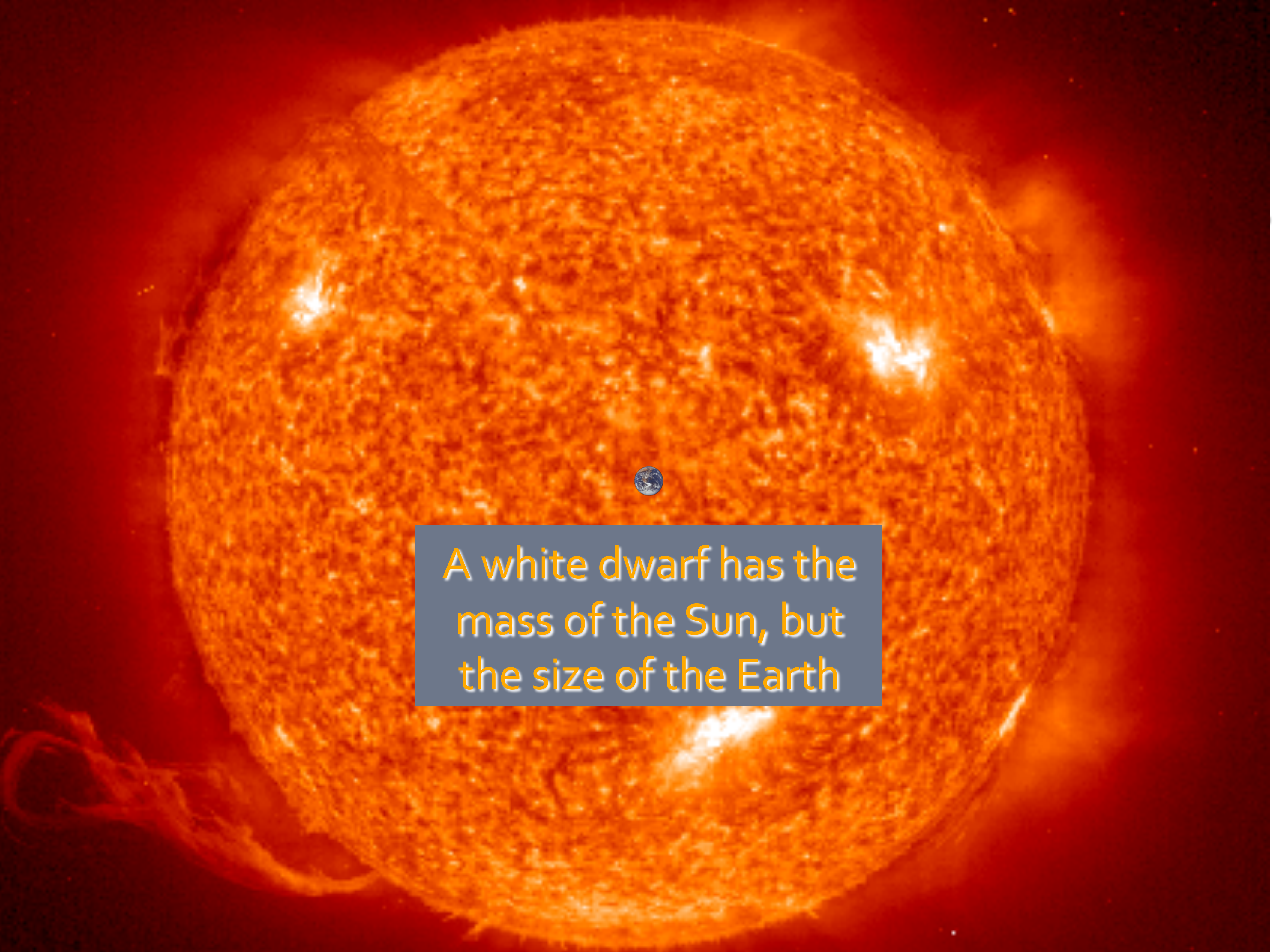
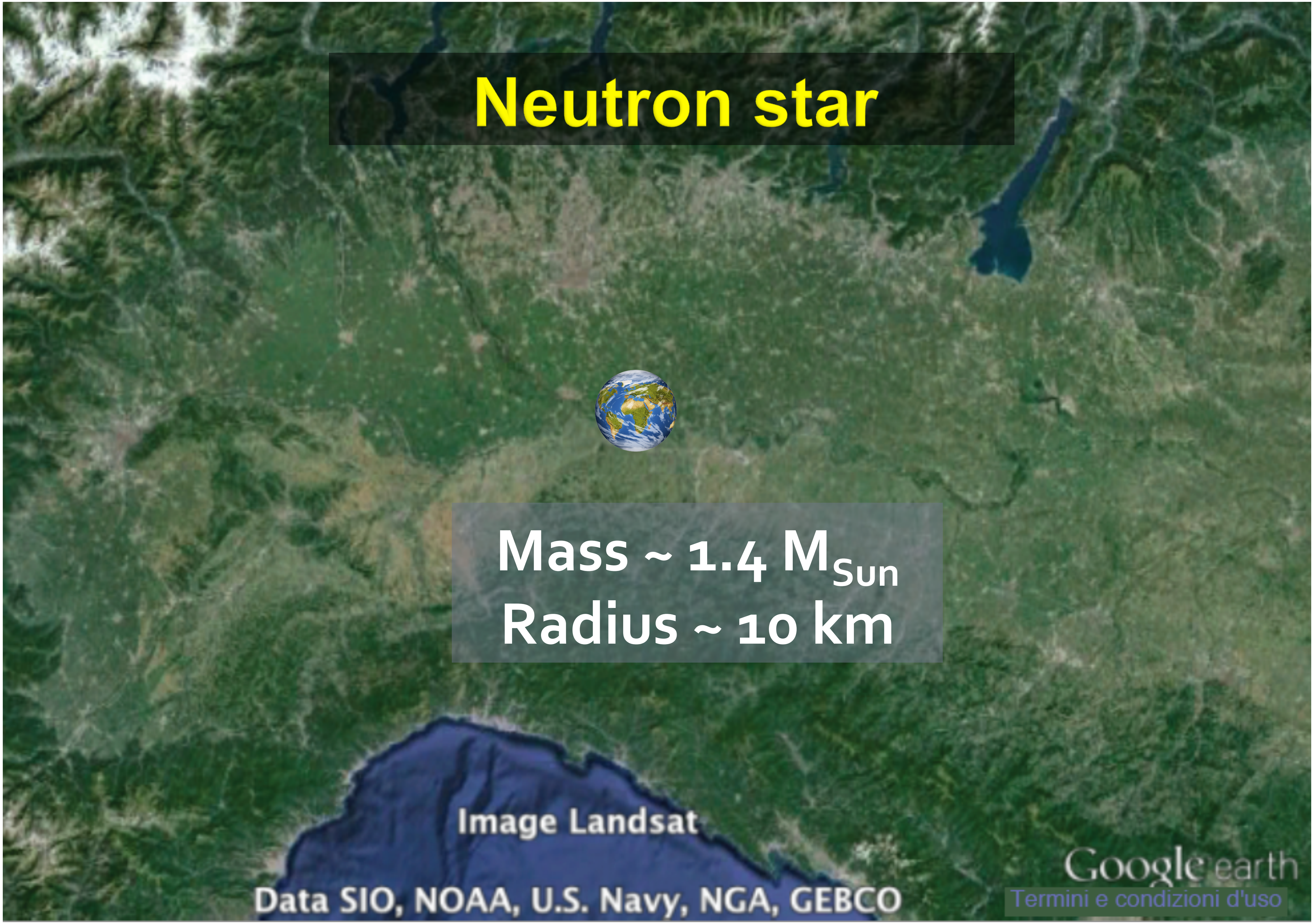

A white dwarf has the mass of the Sun, but the size of the Earth

Neutron star



How compact are compact objects?

Escape velocity:

$$\frac{\mathrm{v}^2}{R} = \frac{GM}{R^2} \Rightarrow \mathrm{v}_e = \sqrt{\frac{GM}{R}}$$

Black Hole when $\mathrm{v}_e \approx c$

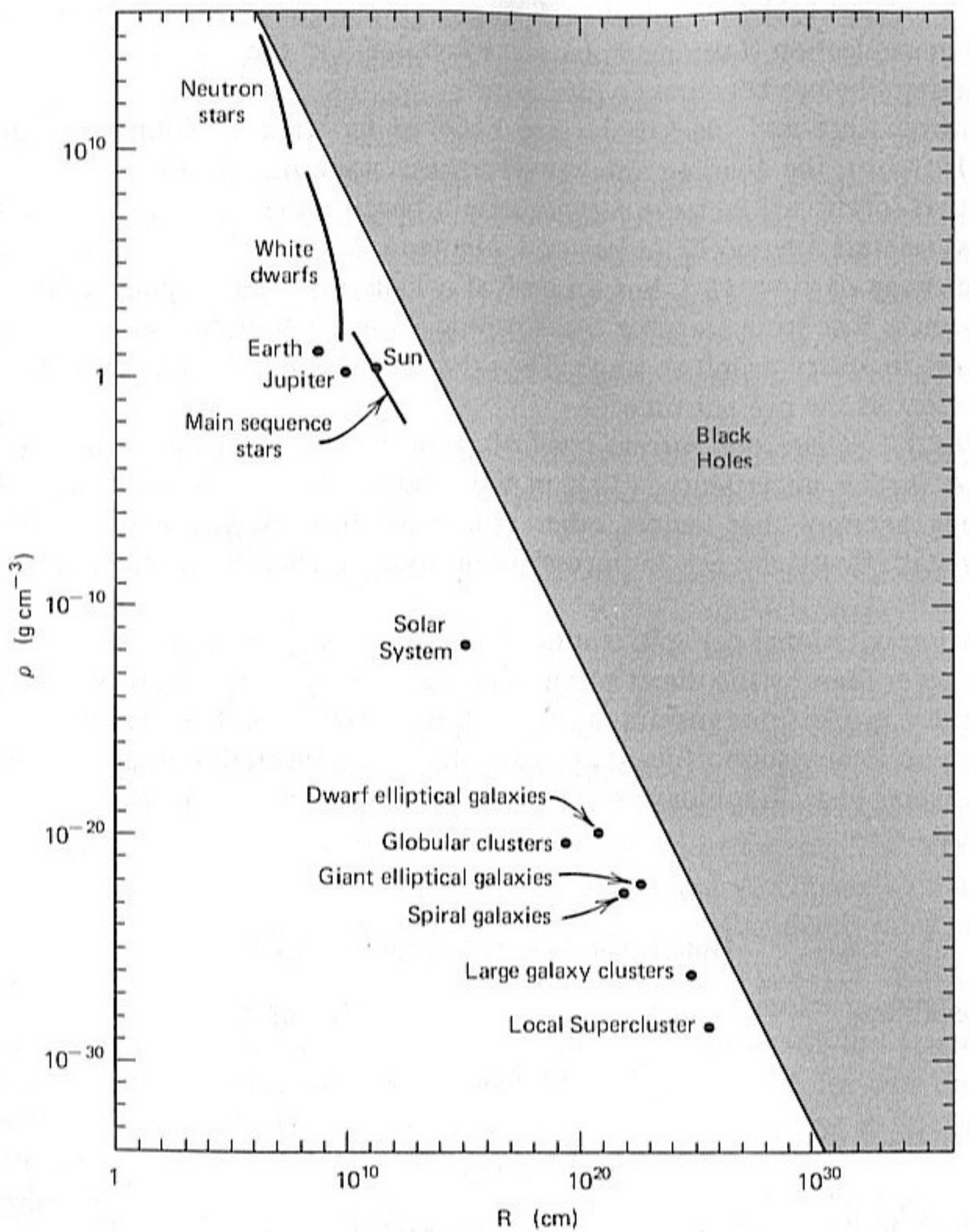
Strong gravity when

$$E_G \approx mc^2$$

$$\Rightarrow \frac{GM}{Rc^2} \approx 1$$

Compactness = 1 for BH

Sun? WD? NS?



Degeneracy Pressure

- Two particles cannot occupy the same space with the same **momentum** (energy).

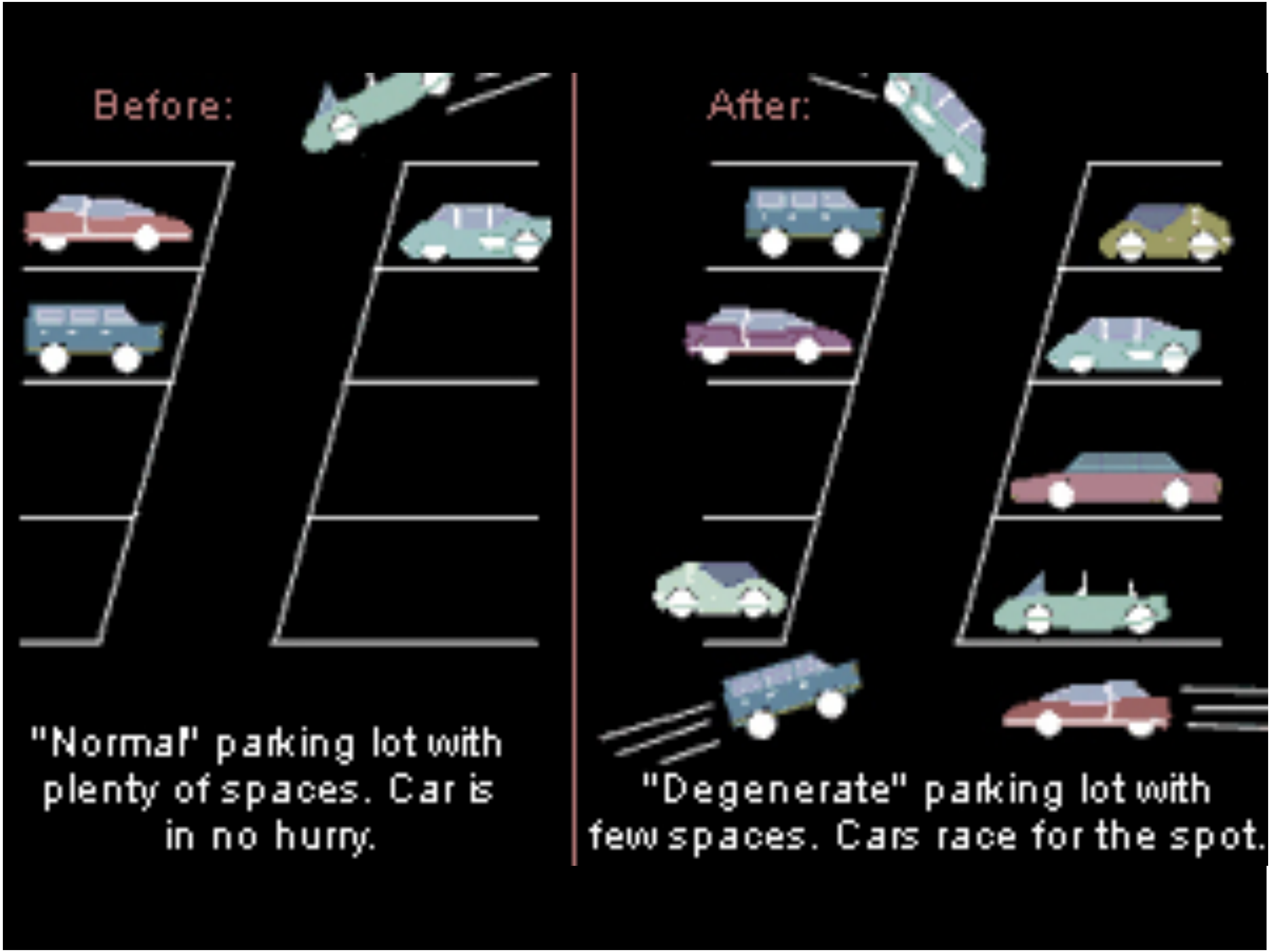
- For **very dense solids**, electrons cannot be in their ground states, they become very energetic ⇒ approaching the speed of light.

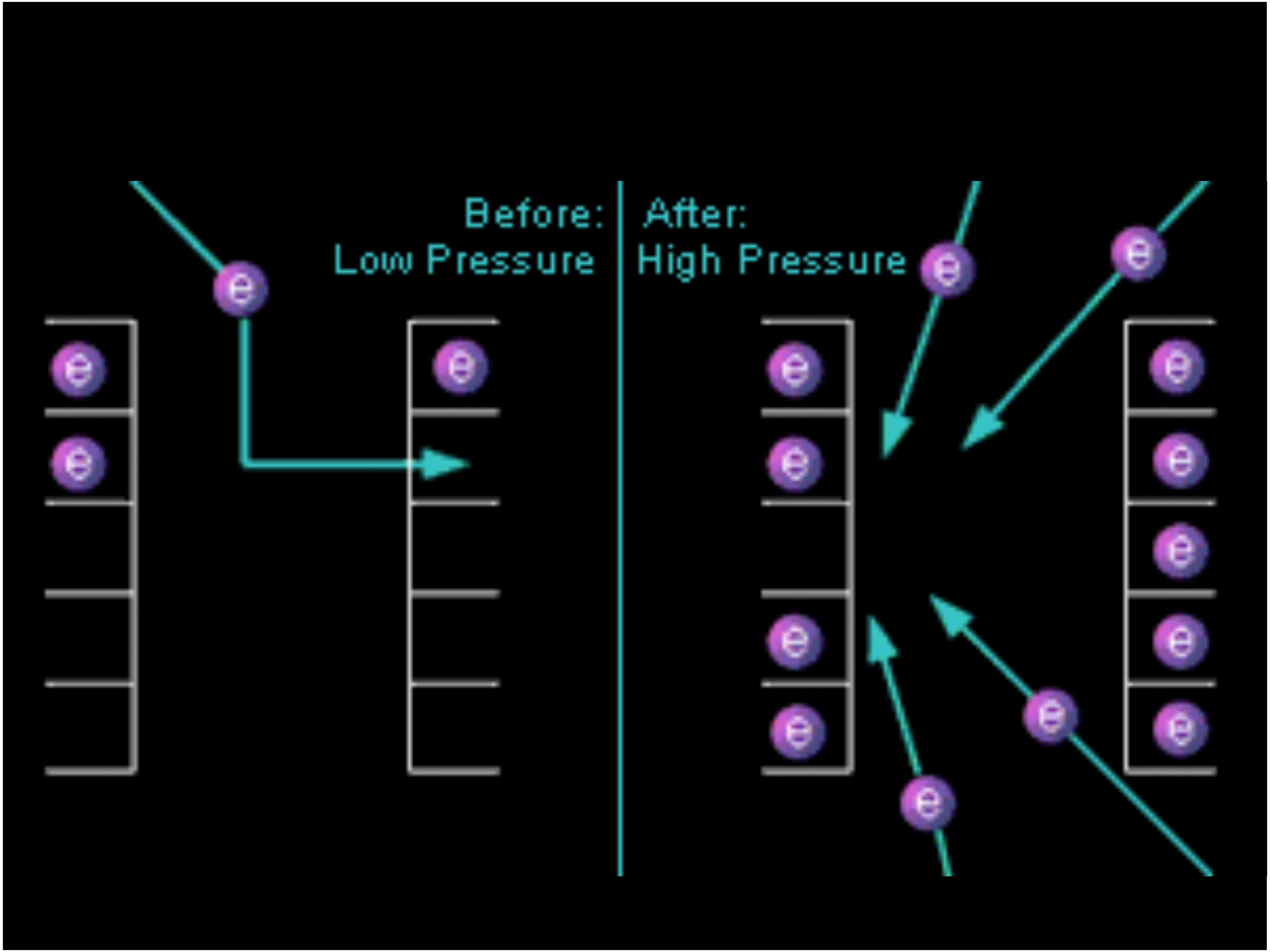
- ***Pressure** holding up star no longer depends on **temperature:***

$$P \propto \rho^{\gamma}$$

$\gamma=5/3$ for **non-relativistic** degenerate gas

$\gamma=4/3$ for **relativistic** degenerate gas


Before:
After:
"Normal" parking lot with plenty of spaces. Car is in no hurry.
"Degenerate" parking lot with few spaces. Cars race for the spot.


Before:
Low Pressure
After:
High Pressure

The Chandrasekhar's limit:

General argument by **Landau** (1932) on limiting mass for a degenerate gas of **electrons** (WDs) or **neutrons** (NSs)

N fermions in star of radius $R \Rightarrow n \sim N/R^3$

Volume per fermion $\sim 1/n$ (**Pauli exclusion principle**) and momentum $\sim \hbar n^{1/3}$ (**Heisenberg principle**)

Fermi energy of fermionic gas in relativistic regime:

$$E_F = p_F c \sim \hbar n^{1/3} c \sim \hbar c N^{1/3}/R$$

Gravitational energy per fermion:

$E_G \sim -GMm_B/R$ ($M=Nm_B$, most of the mass in baryons)

Equilibrium at a minimum of the total energy function:

$$E = E_F + E_G = \hbar c N^{1/3}/R - GNm_B^2/R$$

$$E(N) = E_F + E_G = \hbar c N^{1/3}/R - GNm_B^2/R$$

For arbitrary large N, E is always negative ⇒ if R decreases, E continues to decrease ⇒ collapse continues indefinitely ⇒ M_{max}

For small N, first term dominates ($E > 0$) ⇒ minimum at $E(N)=0$

$$N_{max} \sim (\hbar c/Gm_B^2)^{3/2} \sim 2 \times 10^{57} \Rightarrow M_{max} \sim N_{max}\, m_B \sim 1.7\, M_\odot$$

From this simplified calculation, same M_{max} for WDs and NSs.

Equilibrium radius: $E_F \sim mc^2$ in the relativistic regime and m is the mass of electrons or neutrons, giving WD and NS radius, respectively

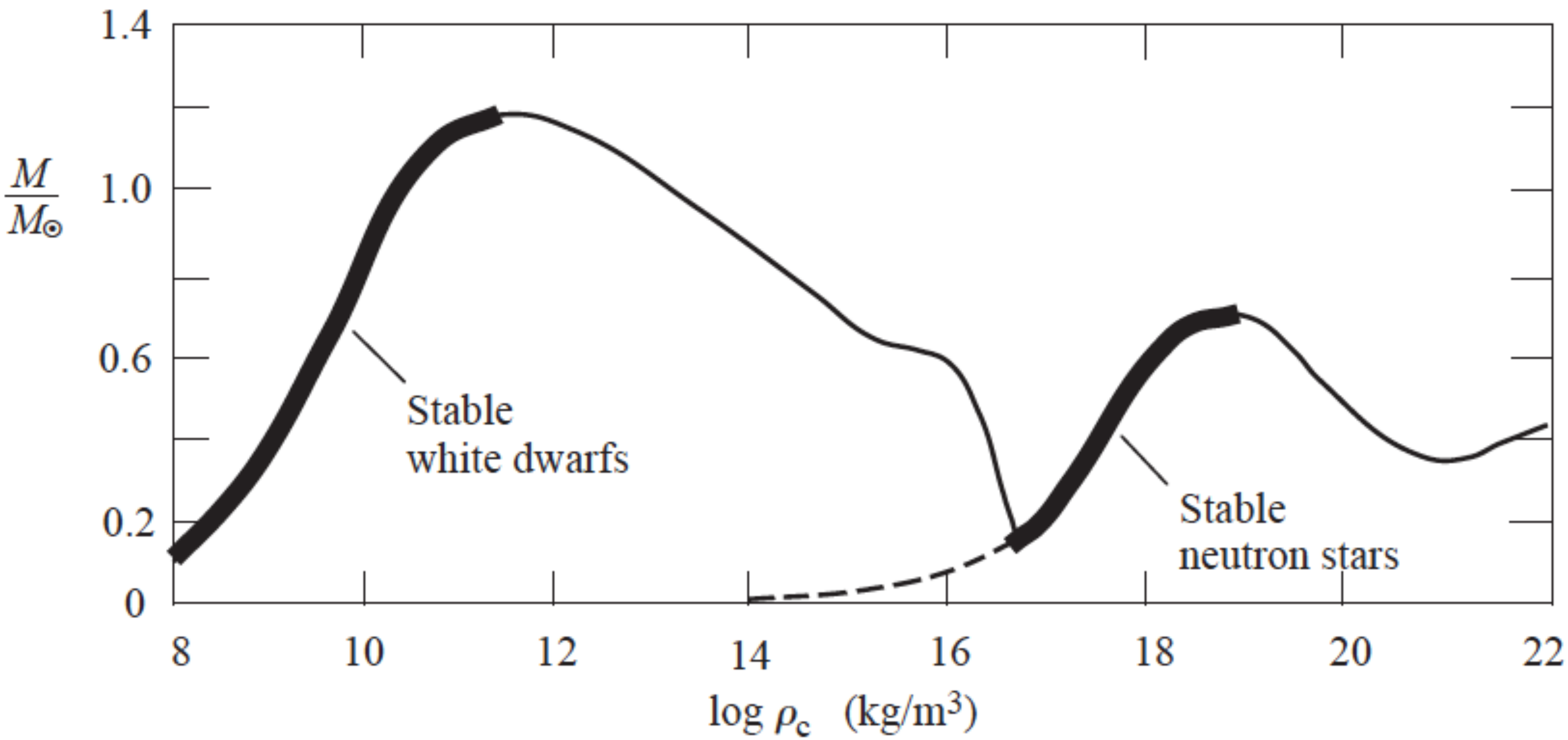
$$E_F \sim \hbar c N^{1/3}/R \sim mc^2 \quad R \sim \hbar/mc(N_{max})^{1/3} \sim \hbar/mc\ (\hbar c/Gm_B^2)^{1/2}$$

$R_{WD} \sim 5 \times 10^8$ cm for $m=m_e$; $R_{NS} \sim 3 \times 10^5$ cm for $m=m_n$

NS radii m_n/m_e times smaller than WD radii

Stable WDs and NSs

HW (1958) and OV (1939) equations of state, *ignoring nuclear forces*.

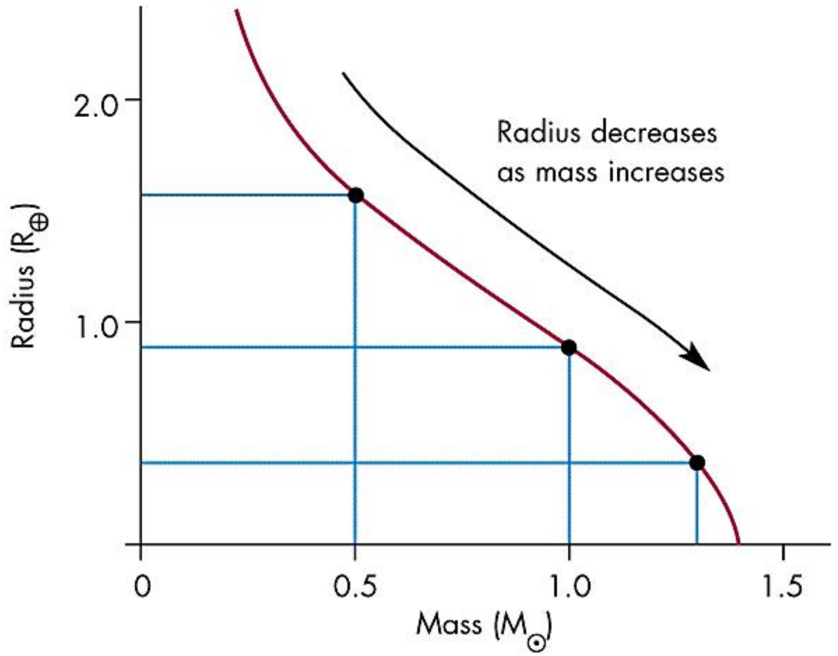


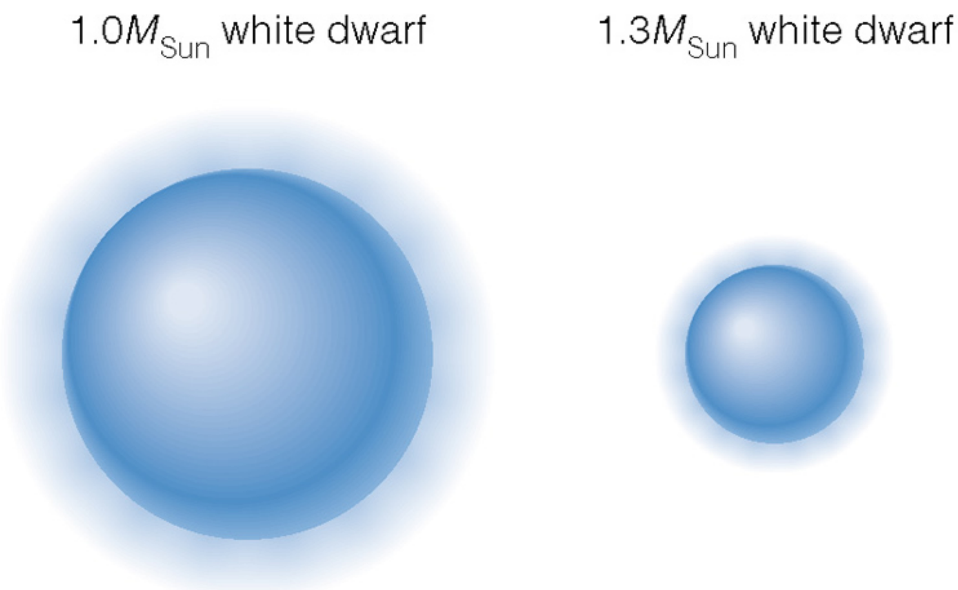
Stability only if mass increase implies larger density

$\Rightarrow$larger pressure to contrast gravity ($P \propto \rho^\gamma$)

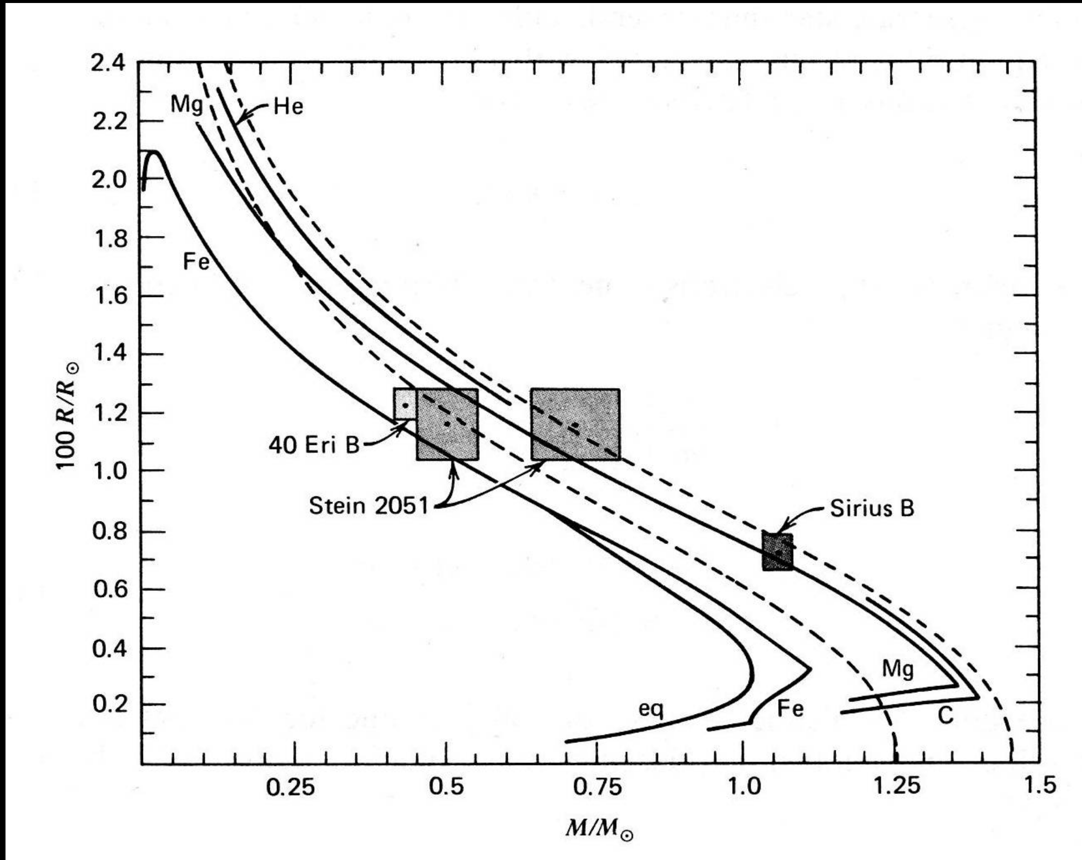
White Dwarfs

- The more mass the star has, the *smaller* the star becomes!
  - increased gravity makes the star denser
  - greater density increases degeneracy pressure to balance gravity





White dwarfs: mass-radius relation



Chandrasekhar's model (dashed line) agrees quite well with better models based on equations of state (with a dominating element, different fermions, particle interactions and electrostatic corrections).

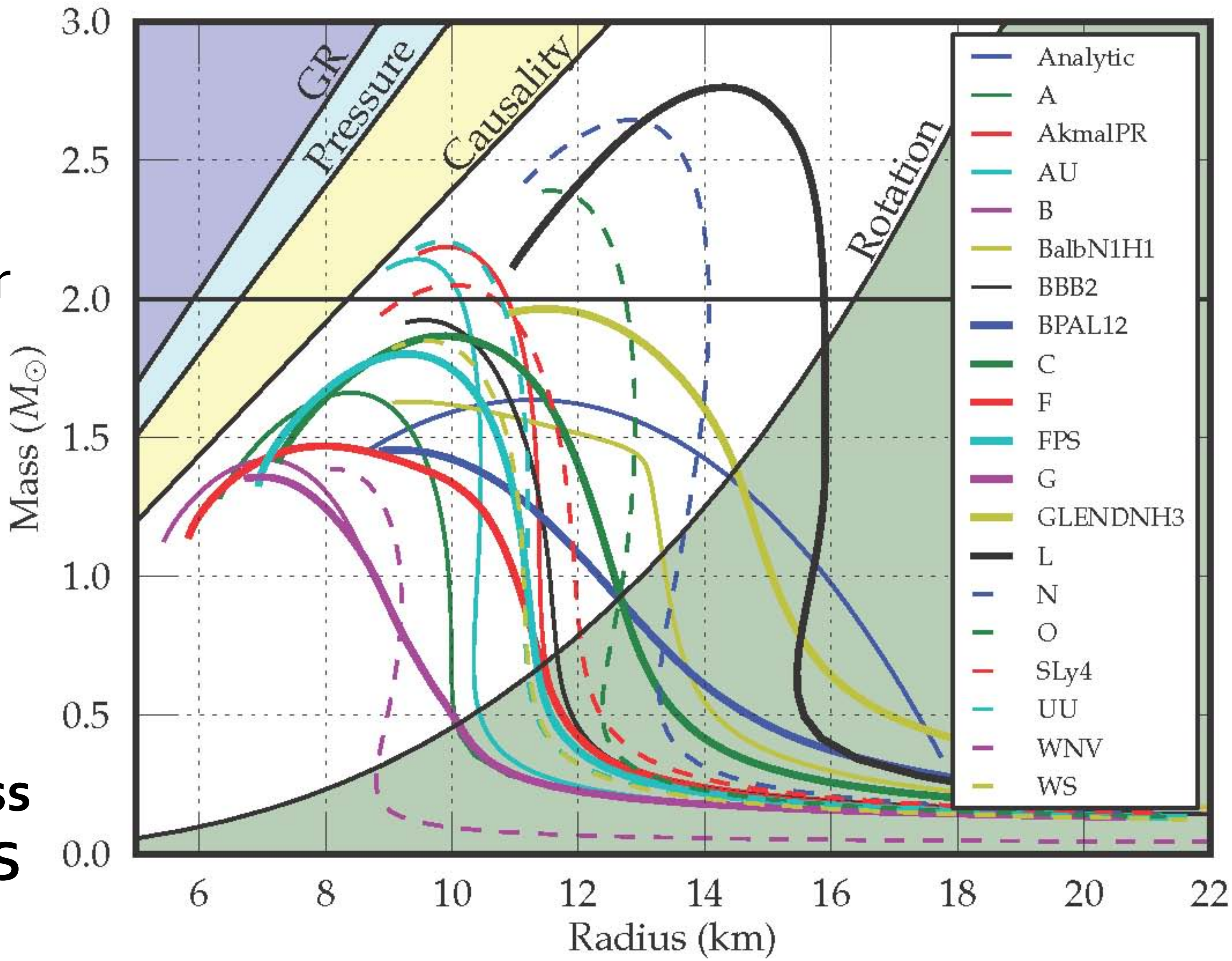
Maximum mass varies in the range 1-1.45 $M_\odot$

Neutron star: mass-radius relation

To determine NS Equation of State (**EoS**) we need to know the behavior of matter at supranuclear density and use General Relativity

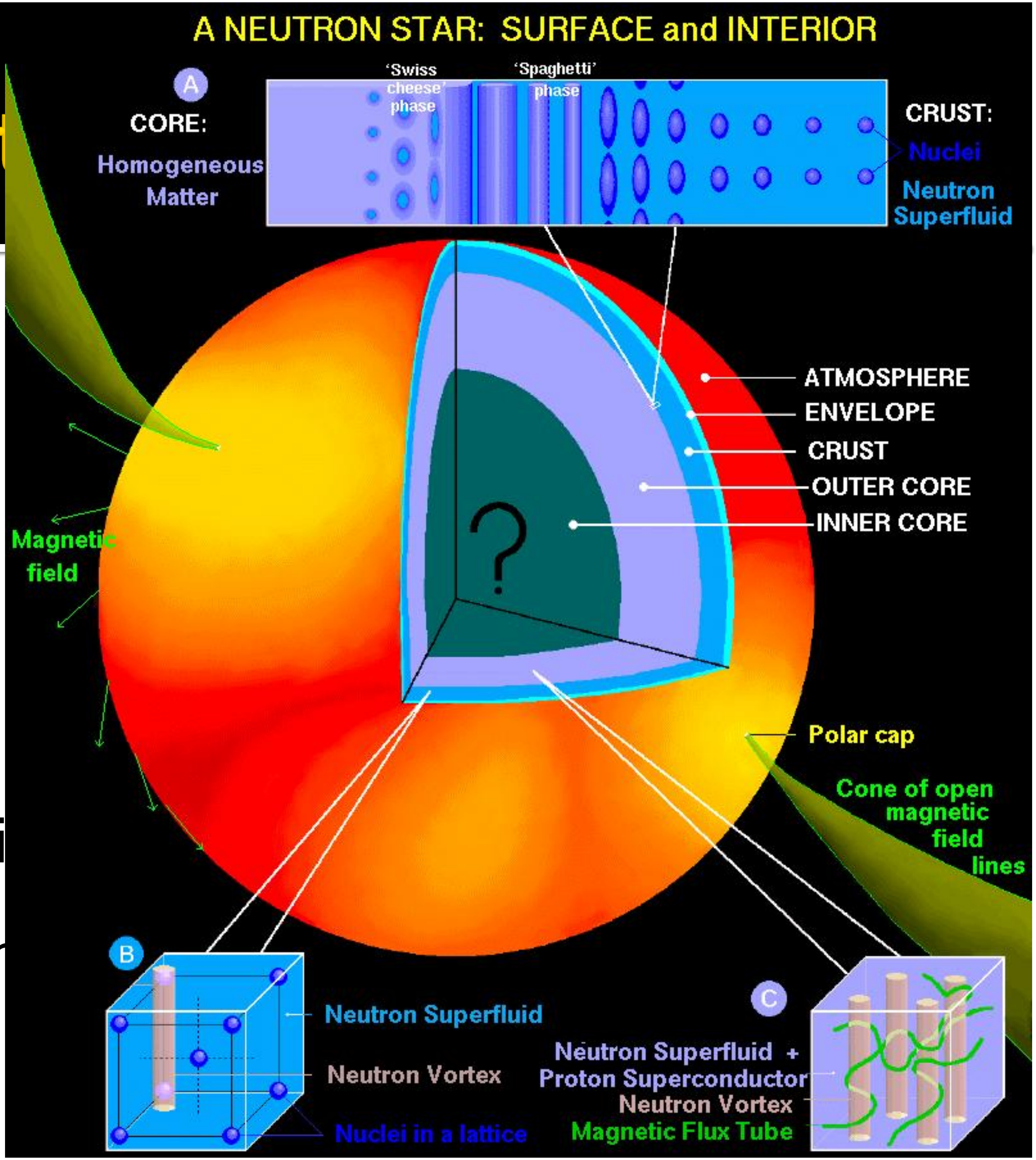
$$\left(\frac{GM_{NS}}{R_{NS}c^2} \approx 0.1\right)$$

Maximum NS mass <3 $M_{\odot}$ for any EoS

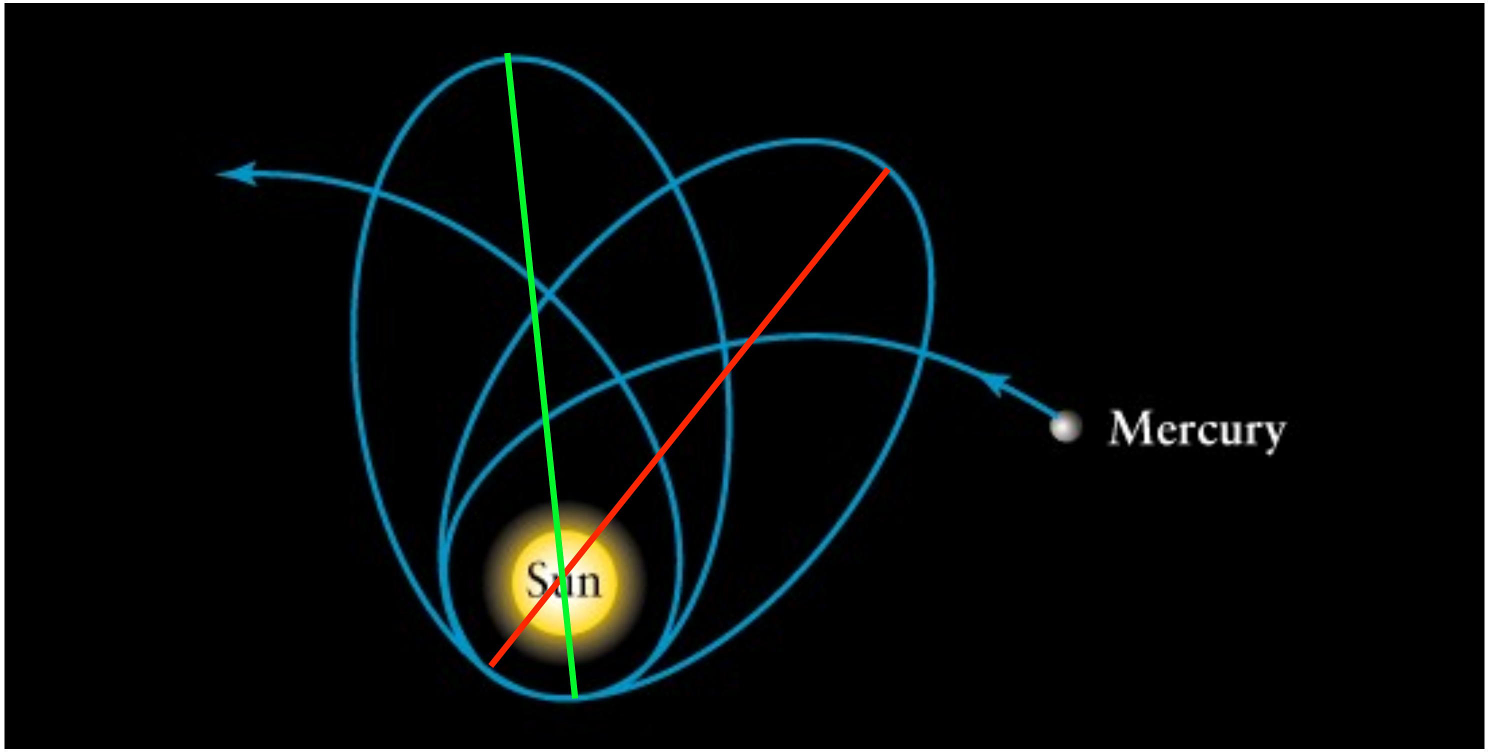


Neutron st

- Atmosphere:
- Crust: Fe
- Neutron drip:
- Superfluidity
- Nuclear densi
- Core: quark m



GR: Mercury orbit precession



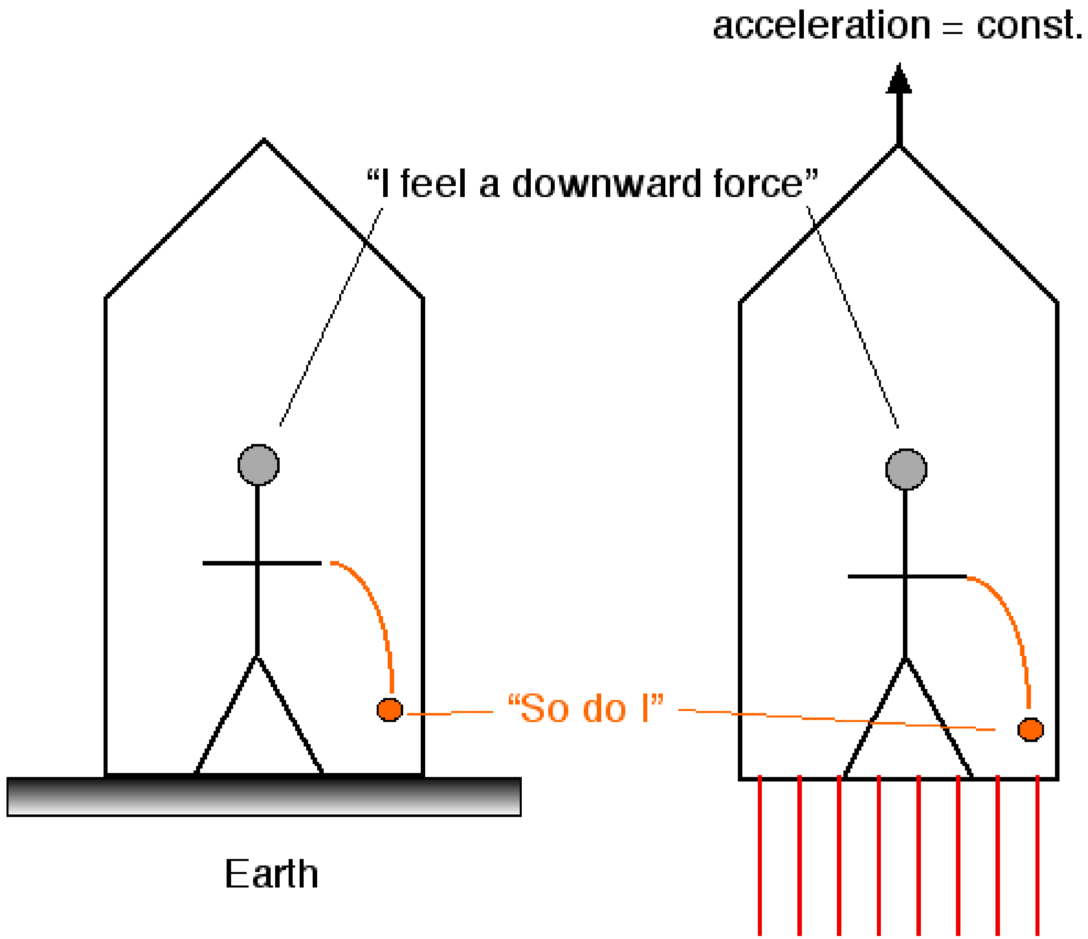
Newtonian Gravity Predicts: 5557.62 arcsec/century

Observed Value: 5600.73 arcsec/century

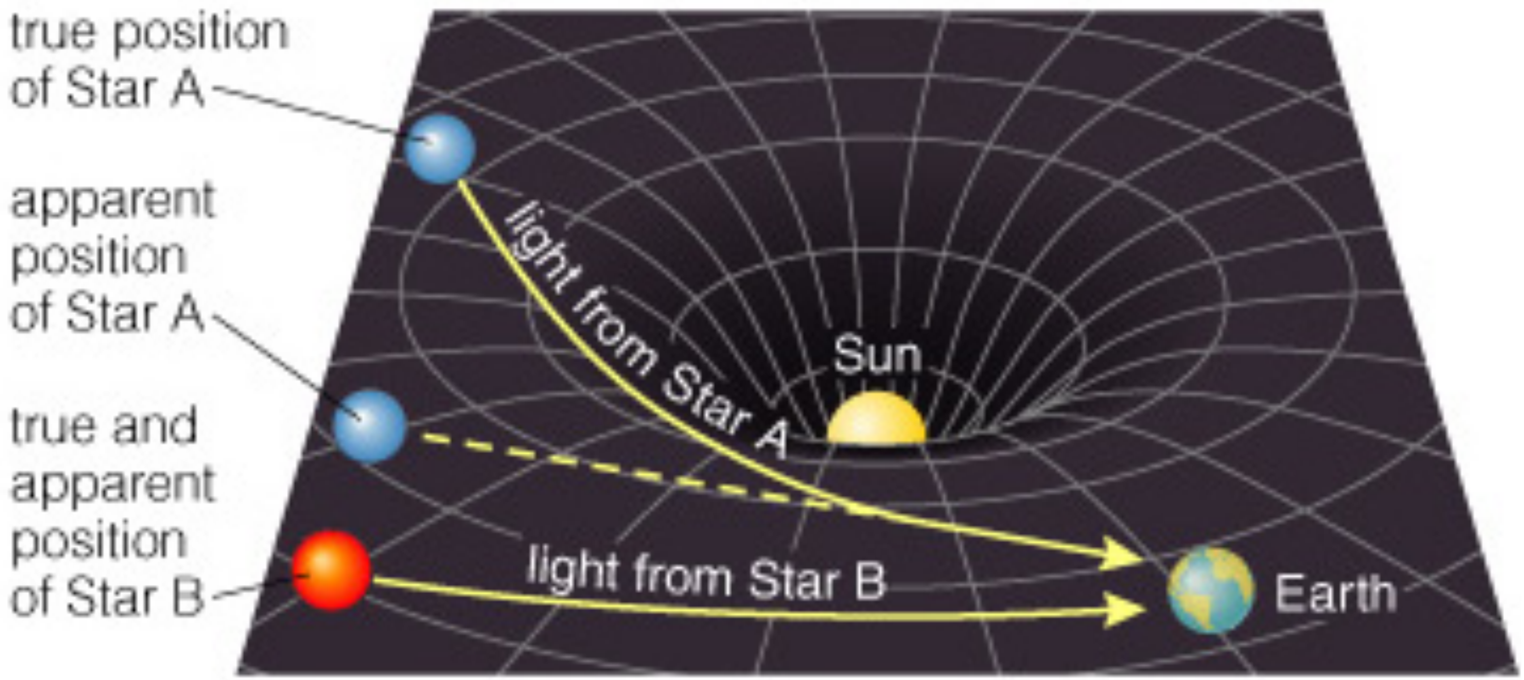
Difference: 43.11 ± 0.45 arcsec/century too fast!!

GR: The Equivalence Principle

The force of gravity is indistinguishable from the force due to accelerated motion.



GR: Deflection of Starlight

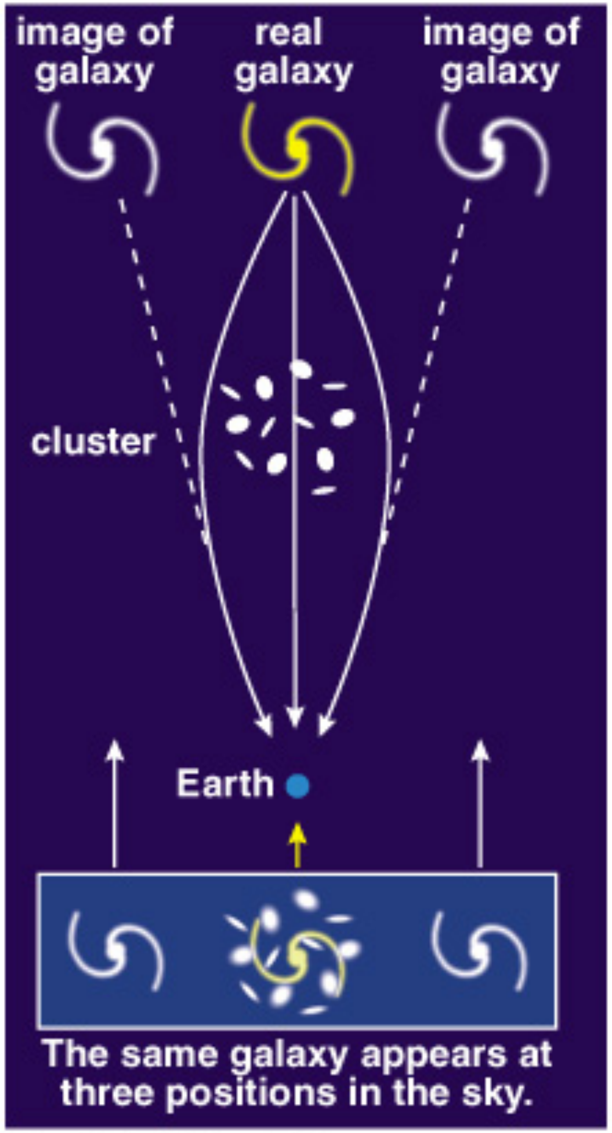


How can we measure this effect?

GR: Deflection of Starlight

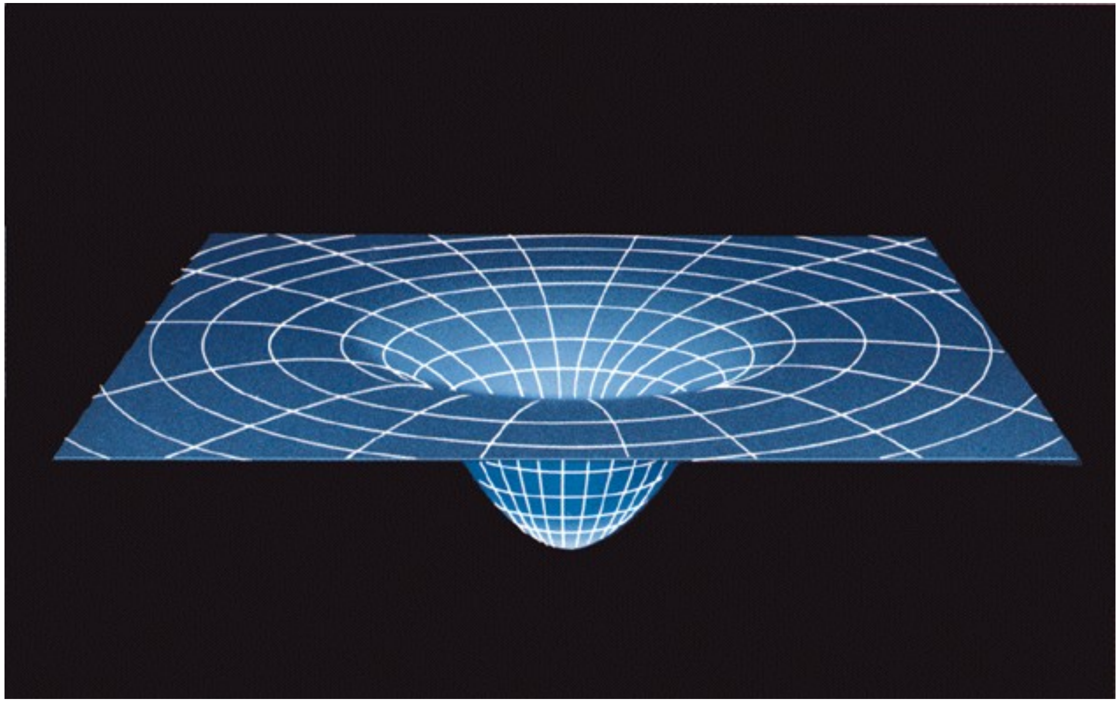
Observation by Eddington during Solar eclipse in 1919

GR: Gravitational Lensing



Distant galaxies lensed/warped in appearance by close galaxy masses (mainly Dark Matter)

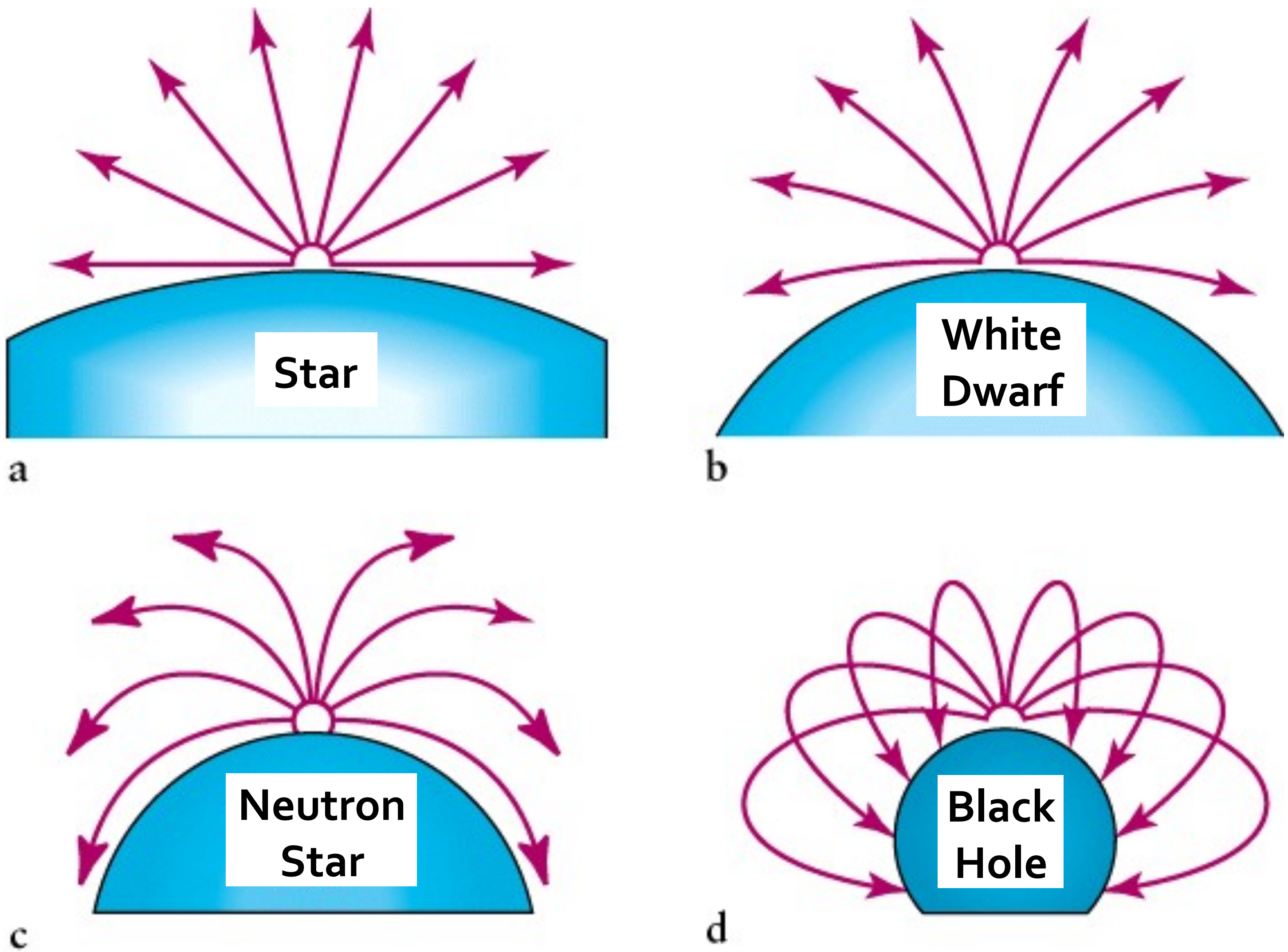
GR: Light travels along "straight" lines in a curved "space-time"



If this were a soccer field, how would a soccer ball "roll" on it?

Light behaves similarly traveling through curved 3D space

Black Holes

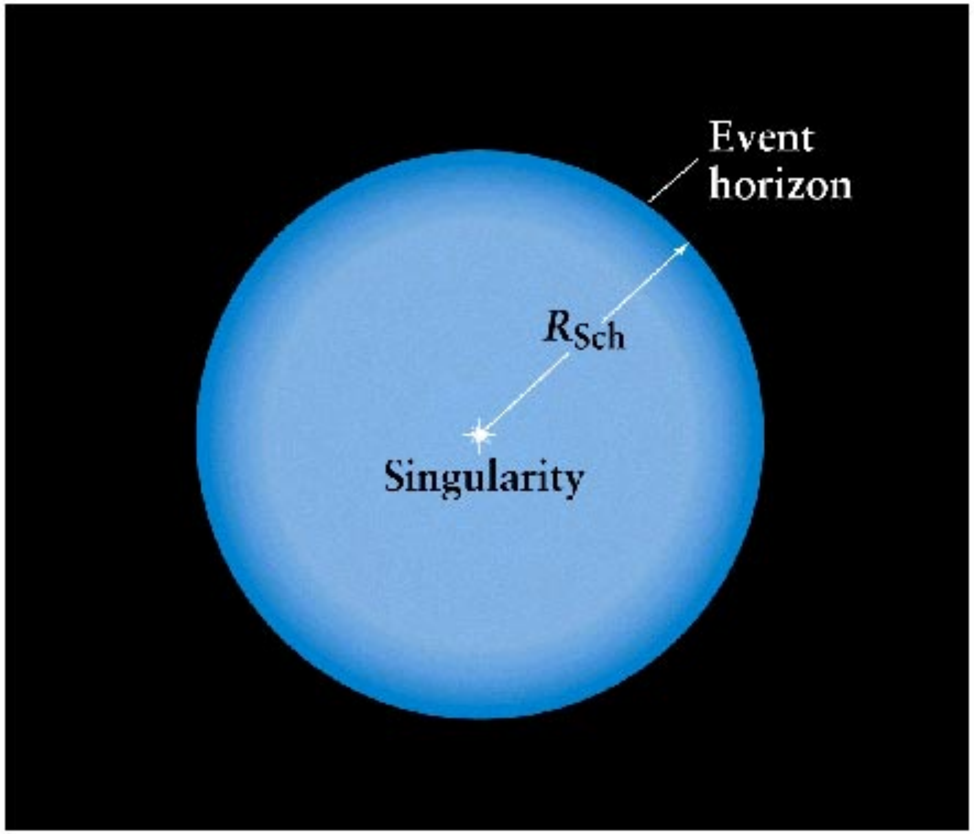


A nonrotating black hole has only a "center" and a "surface"

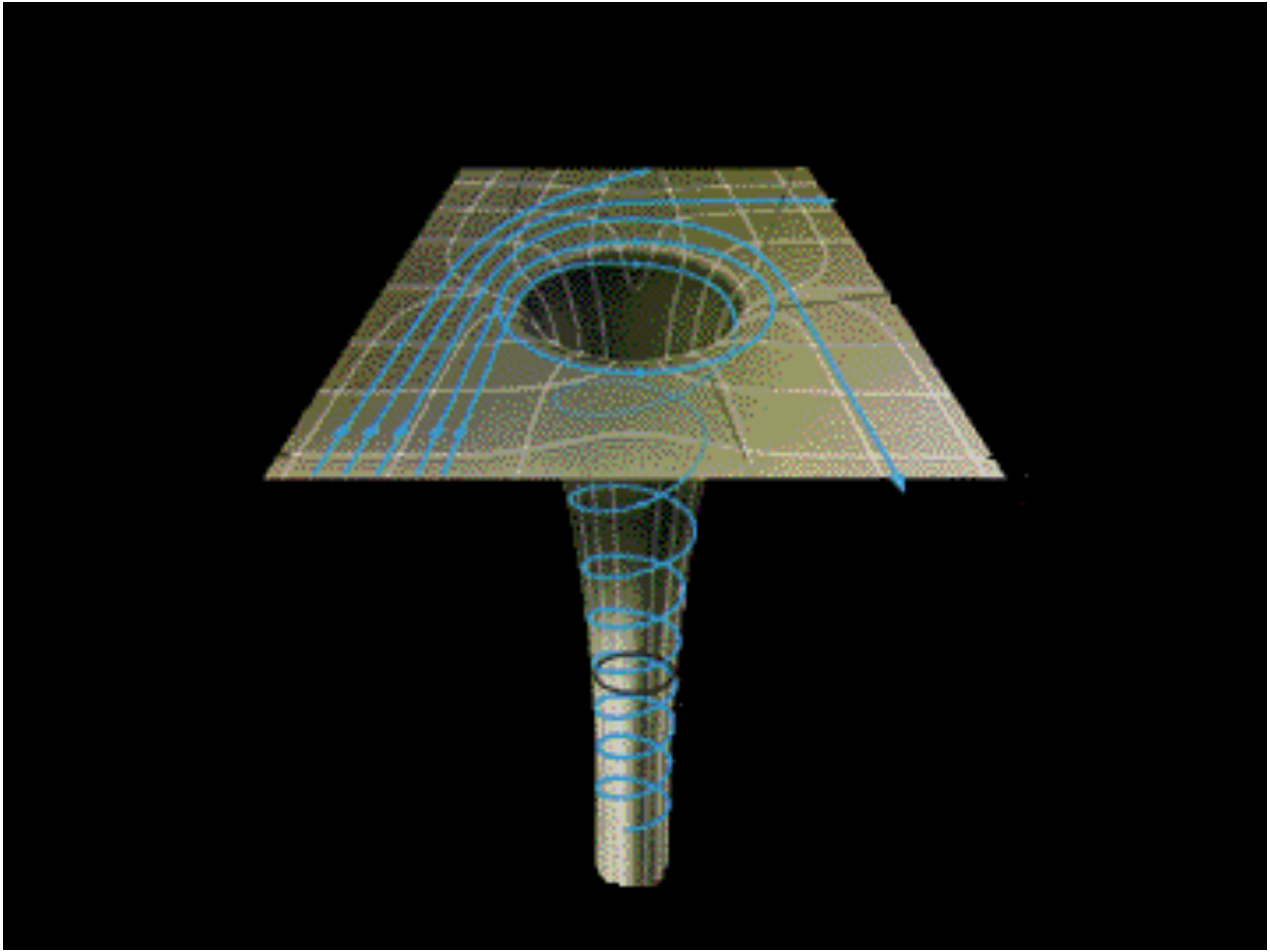
- The ***event horizon*** is the sphere from which light cannot escape
- The distance between the BH and its event horizon is the ***Schwarzschild radius***:

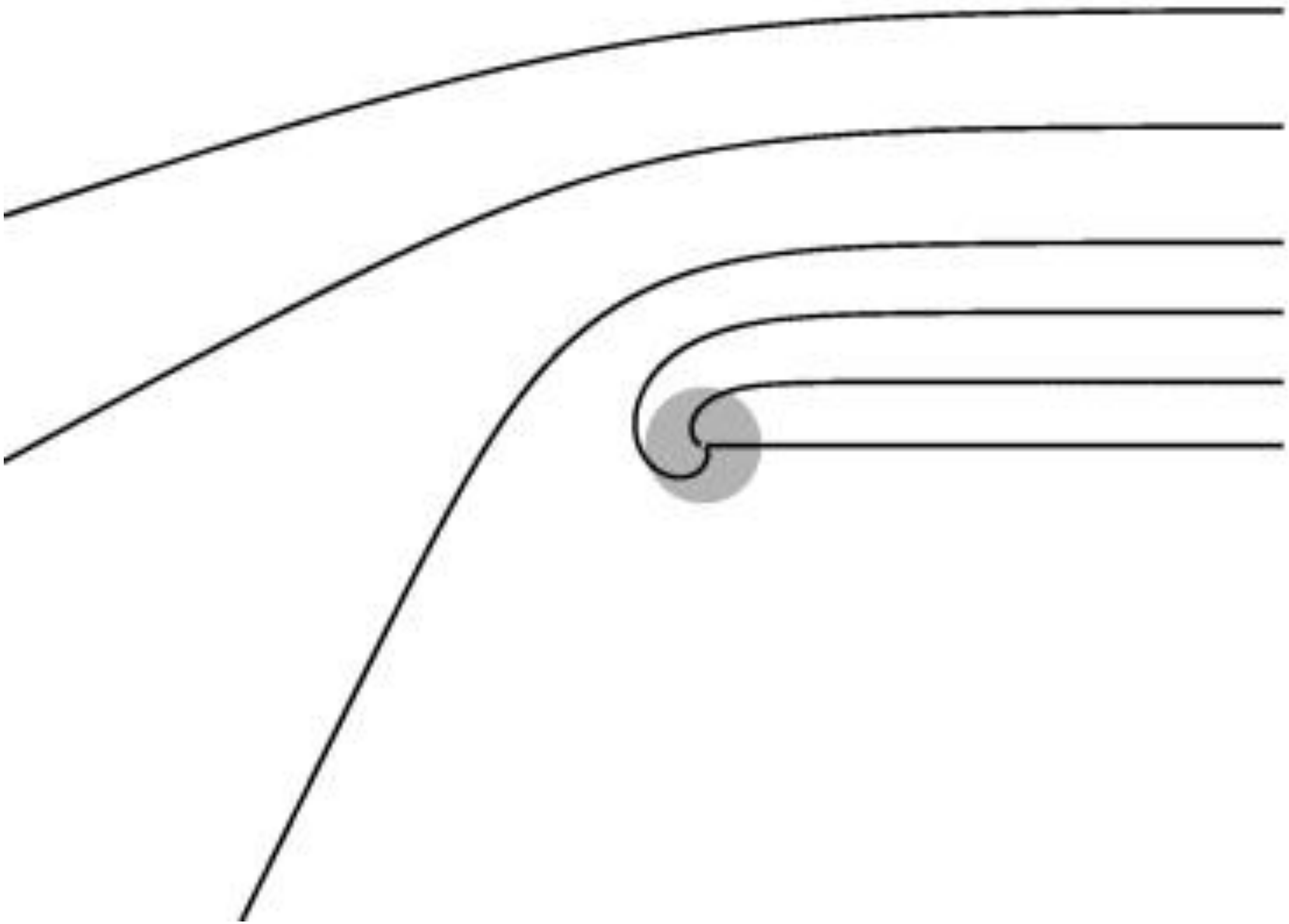
$$R_S = \frac{2GM}{c^2} \approx 3\frac{M}{M_{Sun}}\ km$$

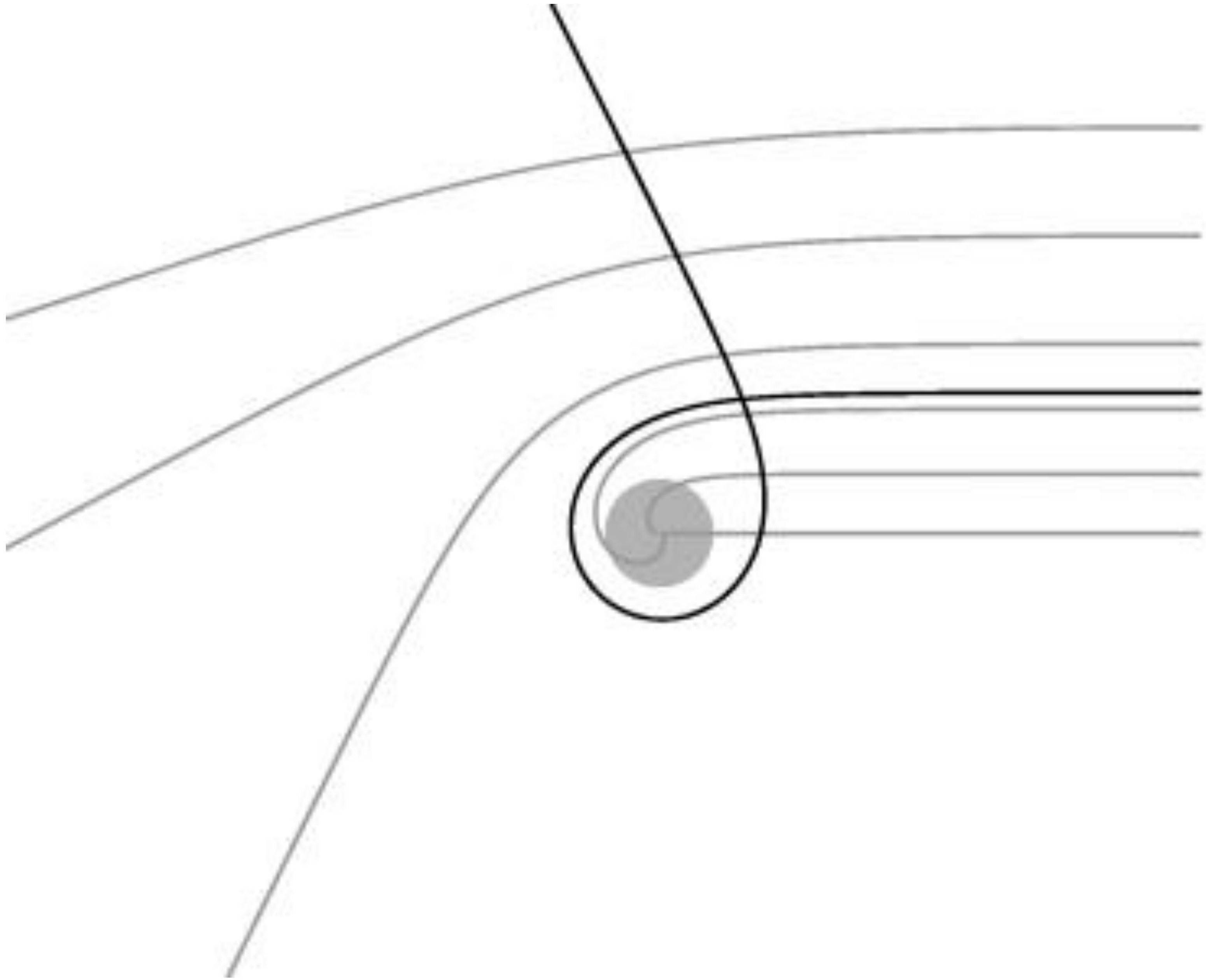
- The center of the BH is a point of infinite density and zero volume, called a ***singularity***

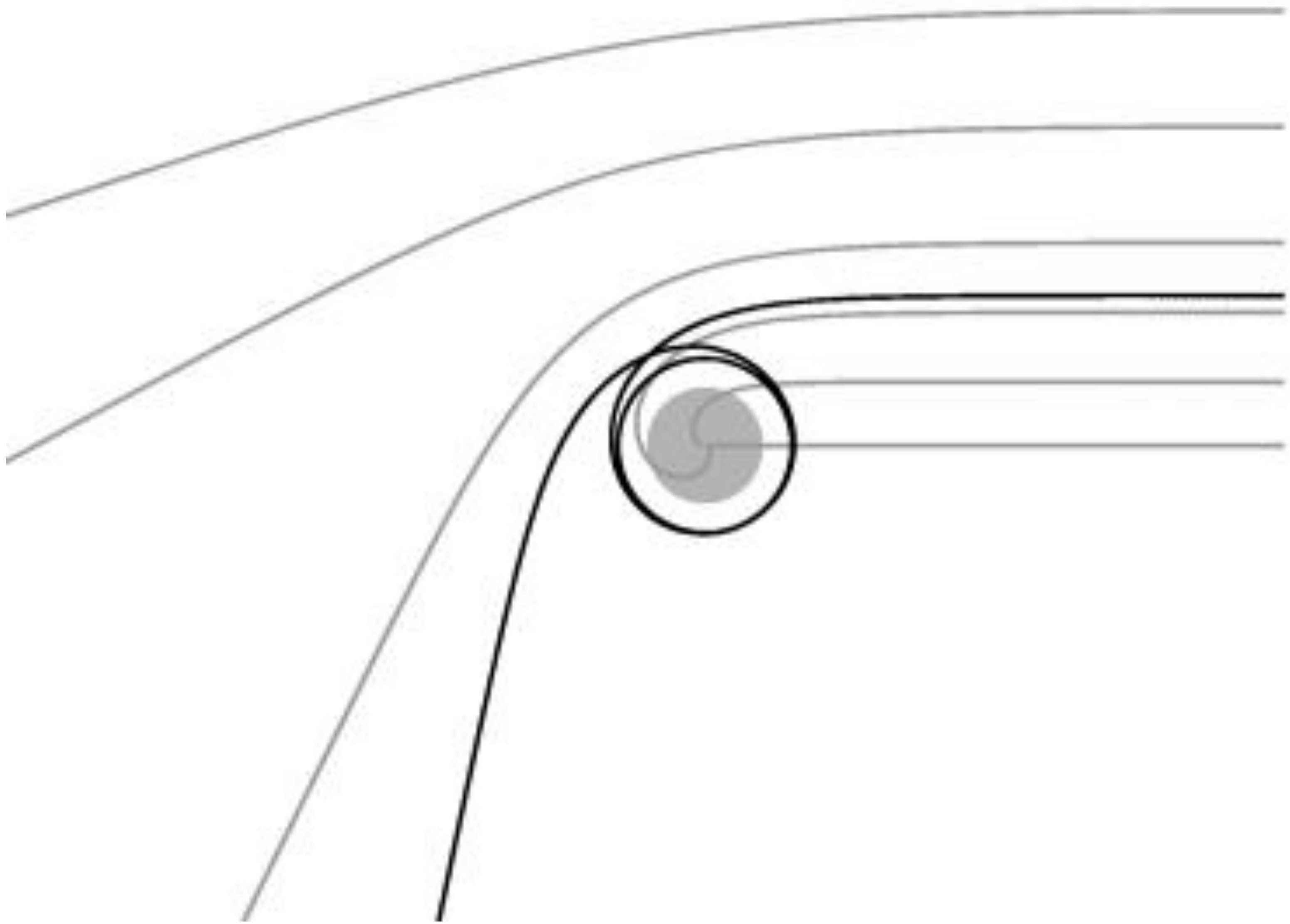


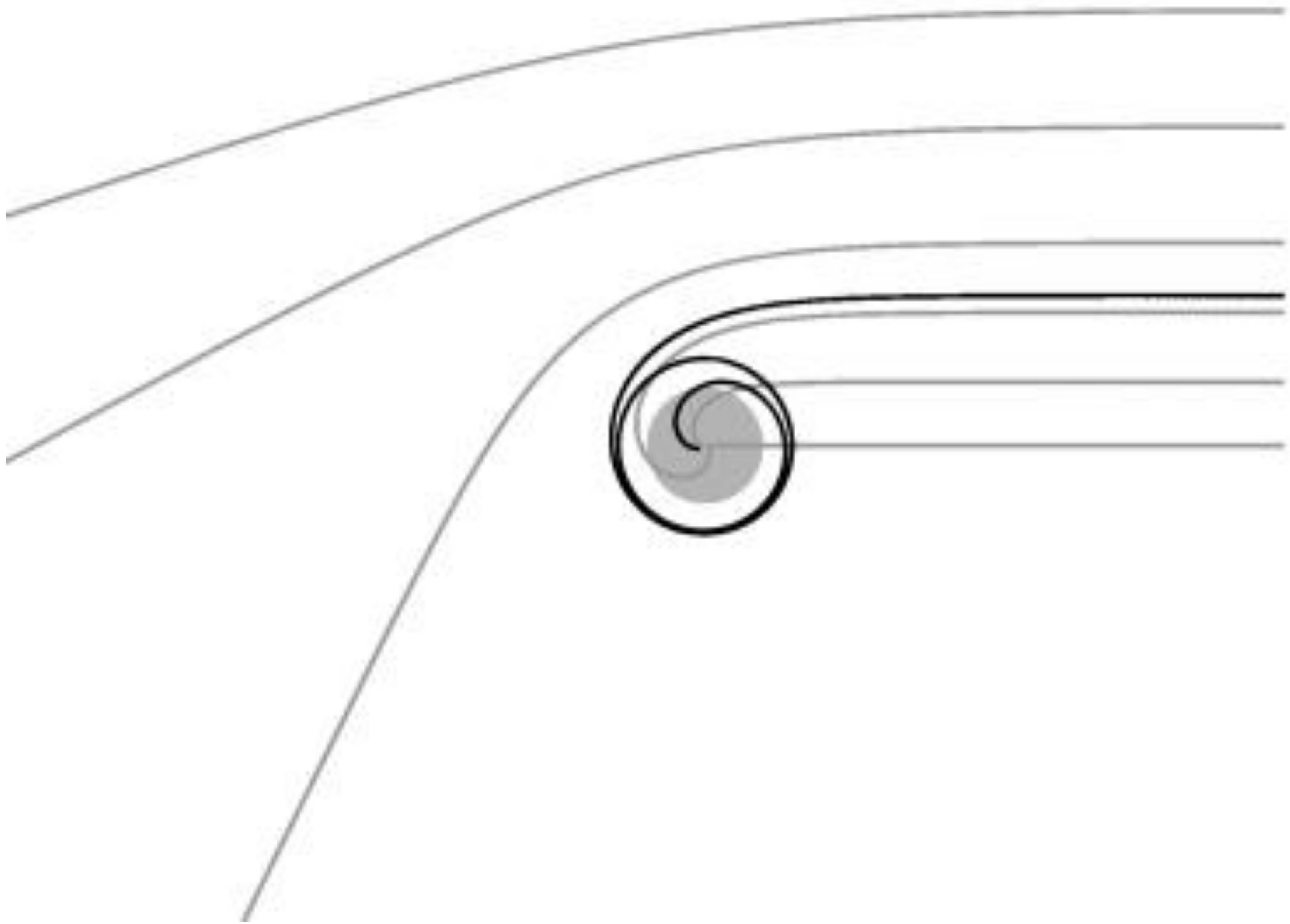
- Every object in the universe has a Schwarzschild radius, but they become a BH only if their mass is contained within R_S.

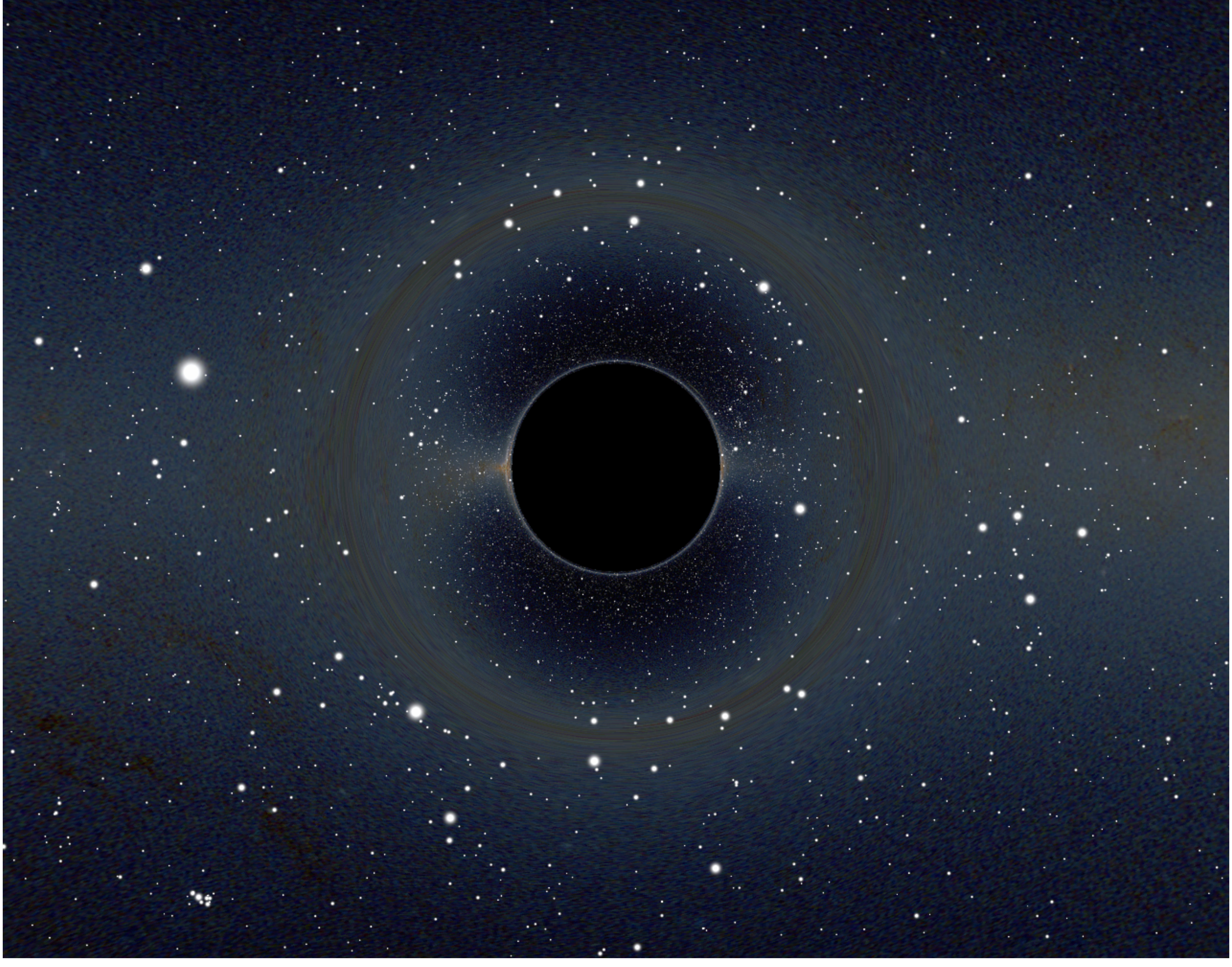


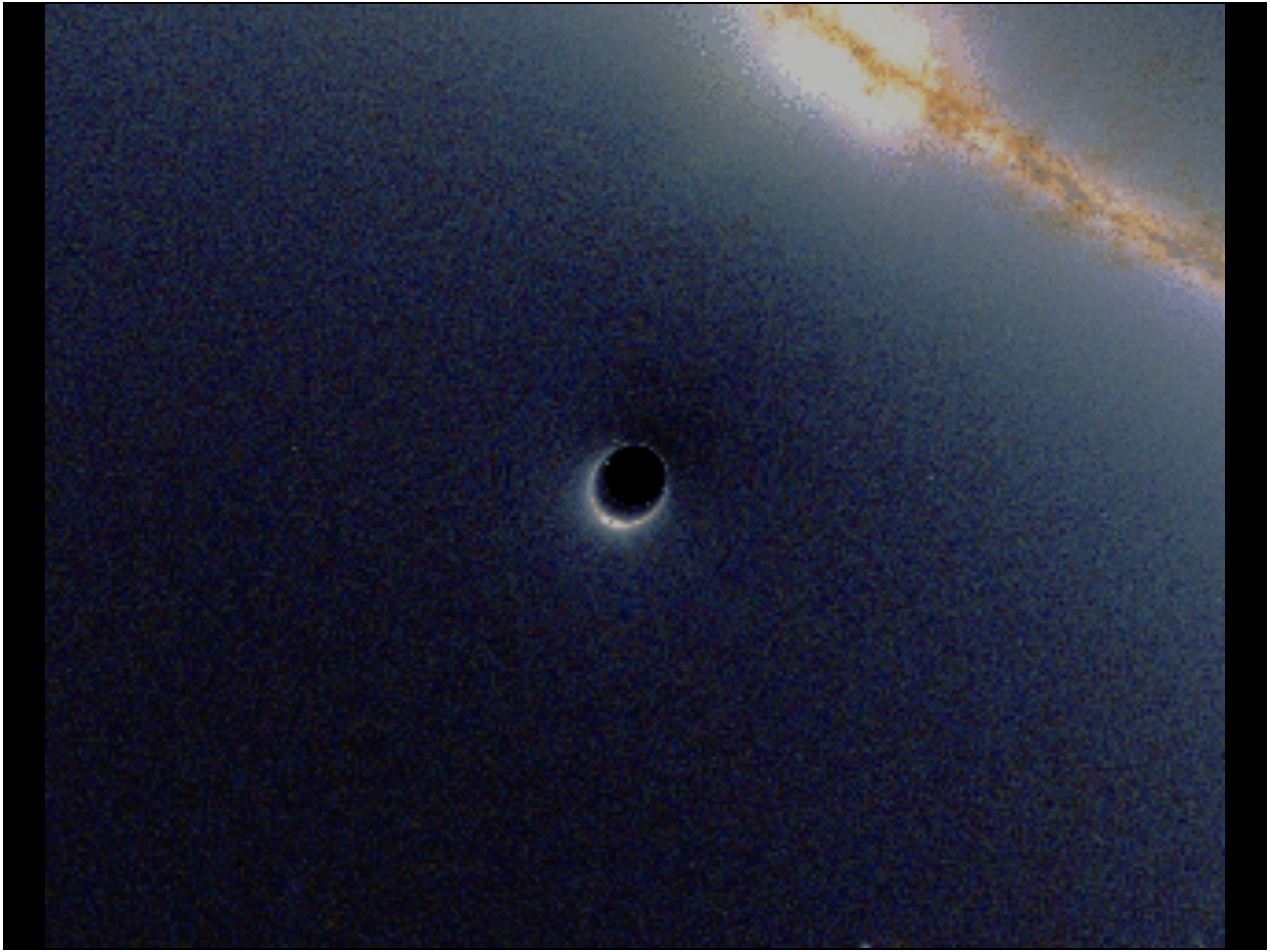






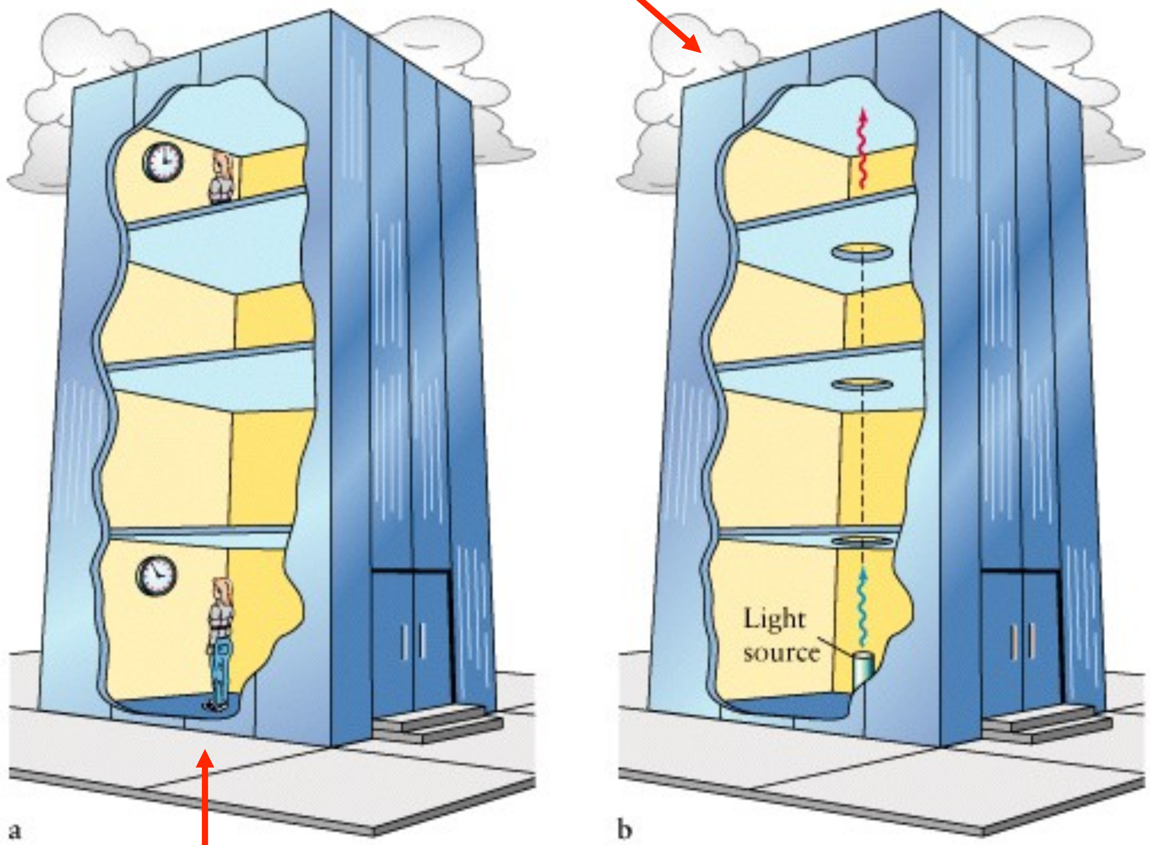






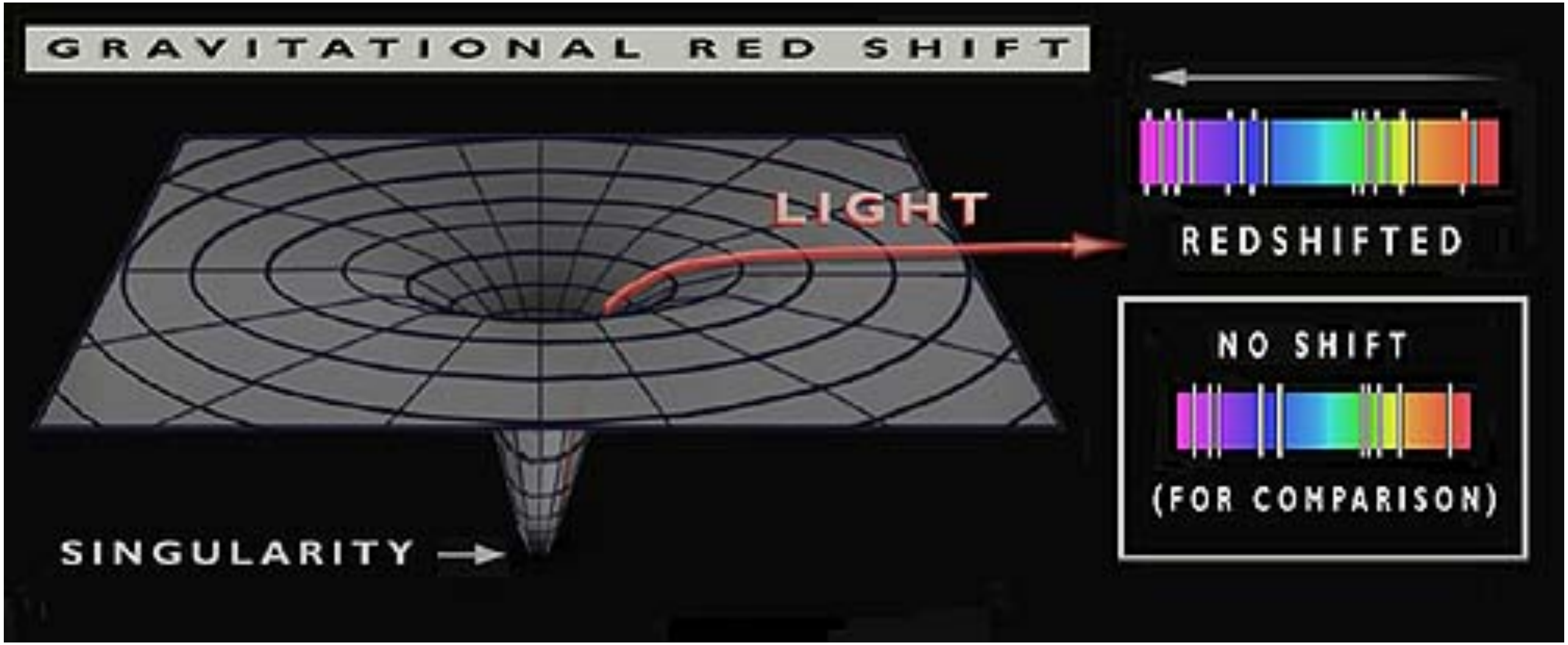
GR: Gravitational Redshift

Light loses energy as it travels away from a source of gravity



Equivalent viewpoint: time runs more slowly the closer you are to a source of gravity!

Gravitational redshift

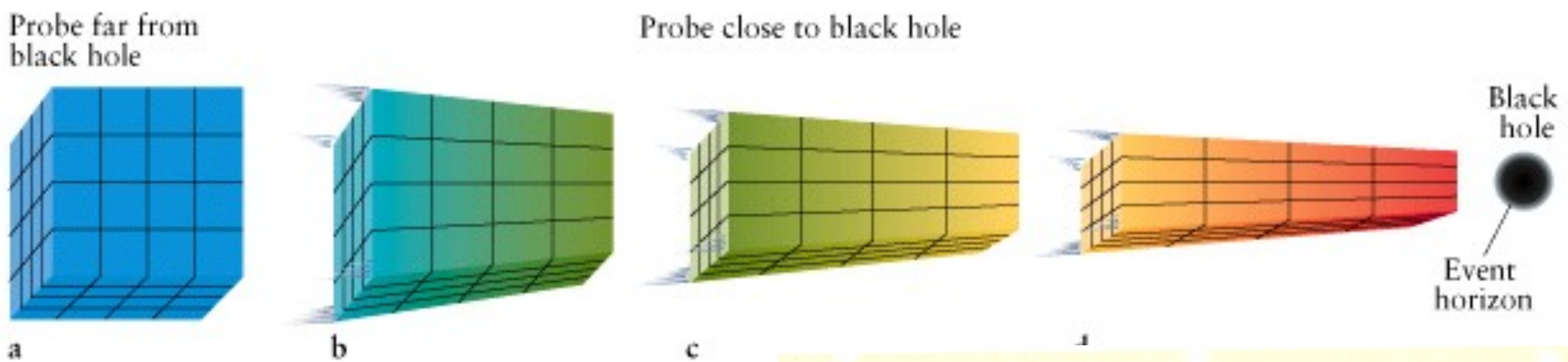


$$\frac{\nu}{\nu_{\mathrm{r}}} = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} \Rightarrow \nu \to 0 \text{ for } r \to R_S$$

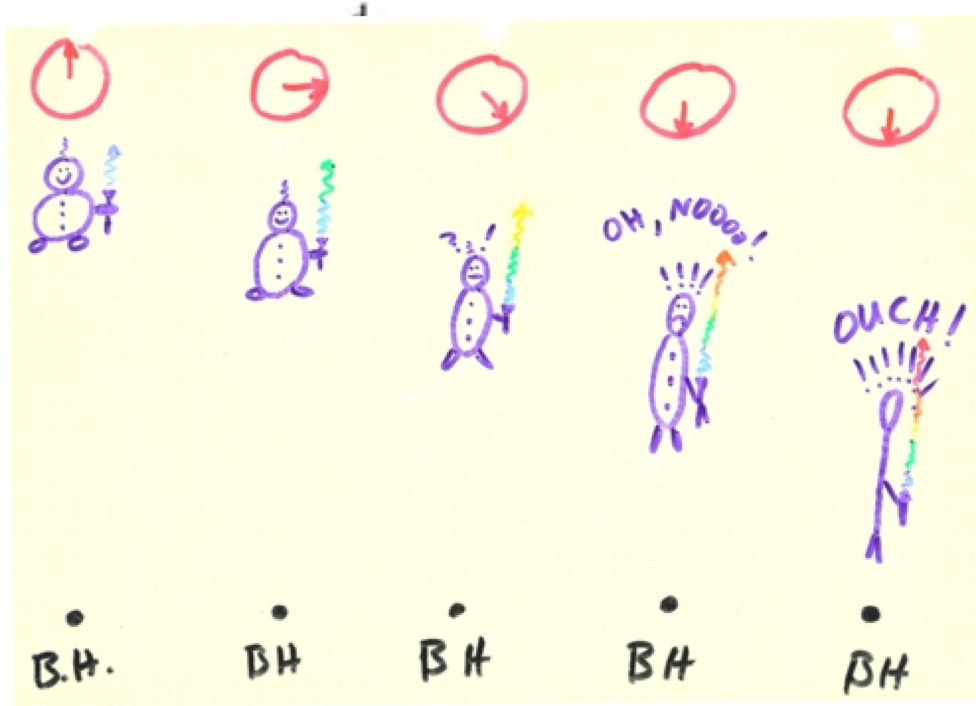
Falling into a black hole

- With a sufficiently ***large*** black hole, a **freely falling observer** would pass right through the event horizon in a finite time, **would not feel the event horizon**.
- A **distant observer** watching the freely falling observer would never see him/her fall through the event horizon (takes an **infinite time**).
- Signals sent from the freely falling observer would be **time dilated** and **redshifted**.

Falling into a black hole



Falling into smaller BH, the freely falling observer would be ripped apart by **tidal effects**.



Falling into a black hole

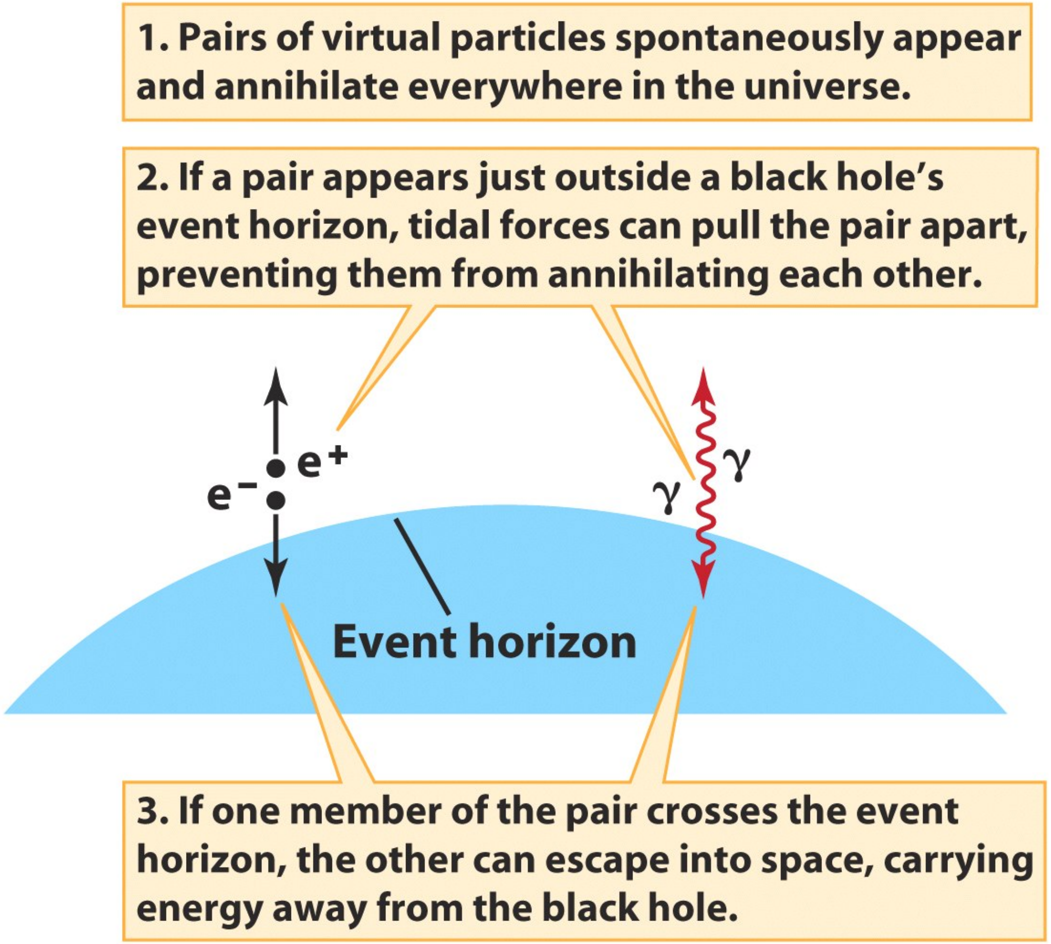
- Once **inside the event horizon**, **no communication** with the universe outside the event horizon is possible.
- But **incoming** signals from external world can enter.
- A black hole of mass M has exactly the same gravitational field as an ordinary mass M **at large distances**.

No-hair theorem

Three parameters completely describe the structure of a BH

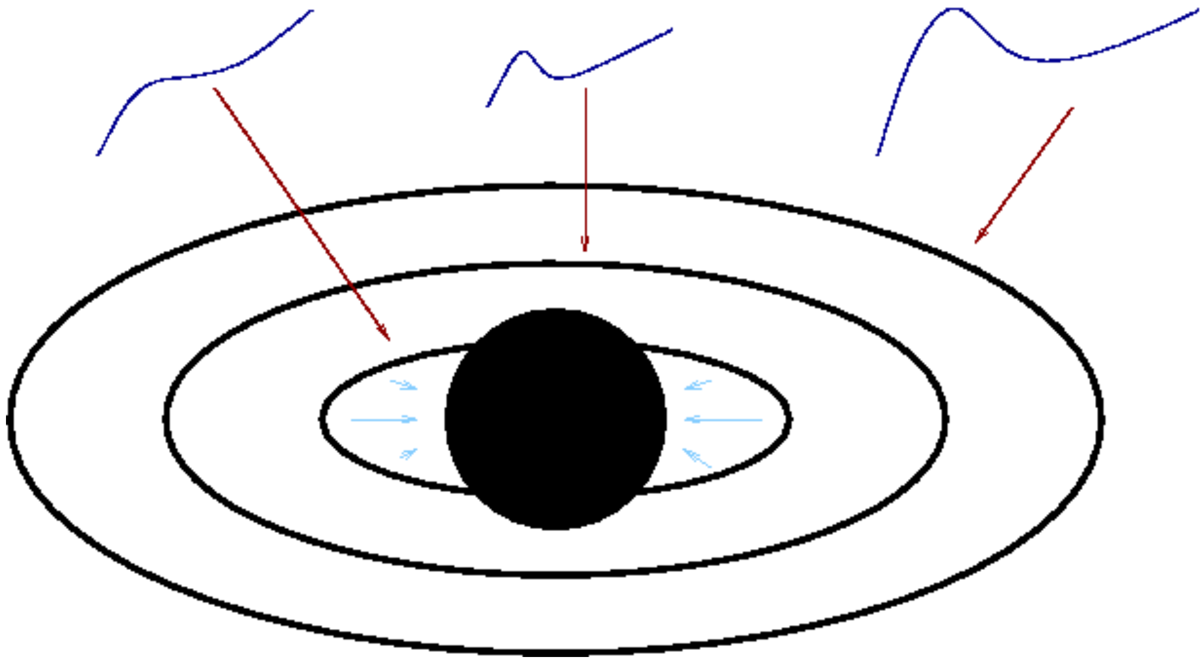
- Mass ($\boldsymbol{M}$)
  - As measured by the black hole's effect on orbiting bodies, such as another star
- Total electric charge ($\boldsymbol{Q}$)
  - As measured by the strength of the electric force (Q = 0)
- Spin = angular momentum ($\boldsymbol{a_*}$)
  - How fast the black hole is spinning ($a_* < 1$)

Hawking radiation



Extremely low luminosity (undetectable), but may cause evaporation of micro-BH (formed at Big Bang?)

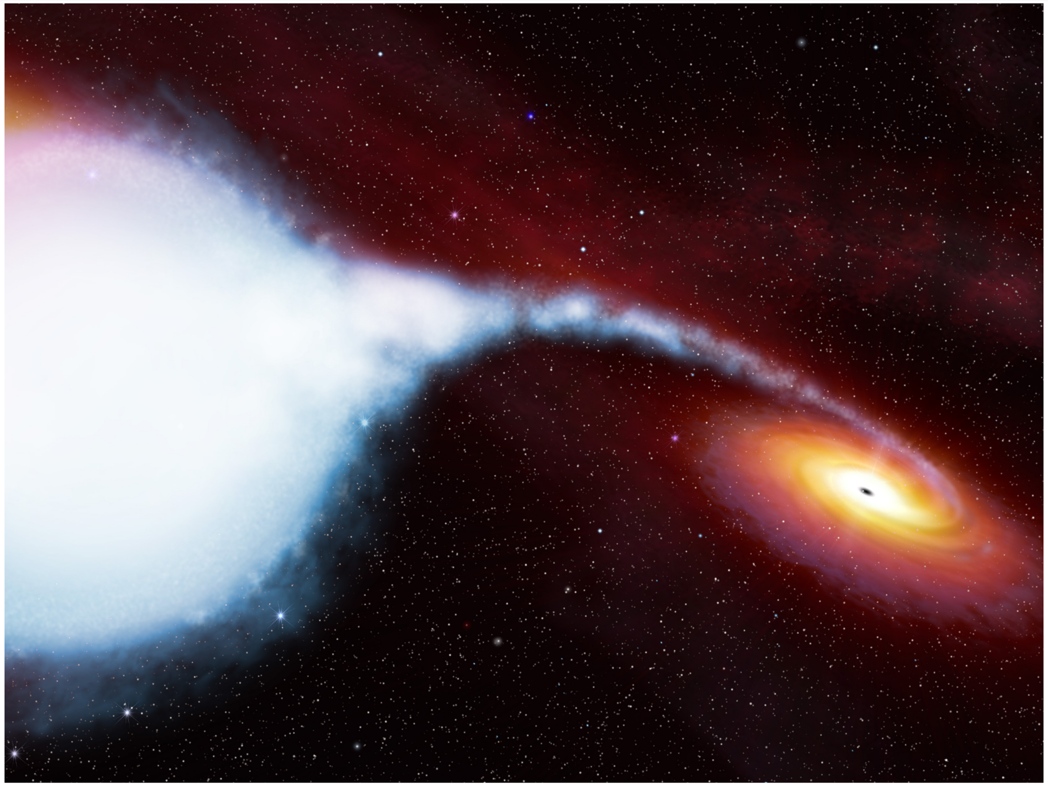
Some orbits and their potentials



By altering angular momentum, we get **stable orbits** at different radii: stable circular orbit at a minimum of potential.

At $R=6GM/c^2=3R_S$ the minimum becomes a point of inflection ⇒ **Innermost Stable Circular Orbit (ISCO)**

Accretion efficiency



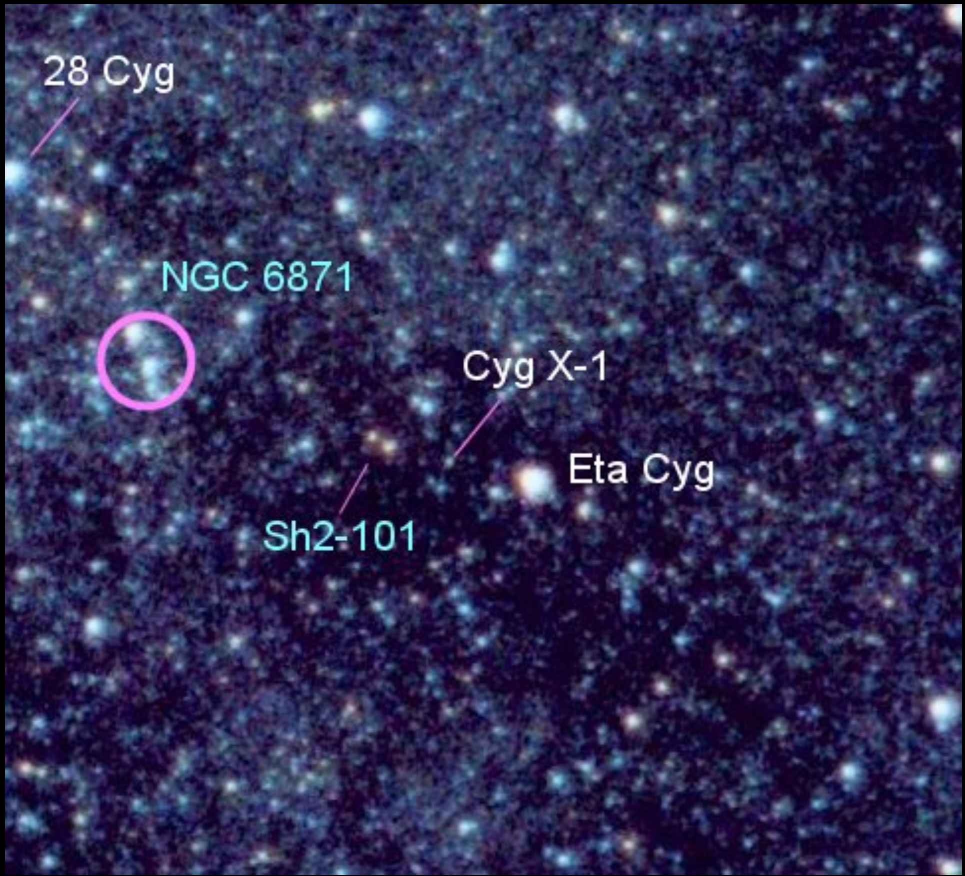
Gravitational energy at ISCO ($R_{ISCO} = 3R_S \sim 100$ *km for a* $10\ M_{\odot}$ *BH*):

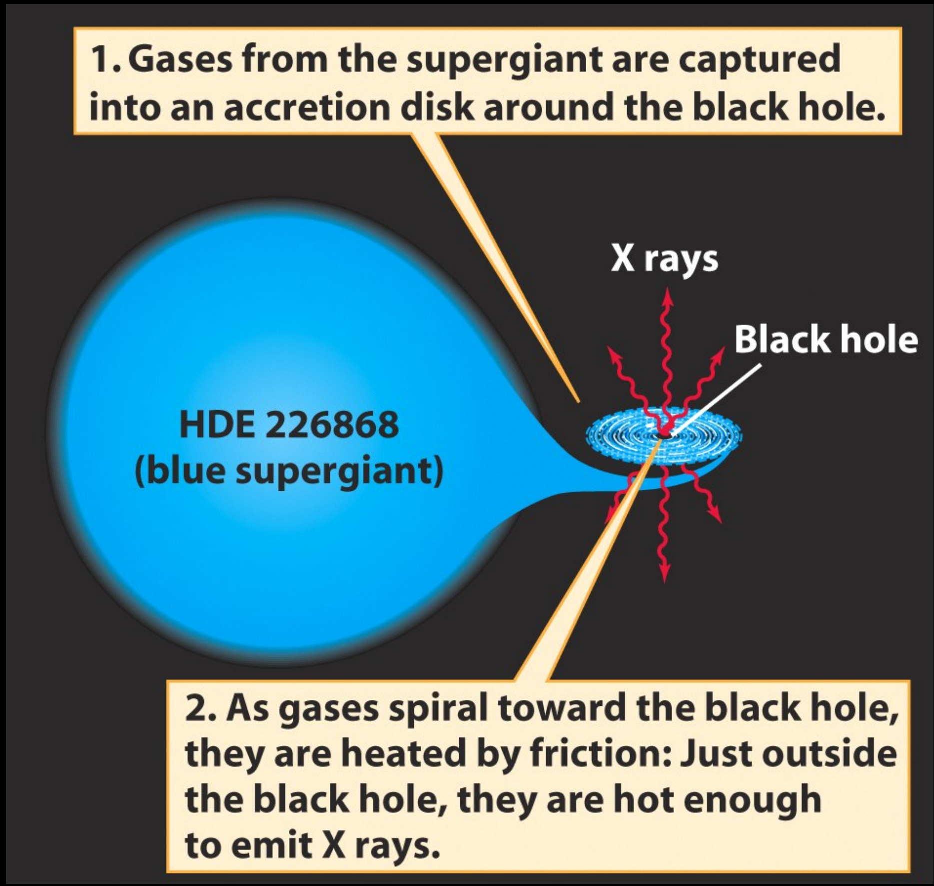
$$E_G \sim GmM/3R_s = GmMc^2/6GM = mc^2/6$$

Efficiency: $E_G/mc^2 \sim$ **1/6 ~ 20%** » 0.7% (nuclear fusion)

Stellar mass BHs

Bright X-ray sources when in accreting binary systems





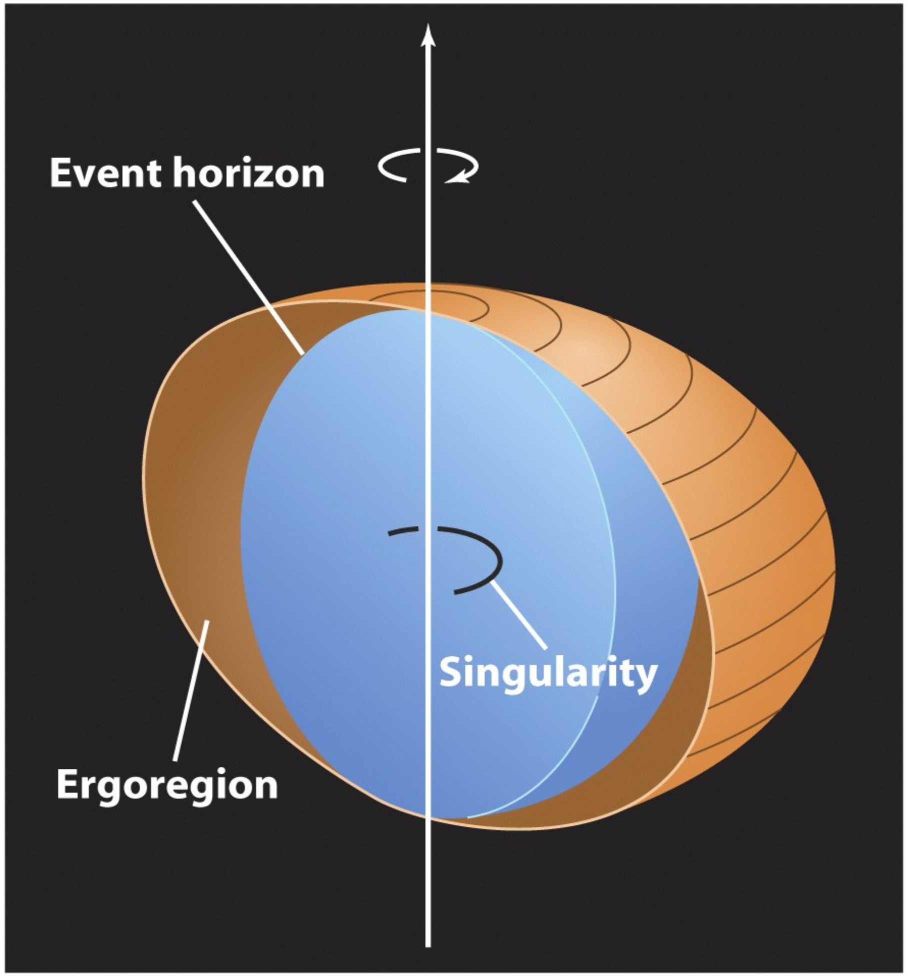
Cyg X-1: X-ray variability on <1 s timescale; $M \sim 15\ M_\odot$

Rotating black holes (Kerr BH)

- A rotating black hole has an **ergosphere** around the outside of the event horizon
- In the ergosphere, space and time themselves are dragged along with the rotation of the black hole
- If maximum spin ($a_*=1$): event horizon at

$$R=GM/c^2=1/2\ R_S\ ;$$

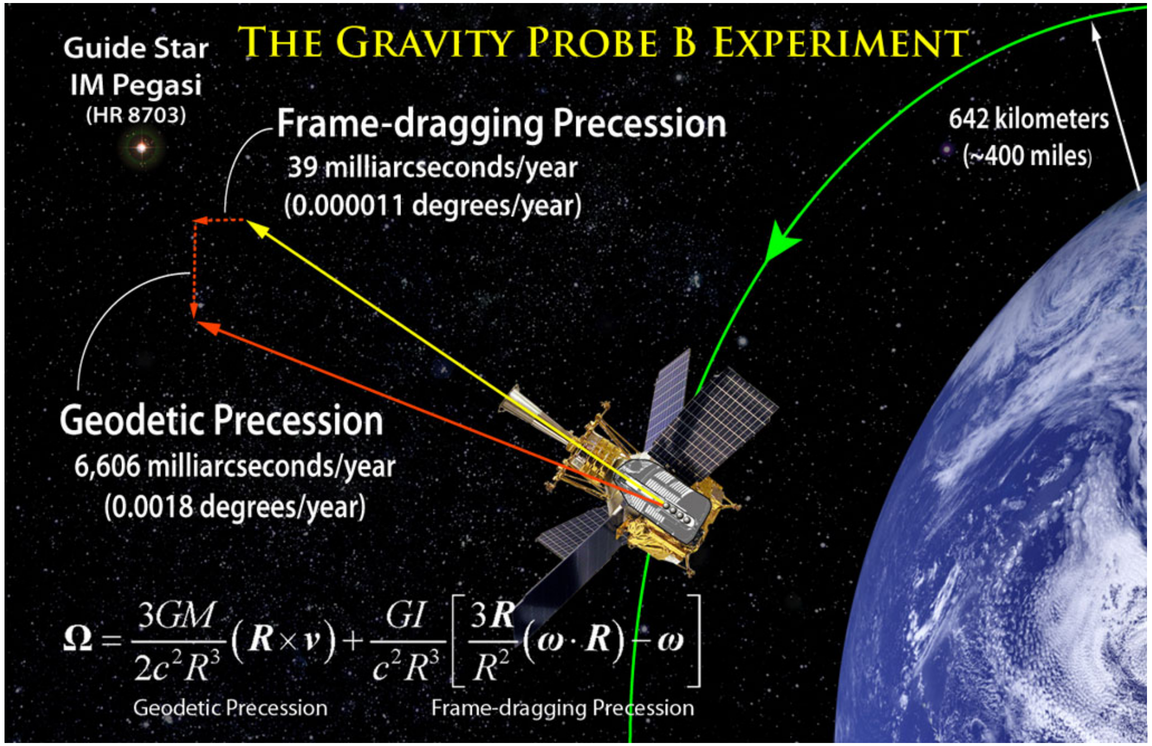
$$R_{ISCO}=GM/c^2=R$$

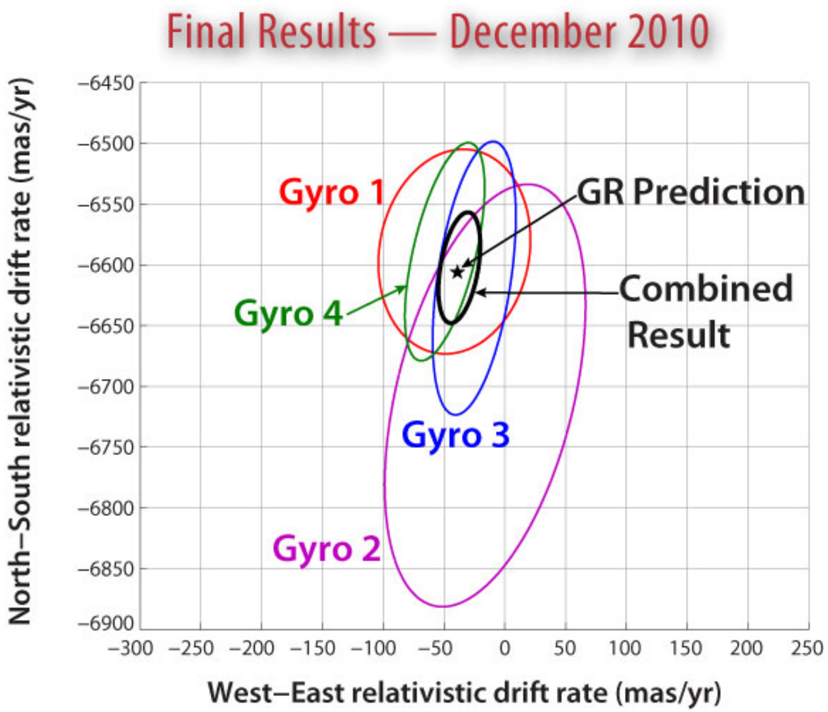


A rotating mass has a tendency to pull space-time along with it

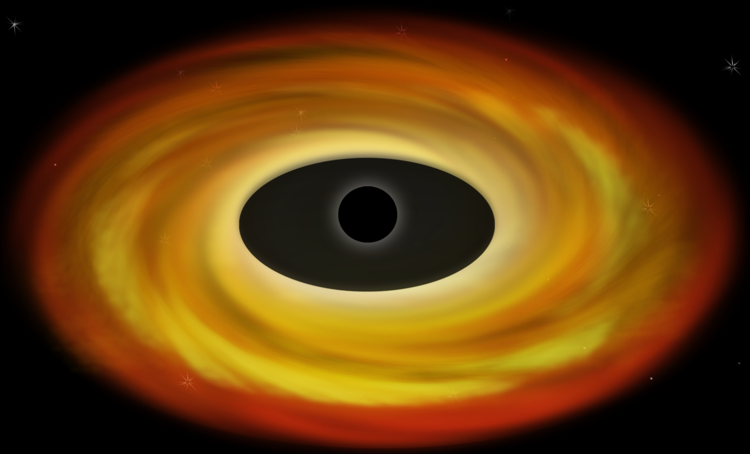
Gravity Probe B

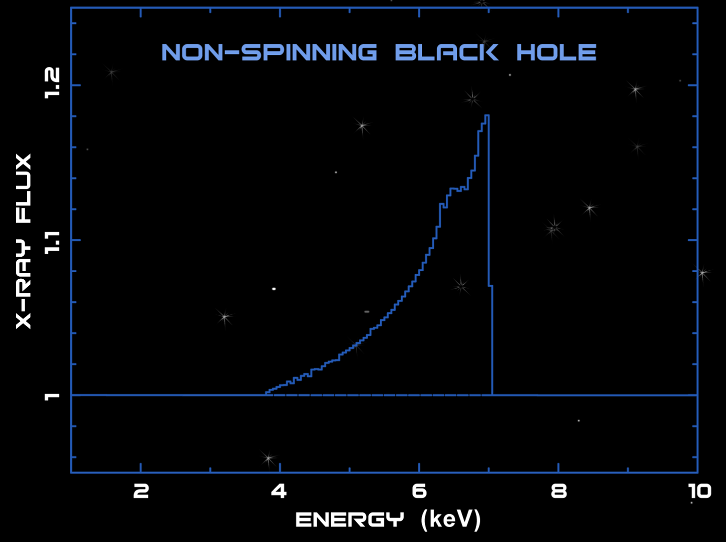
Launched 20 April 2004 to test geodetic and frame-dragging GR effects, by means of cryogenic gyroscopes in Earth orbit



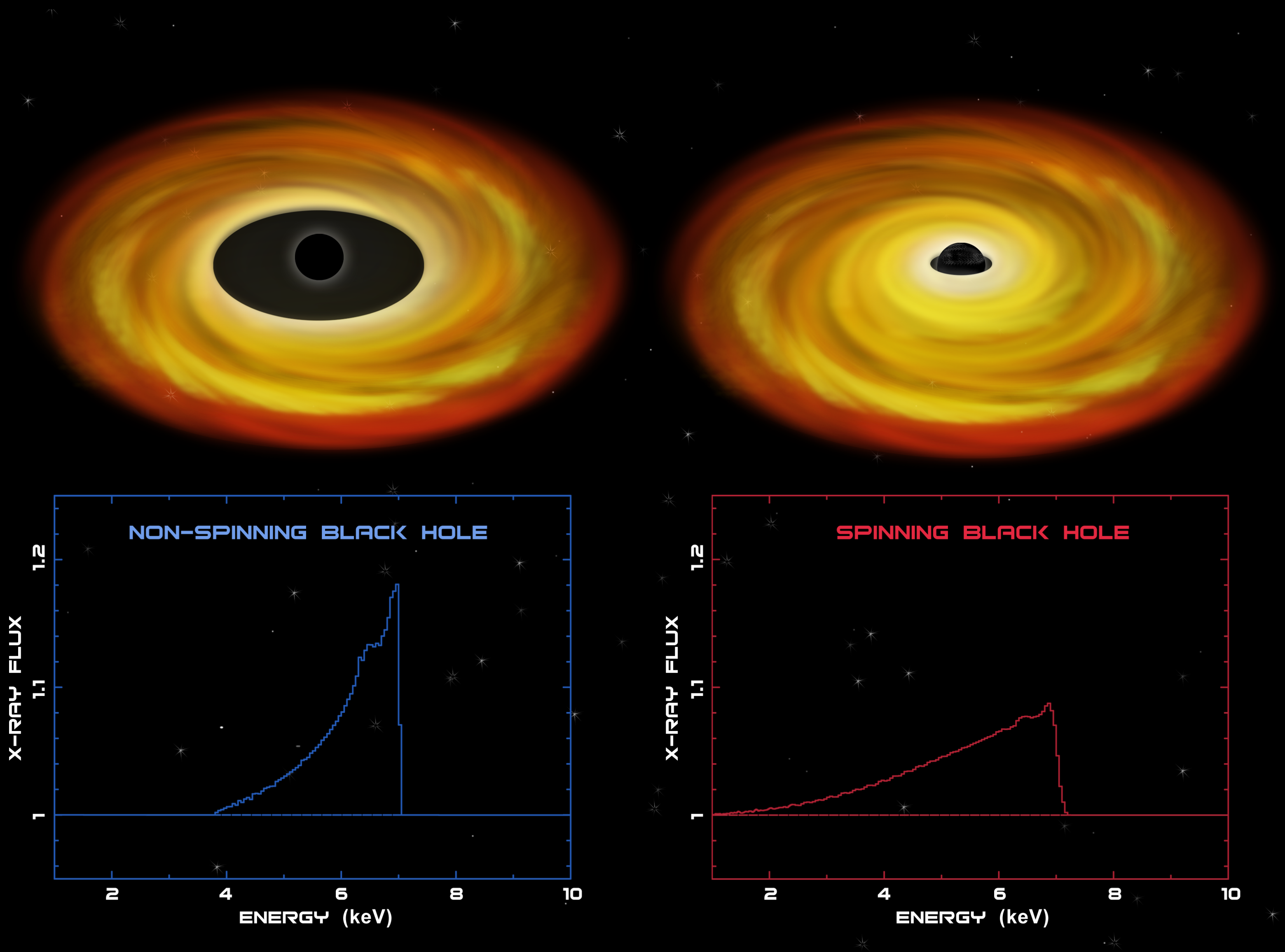


Relativistic line

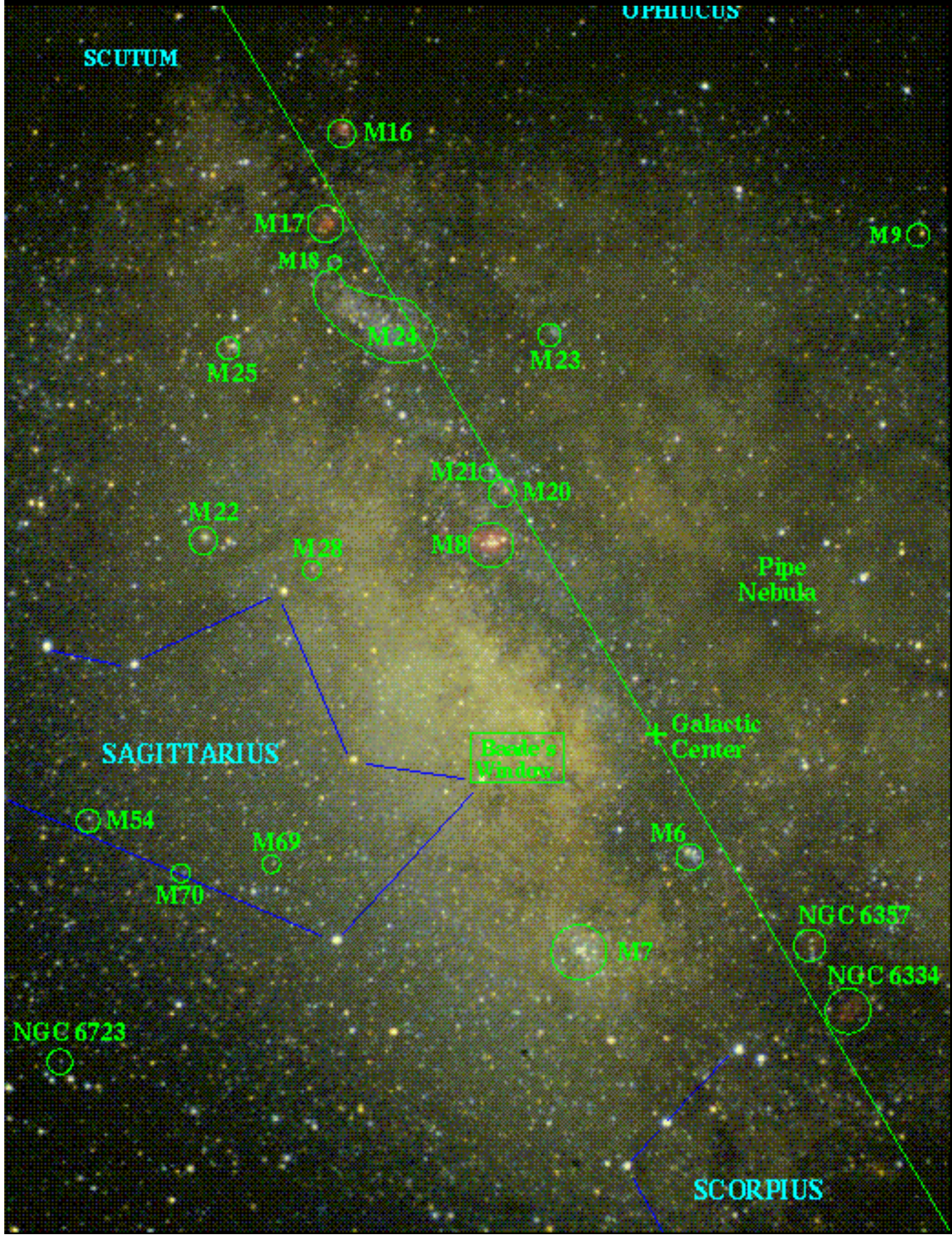




Relativistic line



The biggest nearby BH: Sgr A*



Center of our galaxy: radio source Sgr A*

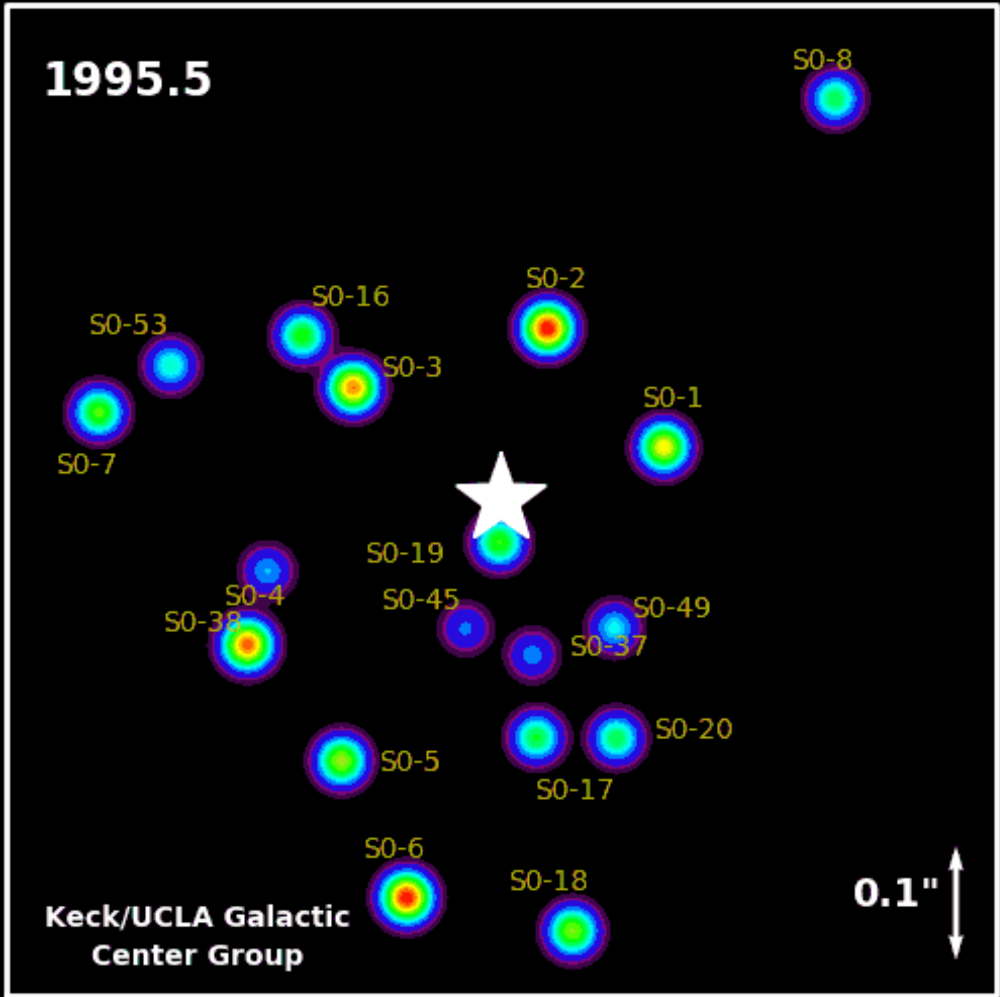
Distance: 8 kpc

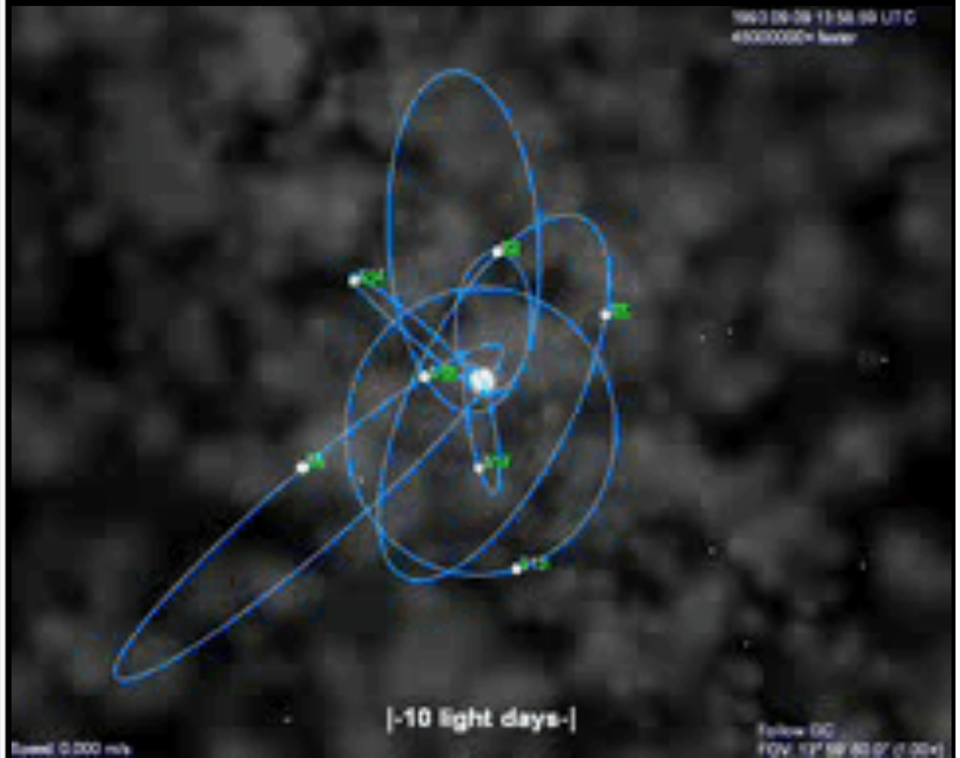
Highly obscured in optical

Dense central star cluster visible in infrared

Photo/illustration from A. Tanner, UCLA

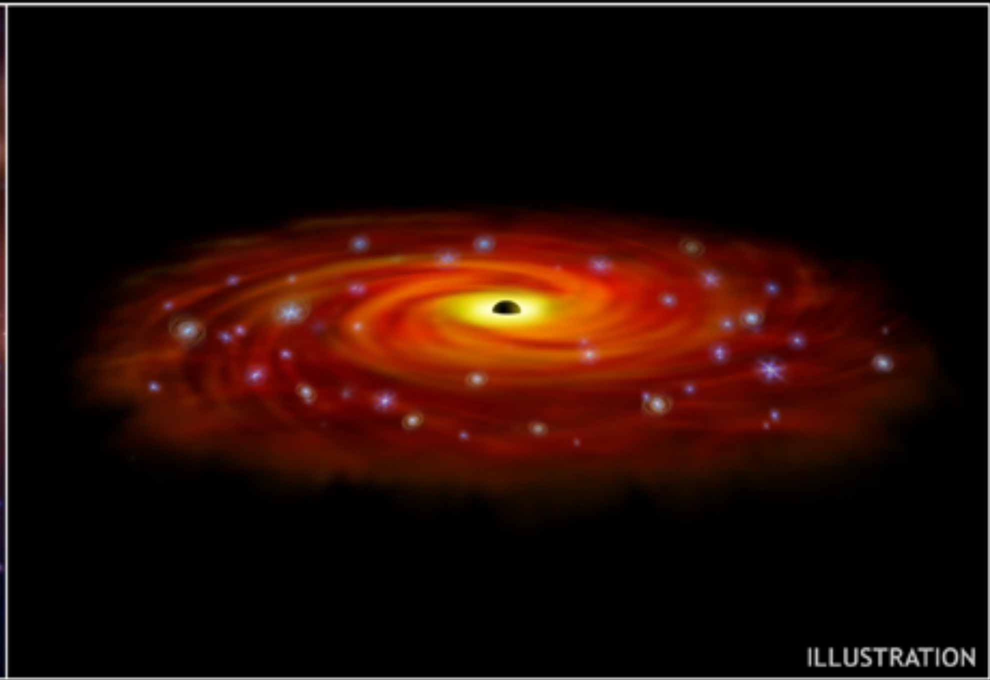
The mass of Sgr A*

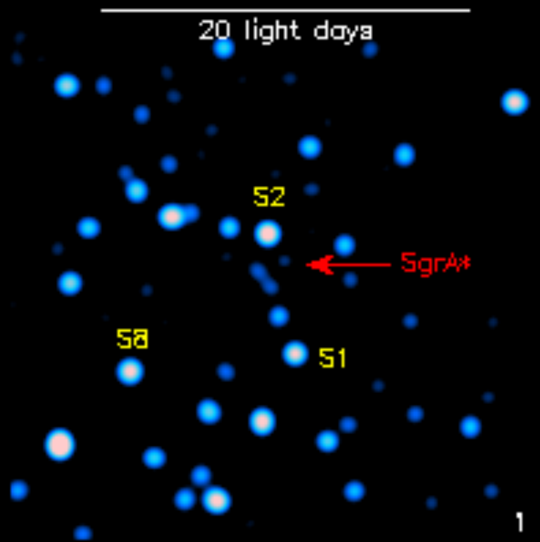




Mass = 4 x 10^6 $M_\odot$; R_S ~ 10^7 km ~ 10 $R_\odot$ ~ 1/15 AU

Is Sgr A* "black"?





SgrA is a variable point source in radio, IR, X-rays (obscured in optical/UV)

Accreting at a very low rate
⇒ very low luminosity

LETTER

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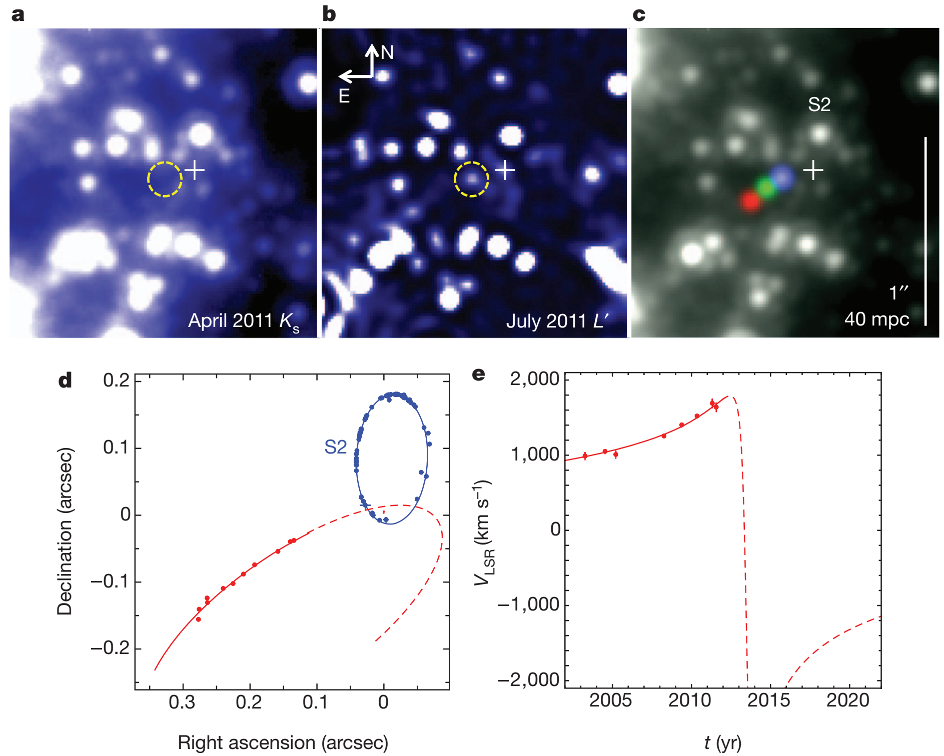
A gas cloud on its way towards the supermassive black hole at the Galactic Centre

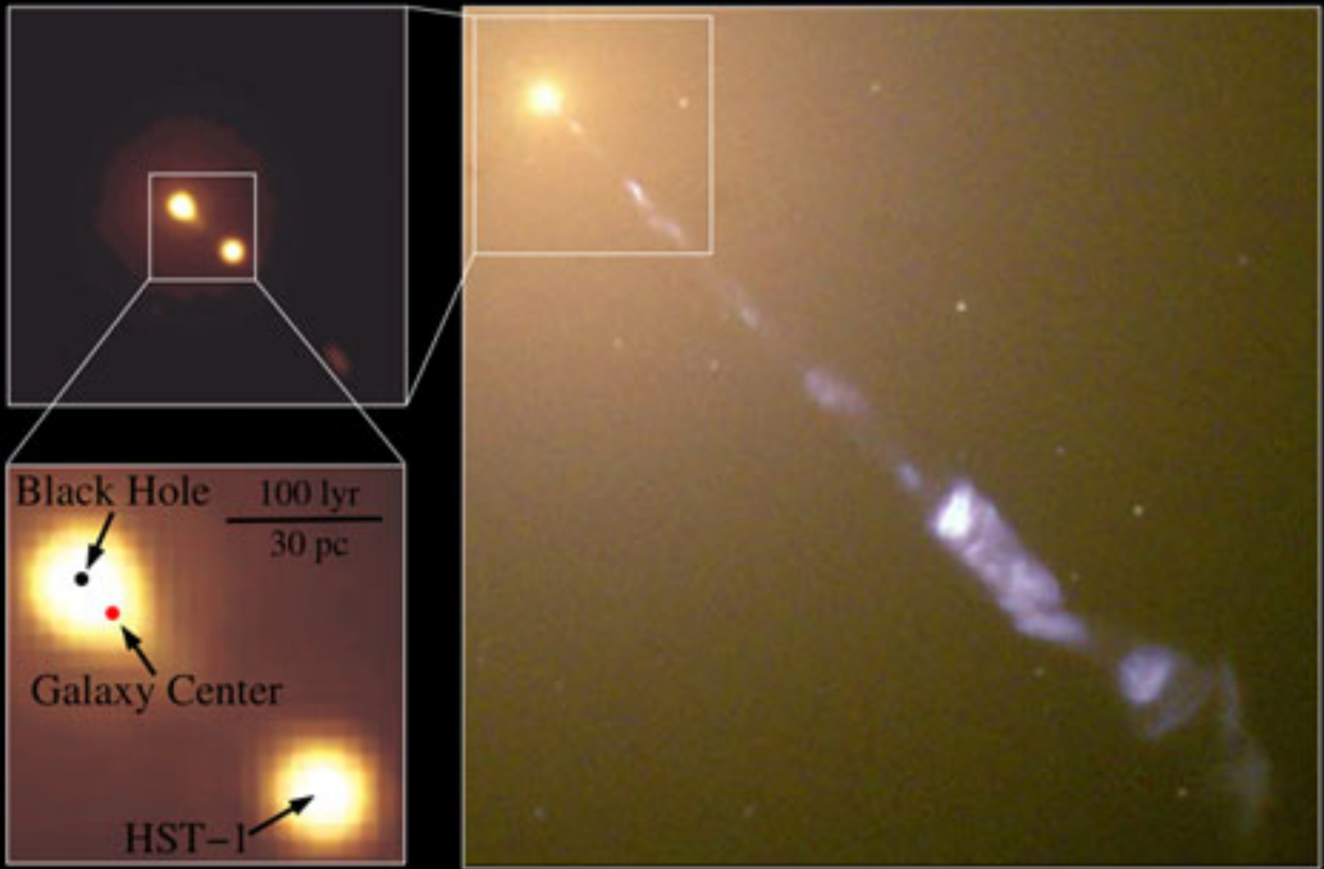
S. Gillessen[1], R. Genzel[1,2], T. K. Fritz[1], E. Quataert[3], C. Alig[4], A. Burkert[4,1], J. Cuadra[5], F. Eisenhauer[1], O. Pfuhl[1], K. Dodds-Eden[1], C. F. Gammie[6] & T. Ott[1]

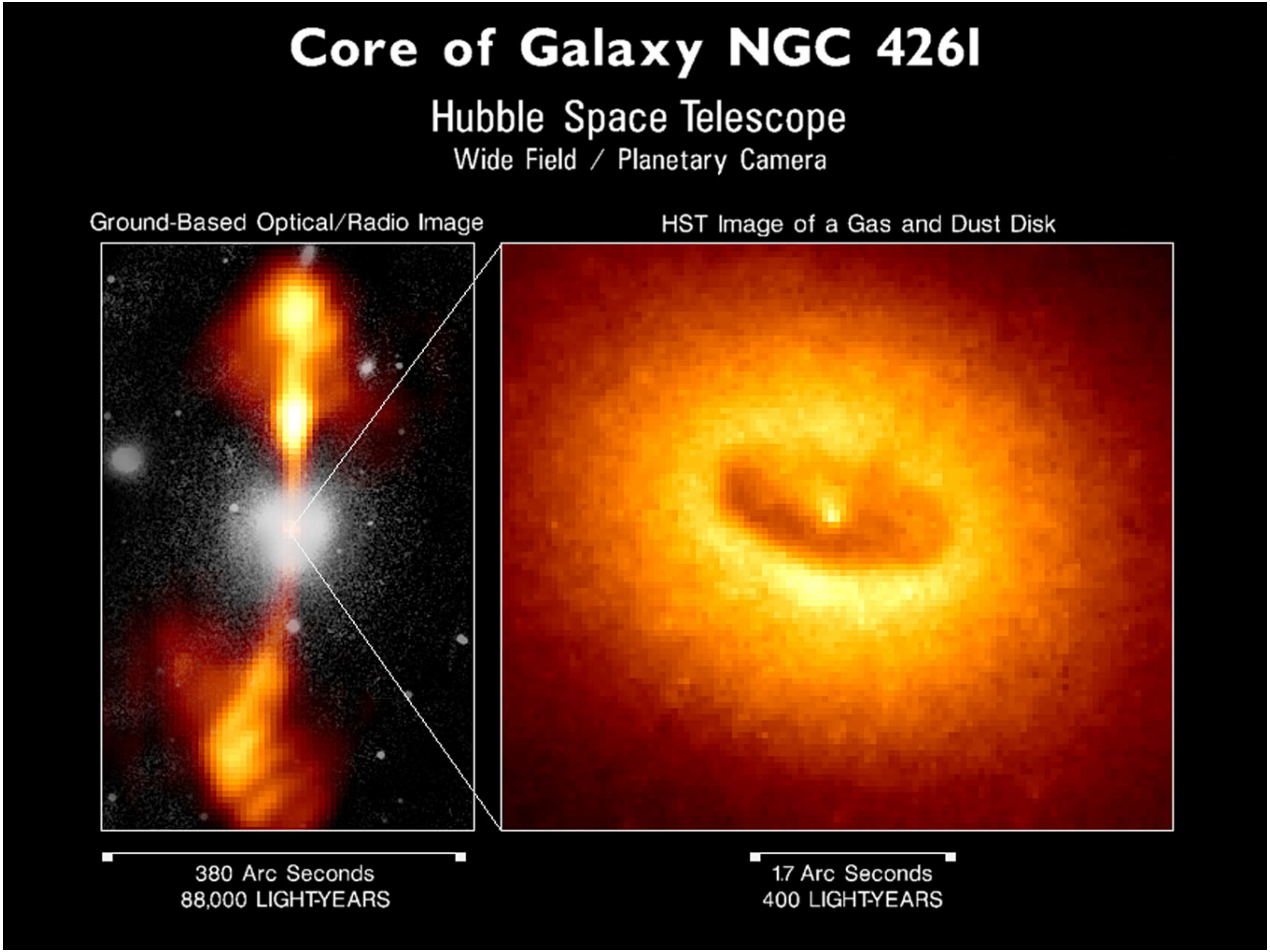
Measurements of stellar orbits[1–3] provide compelling evidence[4,5] that the compact radio source Sagittarius A* at the Galactic Centre is a black hole four million times the mass of the Sun. With the exception of modest X-ray and infrared flares[6,7], Sgr A* is surprisingly faint, suggesting that the accretion rate and radiation efficiency near the event horizon are currently very low[3,8]. Here we report the presence of a dense gas cloud approximately three times the mass of Earth that is falling into the accretion zone of Sgr A*. Our observations tightly constrain the cloud's orbit to be highly eccentric, with an innermost radius of approach of only ∼3,100 times the event horizon that will be reached in 2013. Over the past three years the cloud has begun to disrupt, probably mainly through tidal shearing arising from the black hole's gravitational force. The cloud's dynamic evolution and radiation in the next few years will probe the properties of the accretion flow and the feeding processes of the supermassive black hole. The kilo-electronvolt X-ray emission of Sgr A* may brighten significantly when the cloud reaches pericentre. There may also be a giant radiation flare several years from now if the cloud breaks up and its fragments feed gas into the central accretion zone.



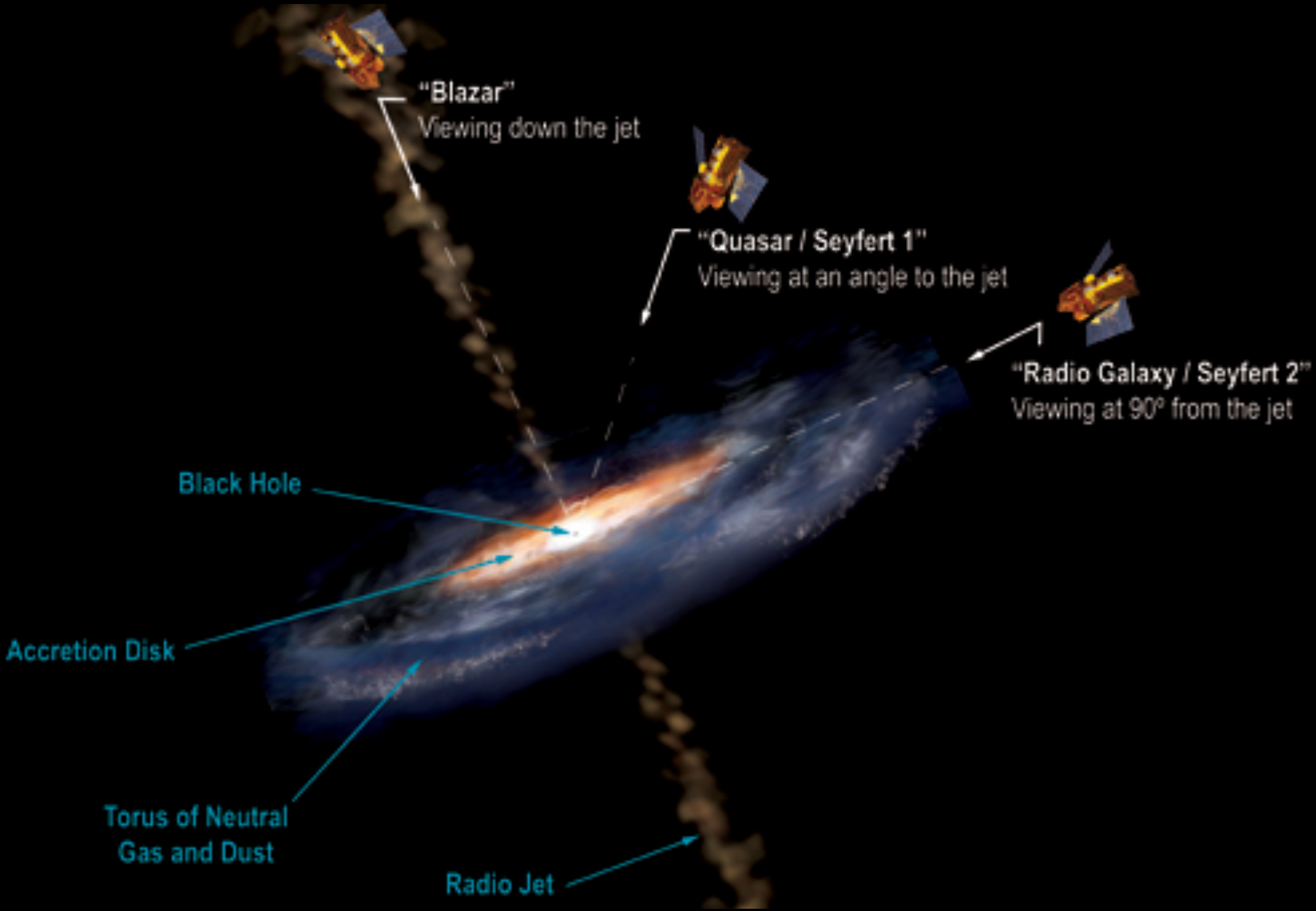
Active Galactic Nuclei (AGN)

Accreting supermassive BHs (up to billions of Solar masses) at the center of galaxies

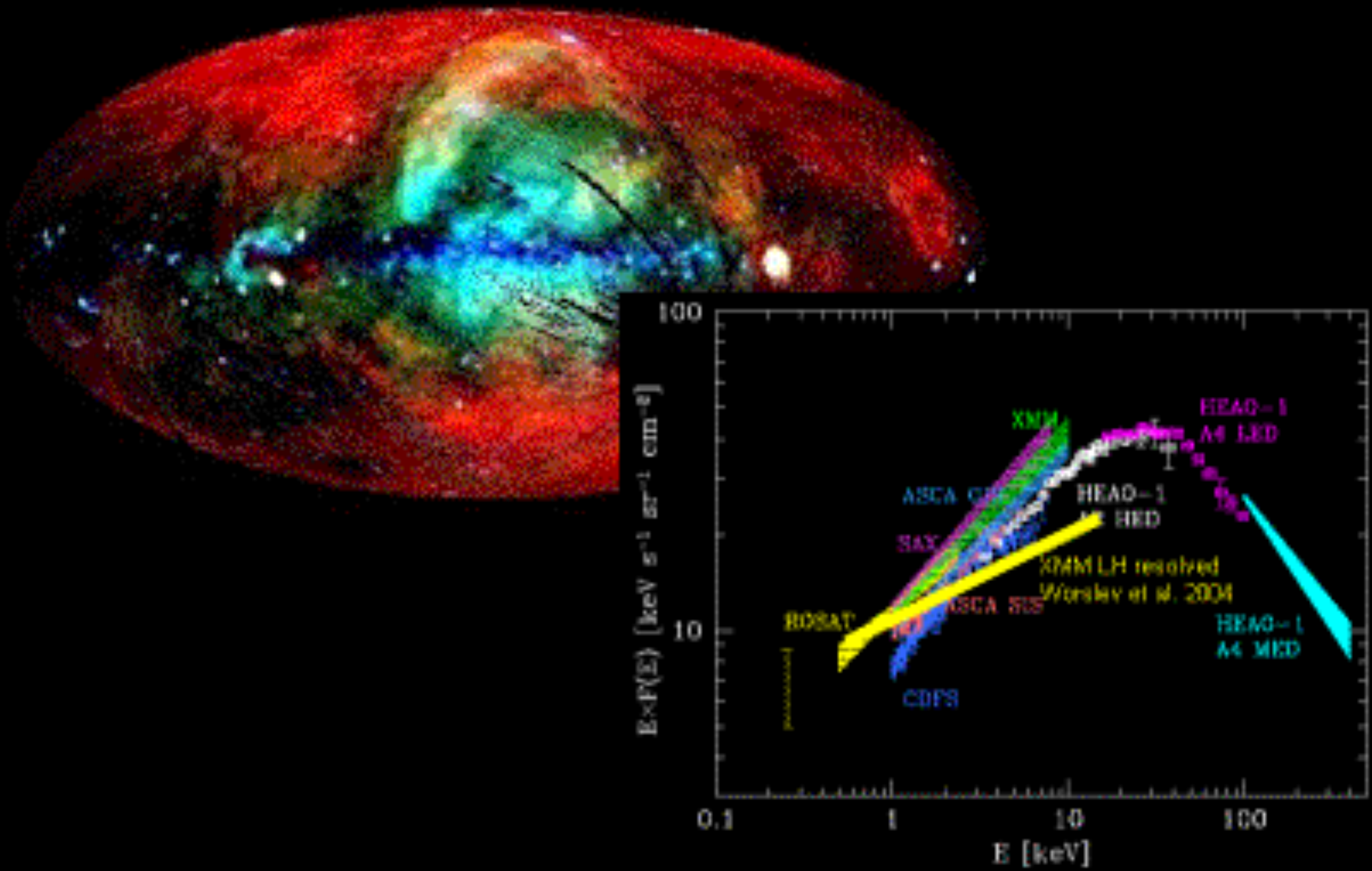



Core of Galaxy NGC 4261
Hubble Space Telescope
Wide Field / Planetary Camera
Ground-Based Optical/Radio Image
HST Image of a Gas and Dust Disk
380 Arc Seconds
88,000 LIGHT-YEARS
1.7 Arc Seconds
400 LIGHT-YEARS

AGN classes

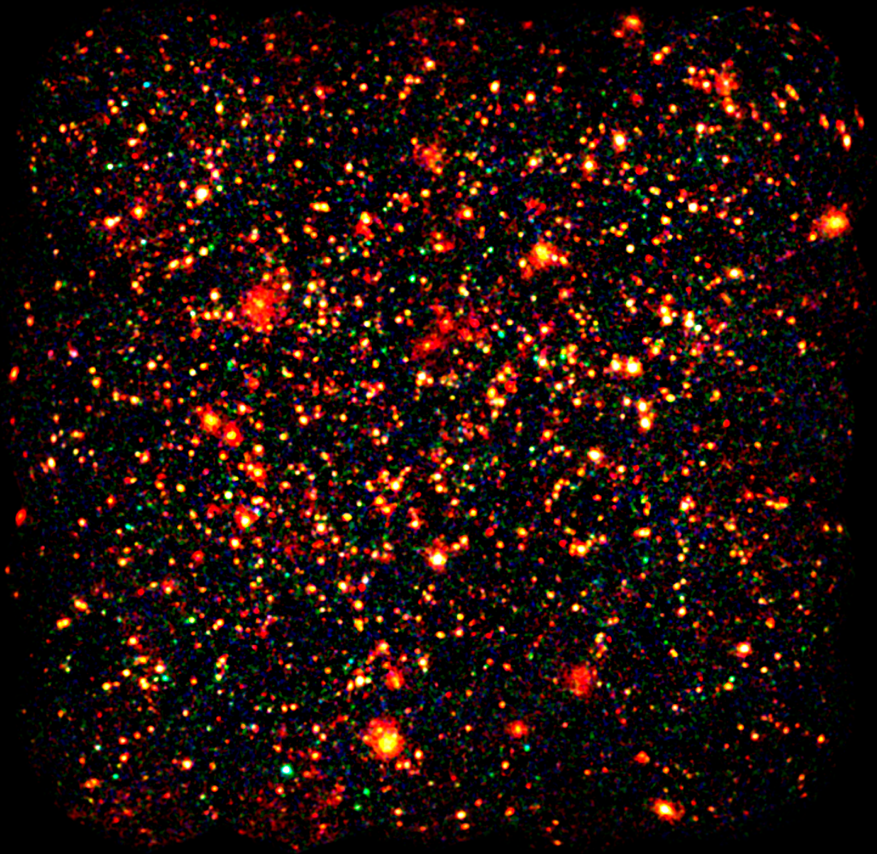

"Blazar"
Viewing down the jet
"Quasar / Seyfert 1"
Viewing at an angle to the jet
"Radio Galaxy / Seyfert 2"
Viewing at 90° from the jet
Black Hole
Accretion Disk
Torus of Neutral
Gas and Dust
Radio Jet

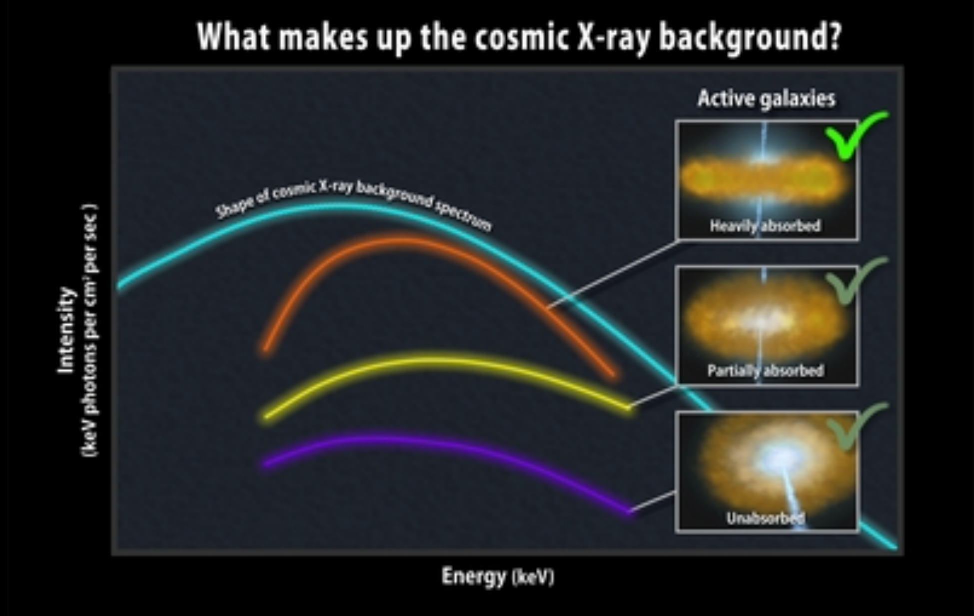
The Cosmic X-ray Background (CXB)



Removing discrete X-ray sources, residual diffuse emisssion

CXB and AGNs





CXB is due to unresolved X-ray emission from distant AGNs