Radiation from relativistic particles

Special relativity

$$V = (x, y, z, ct)$$

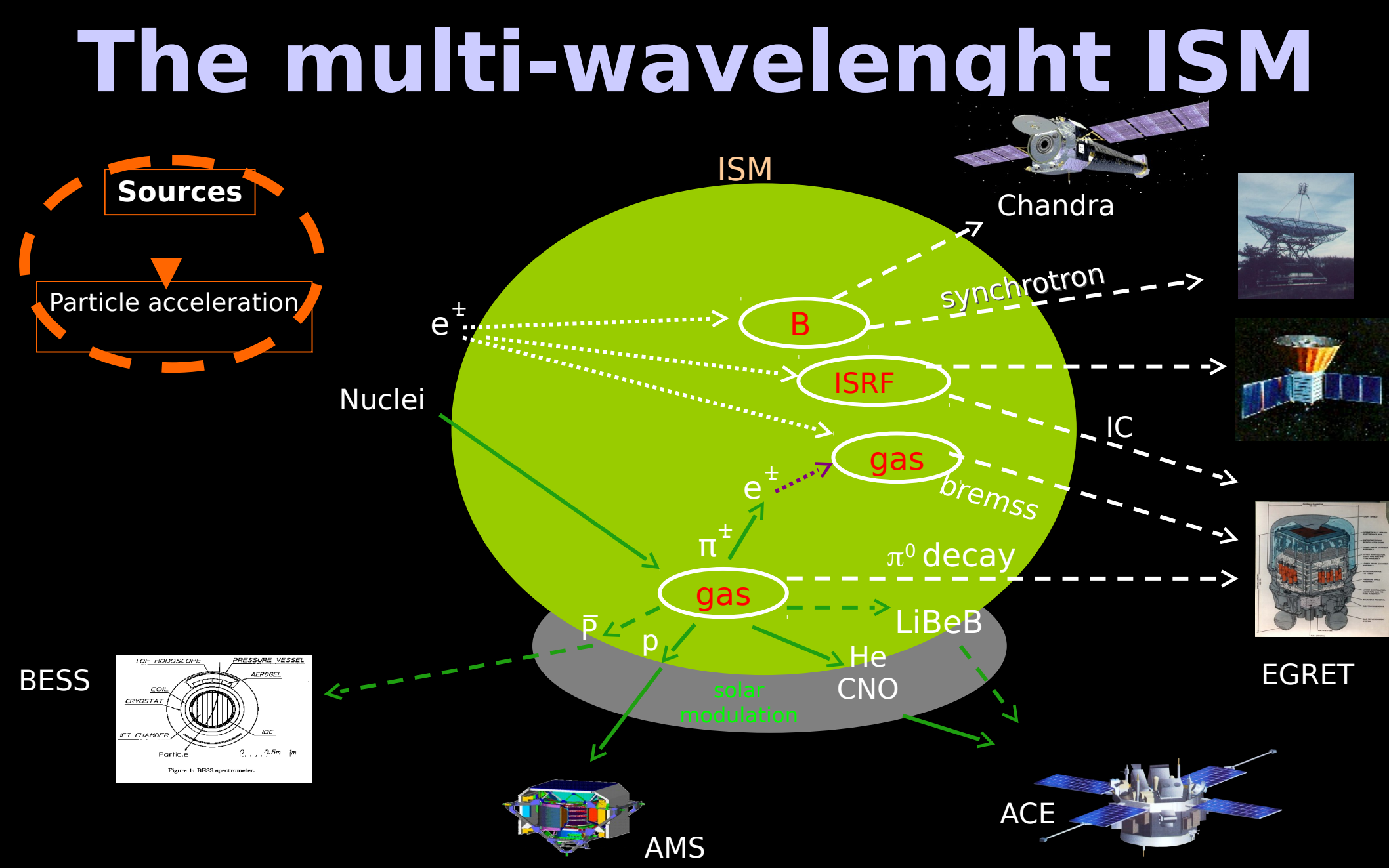
$$V^2 = V^T \eta V = x^2 + y^2 + z^2 - c^2 t^2 = r^2 - c^2 t^2$$

$$\eta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

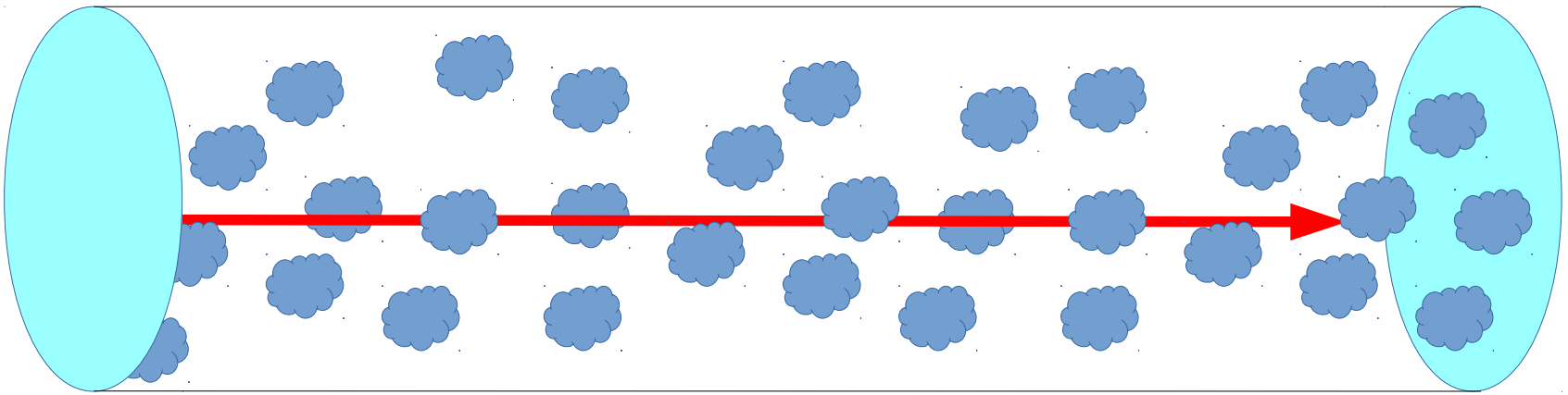
$$P = (p_x, p_y, p_z, \frac{E}{c})$$

$$P^2 = p_x^2 + p_y^2 + p_z^2 - \frac{E^2}{c^2} = p^2 - \frac{E^2}{c^2}$$

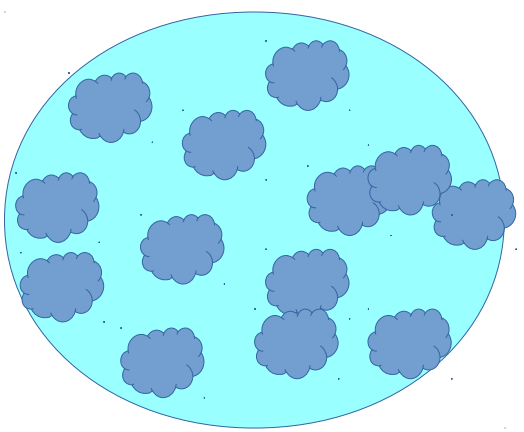
The multi-wavelenght ISM



Interaction Rate



$$P = \frac{\sigma\, N_t}{S} = c\, \sigma\, n_t$$



Emissivity and E. losses

$$P(E_p) = c\,\sigma(E_p)\,n_t \qquad \left[\frac{interactions}{s \cdot particle}\right]$$

$$\epsilon_{ic}(E_\gamma) = \int P(E_p)\,n_p(E_p)\,dE_p = \int c\,\sigma(E_p, E_\gamma)\,n_t\,n_p(E_p)\,dE_p \qquad \left[\frac{ph}{cm^3 s}\right]$$

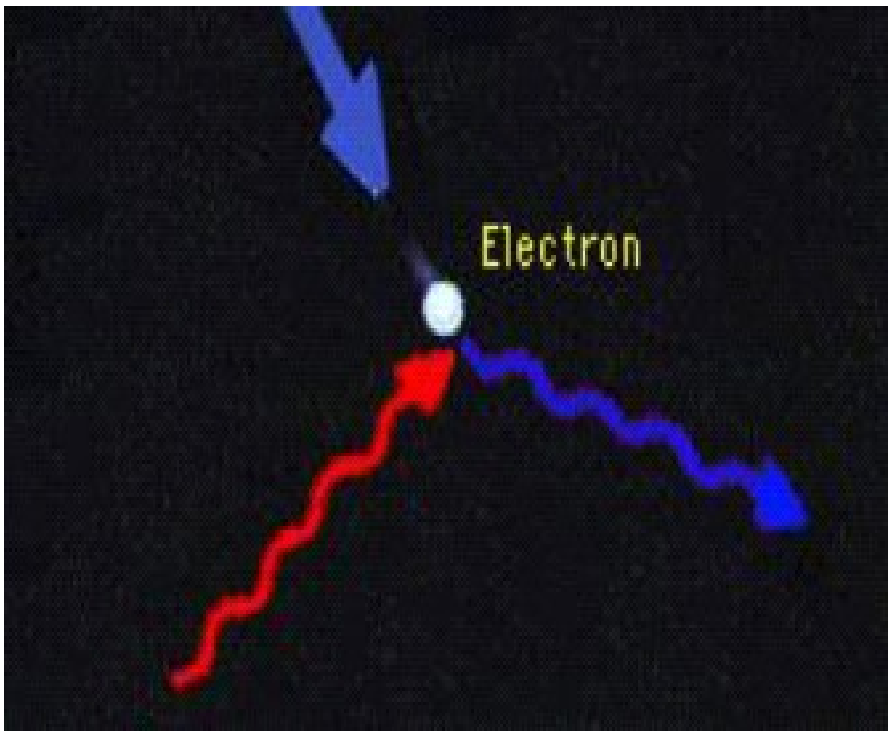
$$\frac{dE_p}{dt} = P(E_p)\,\delta(E_p) = c\,\sigma(E_p)\,n_t\,\delta(E_p) \qquad \left[\frac{erg}{s}\right]$$

Inverse Compton

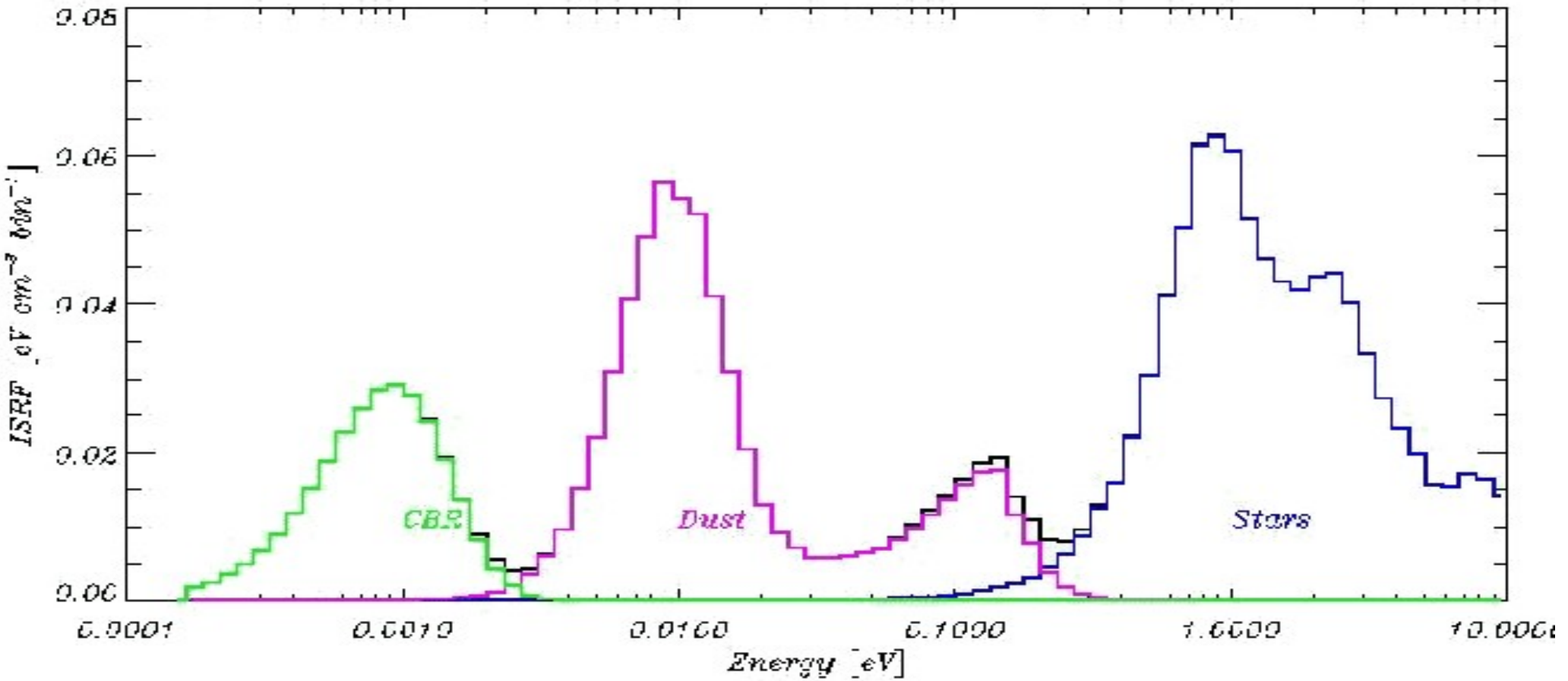
$$E_\gamma = \frac{4}{3}\gamma^2\beta^2 E_{ph} \xrightarrow{\gamma>>1} \frac{4}{3}\gamma^2 E_{ph}$$

$$\epsilon_{ic}(E_\gamma)dE_\gamma = 4\pi\sigma_t n_{ph} I(E_e)dE_e$$

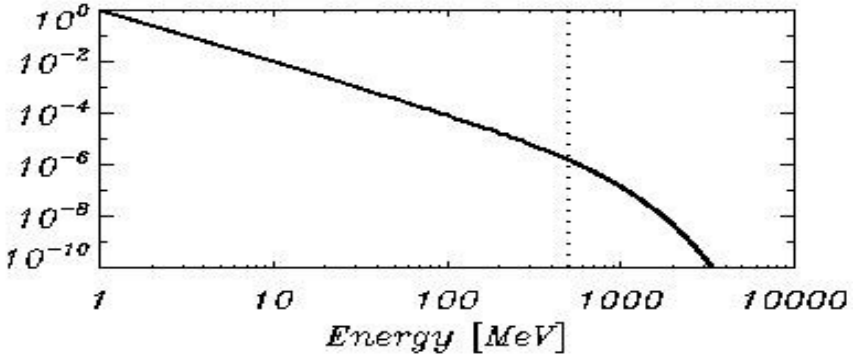
$$\frac{dE_p}{dt} = cn_{ph}\sigma_t E_\gamma = \frac{4}{3}cU_{ph}\sigma_t\gamma^2$$



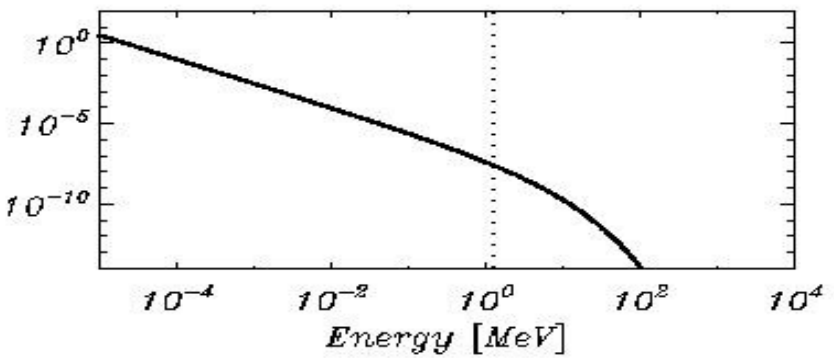
The Interstellar Radiation Field



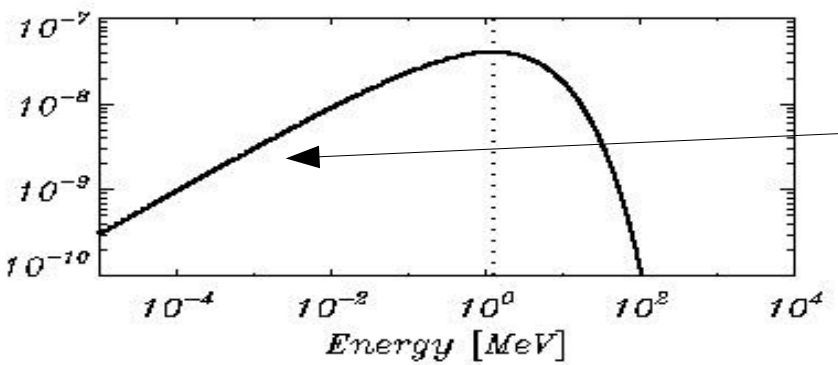
Strong, Moskalenko, & Reimer (2000)



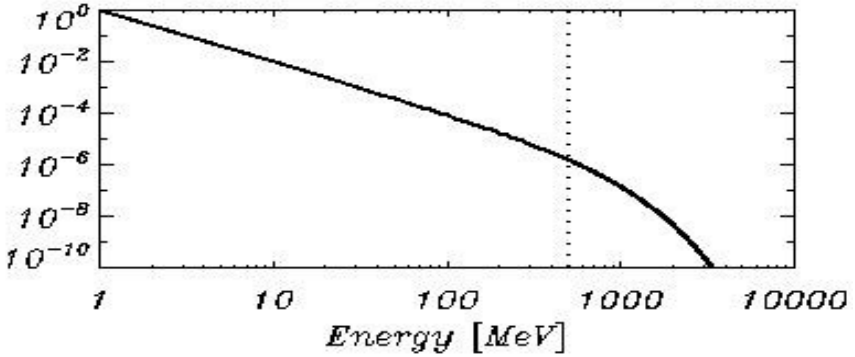
$$I_e \sim E^{-a}$$

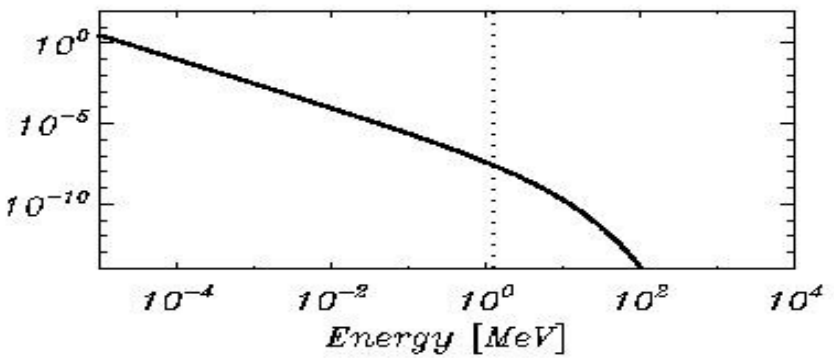


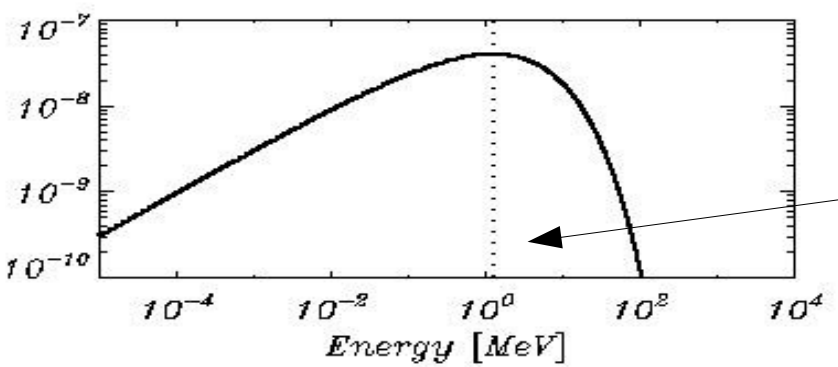
$$I_\gamma \sim E^{-(a+1)/2}$$



$$I_\gamma E^2 \sim E^{(3-a)/2}$$

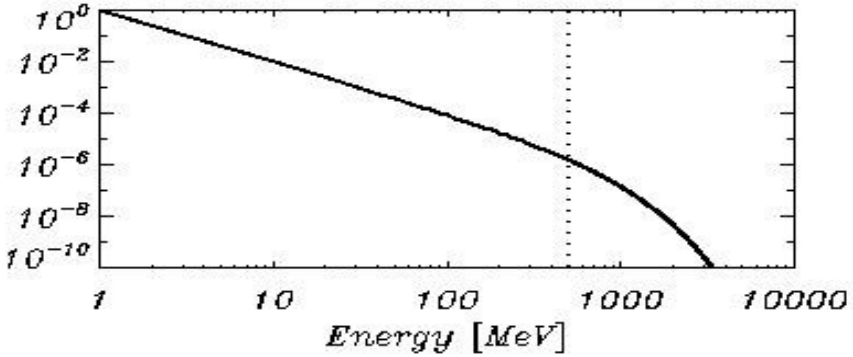


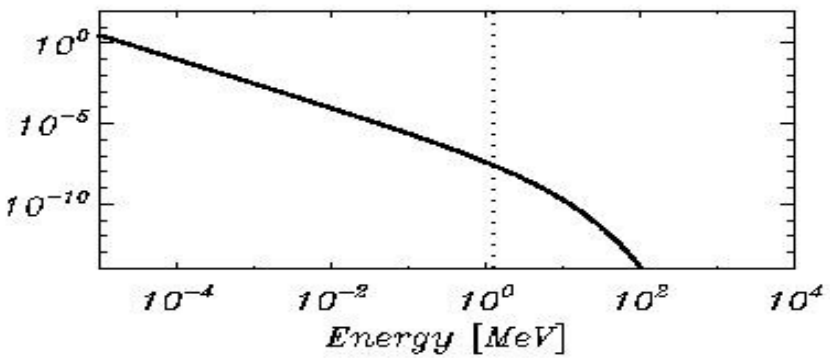


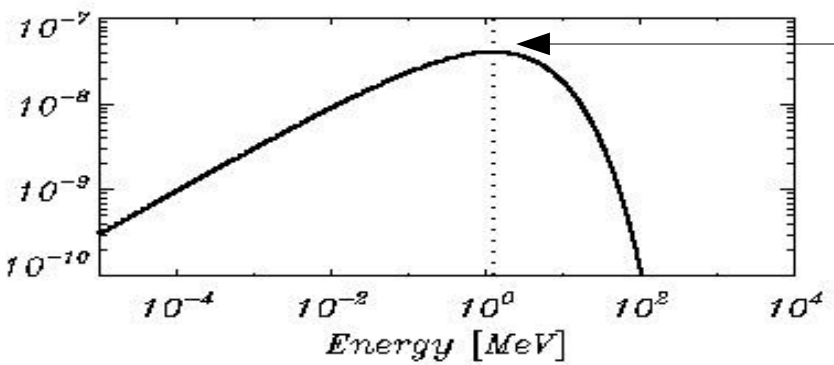


Cut off !

$$\nu_{ic} = \frac{4}{3}\gamma^2 \nu_{ph}$$


10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1
10
100
1000
10000
Energy [MeV]


10^{0}
10^{-5}
10^{-10}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
Energy [MeV]

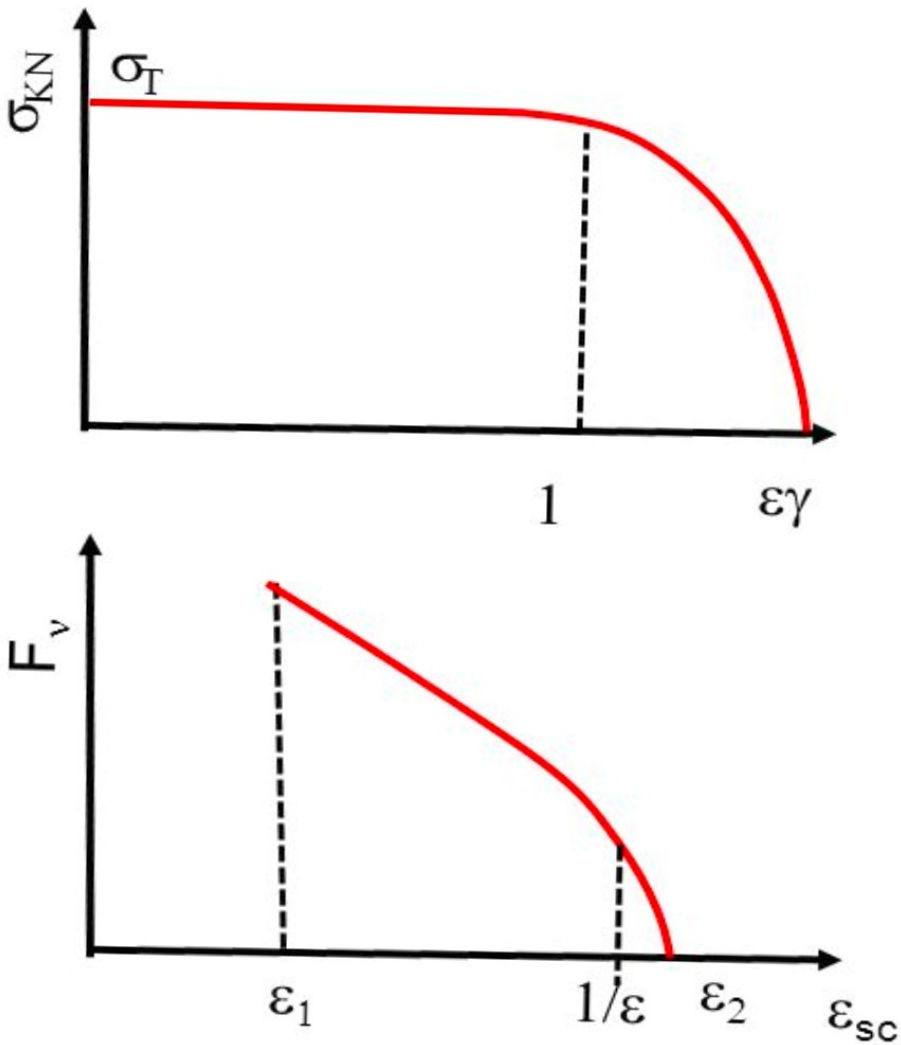

10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
Energy [MeV]

~ *U*

Klein–Nishina cross section

$$\gamma E_{ph} \sim m_e c^2$$

$$\sigma_{KN} = \frac{\pi^2 r_e^2}{h\nu}\left(\ln 2h\nu + \frac{1}{2}\right).$$

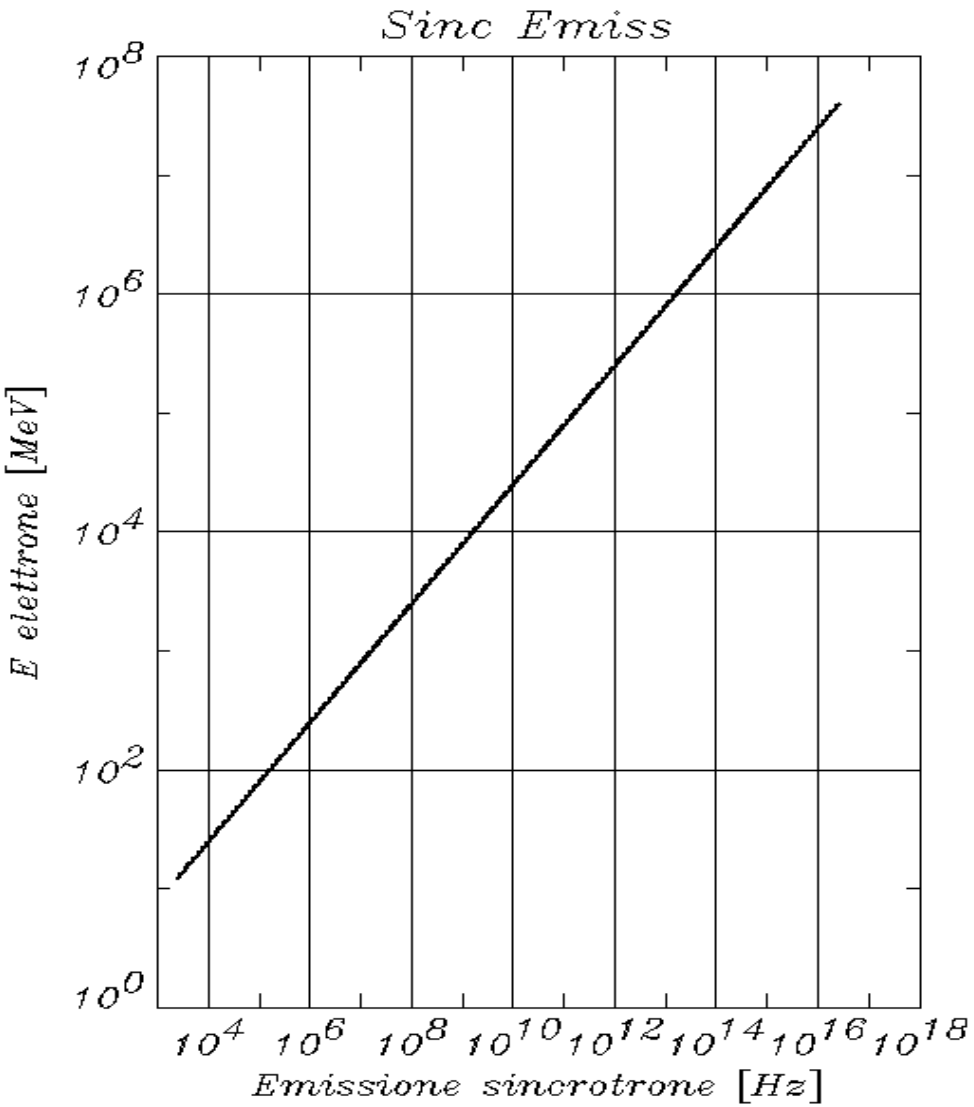


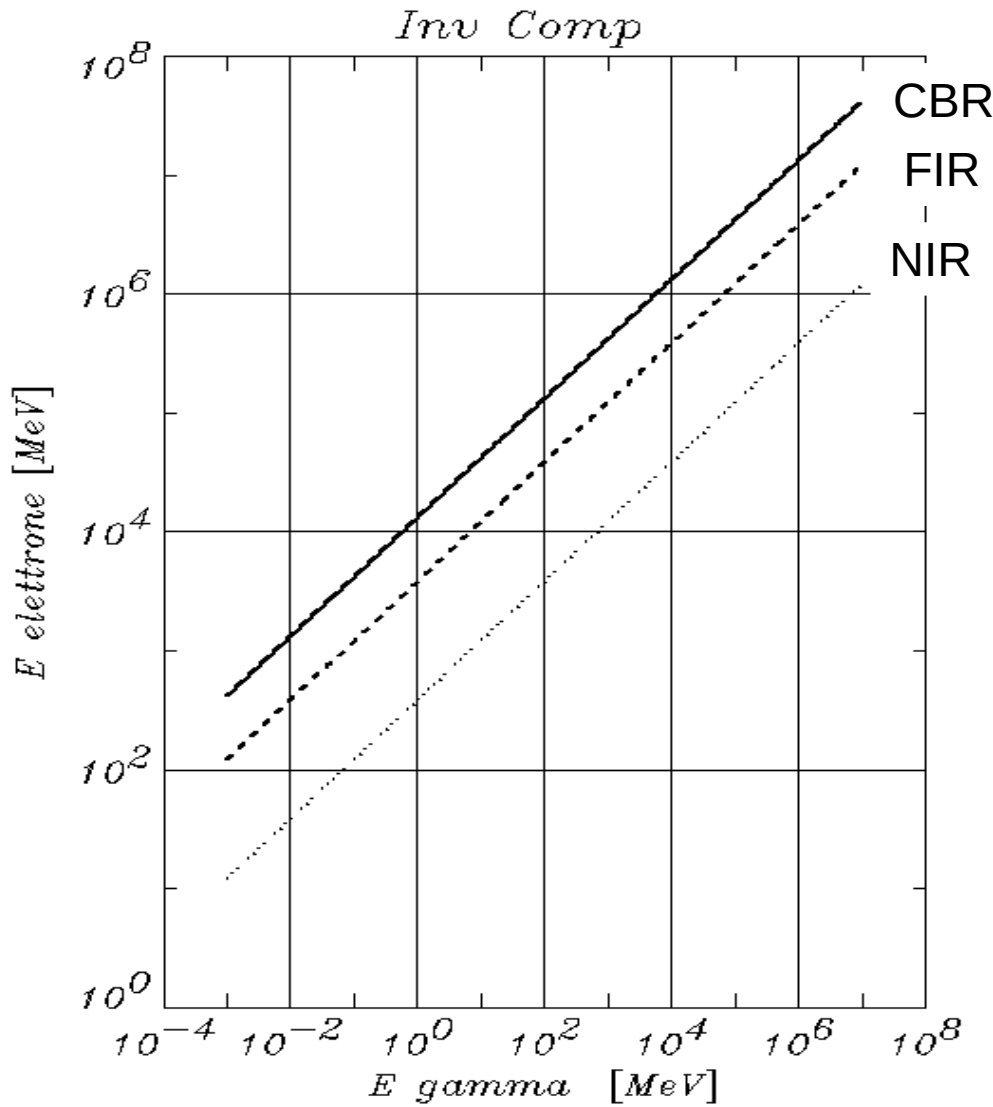
Synchrotron

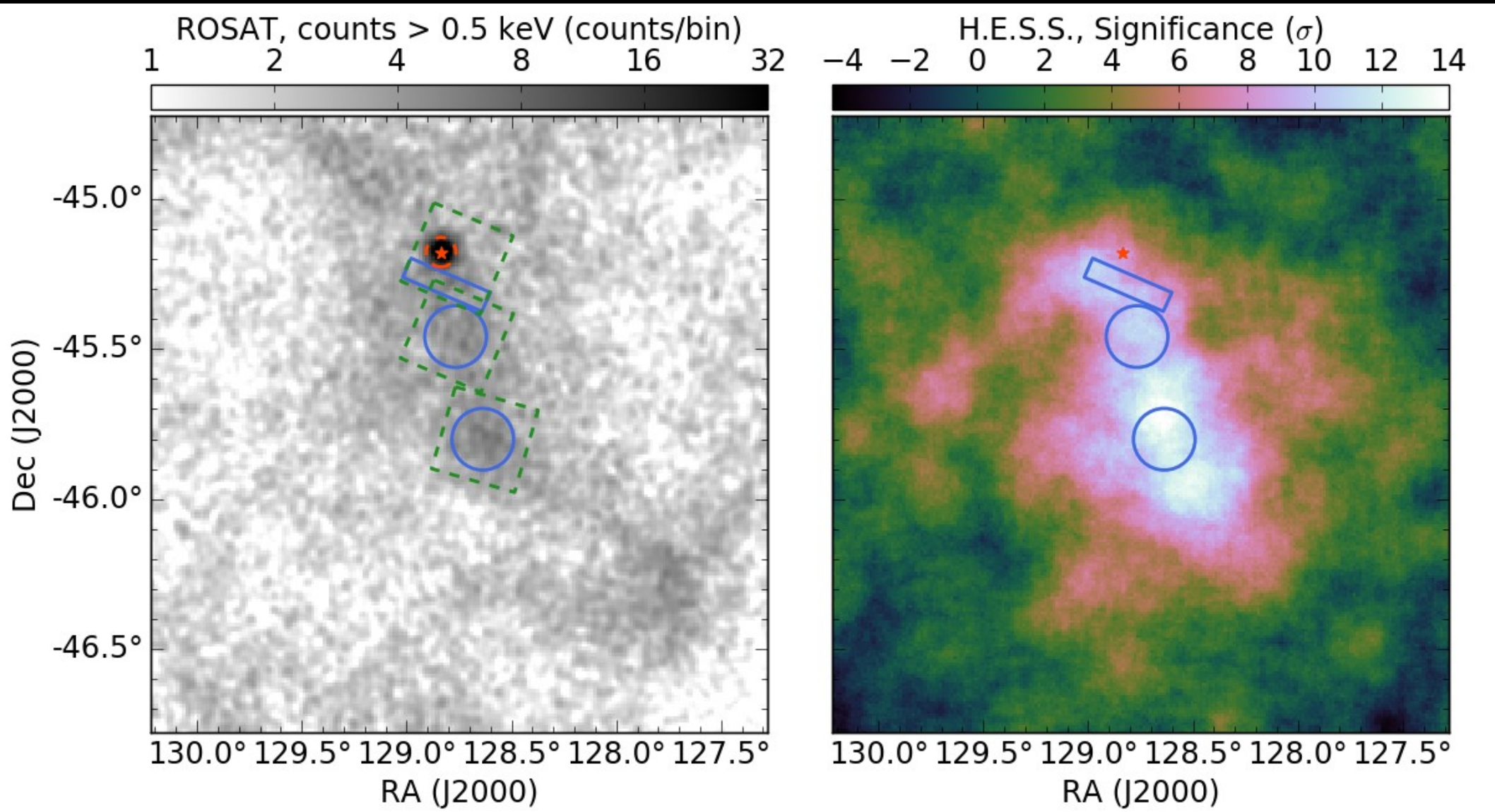
$$\nu_{syn} = \frac{4}{3}\gamma^2 \nu_{gyr} \qquad \nu_{gyr} = \frac{e\ B}{2\ \pi\ m_e\ c}$$

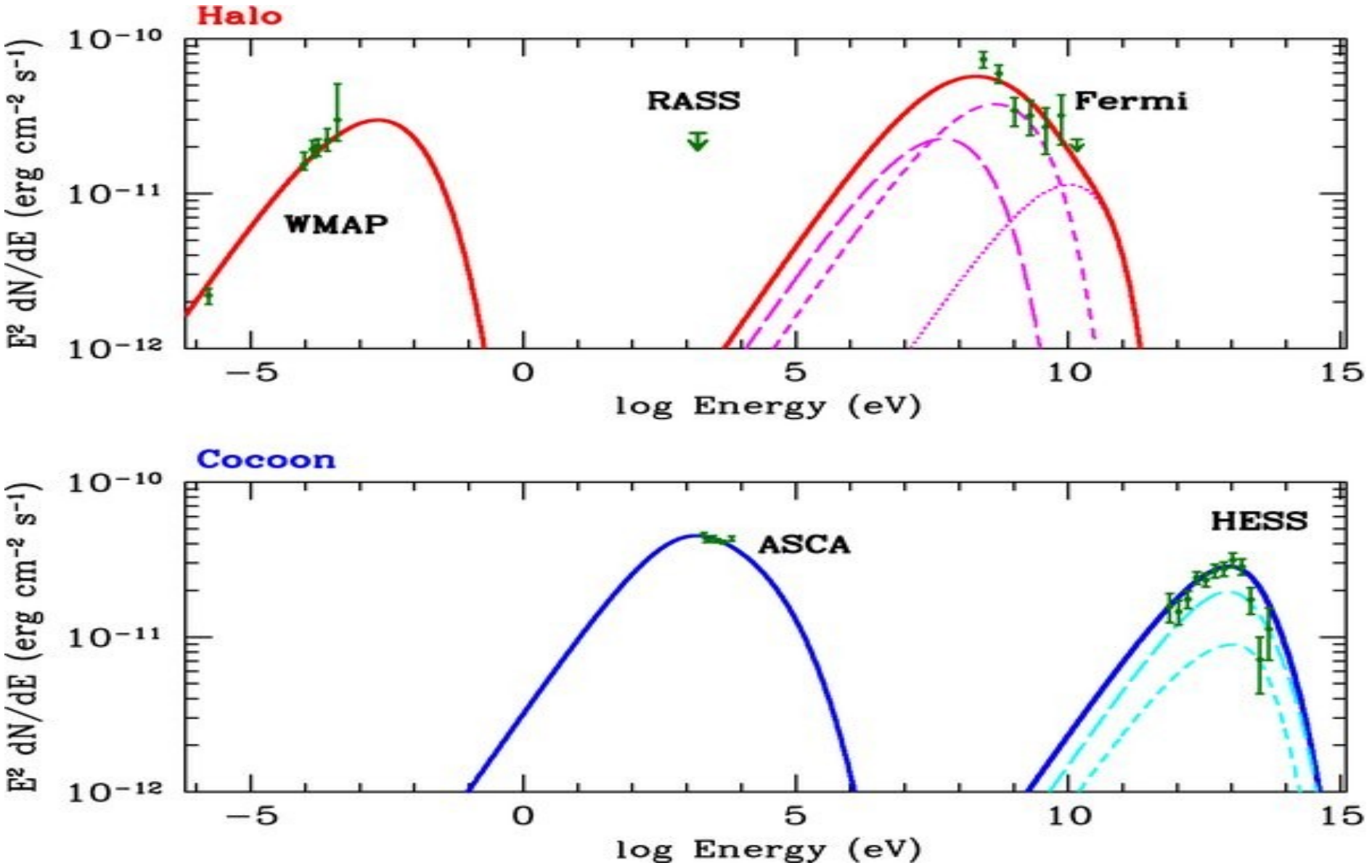
$$\frac{dE}{dt} = \frac{4}{3}\ \sigma_T\ U_B\ c\ \gamma^2$$

Inverse Compton emissivity



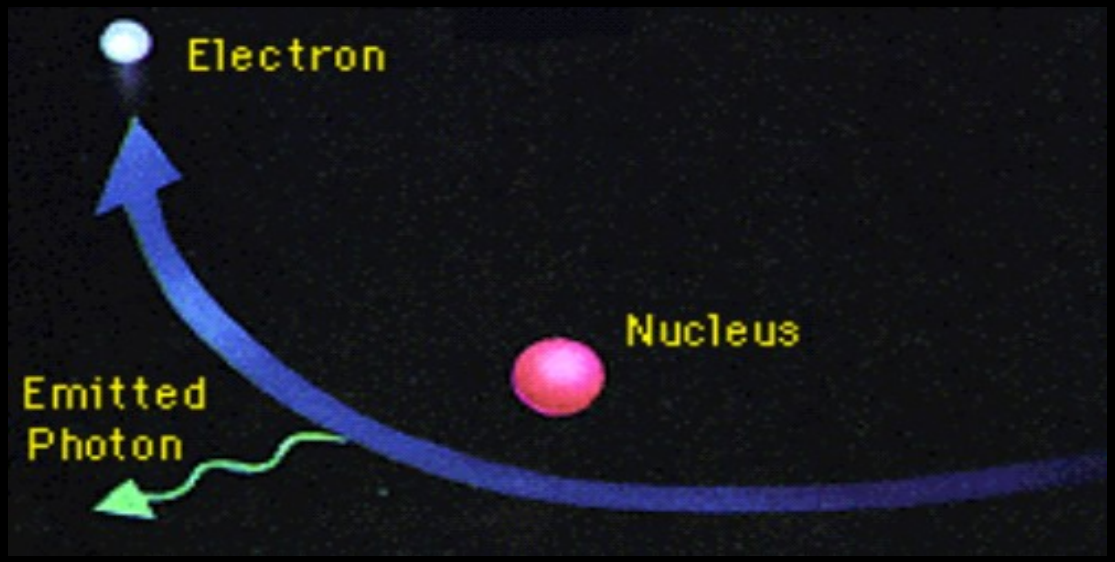



ROSAT, counts > 0.5 keV (counts/bin)
1
2
4
8
16
32
H.E.S.S., Significance (σ)
−4
−2
0
2
4
6
8
10
12
14
-45.0°
-45.5°
-46.0°
-46.5°
Dec (J2000)
130.0° 129.5° 129.0° 128.5° 128.0° 127.5°
RA (J2000)
130.0° 129.5° 129.0° 128.5° 128.0° 127.5°
RA (J2000)


Halo
WMAP
RASS
Fermi
Cocoon
ASCA
HESS
E² dN/dE (erg cm⁻² s⁻¹)
log Energy (eV)
10⁻¹⁰
10⁻¹¹
10⁻¹²
−5
0
5
10
15

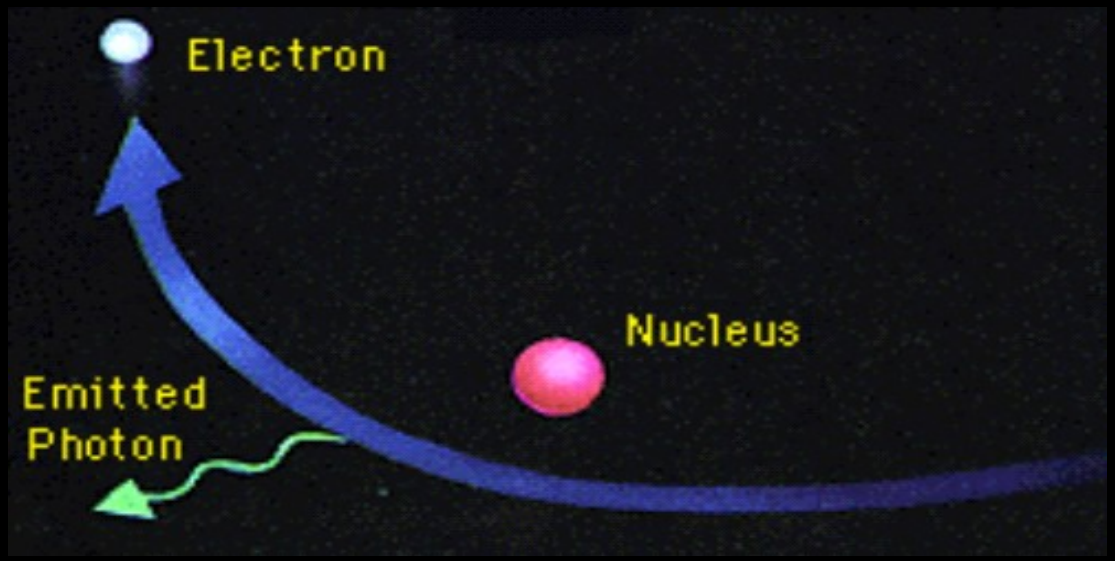
Bremsstrahlung

CR electrons + ISM nuclei → γ rays



Bremsstrahlung

CR electrons + ISM nuclei → γ rays

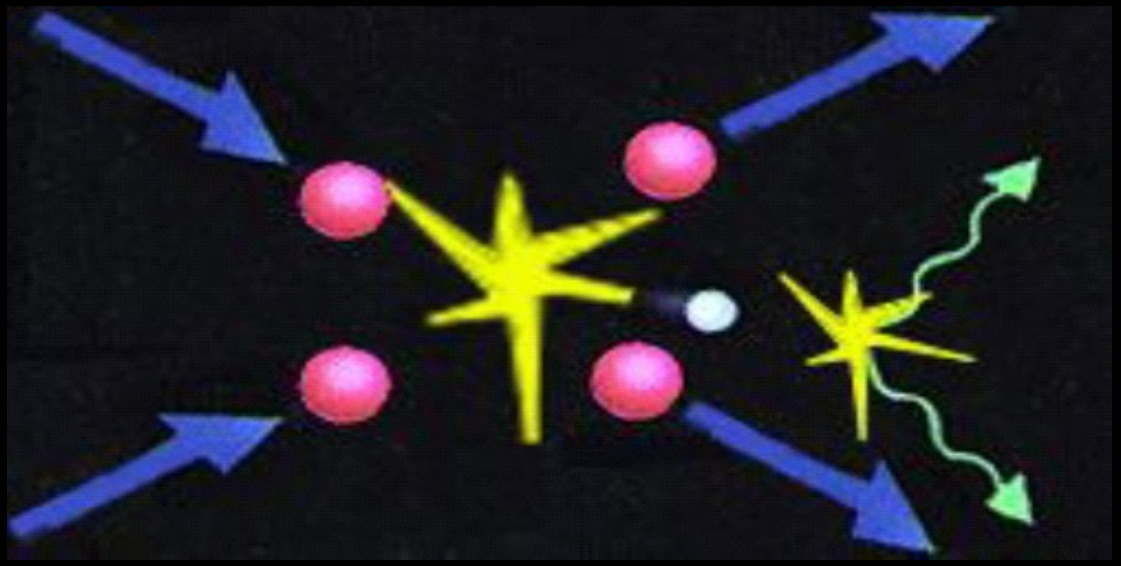


$$-\frac{dE}{dt} = 4c\alpha r_e^2 N G_a \, E$$

$$N_e(E) = kE^{-a}$$

$$I(\hbar\omega) = 4Z^2 c\alpha r_e^2 N G_a k \frac{(\hbar\omega)^{-a}}{a-1}$$

Emission processes: Neutral π decay

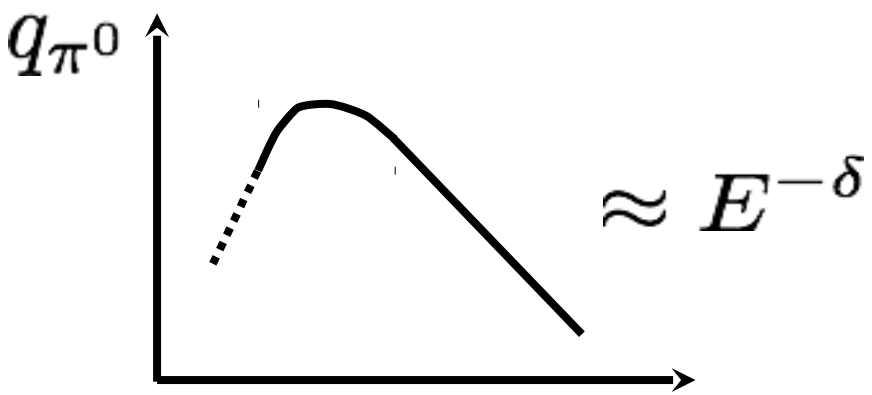


CR protons + ISM nuclei → π^0 → γ rays

p-p interactions

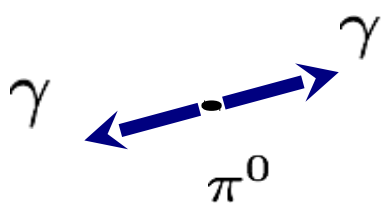
We assume a power law spectrum for CRs: $N_p(E_p) \propto E_p^{-\delta}$

Fraction of proton kinetic energy transferred to pion (from data): $f_{\pi^0} \approx 0.17$



p-p interactions

Pion rest frame:



$$E_\gamma^* = \frac{m_{\pi^0}}{2}$$

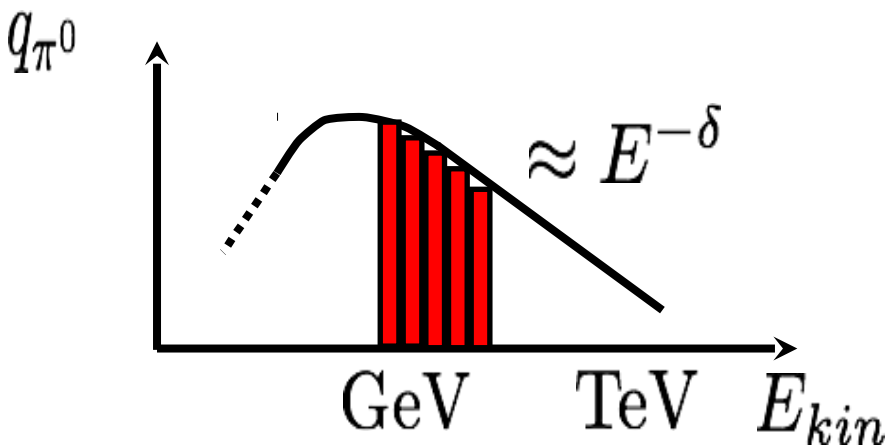
Lab frame:

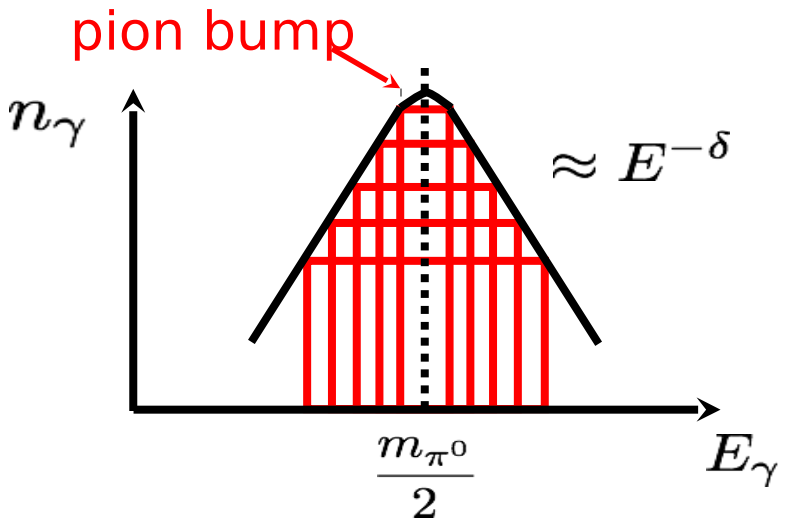
$$E_\gamma = \gamma \left(E_\gamma^* + v p_\gamma^* \cos\theta^* \right)$$

max and min energies -> $\cos\theta^* = \pm 1$

$$\frac{m_{\pi^0}}{2}\sqrt{\frac{1-\beta}{1+\beta}} \leq E_\gamma \leq \frac{m_{\pi^0}}{2}\sqrt{\frac{1+\beta}{1-\beta}}$$

p-p interactions

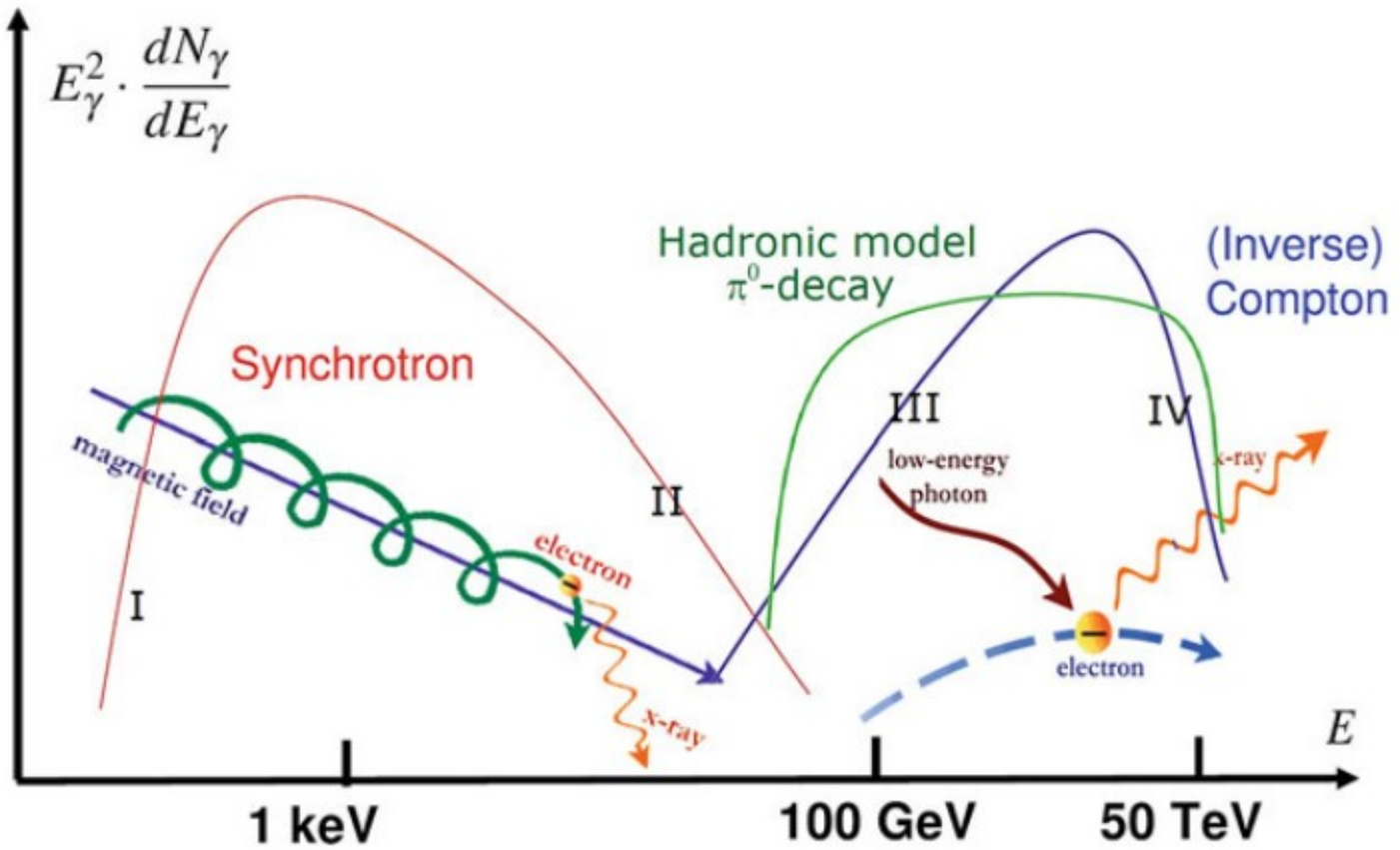




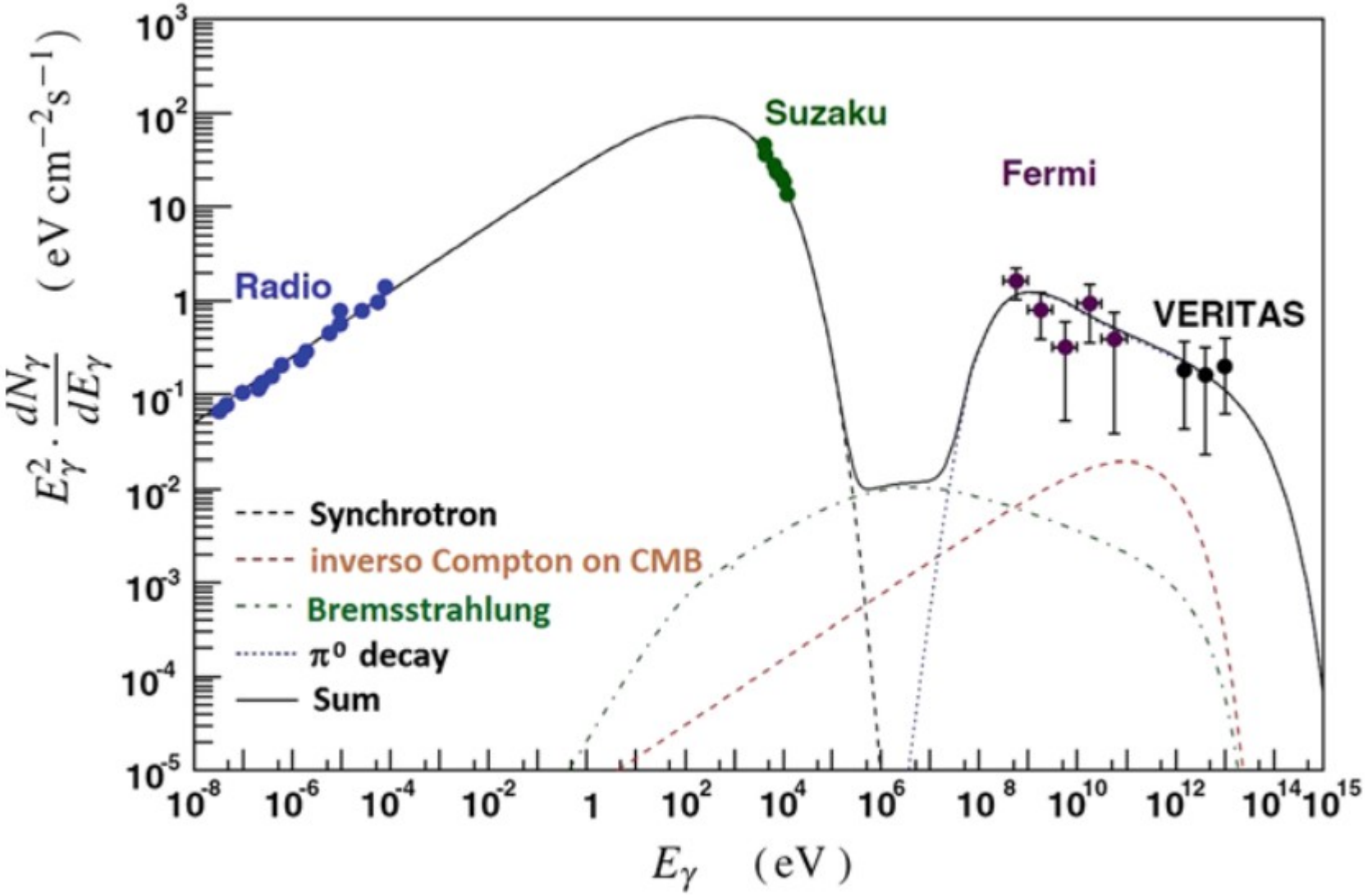
the gamma ray spectrum is symmetric (in log-log) with respect to: $\frac{m_{\pi^0}}{2} \sim 70\ \mathrm{MeV}$

at high energy the spectrum mimic the CR spectrum, with (roughly): $E_\gamma \approx \frac{E_{CR}}{10}$

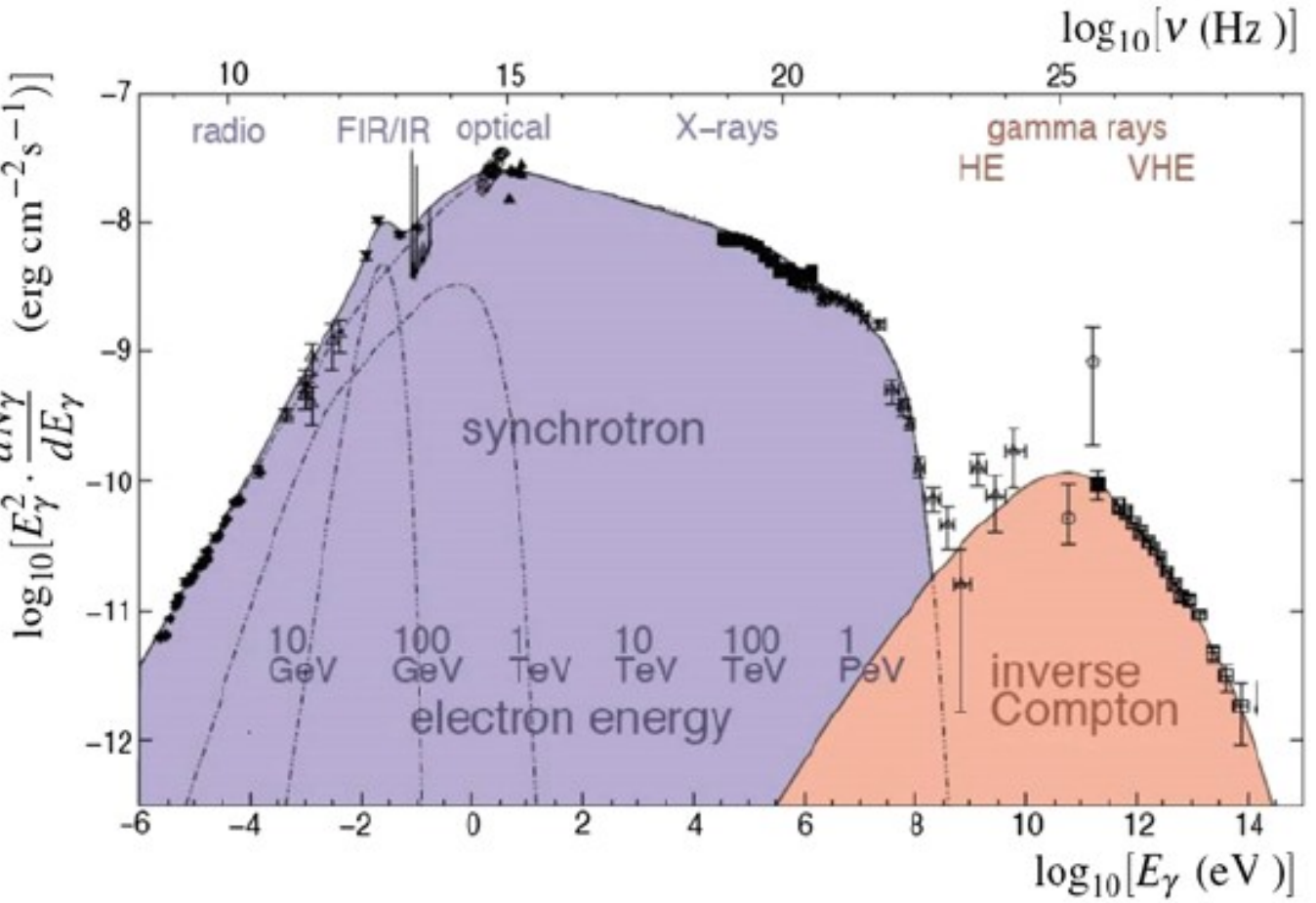
SED Modeling



SED of the Tycho's SNR



Crab SED


log10[ν (Hz)]
10
15
20
25
radio
FIR/IR
optical
X-rays
gamma rays
HE
VHE
synchrotron
inverse
Compton
10
GeV
100
GeV
1
TeV
10
TeV
100
TeV
1
PeV
electron energy
log10[Eγ (eV)]
log10[Eγ2 · dNγ/dEγ (erg cm−2s−1)]

SED of the AGN Mrk 421