einstein equation:

$$\underbrace{R_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu}}_{G_{\mu\nu}} = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu} \quad (8.34)$$

where

$g_{\mu\nu}$ : Metric tensor ( $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ )

$R_{\mu\nu}$ : Ricci tensor (function of  $g_{\mu\nu}$ )

$\mathcal{R}$ : Ricci scalar (function of  $g_{\mu\nu}$ )

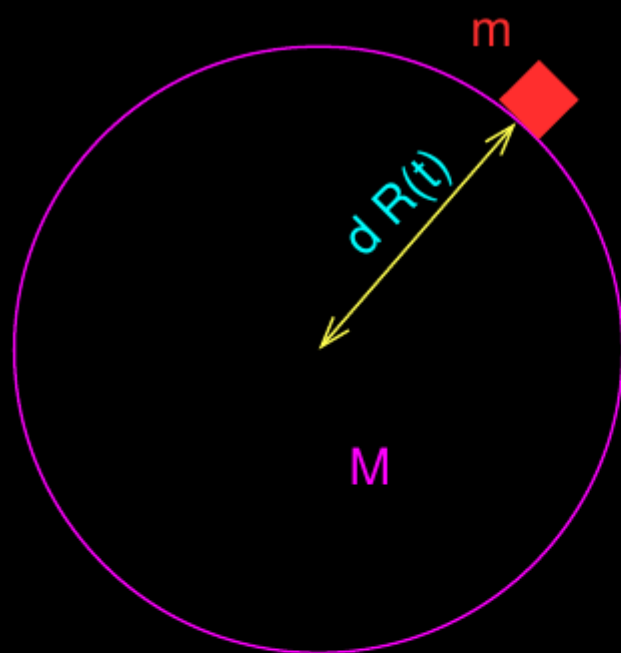
$G_{\mu\nu}$ : Einstein tensor (function of  $g_{\mu\nu}$ )

$T_{\mu\nu}$ : Stress-energy tensor, describing curvature of space due to fields present (matter, radiation, ...)

$\Lambda$ : Cosmological constant

$\Rightarrow$  Messy, but doable





Here, Newtonian derivation of **Friedmann equations**: Dynamics of a mass element on the surface of sphere of density  $\rho(t)$  and **comoving radius**  $d$ , i.e., **proper radius**  $d \cdot R(t)$  (McCrea, 1937)

**Mass of sphere:**

$$M = \frac{4\pi}{3}(dR)^3 \rho(t) = \frac{4\pi}{3}d^3 \rho_0 \quad \text{where} \quad \rho(t) = \frac{\rho_0}{R(t)^3} \quad (8.35)$$

**Force on mass element:**

$$m \frac{d^2}{dt^2}(dR(t)) = -\frac{GMm}{(dR(t))^2} = -\frac{4\pi G}{3} \frac{d\rho_0}{R^2(t)} m \quad (8.36)$$

Canceling  $m \cdot d$  gives **momentum equation**:

$$\ddot{R}(t) = -\frac{4\pi G}{3} \frac{\rho_0}{R(t)^2} = -\frac{4\pi G}{3} \rho(t) R(t) \quad (8.37)$$

Multiplying Eq. (8.37) with  $\dot{R}$  and integrating yields the **energy equation**:

$$\frac{1}{2}\dot{R}(t)^2 = +\frac{4\pi G}{3} \frac{\rho_0}{R(t)} + \text{const.} = +\frac{4\pi G}{3} \rho(t) R^2(t) + \text{const.} \quad (8.38)$$

where the constant can only be obtained from GR.

## Problems with the Newtonian derivation:

1. Cloud is implicitly assumed to have  $r_{\text{cloud}} < \infty$

(for  $r_{\text{cloud}} \rightarrow \infty$  the force is undefined)

$\Rightarrow$  violates cosmological principle.

2. Particles move *through* space

$\Rightarrow v > c$  possible

$\Rightarrow$  violates SRT.

## Why do we get correct result?

GRT  $\longrightarrow$  Newton for small scales and mass densities

Since universe is isotropic: scale invariance on Mpc scales

$\Rightarrow$  Newton sufficient (classical limit of GR).

(In fact, point 1 above *does* hold in GR: Birkhoff's theorem).

The exact GR derivation of Friedmanns equation gives:

$$\begin{aligned}\ddot{R} &= -\frac{4\pi G}{3}R\left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right] \\ \dot{R}^2 &= +\frac{8\pi G\rho}{3}R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right]\end{aligned}\tag{8.39}$$

*Notes:*

1. For  $\Lambda = 0$ : Eq. (8.39)  $\longrightarrow$  Eq. (8.38).
2.  $k$  determines the **curvature of space** (and is *not* an integer here!).
3. The **density**,  $\rho$ , includes the contribution of all different kinds of energy (remember mass-energy equivalence!).
4. There is **energy associated with the vacuum**, parameterized by the parameter  $\Lambda$ .

In Eq. 8.49, it will be shown that the Hubble parameter can be expressed as  $H(t) = \frac{\dot{R}}{R}$  .

For  $\Lambda = 0$ , it evolves as:

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2(t) = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2}\tag{8.40}$$

# The critical density

Solving Eq. (8.40) for  $k$ :

$$\frac{R^2}{c} \left( \frac{8\pi G}{3} \rho - H^2 \right) = k \quad (8.41)$$

$\Rightarrow$  Sign of curvature parameter  $k$  only depends on density,  $\rho$ . With

$$\rho_c = \frac{3H^2}{8\pi G} \quad \text{and} \quad \Omega = \frac{\rho}{\rho_c} \quad (8.42)$$

$$\Omega > 1 \Rightarrow k > 0 \Rightarrow \text{closed universe}$$

it is easy to see that:  $\Omega = 1 \Rightarrow k = 0 \Rightarrow \text{flat universe}$

$$\Omega < 1 \Rightarrow k < 0 \Rightarrow \text{open universe}$$

$\rho_c$  is called the critical density.

For  $\Omega \leq 1$  the universe will expand until  $\infty$ ,

For  $\Omega > 1$  we will see the “big crunch”.

Current value of  $\rho_c$ :  $\sim 1 \times 10^{-23} \text{ g cm}^{-3}$  (3...10 H-atoms  $\text{m}^{-3}$ ).

# Cosmology and General Relativity

Einstein's **principle of equivalence** found that space-time is **curved** in the presence of a gravitational field. The mass density of the Universe tells us about the geometry of space-time.



Curvature of space (in 2-d) can be described in terms of angles/areas of a triangle. If the 3 angles on the triangles drawn below are  $\alpha, \beta, \gamma$  then

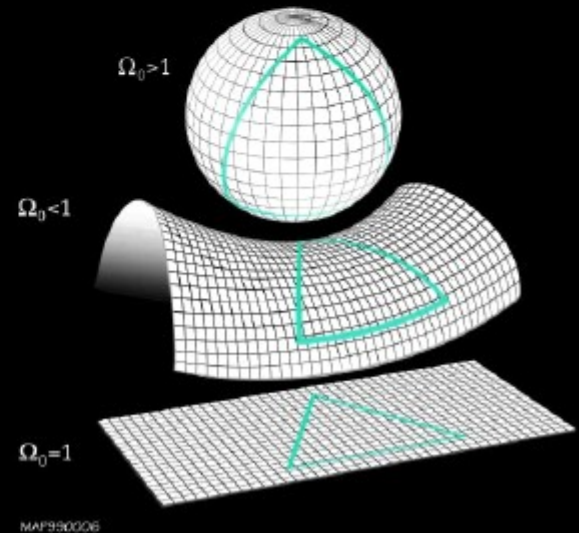
$$\alpha + \beta + \gamma = 180^\circ + (\kappa A / r_c^2)$$

where  $\kappa$  is the curvature constant and  $r_c$  is the radius of curvature

**$\kappa = +1$  or  $\Omega > 1$ , Bound/Closed Universe**

**$\kappa = -1$  or  $\Omega < 1$ , Open Universe**

**$\kappa = 0$  or  $\Omega = 1$ , Flat/Critical Universe**



# The Cosmological Constant

Einstein used **General Relativity** to improve upon Newtonian cosmology and provide a description of the Universe as a whole. He introduced the **cosmological constant  $\Lambda$**  into his **GR equations** since he found (as in the Newtonian case) that if the density of the Universe is non-zero, the Universe must be expanding. As this was before Hubble's work and most believed in a Steady State Universe, this constant allowed for a **non-zero density** and a **static Universe**.

Newtonian equation of motion:

$$\ddot{R} = -\frac{4\pi}{3} \frac{G\rho_0}{R^2}$$

Let  $\rho = \rho_0 - \Lambda/8\pi G$  and a non-zero density is possible with  $\dot{R}=0$

After Hubble's discovery, Einstein called this his "greatest blunder".

# Einstein Equation and models of the Universe

Einstein's equation (analogous to Newtonian case):

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

In General Relativity, not only density, but also pressure is a source of gravity.

$$q_0 = -\frac{1}{H_0^2} \left[ \frac{\Lambda c^2}{3} - 4\pi G \left( \frac{\rho_0}{3} + \frac{P_0}{3c^2} \right) \right]$$

This reduces to classical case when  $\Lambda=P=0$

Several cosmological theories were developed upon GR principles

- de Sitter: flat Universe and positive  $\Lambda$
- Friedmann:  $\Lambda=0$  and zero pressure (low density)
- Lemaitre: non-zero density and  $\Lambda$



# Expressing Distances in an Expanding Universe

The geometry and expansion rate of the Universe effects angular sizes and distances measured.

$D_A = L(\text{size})/\theta(\text{angular size}) \rightarrow \text{Angular Distance}$

$D_L = \sqrt{\text{Luminosity}/4\pi\text{Flux}} \rightarrow \text{Luminosity Distance}$

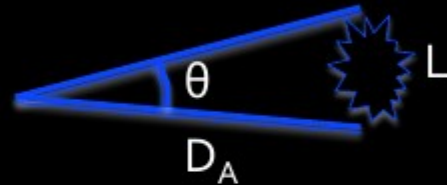
$$D_L = (1+z)^2 D_A$$

If  $\Lambda = 0$ , then

$$D_L = 2c/H_0 [z/(G+1)] \{1+[z/(G+1)]\} \text{ where } G = (1 + 2q_0 z)^{1/2}$$

If non-zero  $\Lambda$ , use a Cosmology Calculator:

<http://www.astro.ucla.edu/~wright/CosmoCalc.html>



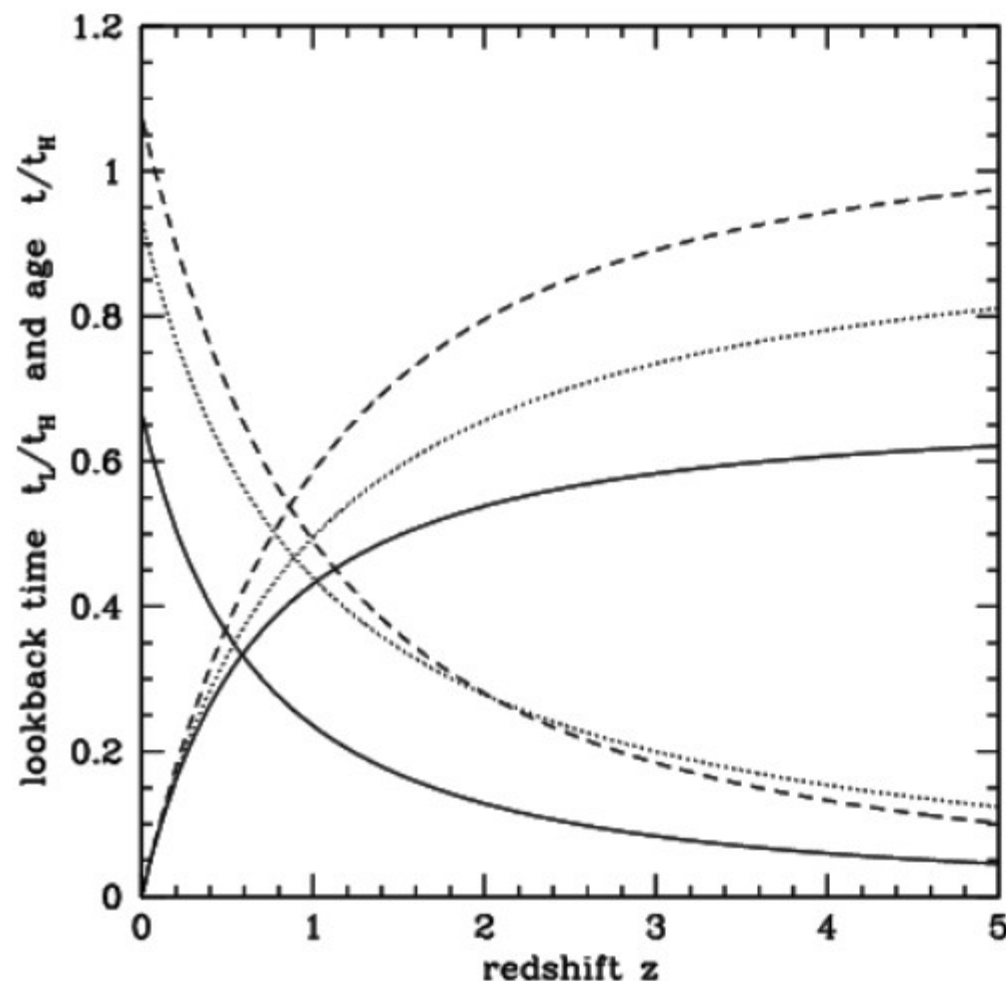


Figure 6: The dimensionless lookback time  $t_L/t_H$  and age  $t/t_H$ . Curves cross at the redshift at which the Universe is half its present age. The three curves are for the three world models,  $(\Omega_M, \Omega_\Lambda) = (1, 0)$ , solid;  $(0.05, 0)$ , dotted; and  $(0.2, 0.8)$ , dashed.

**Lookback time**  
 $t_L$  is difference  
 between age of  
 Universe now  
 and age  $t_e$  when  
 photons left  
 emitting source

*flat,  $\Lambda=0$  – solid*  
*open,  $\Lambda=0$  – dotted*  
*flat, non-zero  $\Lambda$  – dashed*

# How to determine if the Universe is Open, Closed or Flat?

## 1) Add up all matter in the Universe to determine density

Luminous matter: only  $\sim 1\%$  of  $\rho_{\text{crit}}$

Dark matter?

Table 20.2.   Mass-to-light ratios on different scales.	
Scale	$M/L$ (solar units)
Milky Way to Sun	3
Spiral galaxy disk	10
Elliptical galaxy	30
Halo of giant elliptical	40
Rich cluster of galaxies	200
To close the universe	1200

Still a factor of  $\sim 5$  short to close the Universe with “known” DM (where dynamical evidence exists)

2) look for gravitational effects of the mass of the Universe, such as the slowing down of the expansion rate:  $\dot{H}(t_0)$

$$q_0 = - \left[ \frac{\dot{H}(t_0)}{H_0^2} + 1 \right]$$

3) **Measure the curvature of space-time by surveying the Universe on large scales:** galaxy counts and sizes show that the radius of curvature is comparable to or greater than the size of the observable Universe  $\Rightarrow$  **essentially flat!**

*Why should we be so close to the boundary ( $\rho_o = \rho_{crit} = 3H^2/(8\pi G)$ )?*

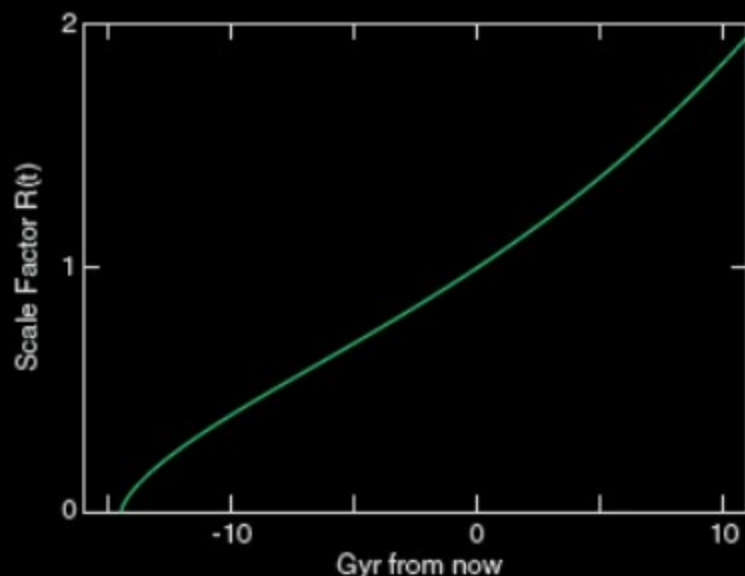
With a non-zero  $\Lambda$ , it is still possible to have a flat universe and low mass density

$$\Omega_M = \rho_M / \rho_{crit}$$

Let  $\rho_{\Lambda, eff} = \Lambda / 8\pi G$   
then  $\Omega_\Lambda = \Lambda / 8\pi G \rho_{crit}$

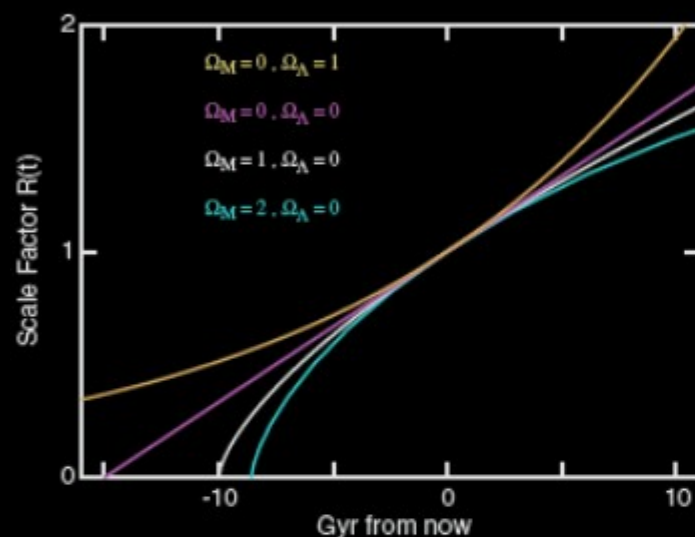
Total density parameter for the Universe is then

$$\Omega_{tot} = \Omega_M + \Omega_\Lambda$$



**Fig 20.10.** Scale factor vs. time for the cosmological model which best fits current data. The model has a Hubble constant of 65 km/s/Mpc and  $\Omega_M = 0.3$ ,  $\Omega_\Lambda = 0.7$ . [© Edward L. Wright, used with permission]

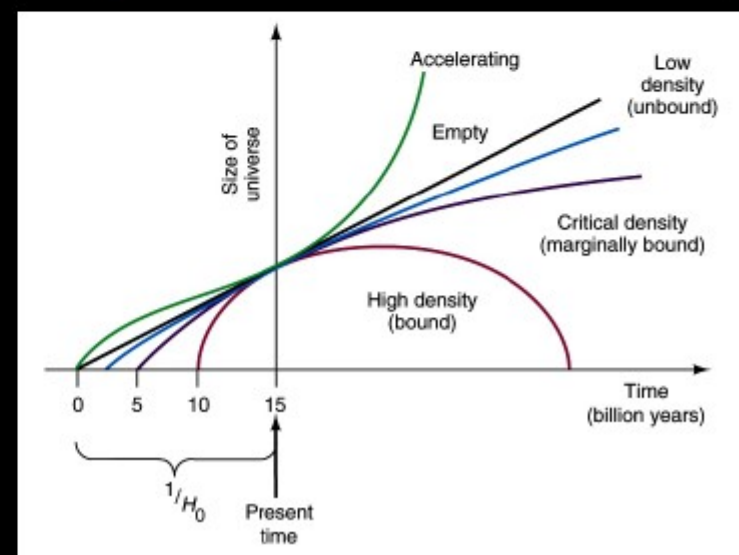
Einstein equations are integrated to give  $R(t)$  by looking at limiting cases (i.e. zero cosmological constant and zero density). Each case must be evaluated at zero, + and - curvature



**Fig 20.9.** Scale factor vs. time for various cosmological models. Models are all chosen to have  $R = 1$ , now, and a Hubble constant = 65 km/s/Mpc (so they all have the same slope now). In terms of the density parameter (defined in equation 20.45), the models are (from top to bottom): ( $\Omega_M = 0, \Omega_\Lambda = 1$ ), ( $\Omega_M = 0, \Omega_\Lambda = 0$ ) ( $\Omega_M = 1, \Omega_\Lambda = 0$ ) ( $\Omega_M = 2, \Omega_\Lambda = 0$ ). [© Edward L. Wright, used with permission]

Table 1. Comparison of Different Cosmological Models

Name	$\Omega_0$	$\Omega_\Lambda$	$t_0$	$q_0$
Einstein-de Sitter	1	0	$\frac{2}{3}t_H$	$\frac{1}{2}$
Empty, no $\Lambda$	0	0	$t_H$	0
Example Open	0.3	0	$0.82t_H$	0.15
Example Closed	2	0	$0.58t_H$	1
Example Flat, Lambda	0.5	0.5	$0.84t_H$	-0.25
Concordance	.27	.73	$\approx 1.001t_H$	-0.6
Steady State	1	0	$\infty$	-1



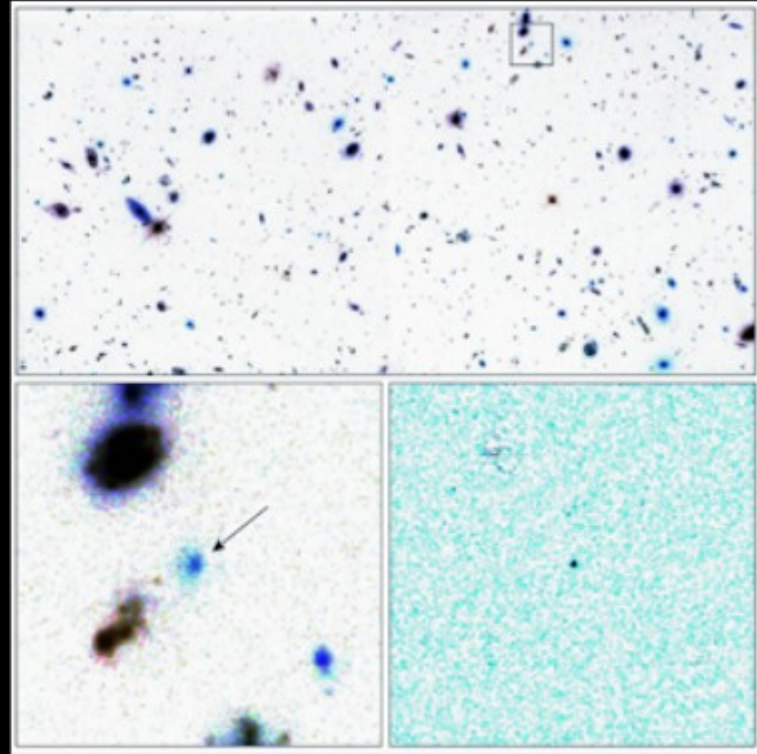
# SN Ia

- Standard candles
- Their intrinsic luminosity is known
- Their apparent luminosity can be measured
- The ratio of the two can provide the luminosity-distance ( $d_L$ ) of the supernova
- The redshift  $z$  can be measured independently from spectroscopy
- Finally, one can obtain  $d_L(z)$  or equivalently the magnitude( $z$ ) and draw a Hubble diagram

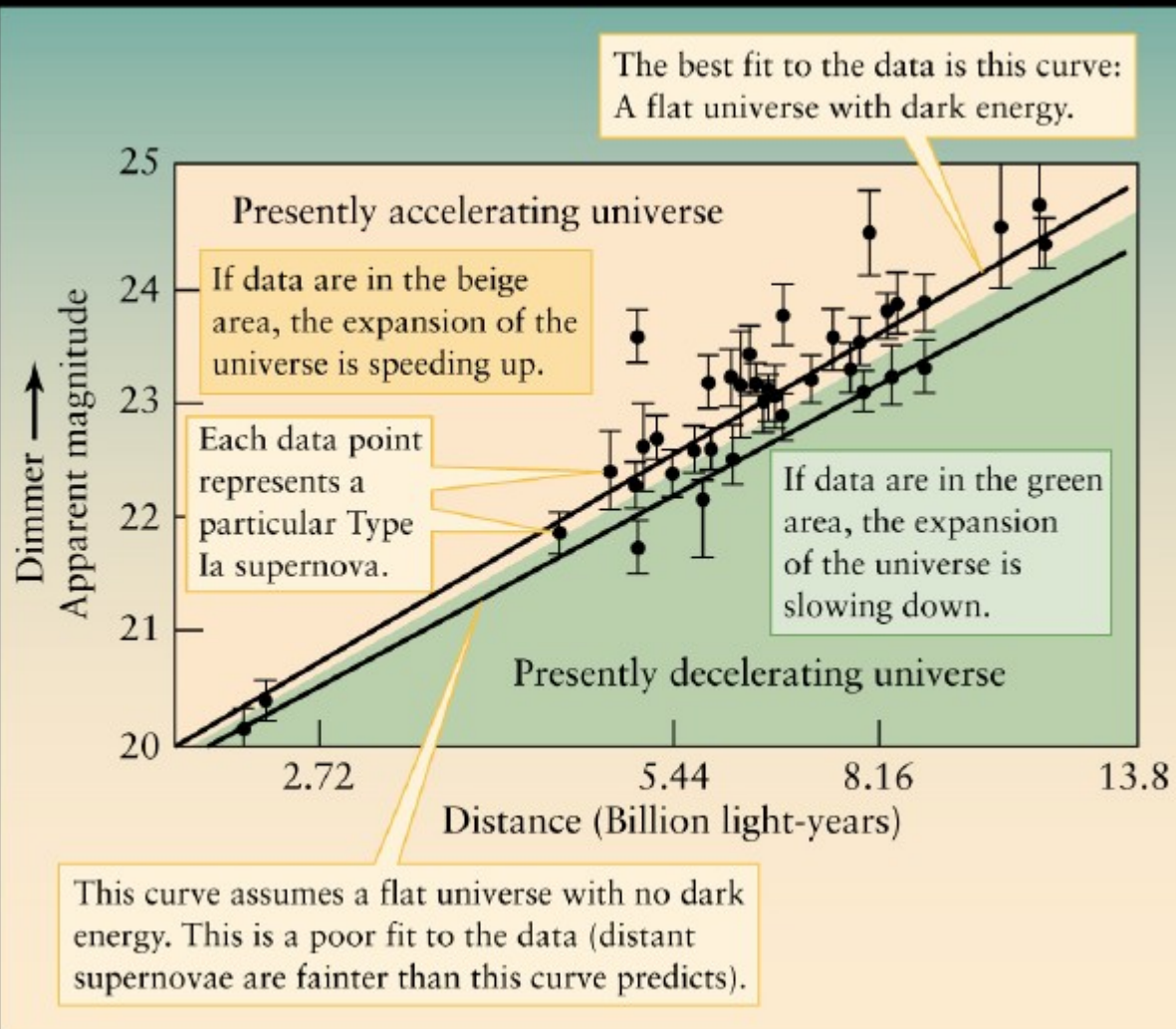




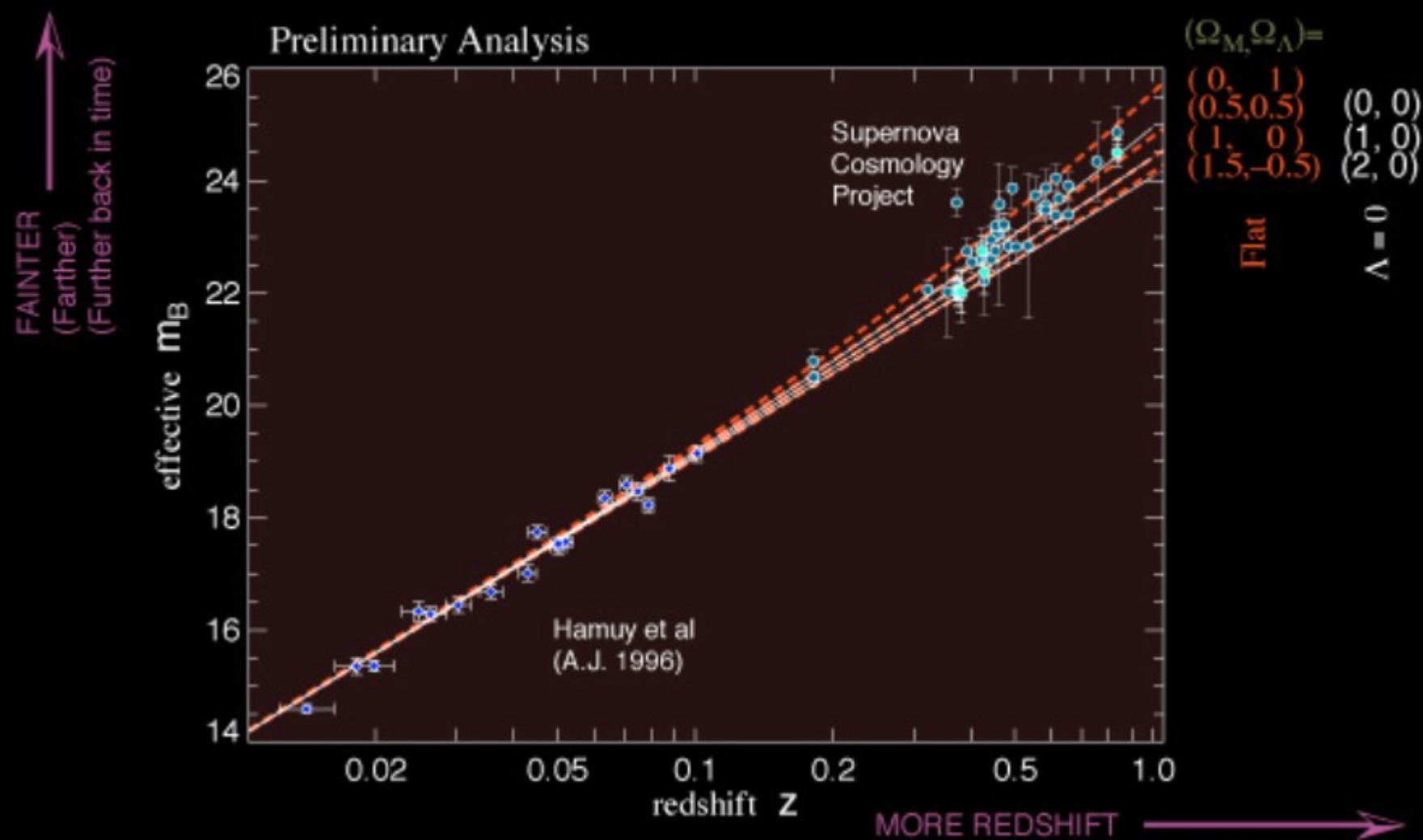
- These HST images show the galaxy in which the supernova SN 1997ff occurred.
- This supernova was dimmer than expected.
- The distance to it is greater than it would be if the universe had been continually slowing down.
- An outward force is acting over vast distances in the universe.

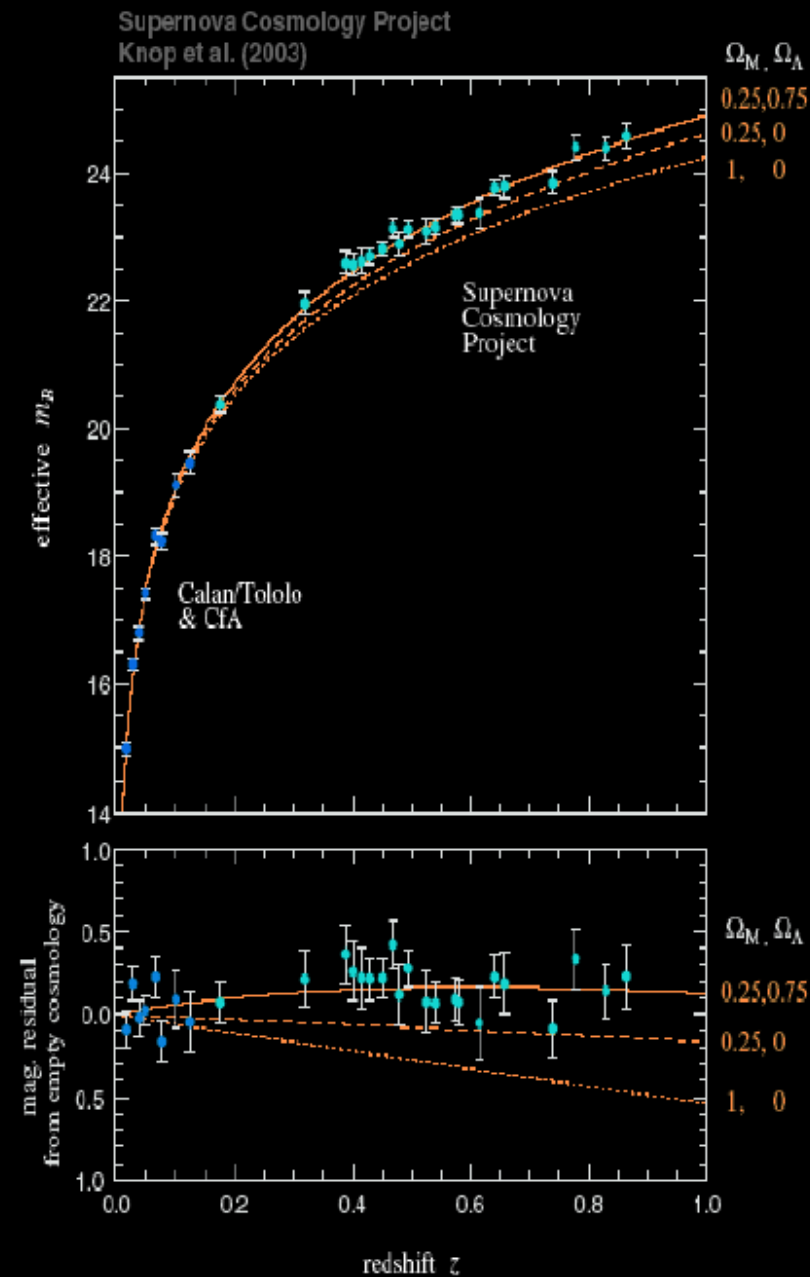
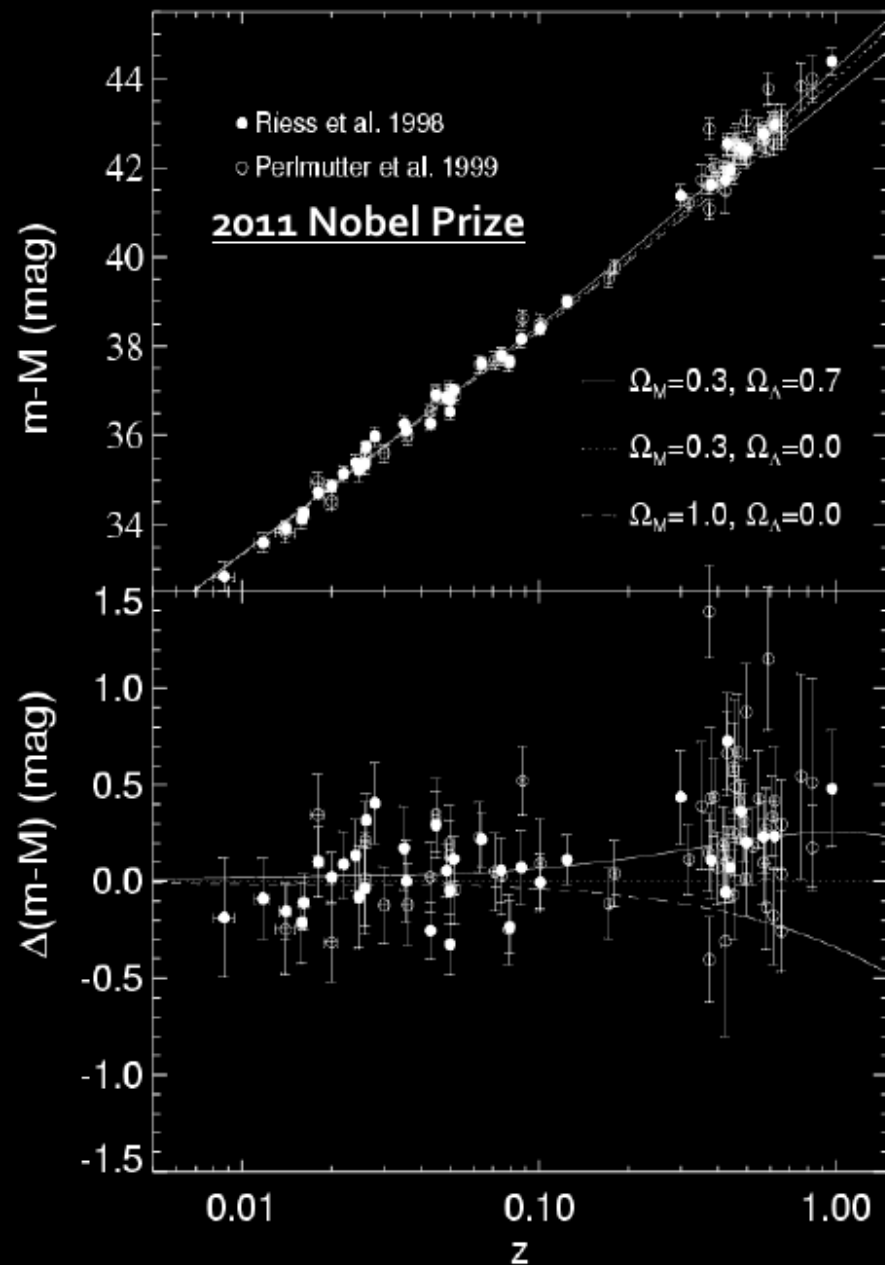




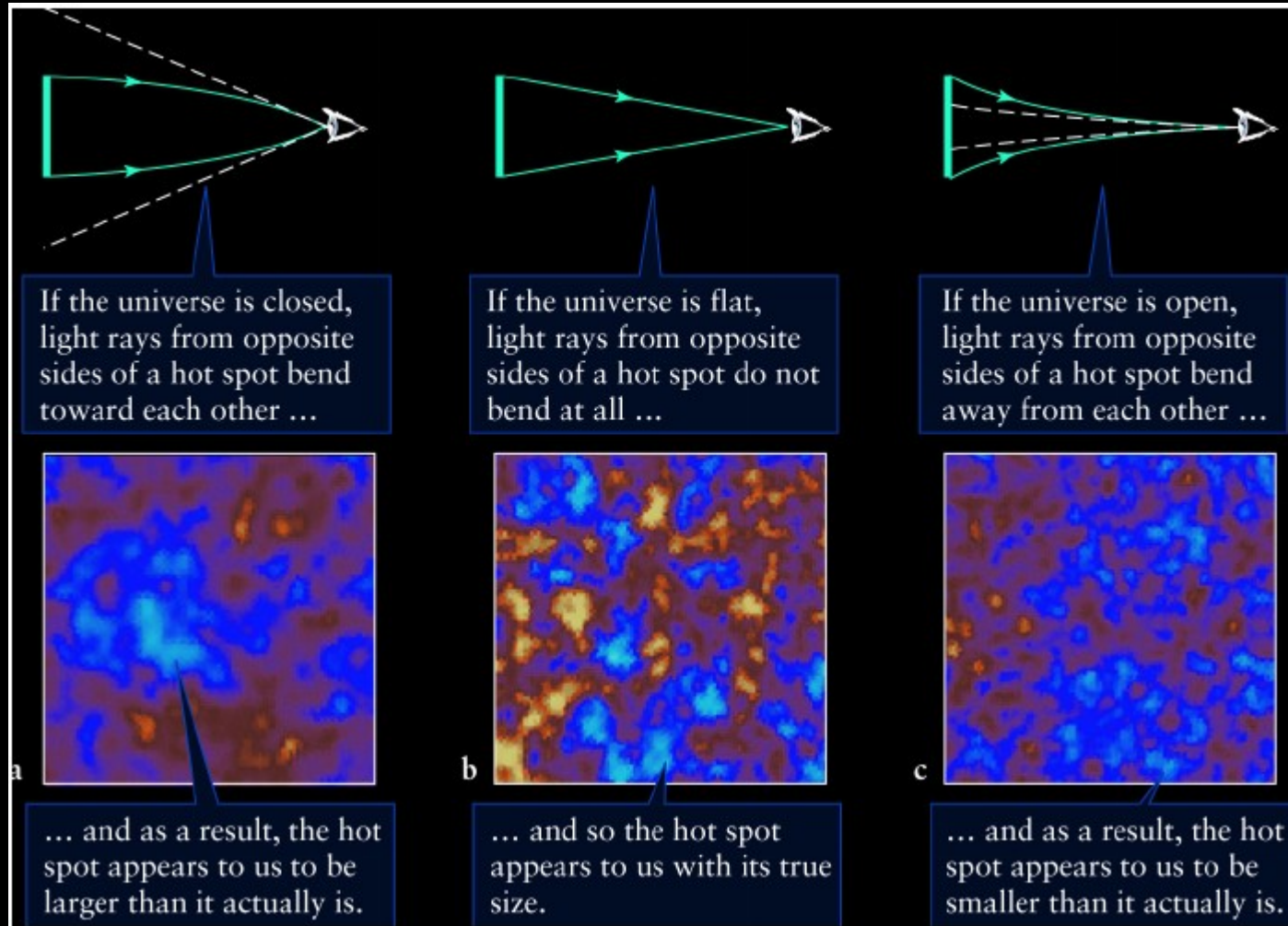


# Hubble Plots





# CBM and curvature

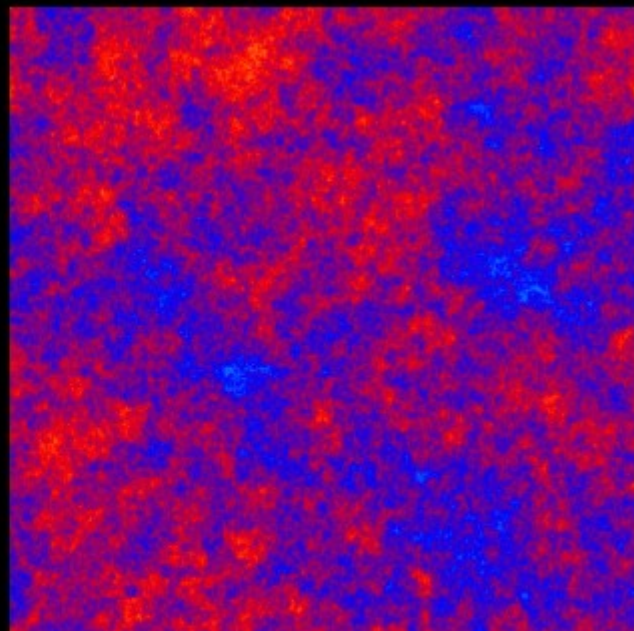


Maybe..

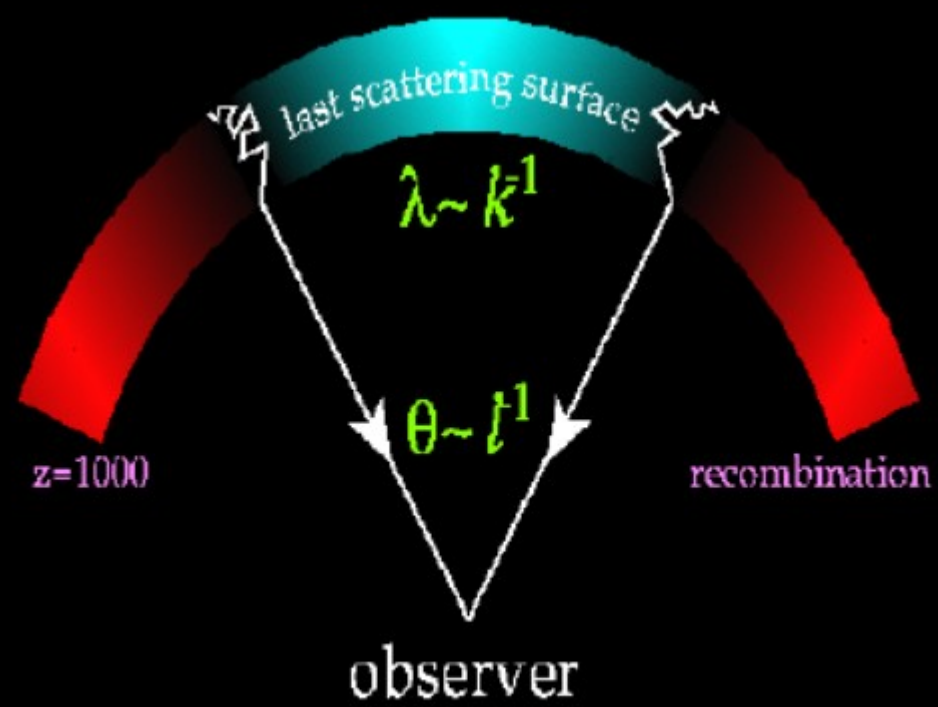


Quantum Mechanics

**Inflation**  
make  $>10^{30}$  times bigger



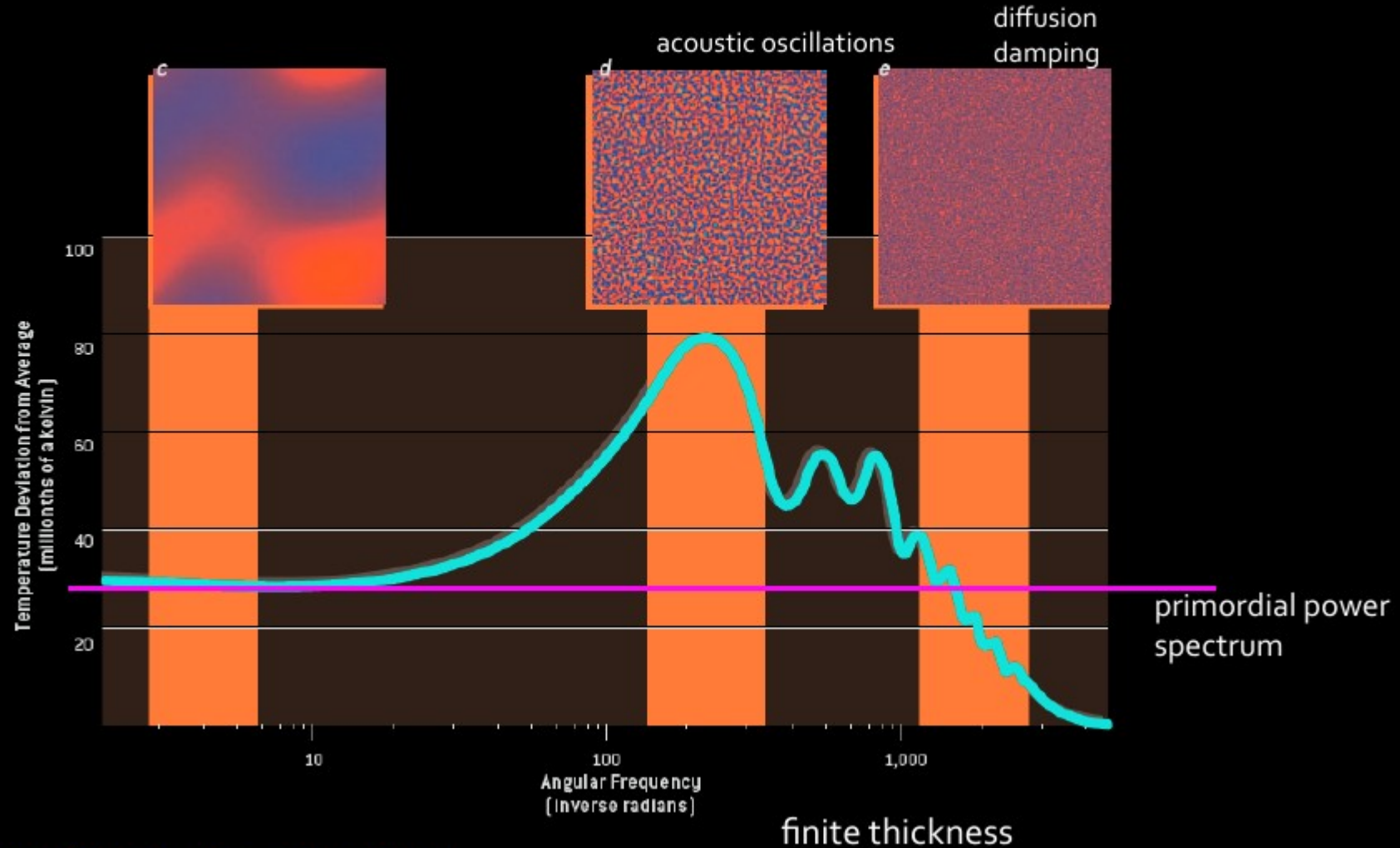
**After inflation**  
Huge size, amplitude  $\sim 10^{-5}$





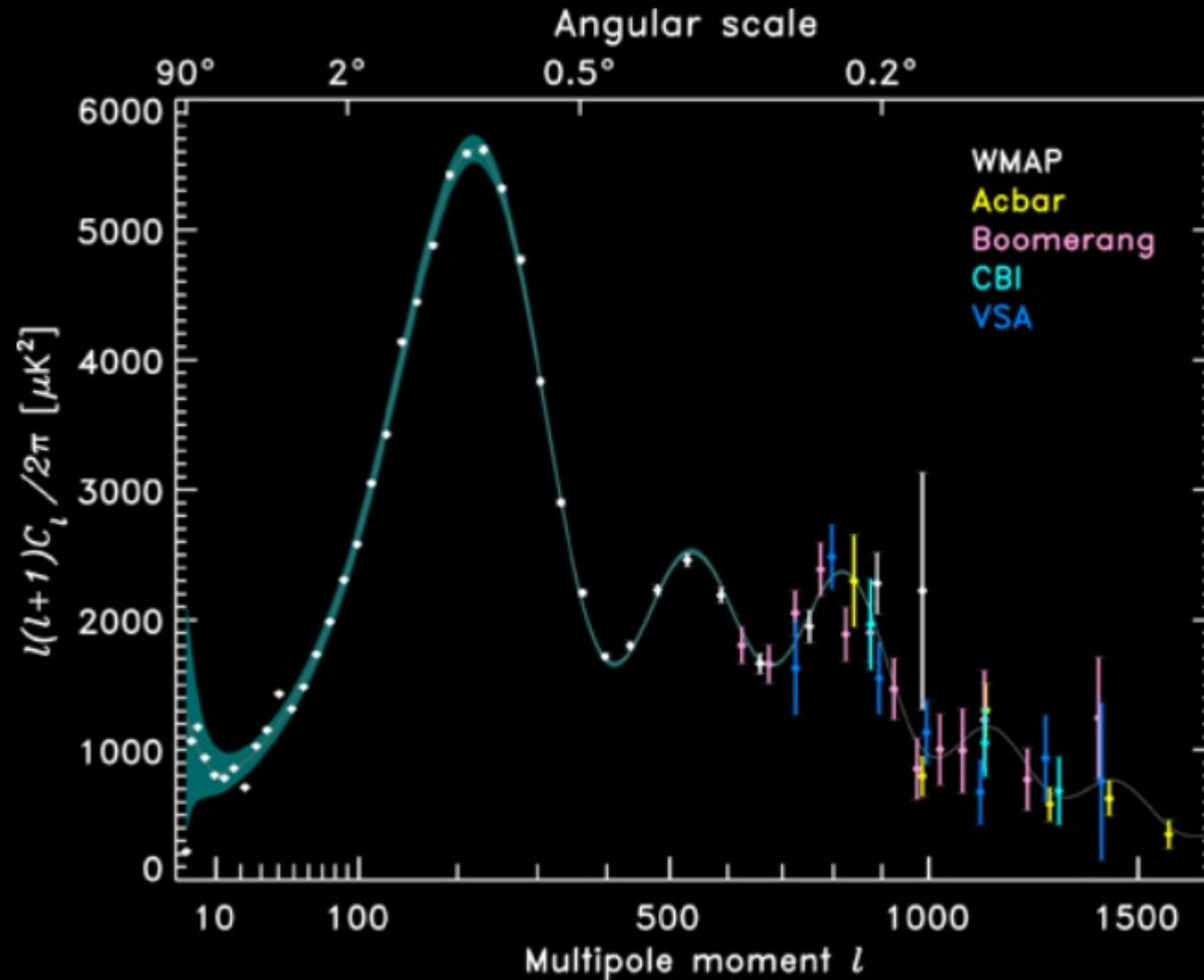
# CMB temperature power spectrum

Primordial perturbations + later physics



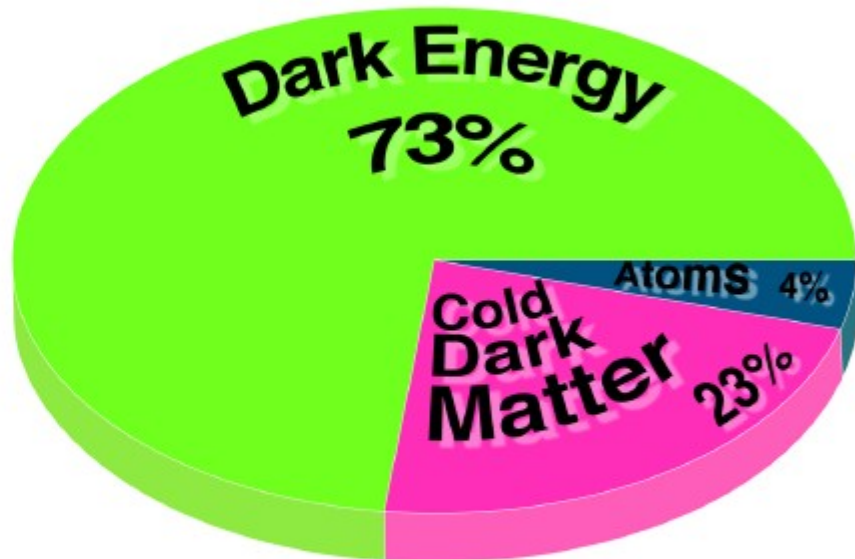
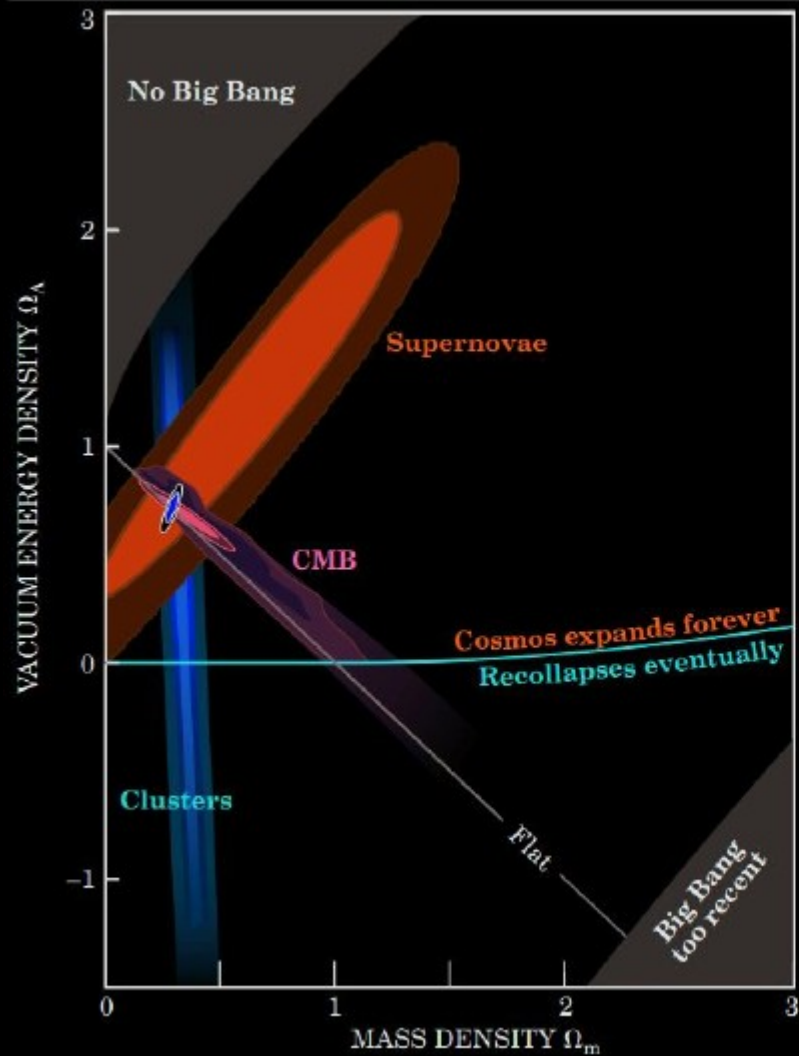


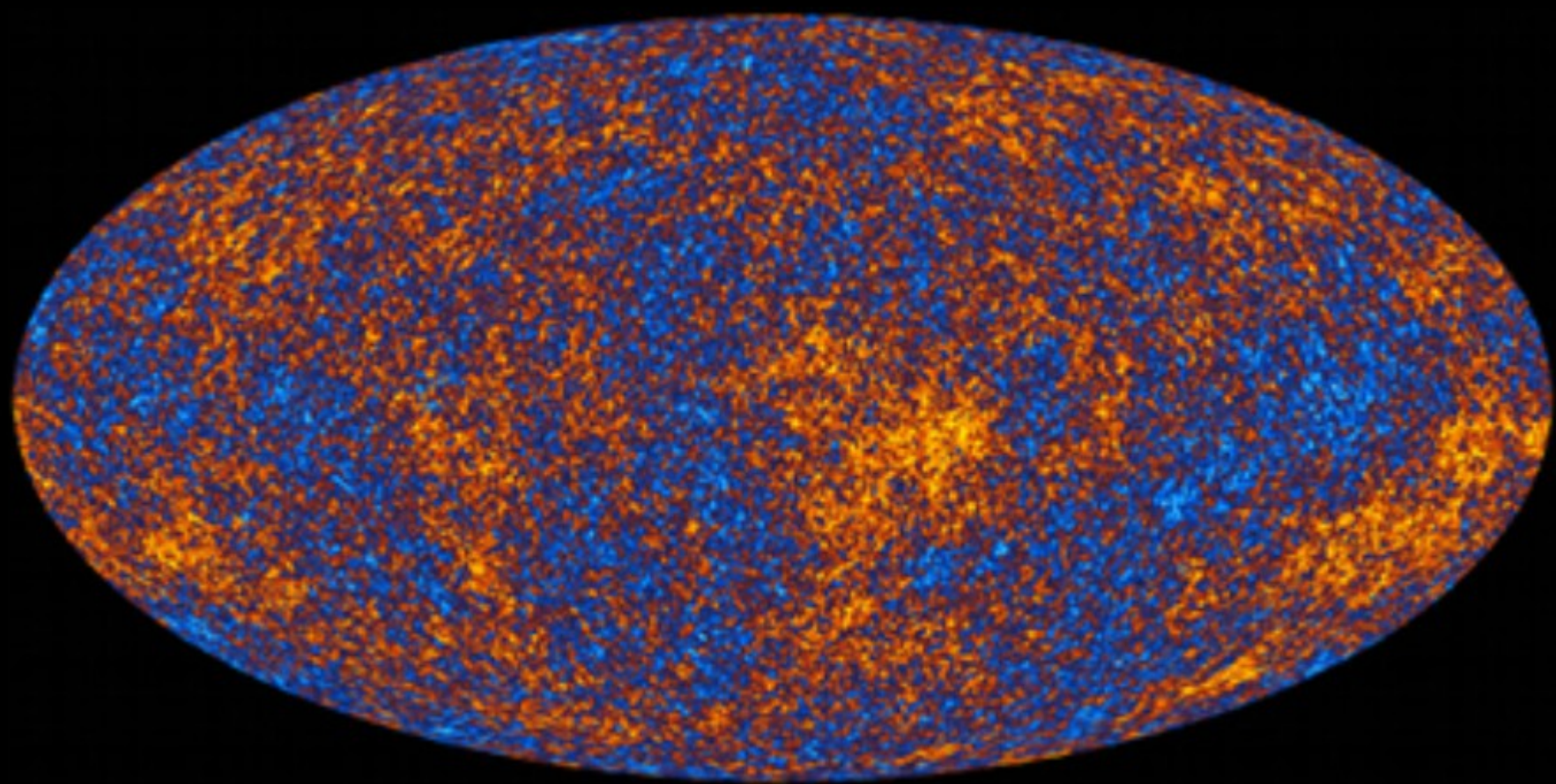
# Anisotropy observations

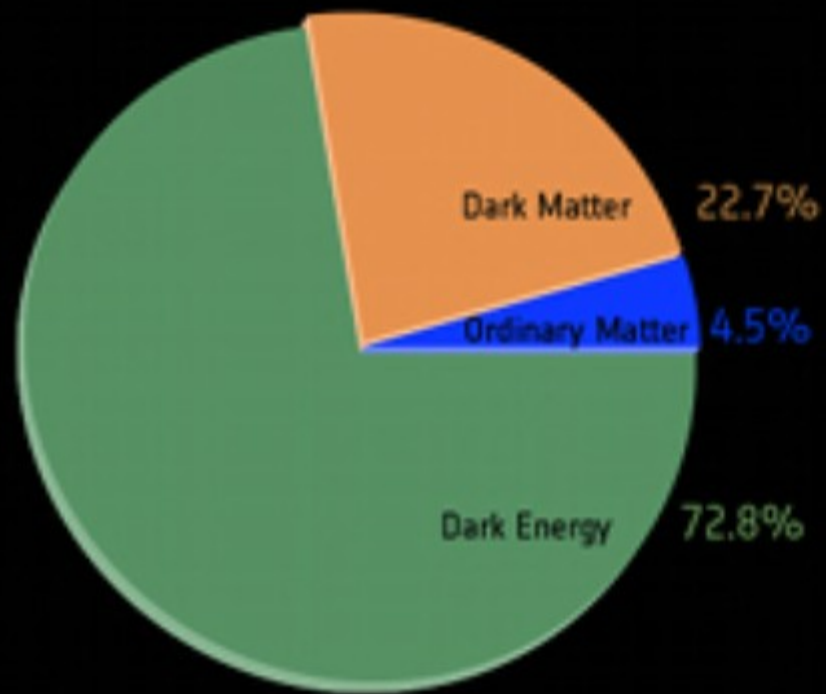


# Dark energy

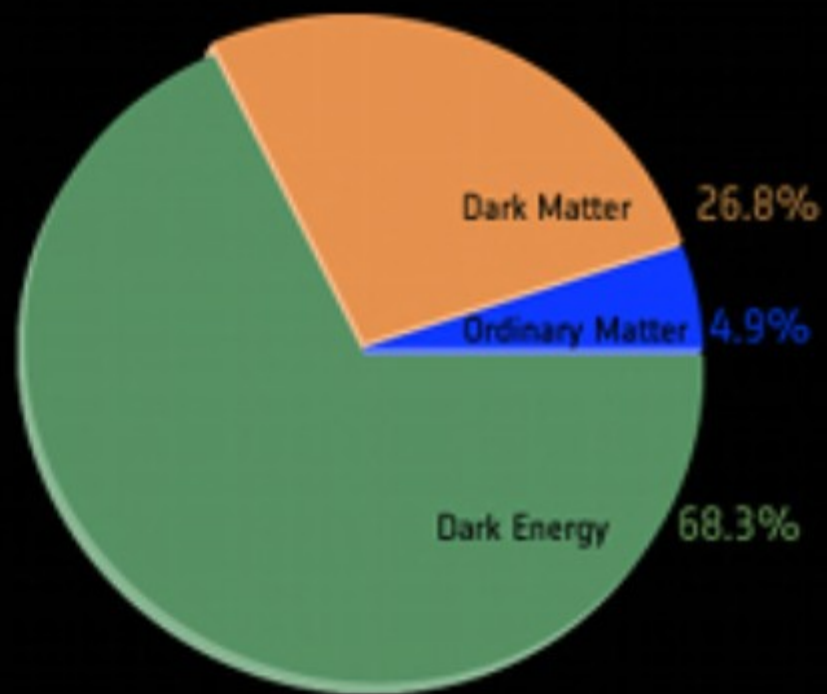
- Counting and weighing clusters of galaxies can infer the matter energy density in the universe
- The matter energy density found is usually around  $\sim 0.3$  the critical density
- CMB best fit model has a total energy density of  $\sim 1$ , so another  $\sim 0.7$  is required
- The same  $\sim 0.7$  is required from combining supernovae data and CMB constraints







Before Planck



After Planck





