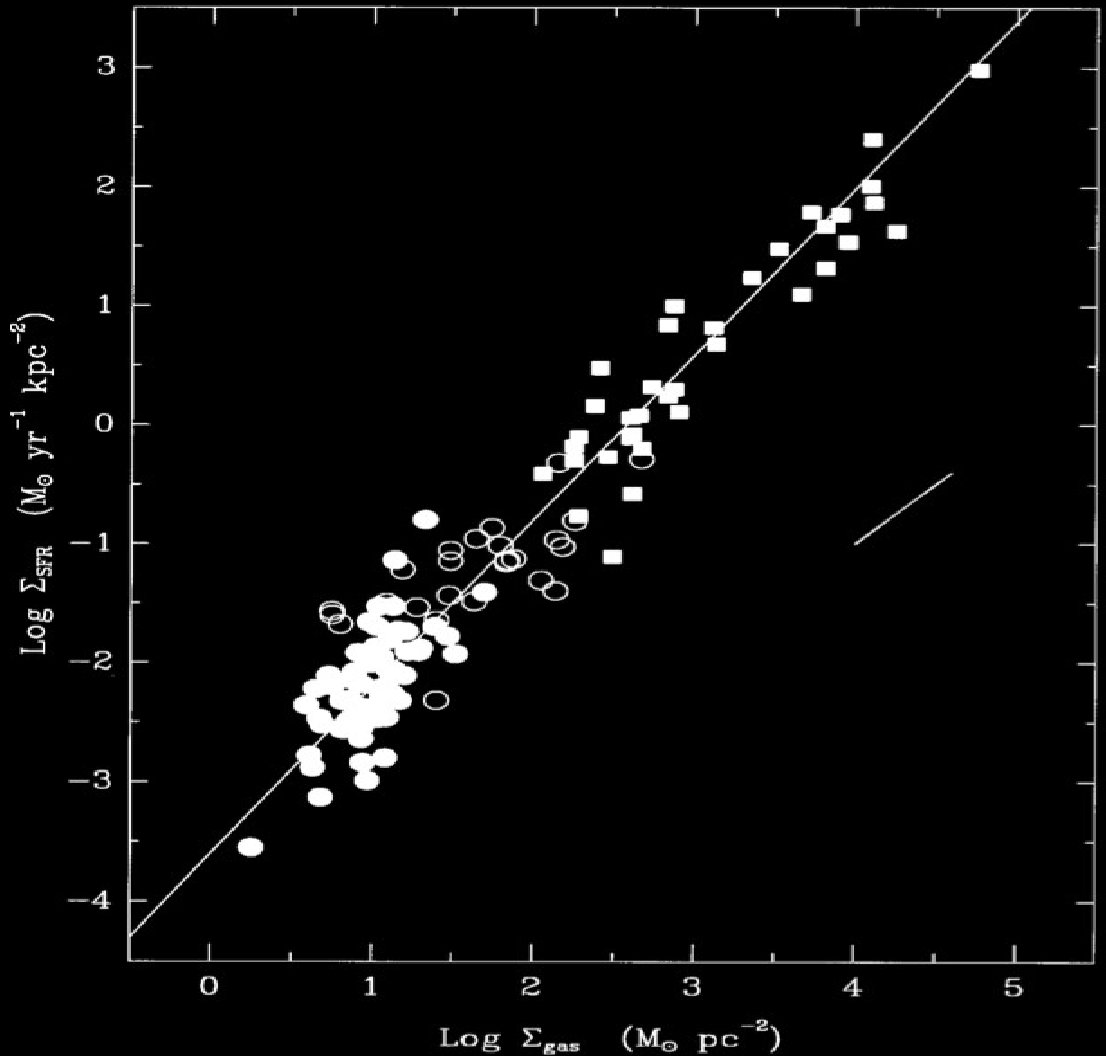
# Stars

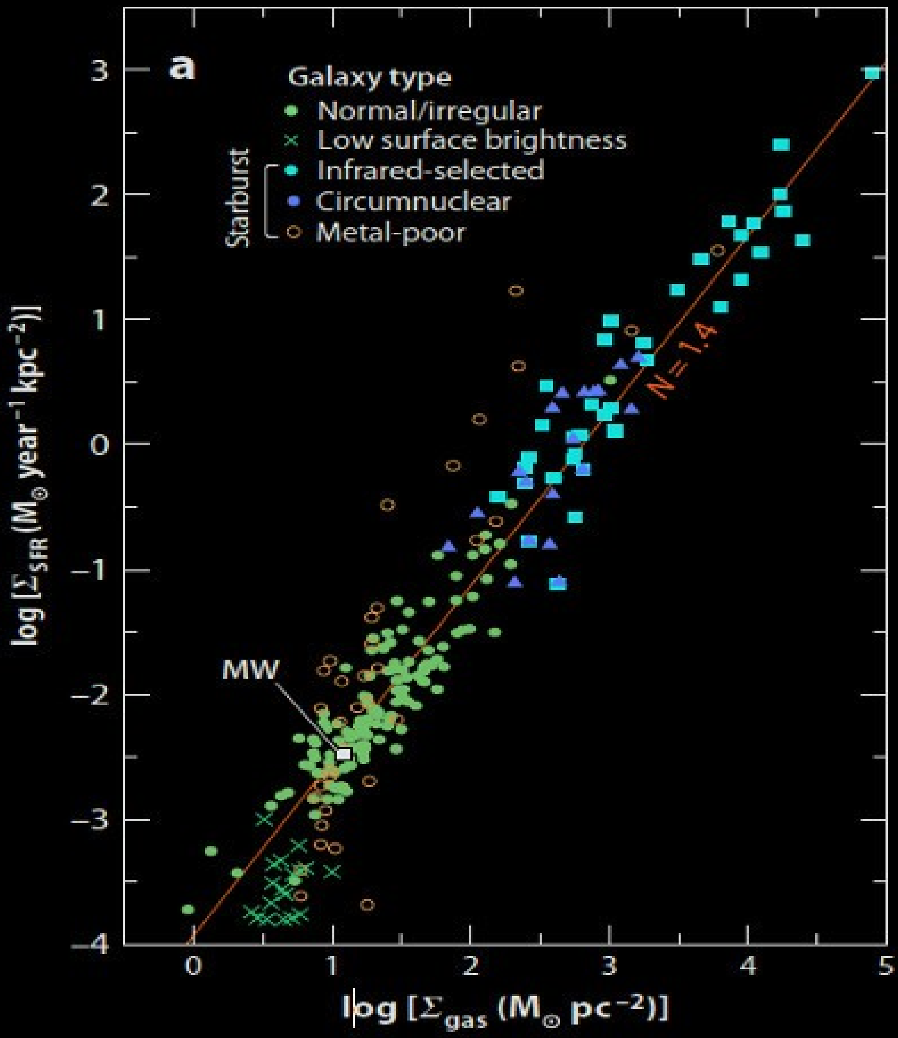
- *Star formation*
- *Stellar equation*
- *Stellar evolution*
- *Final stages of stellar evolution*

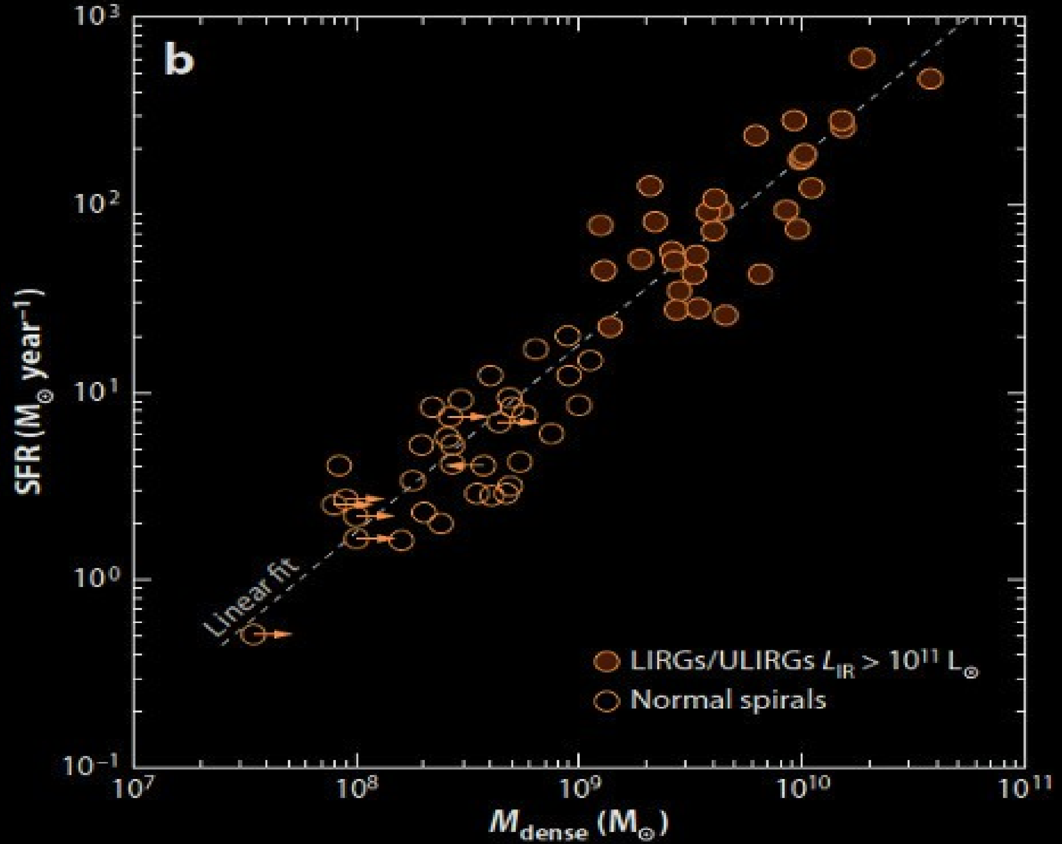
# Star formation

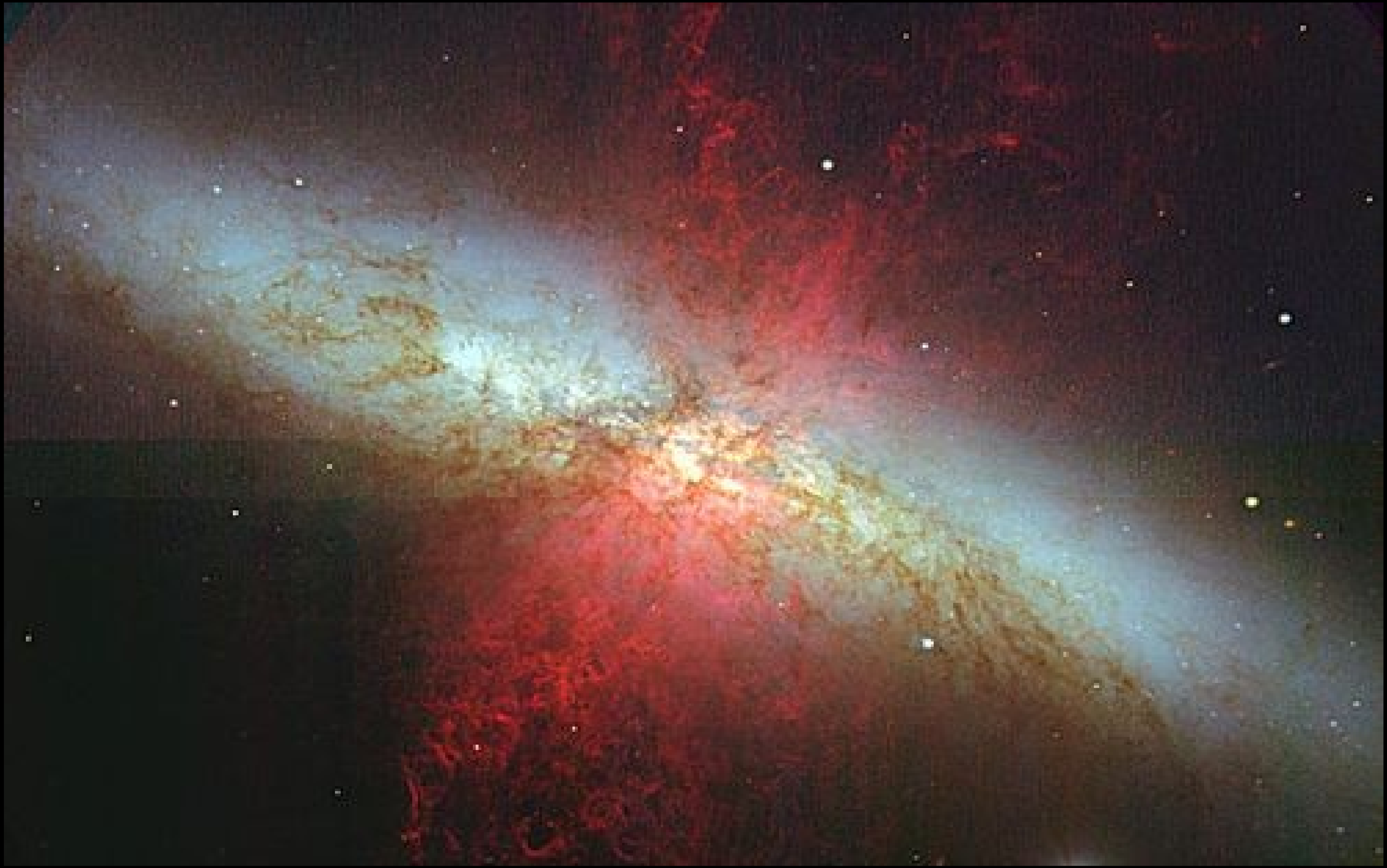
# Schmidt-Kennicut law



# Schmidt-Kennicut law





# Jeans Theory

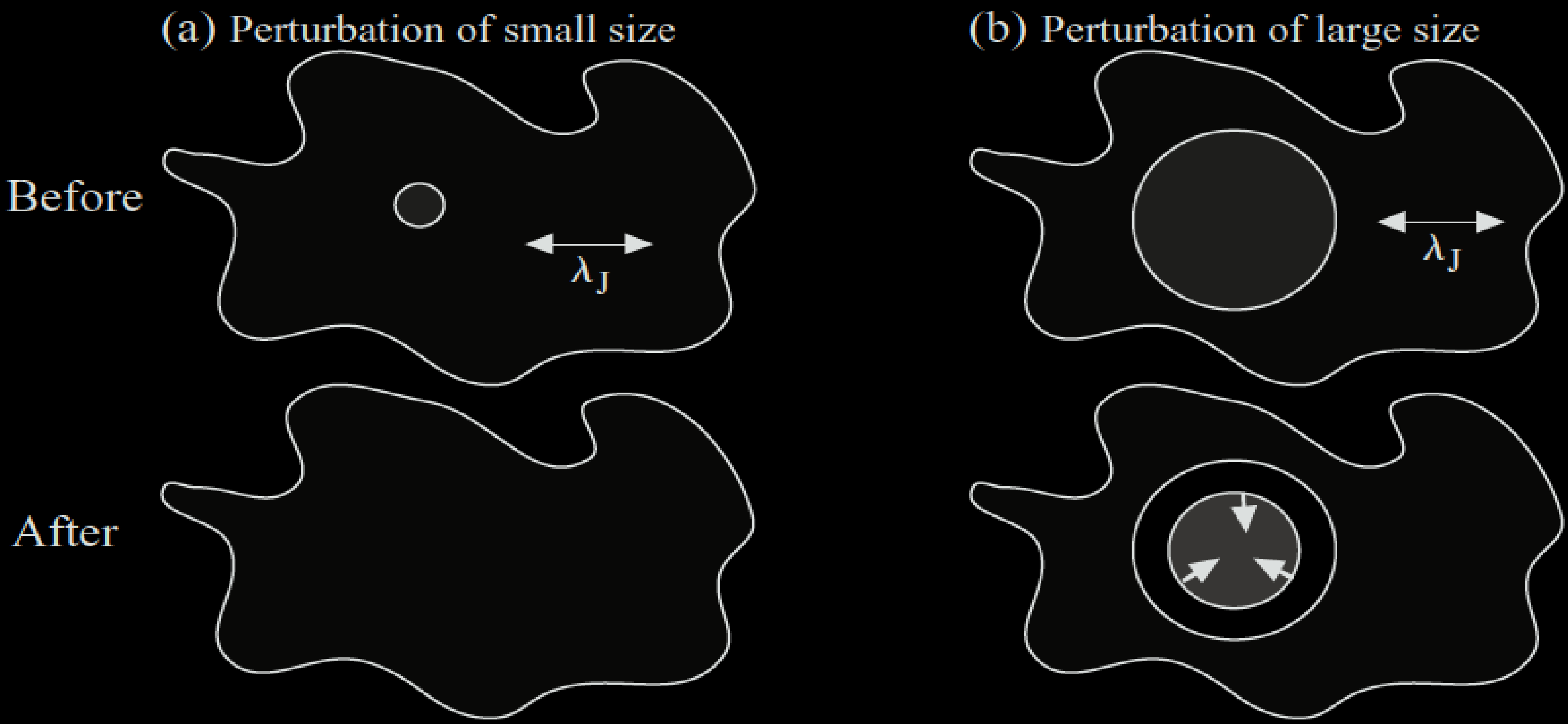
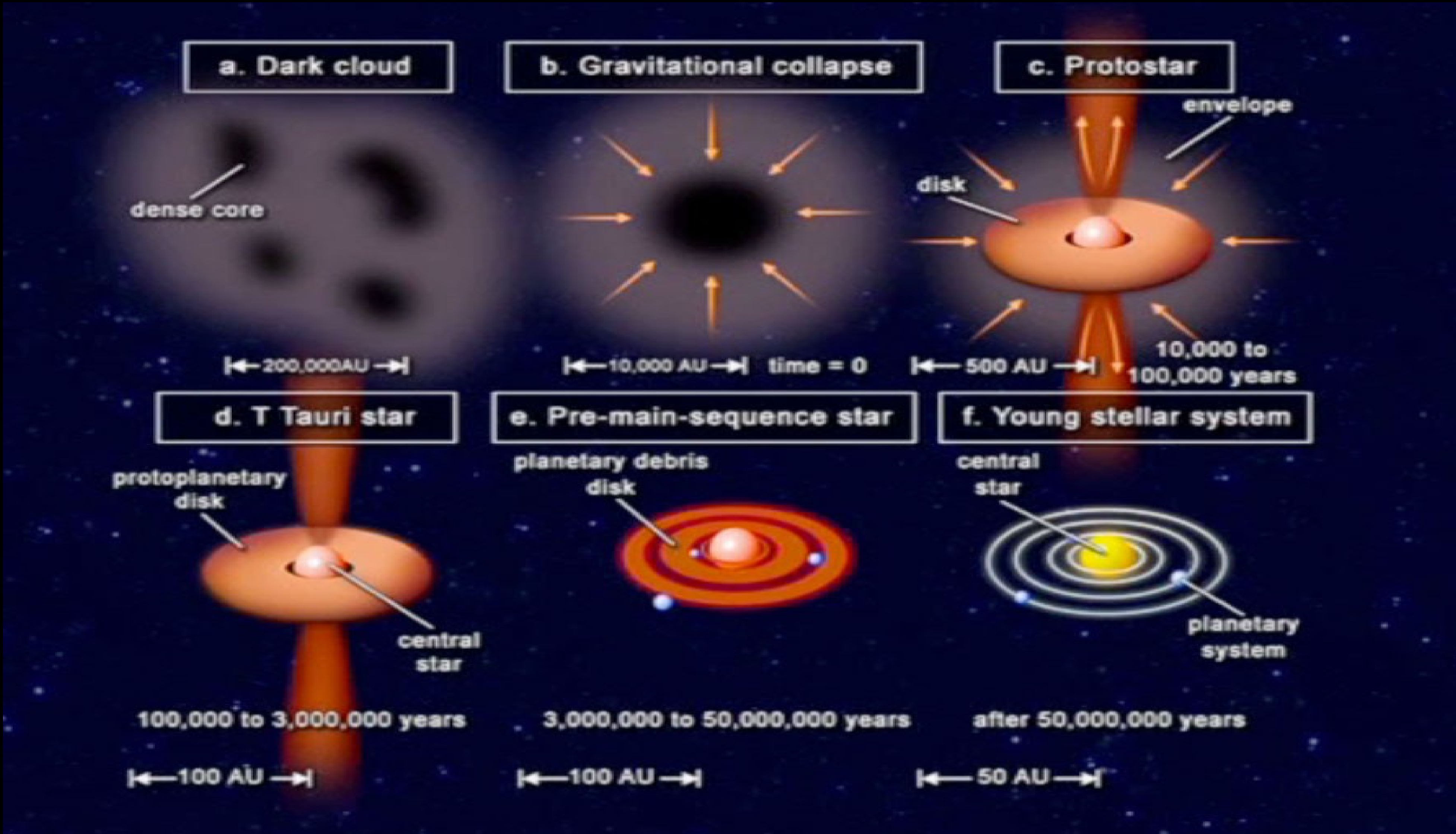
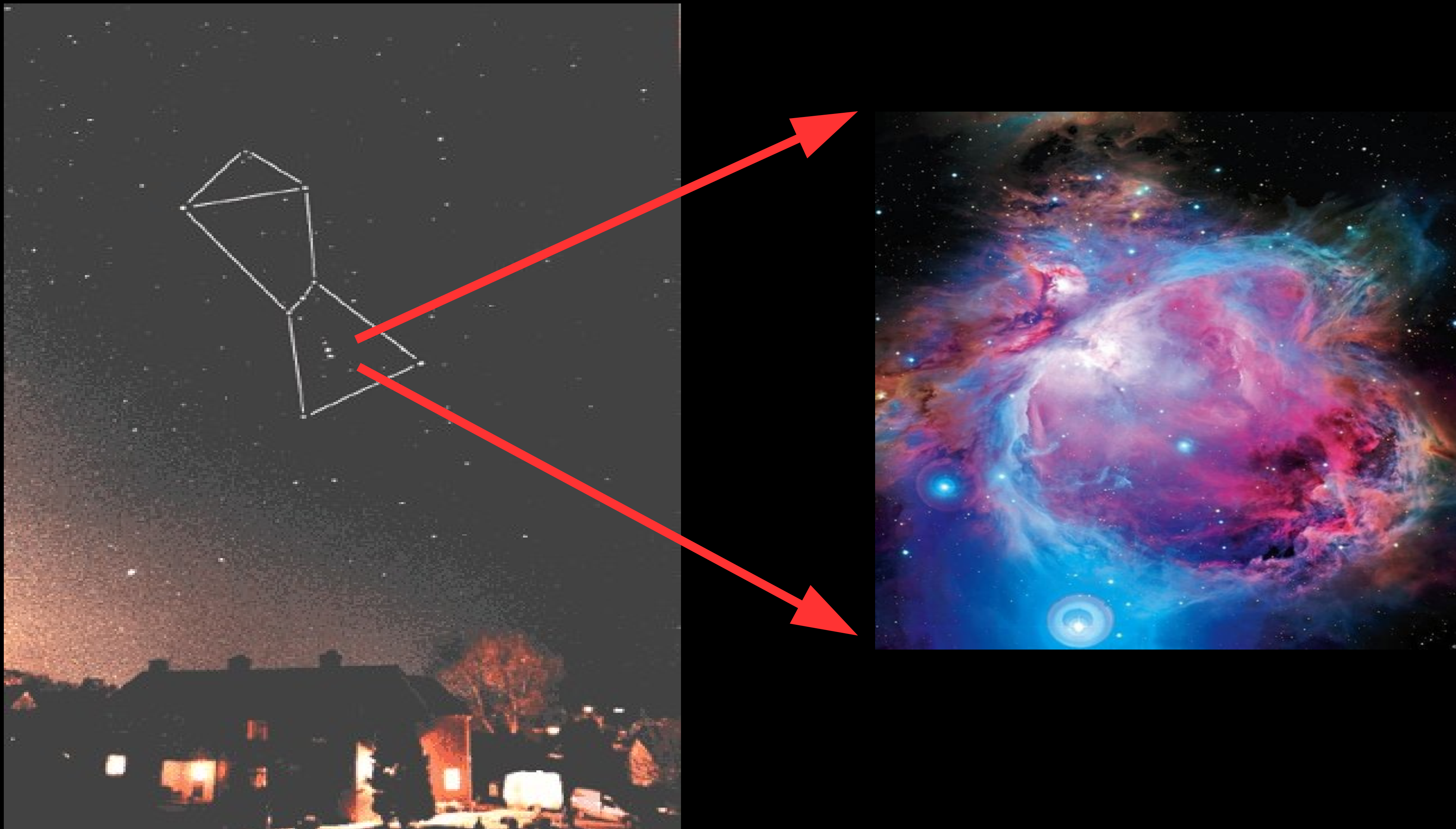


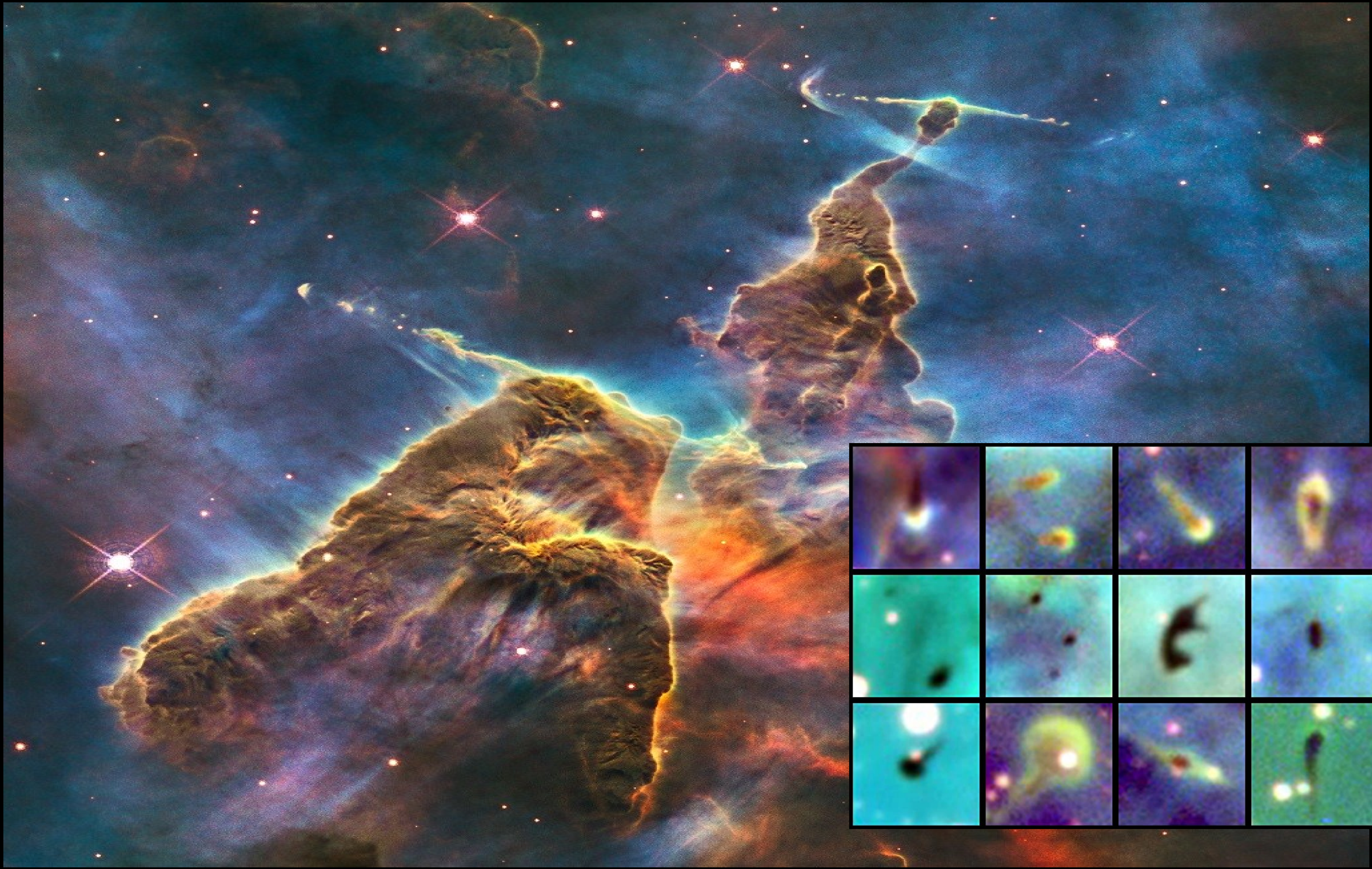
**Table 12.4** The Jeans criterion and the contents of giant molecular clouds.

| | GMC | Clump | Dense core |
|---|---|---|---|
| Size | 50 pc | 10 pc | 0.1 pc |
| Mass | $10^5\ M_\odot$ | $30$–$10^3\ M_\odot$ | $3$–$10\ M_\odot$ |
| Number density | $10^8\ \text{m}^{-3}$ | $5 \times 10^8\ \text{m}^{-3}$ | $5 \times 10^{10}\ \text{m}^{-3}$ |
| Temperature | 15 K | 10 K | 10 K |
| Jeans length | 4 pc | 1.5 pc | 0.15 pc |
| Jeans mass | $600\ M_\odot$ | $100\ M_\odot$ | $30\ M_\odot$ |

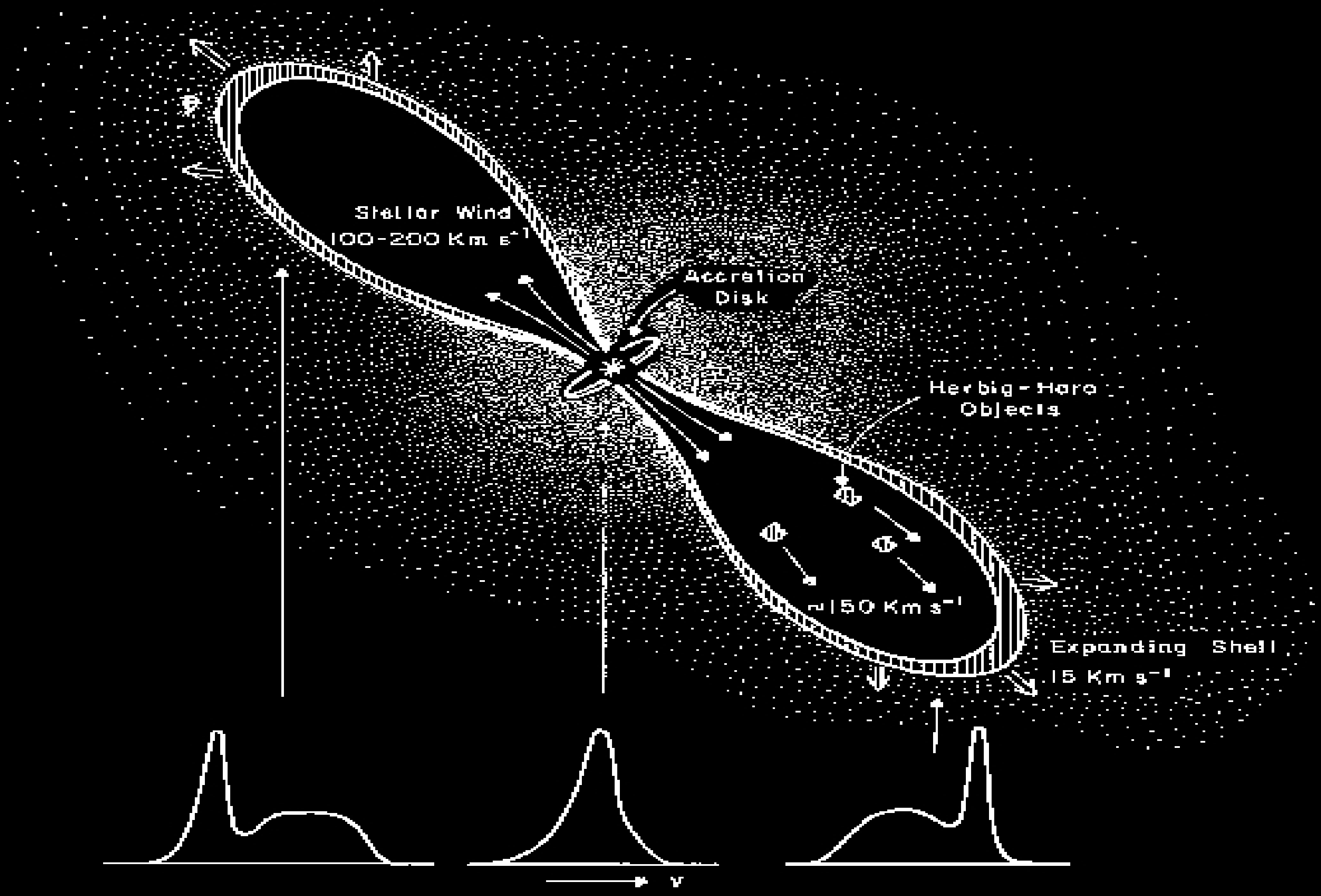

a. Dark cloud
dense core
200,000AU
b. Gravitational collapse
10,000 AU
time = 0
c. Protostar
envelope
disk
500 AU
10,000 to
100,000 years
d. T Tauri star
protoplanetary
disk
central
star
100,000 to 3,000,000 years
100 AU
e. Pre-main-sequence star
planetary debris
disk
3,000,000 to 50,000,000 years
100 AU
f. Young stellar system
central
star
planetary
system
after 50,000,000 years
50 AU

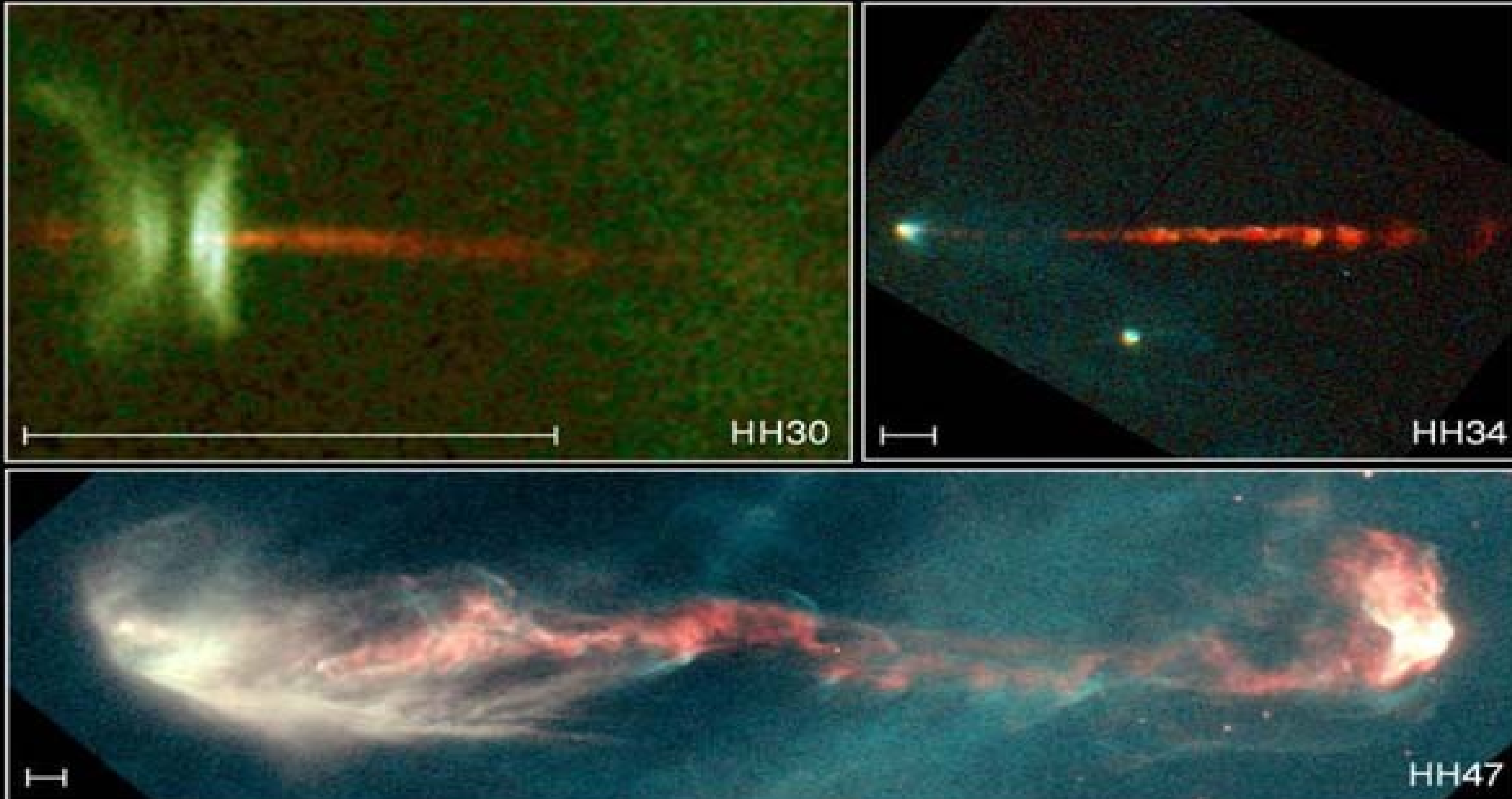



HST–Star Formation in the Orion Nebula
Red = Nitrogen
Green = Hydrogen
Blue = Oxygen
Proplyds
Trapezium



# Bipolar outflows



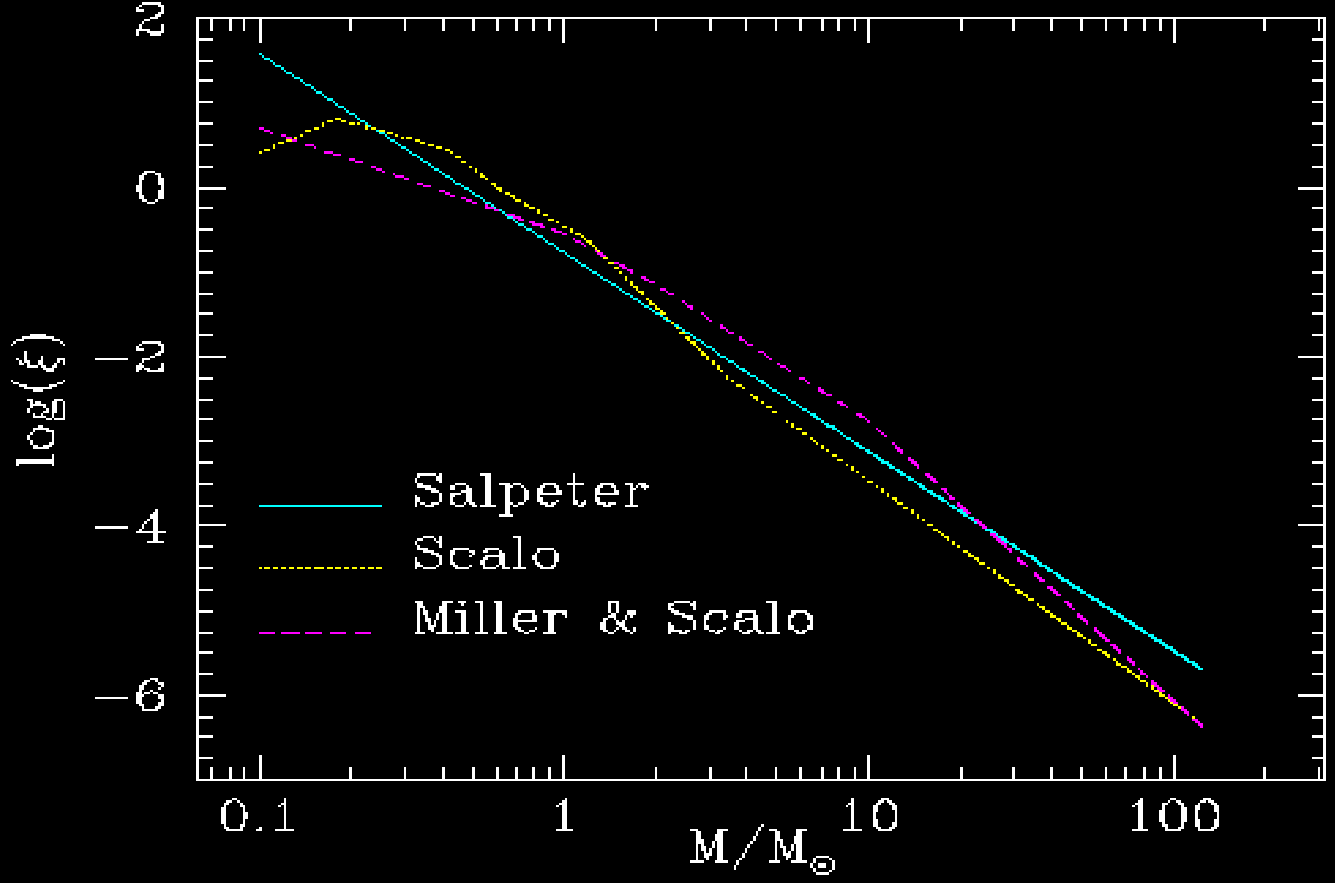



**Jets from Young Stars** HST • WFPC2

PRC95-24a • ST ScI OPO • June 6, 1995

C. Burrows (ST ScI), J. Hester (AZ State U.), J. Morse (ST ScI), NASA

# Initial Mass Function



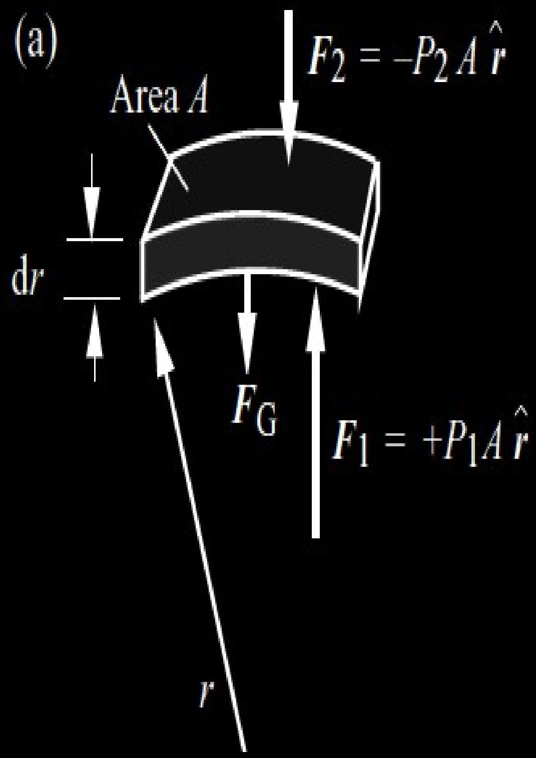
# Hydrostatic equilibrium

balance between gravity and gas pressure

$$F_g = \frac{GM(r)}{r^2}\rho(r) dr dA$$

$$F_p = \left[P(r) - P(r+dr)\right] dA = -\frac{dP}{dr} dr dA$$

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$



# Stellar Equations

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$

*1) Hydrostatic equilibrium*

# Stellar Equations

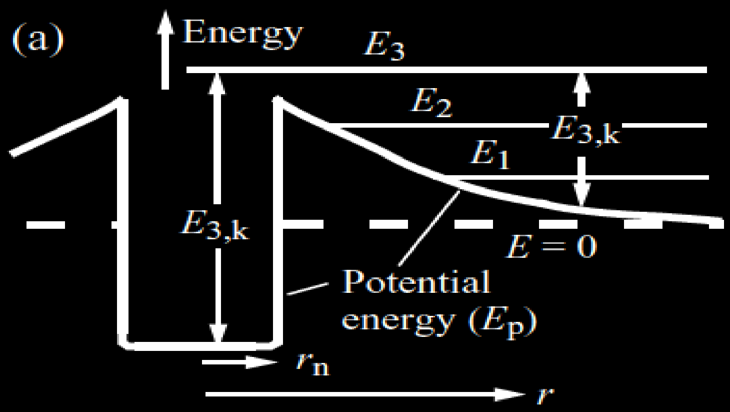
$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$

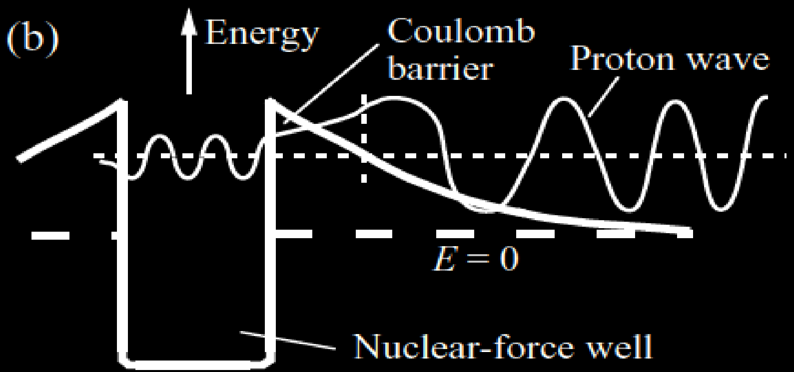
1) *Hydrostatic equilibrium*

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

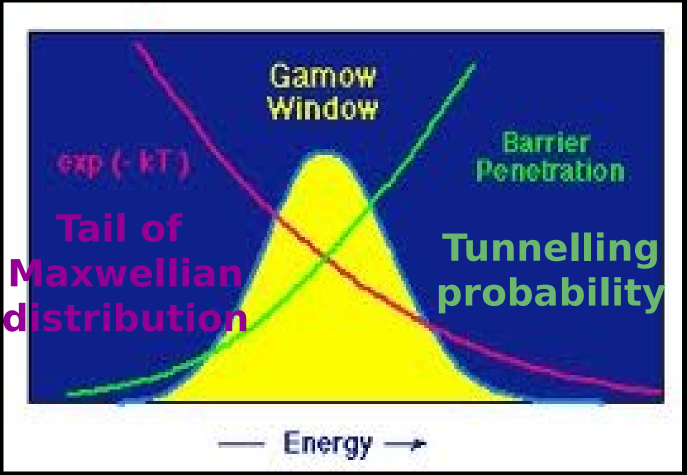
2) *Conservation of mass*

# Nuclear reactions



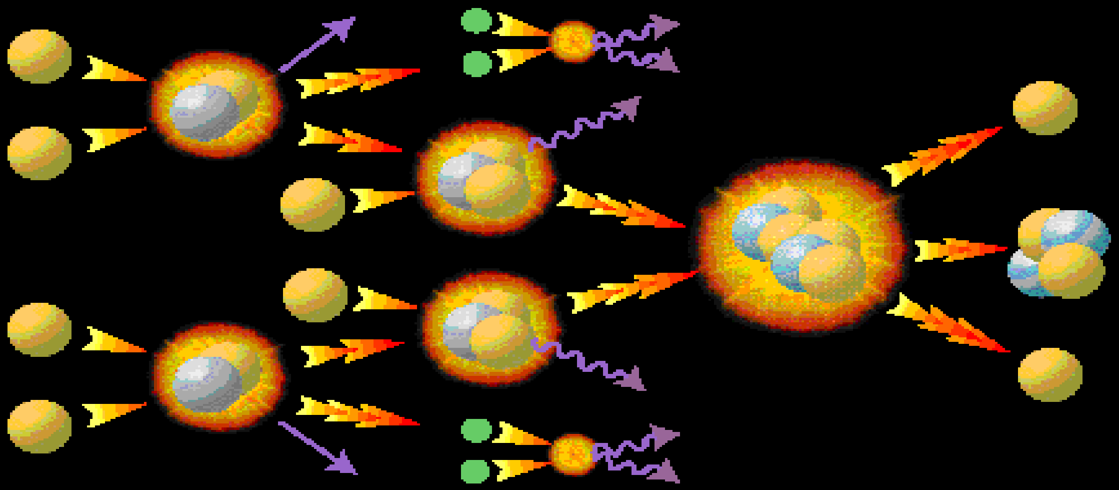




- T > $10^{10}$ K would be required to surmount **Coulomb barrier**
- Quantum effects (**tunnelling**) allow nuclear reactions at much lower temperatures (low, and strongly T-dependent, efficiency)



# Proton-proton (pp) chain

Most of the nuclear energy from stars is produced by the fusion of four hydrogen atoms into a helium nucleus: the pp chain



$$^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}D + e^{+} + \nu_e$$

$$^{2}_{1}D + ^{1}_{1}H \rightarrow ^{3}_{2}He + \gamma$$

$$^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H$$

$$6\,^{1}H^{+} \longrightarrow \,^{4}He^{++} + 2\,^{1}H^{+} + 2e^{+} + 2\nu + 2\gamma$$

# pp Chain

The energy released by the pp chain is simply the mass decrement between the initial and final nuclei

$$6\,{}^{1}H^{+} \longrightarrow {}^{4}He^{++} + 2\,{}^{1}H^{+} + 2e^{+} + 2\nu + 2\gamma$$

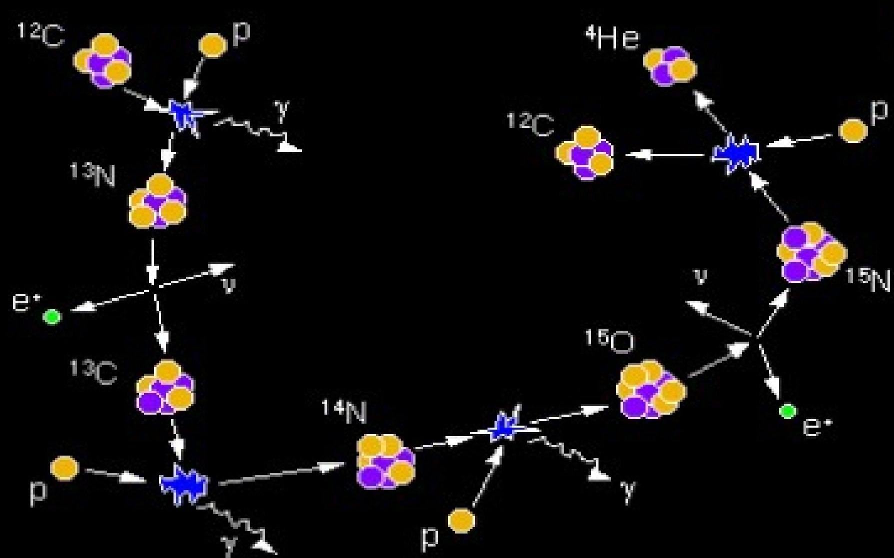
Energy released

Mass difference between initial and final nuclei

$$\begin{aligned} \Delta E &= \Delta m c^2 \\ &= (M_{6H} - M_{2H} - M_{He})c^2 \\ &\sim 26 \text{ MeV} \end{aligned}$$

# CNO Chain

The CNO cycle commences once the stellar core temperature reaches $1.4 \times 10^7$ K and is the primary source of energy in stars of mass **M > 1.5 M$_\odot$**




Hydrogen in

$$^{12}C + {}^1H \longrightarrow {}^{13}N + \gamma$$

$$^{13}N \longrightarrow {}^{13}C + e^+ + \nu$$

$$^{13}C + {}^1H \longrightarrow {}^{14}N + \gamma$$

$$^{14}N + {}^1H \longrightarrow {}^{15}O + \gamma$$

$$^{15}O \longrightarrow {}^{15}N + e^+ + \nu$$

$$^{15}N + {}^1H \longrightarrow {}^{12}C + {}^4He$$

Helium out

C is only a **catalyst** for the CNO reaction

How much energy is released?

# Nuclear reatcions

Many nuclear reactions can occur in stars, with relative efficiencies depending on temperature, density and abundances of chemical elements

⇒ different reactions are dominant in different stages of **stellar evolution**

| *Nuclear Fuel* | *Process* | *Threshold Temperature* | *Products* |
|---|---|---|---|
| ***H*** | p-p chain | ~ $4 \times 10^6$ K | He |
| ***H*** | CNO cycle | $15 \times 10^6$ K | He |
| ***He*** | $3\alpha$ | $100 \times 10^6$ K | C, O |
| ***C*** | C + C | $600 \times 10^6$ K | O, Ne, Na, Mg |
| ***O*** | O + O | $1000 \times 10^6$ K | Mg, S, P, Si |
| ***Si*** | Disintegration | $3000 \times 10^6$ K | Co, Fe, Ni |

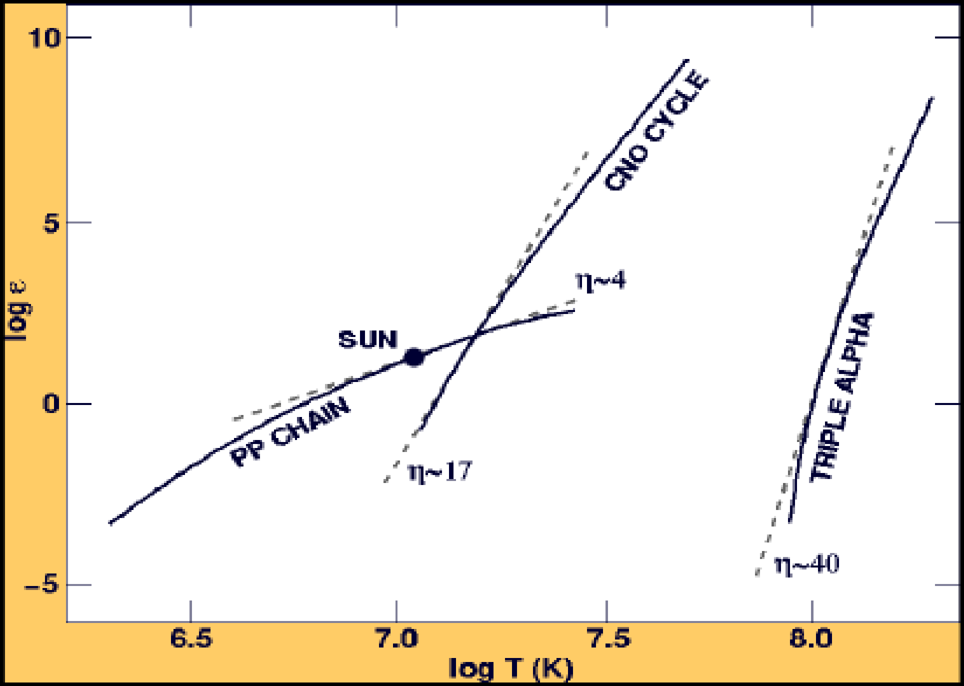
# Nuclear reactions

The energy generation rate **ε (energy/mass)** is proportional to the number of interactions per second and strongly depends on temperature:

$$\varepsilon_{PP} \propto \rho X_H^2 T^{4.6}$$

$$\varepsilon_{CNO} \propto \rho X_H X_{CNO} T^{16.7}$$

$$\varepsilon_{3\alpha} \propto \rho^2 T^{40}$$



# Neutron capture and beta decay

- Interaction between nuclei and free neutrons (**neutron capture**)
- Neutrons capture by heavy nuclei **is not limited by the Coulomb barrier**, so could proceed at relatively **low temperatures**.
- If enough neutrons available, chain of reactions:

$$I(A, Z) + n \rightarrow I_1(A+1, Z)$$
$$I_1(A+1, Z) + n \rightarrow I_2(A+2, Z)$$
$$I_2(A+2, Z) + n \rightarrow I_3(A+3, Z) \quad \ldots etc$$

- If a radioactive isotope is formed it will undergo **β-decay**, creating a new element:

$$I_N(A+N, Z) \rightarrow J(A+N, Z+1) + e^- + \bar{\nu}_e$$

- If new element is stable, it will resume **neutron capture**, otherwise may undergo series of **β-decays**

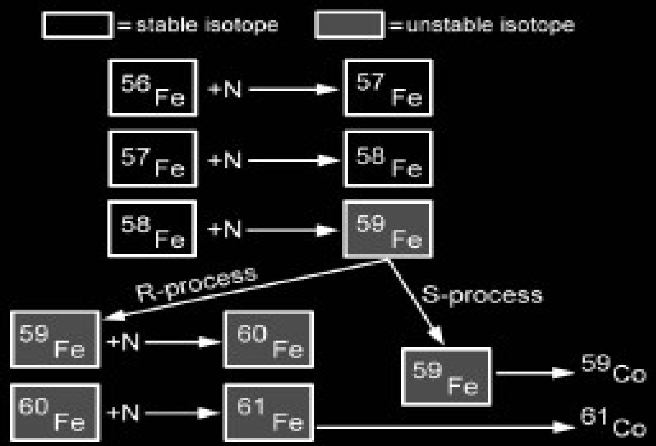
$$J(A+N, Z+1) \rightarrow K(A+N, Z+2) + e^- + \bar{\nu}_e$$
$$K(A+N, Z+2) \rightarrow L(A+N, Z+3) + e^- + \bar{\nu}_e$$

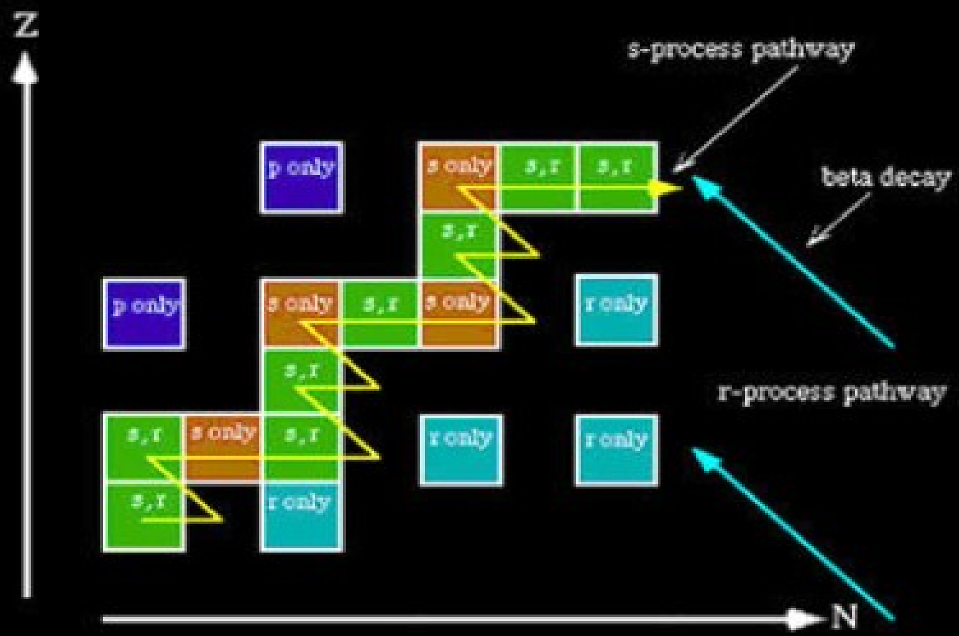
# s-process and r-process

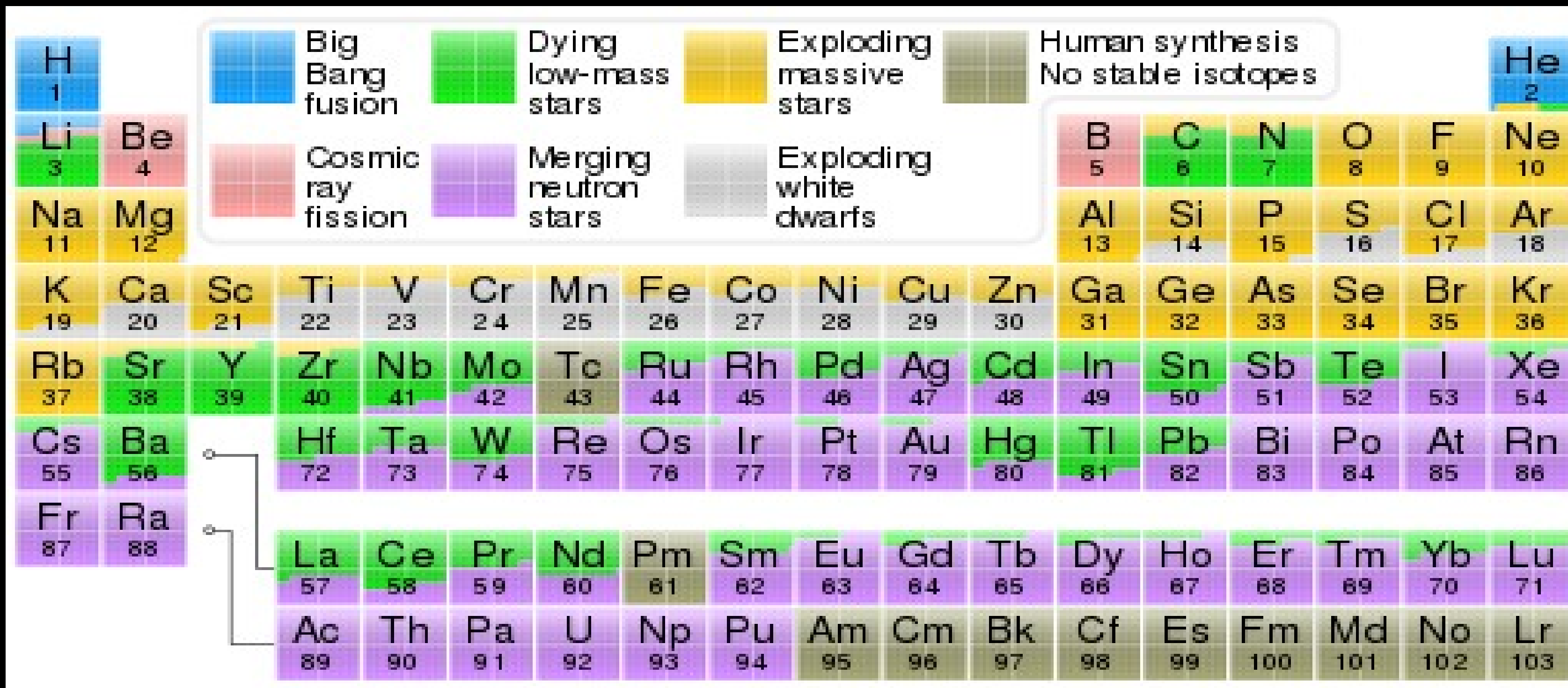
Stable nuclei may undergo only neutron captures, unstable ones may undergo both, with the outcome depending on the timescales for the two processes.

Timescales: neutron capture reactions may proceed more ***slowly*** or more ***rapidly*** (if many neutrons are available) than the competing β-decays: ***s-process*** or ***r-process***.

Formation of Cobalt from Neutron Capture





Big Bang fusion
Dying low-mass stars
Exploding massive stars
Human synthesis No stable isotopes
Cosmic ray fission
Merging neutron stars
Exploding white dwarfs
H 1
He 2
Li 3
Be 4
B 5
C 6
N 7
O 8
F 9
Ne 10
Na 11
Mg 12
Al 13
Si 14
P 15
S 16
Cl 17
Ar 18
K 19
Ca 20
Sc 21
Ti 22
V 23
Cr 24
Mn 25
Fe 26
Co 27
Ni 28
Cu 29
Zn 30
Ga 31
Ge 32
As 33
Se 34
Br 35
Kr 36
Rb 37
Sr 38
Y 39
Zr 40
Nb 41
Mo 42
Tc 43
Ru 44
Rh 45
Pd 46
Ag 47
Cd 48
In 49
Sn 50
Sb 51
Te 52
I 53
Xe 54
Cs 55
Ba 56
Hf 72
Ta 73
W 74
Re 75
Os 76
Ir 77
Pt 78
Au 79
Hg 80
Tl 81
Pb 82
Bi 83
Po 84
At 85
Rn 86
Fr 87
Ra 88
La 57
Ce 58
Pr 59
Nd 60
Pm 61
Sm 62
Eu 63
Gd 64
Tb 65
Dy 66
Ho 67
Er 68
Tm 69
Yb 70
Lu 71
Ac 89
Th 90
Pa 91
U 92
Np 93
Pu 94
Am 95
Cm 96
Bk 97
Cf 98
Es 99
Fm 100
Md 101
No 102
Lr 103

# Stellar Equations

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$

1) Hydrostatic equilibrium

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

2) Conservation of mass

$$\frac{dL}{dr} = 4\pi r^2 \rho(r)\epsilon$$

3) Energy generation

## ***Opacity:*** $\kappa_\nu = \alpha_\nu / \rho$

• Opacity in a star is a function of composition and temperature.

• Determined by the details of how photons interact with particles (atoms, ions, free electrons).

• If the opacity varies slowly with λ it determines the star continuous spectrum (continuum). A rapid variation of opacity with λ produces dark absorption lines in the spectrum.

Optically thin cloud: $\tau << 1$

- Chances are small that a photon will interact with particle

- Can effectively see right through a cloud

- In the optically thin regime, the amount of extinction (absorption plus scattering) is linearly related to the amount of material: double the amount of gas, double the extinction

→ if we can measure the amount of light absorbed (or emitted) by the gas, we can calculate exactly how much gas there is

Optically thick cloud: $\tau >> 1$

- Certain that a photon will interact many times with particles before it finally escapes from the cloud
- Any photon entering the cloud will have its direction changed many times by collisions, which means that its "output" direction has nothing to do with its "input" direction.

→ ***Cloud is opaque***

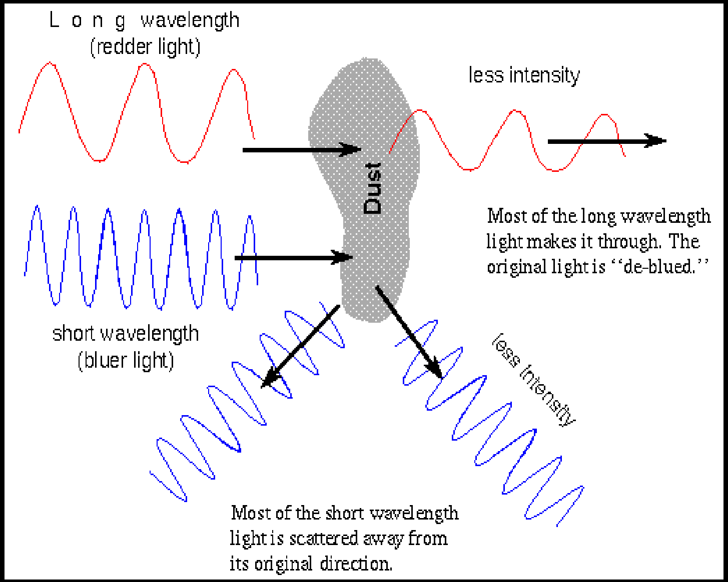
- You can't see through an optically thick medium; you can only see light emitted by the very outermost layers.

→ ***you can't observe interior of a star, but only the surface (photosphere)***

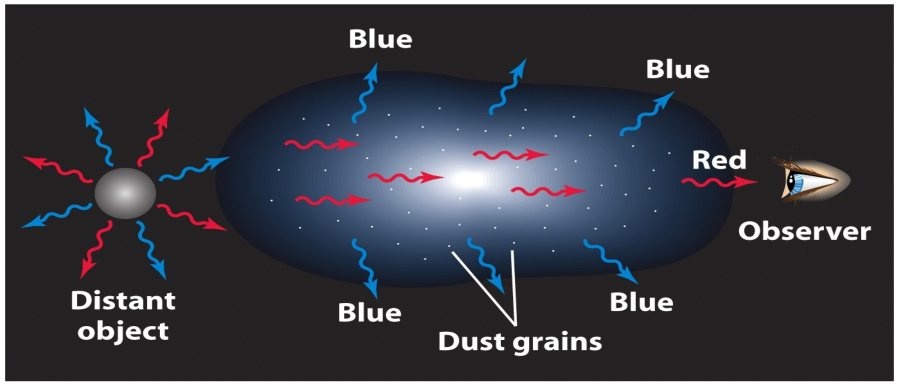
- The spectrum of the radiation emitted by optically thick material is a **blackbody**

- **Bound-Bound absorption:** Small, except at those discrete wavelengths capable of producing a transition (*absorption lines*)
- **Bound-Free absorption:** *Photoionisation*. Occurs when photon has sufficient energy to ionize atom. The freed $e^-$ can have any energy, thus this is a source of continuum opacity
- **Free-Free absorption:** *Bremsstrahlung*. A free electron absorbs a photon, causing its speed to increase. It is a source of continuum opacity and important at high temperatures (it needs free $e^-$).
- **Electron scattering:** *Thomson scattering*. A photon is scattered, but not absorbed by a free electron.
- **Dust extinction:** Only important for very cool stellar atmospheres and cold interstellar medium

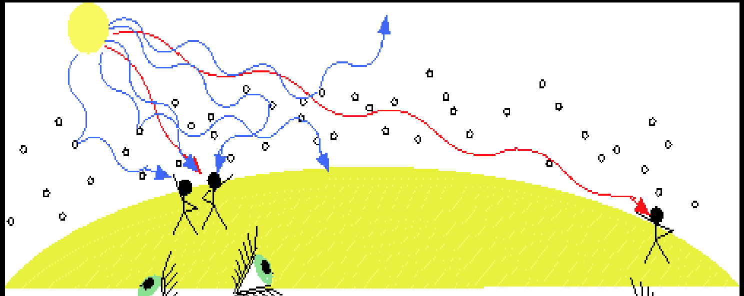
# Dust and light




Dust **extinction** and **reddening** in astronomical optical/UV observations




Why is the sky **blue** (and **red** at sunset)?



# Stellar Equations

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$

1) Hydrostatic equilibrium

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

2) Conservation of mass

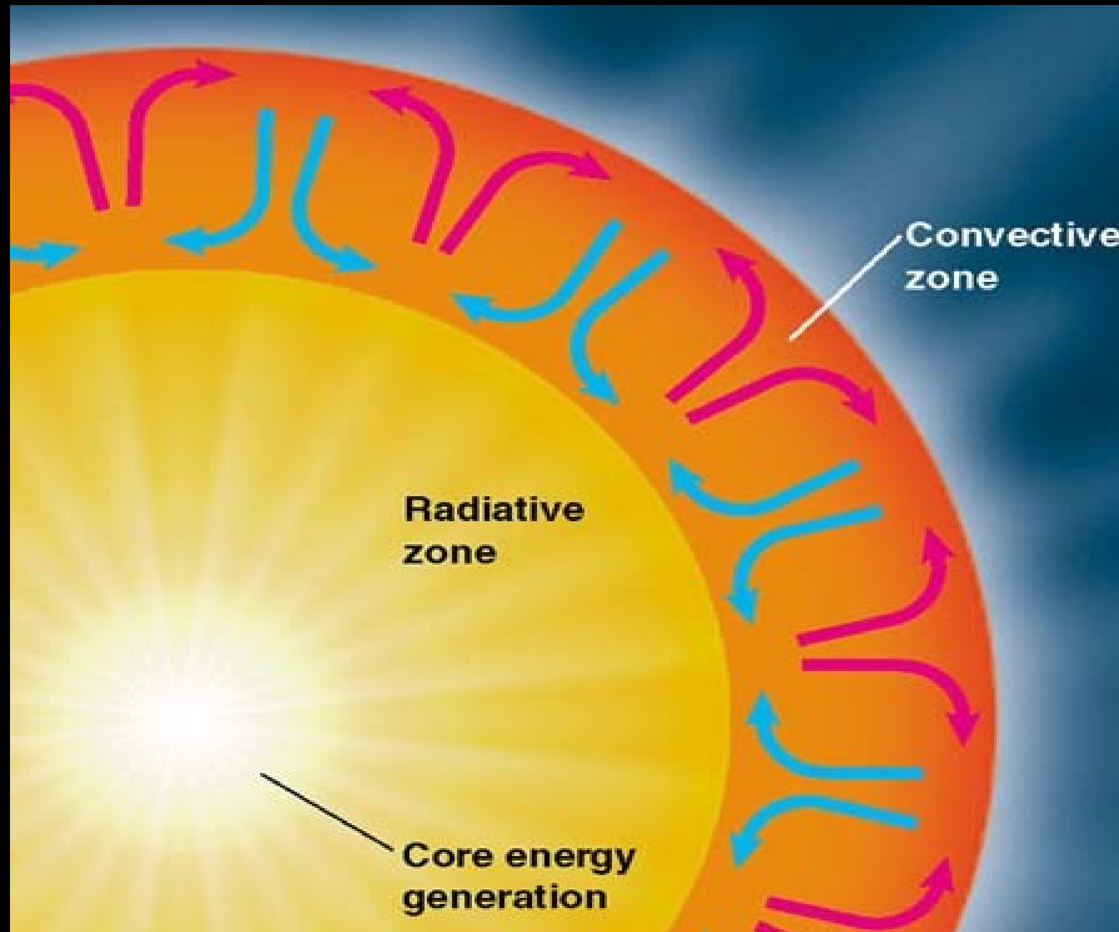
$$\frac{dL}{dr} = 4\pi r^2 \rho(r)\epsilon$$

3) Energy generation

$$\frac{dT}{dr} = -\frac{3k\rho}{16\pi ac\, r^2 T^3} L(r)$$

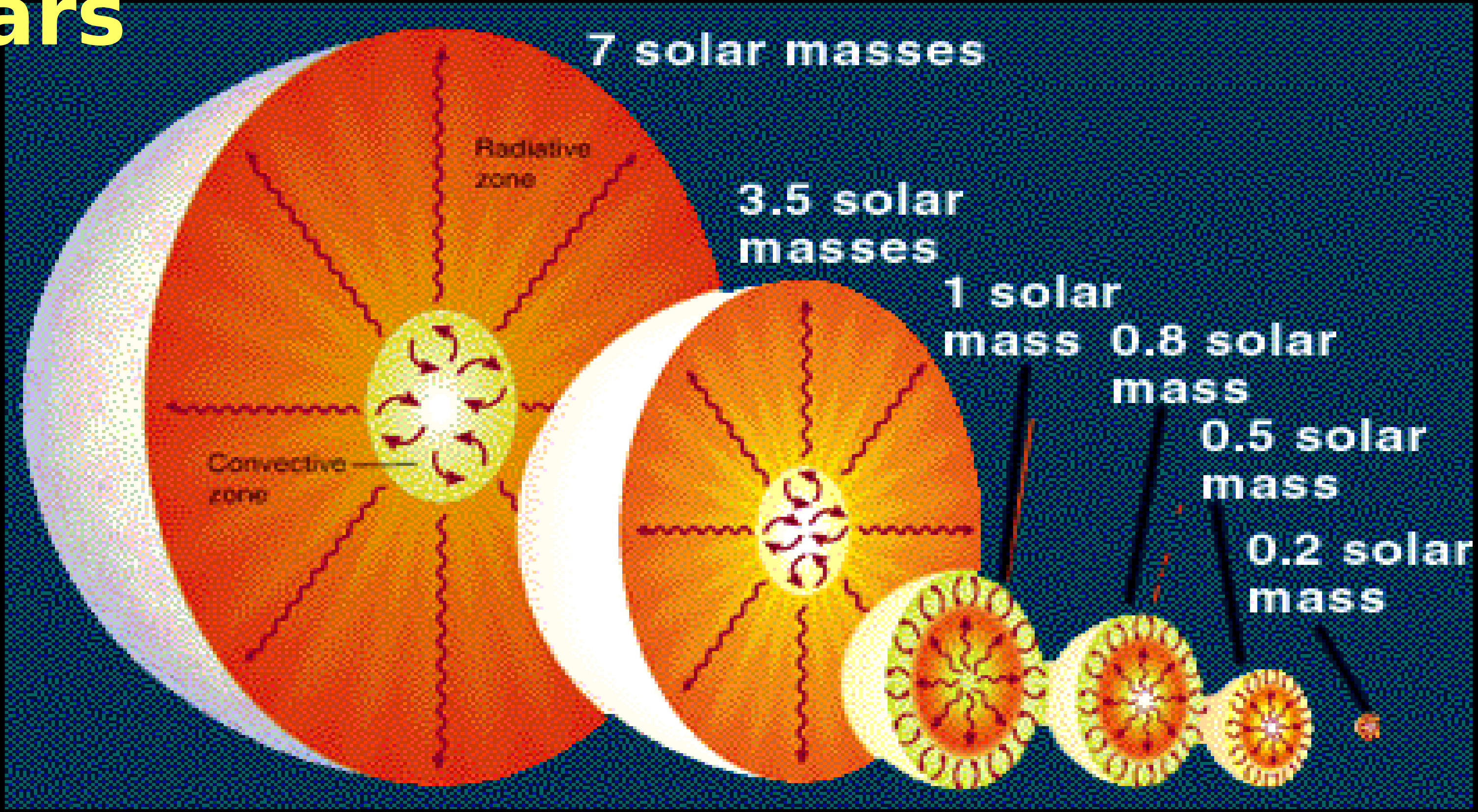
4) Energy transport

# Energy Transport in the Sun




In the sun, energy is transported via radiation in the central regions, but by convection in the outer

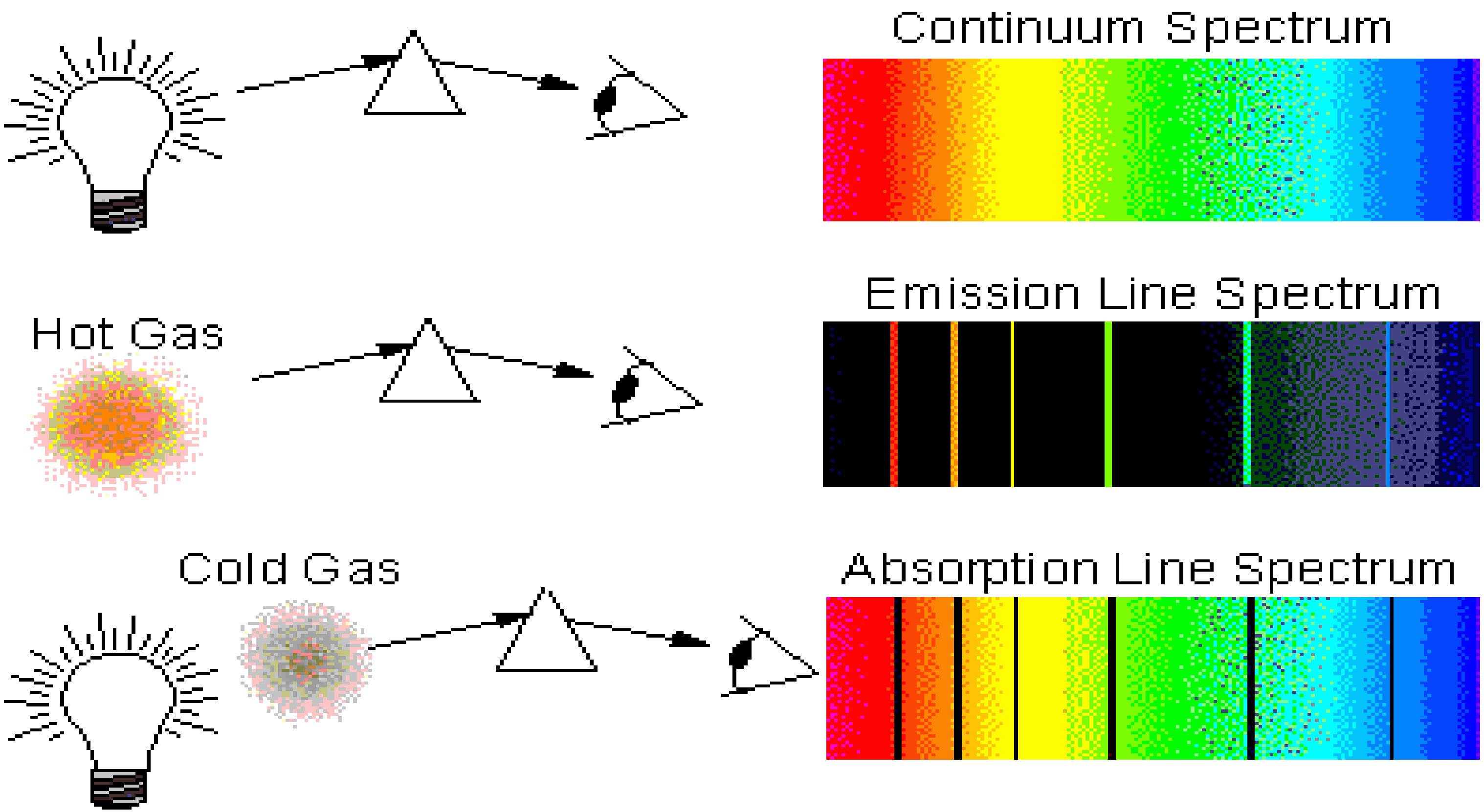
# Energy Transport inside Stars




The structure and evolution of stars is accurately modeled with only a few well understood laws of physics ⇒ stellar models.

# Spectra of stellar photospheres

# Stellar spectra

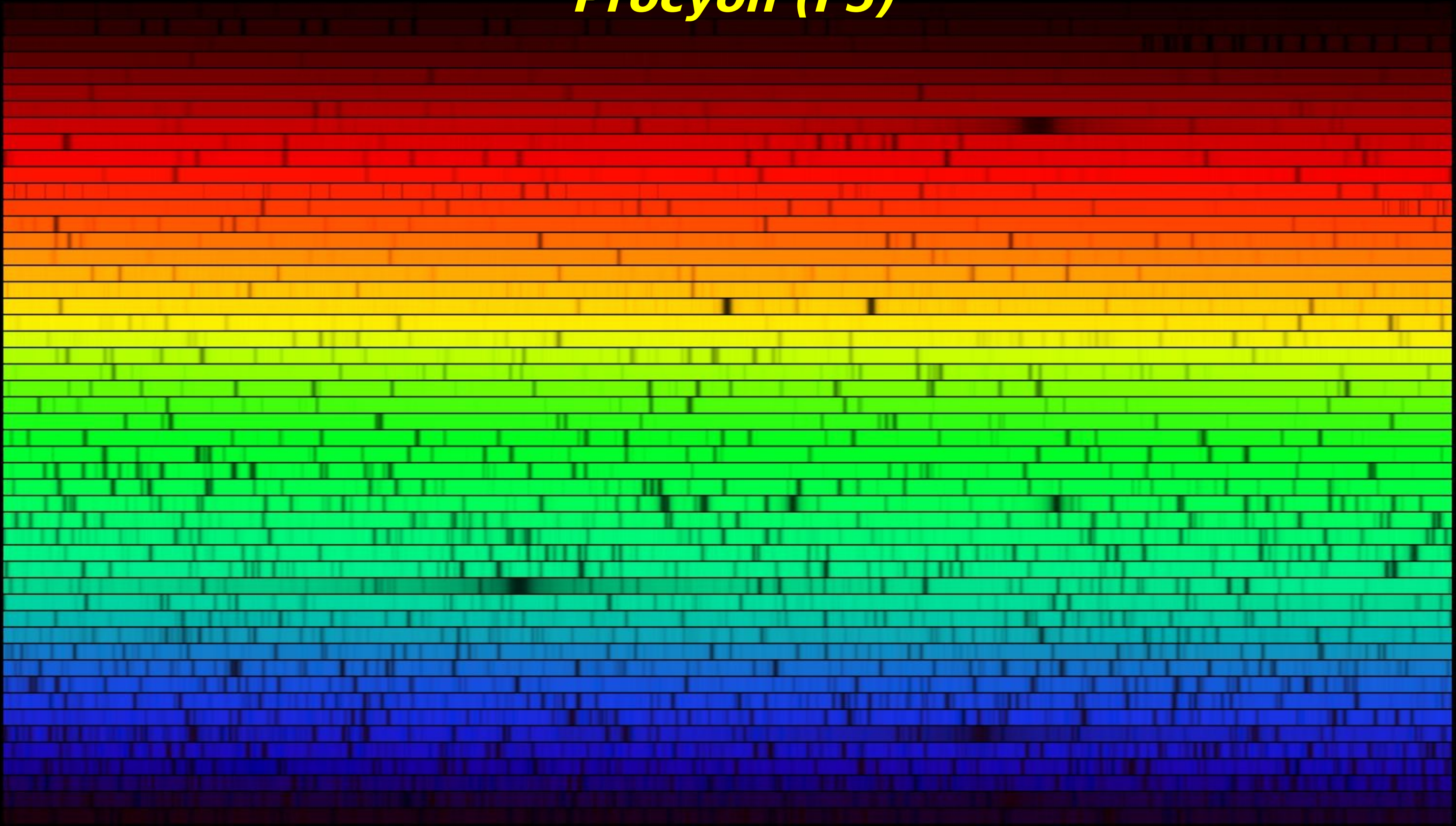



- ***Stellar spectra?***

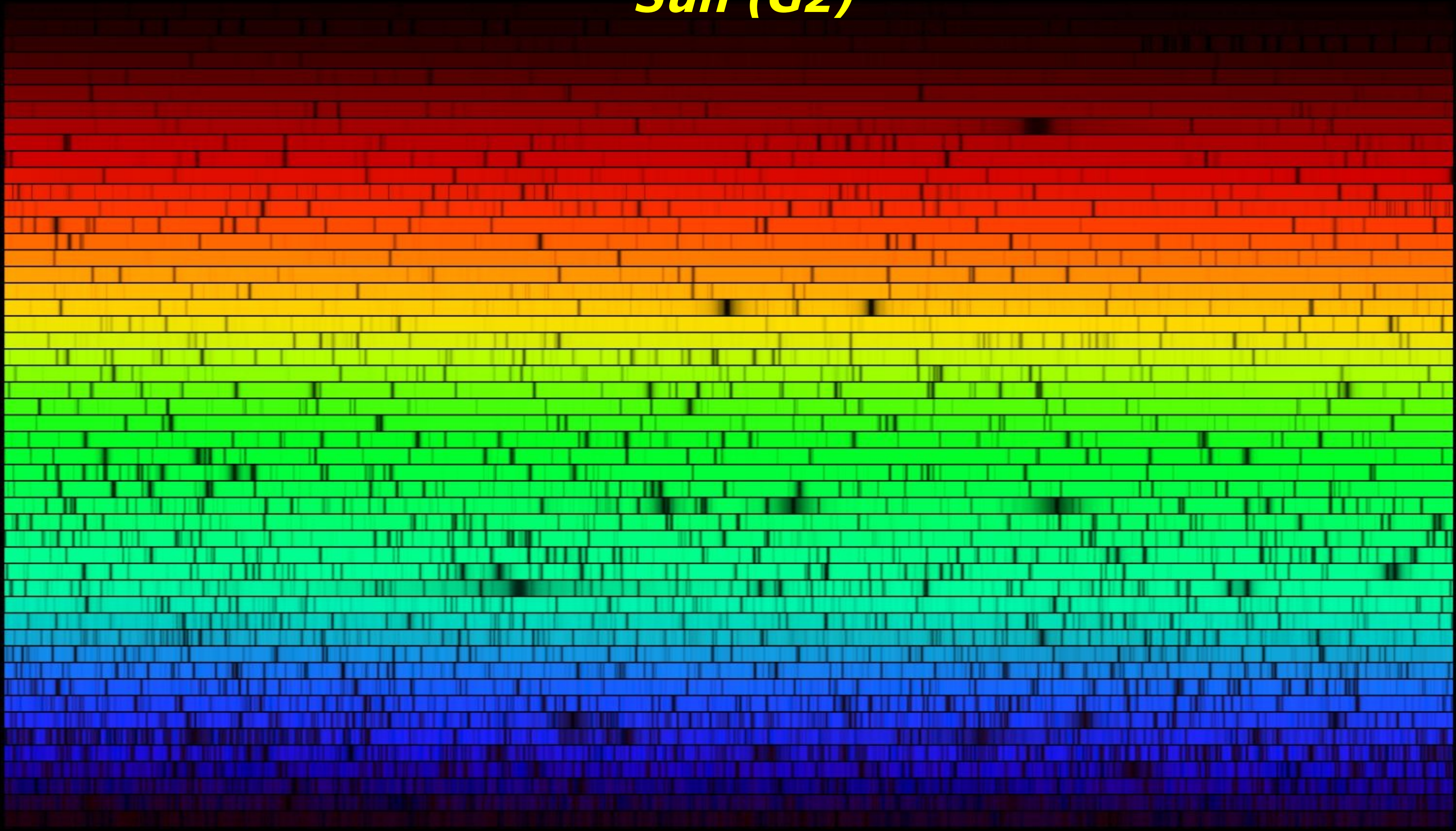
Based on their absorption lines (T indicators) ⇒ spectral types: from warm to cool

"**O**h **B**e **A** **F**ine **G**irl **K**iss **M**

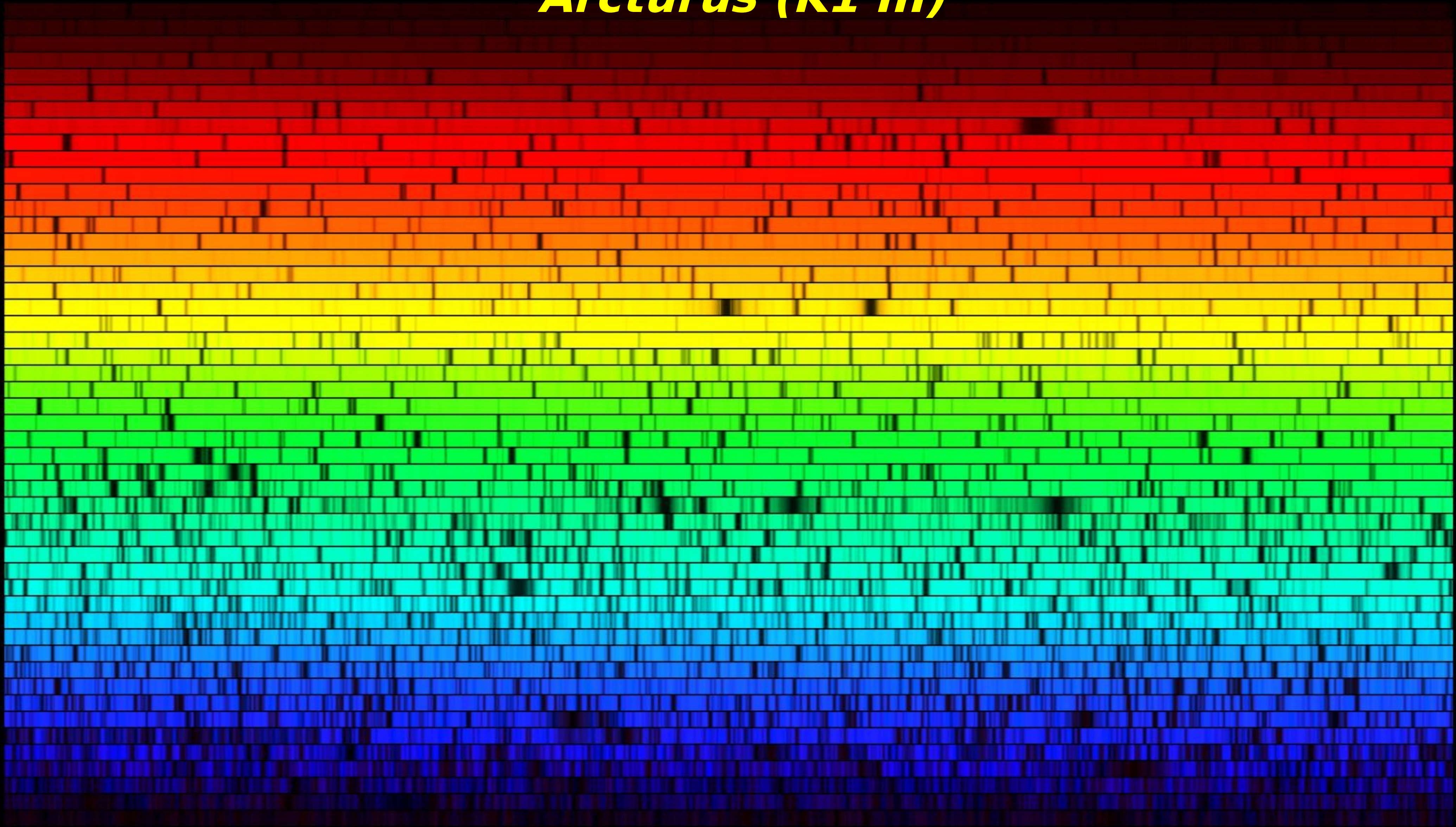
# Procyon (F5)

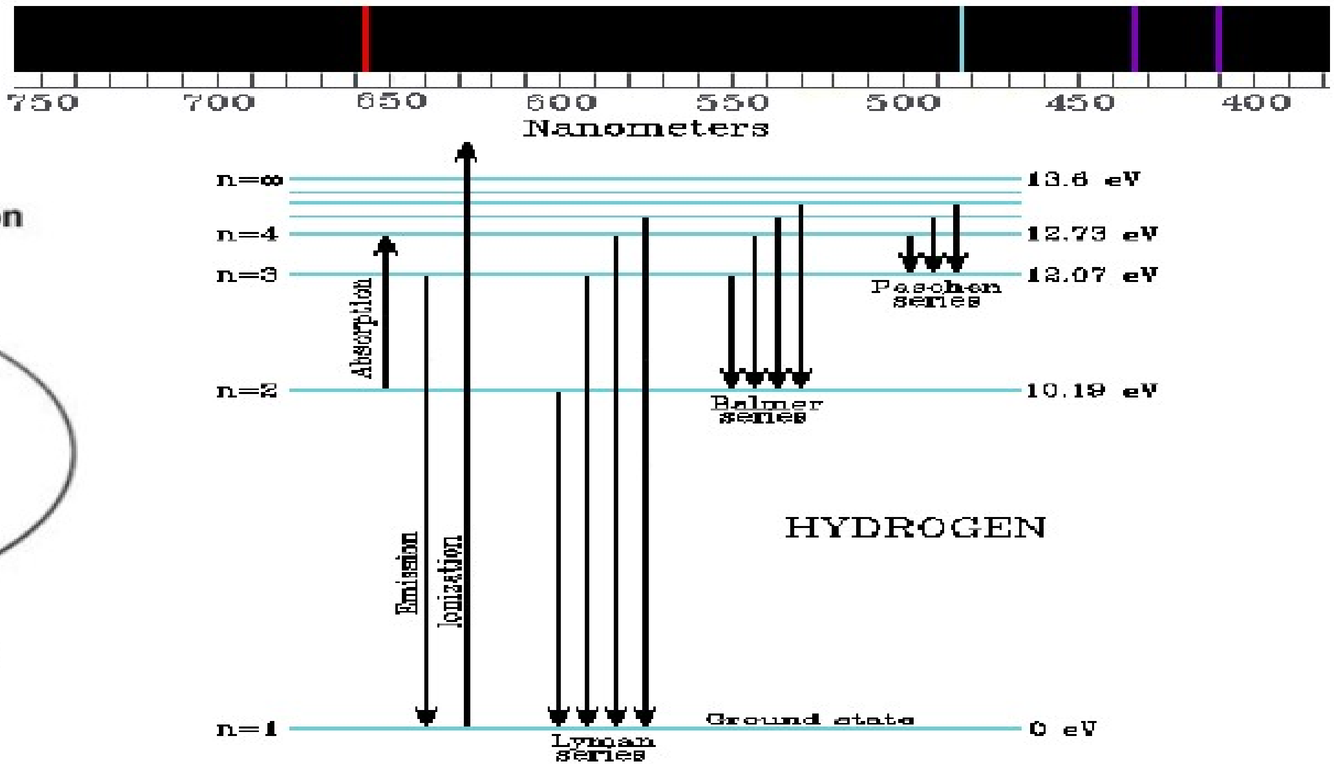


Sun (G2)

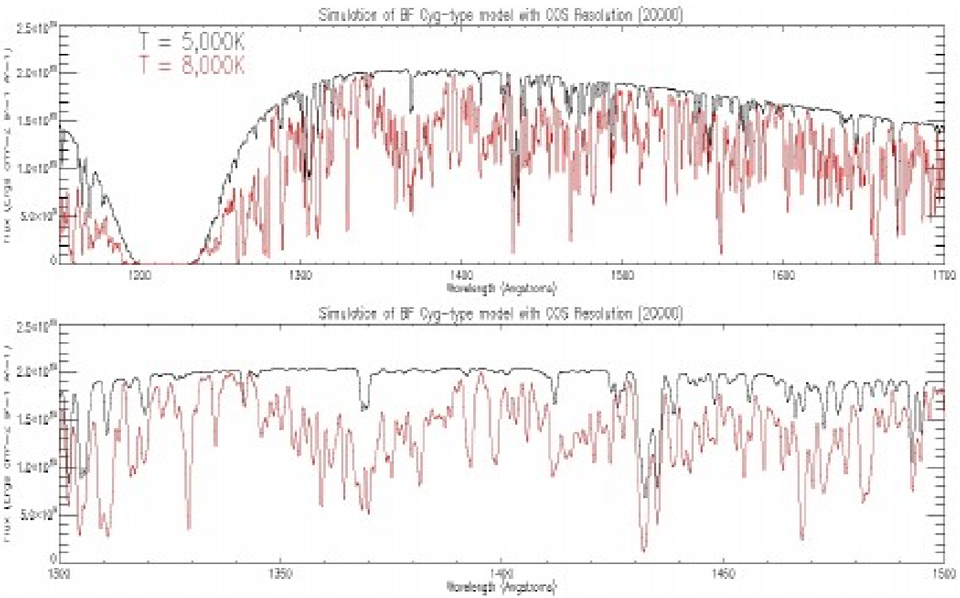


# Arcturus (K1 III)

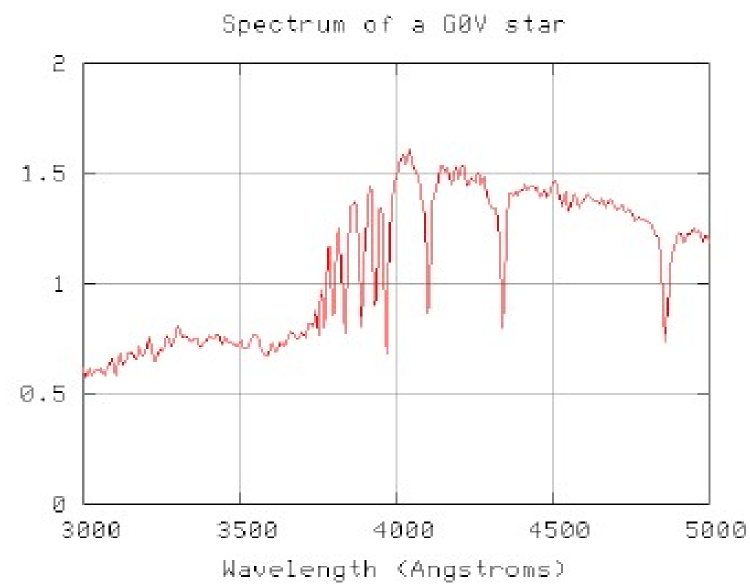



Structure of the H atom → produces spectral features

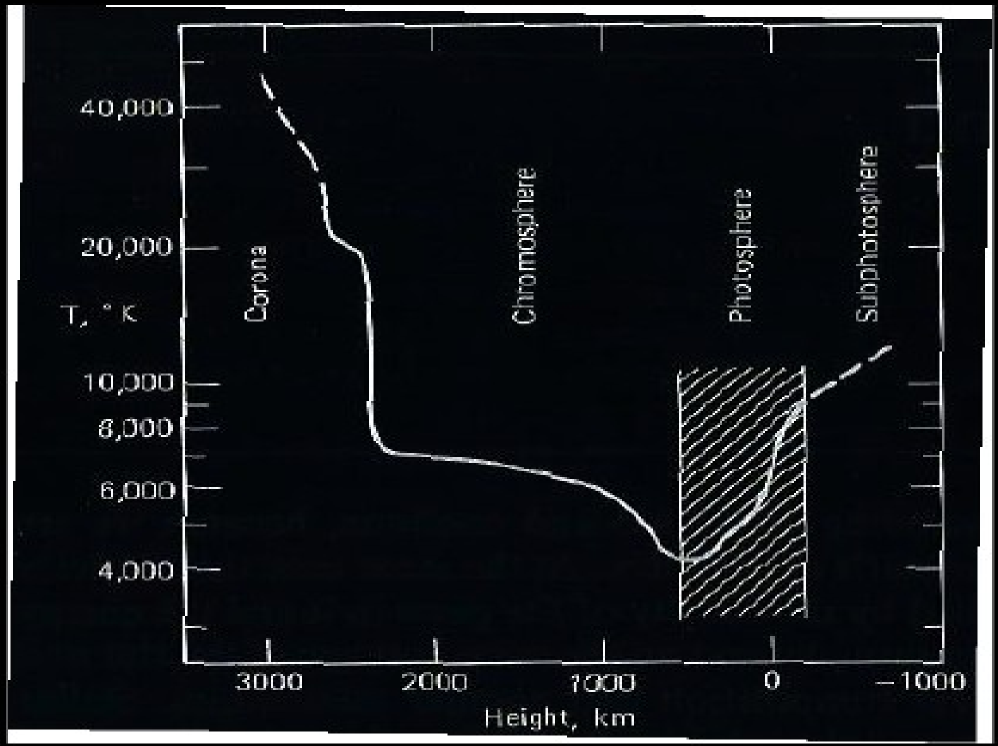



Modelled opacity in the UV due to gas at 5,000K (black) and 8,000K (red). The opacities are due to lines, mostly HI, FeII, SiII, NI, OI, MgII

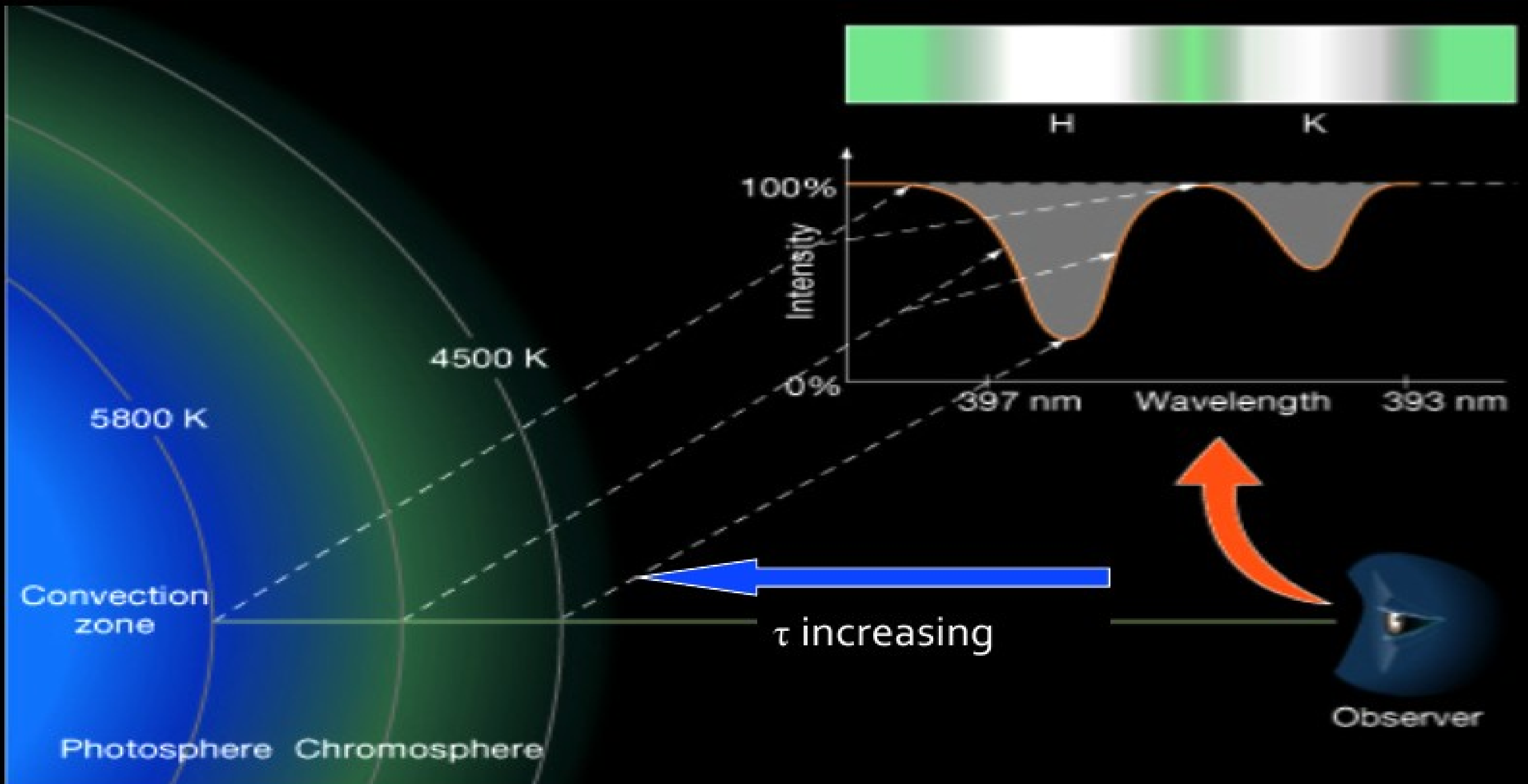



Balmer series bound-bound transitions (note the Balmer edge → continuous, so bound-free)

- The lower the optical depth, the deeper into the star we see
- For weak lines (lower optical depth) the deeper the line formation region
- For strong lines (higher optical depth), the shallower the line formation region

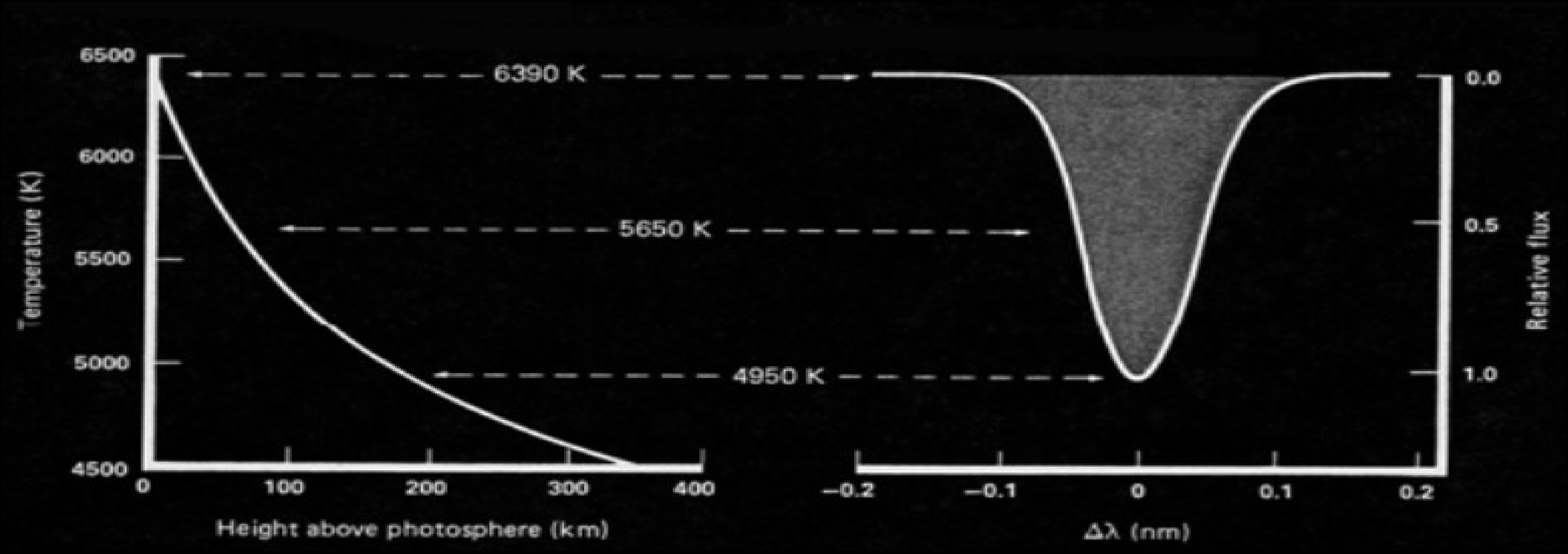



*Temperature structure of solar atmosphere*

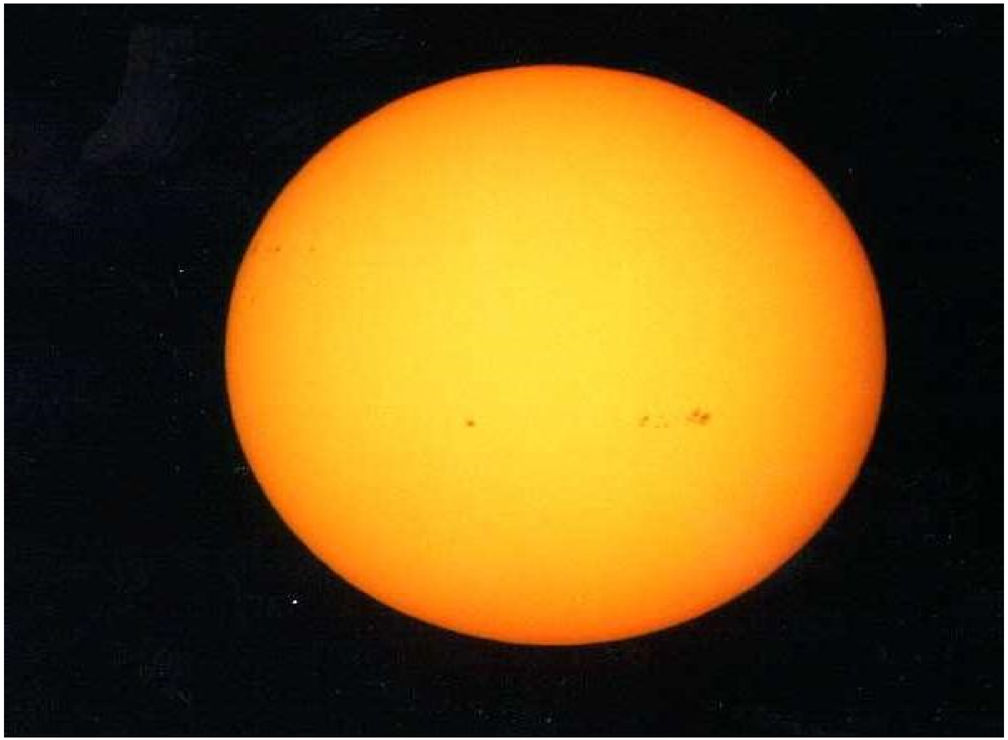


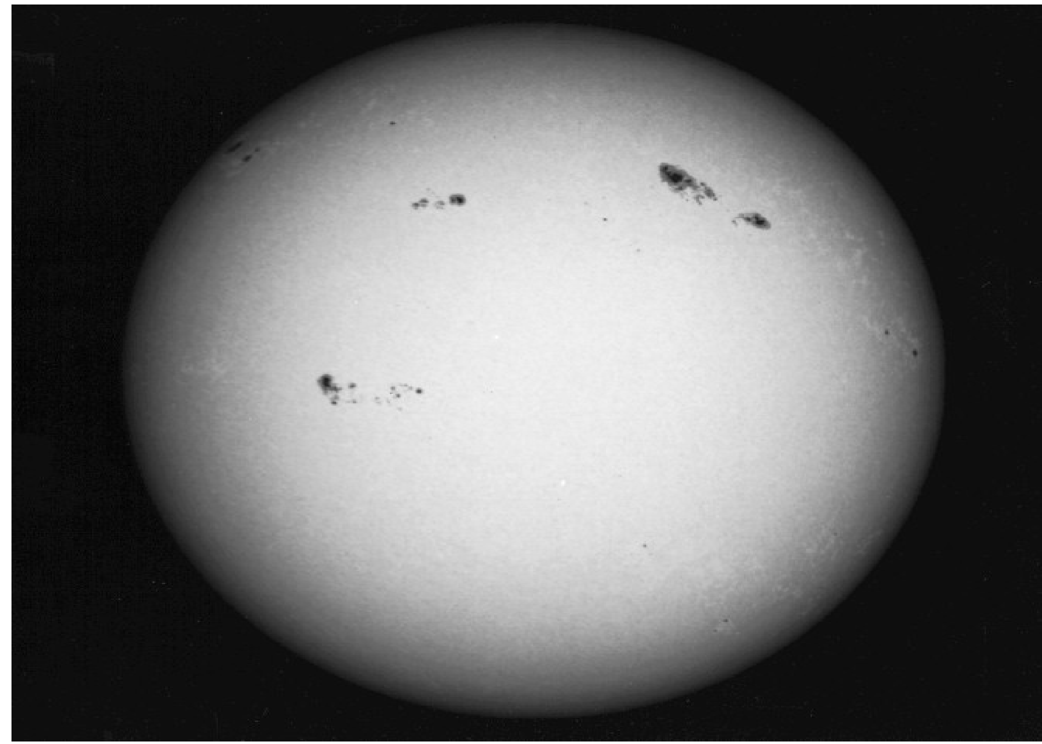

**Formation of absorption lines on the Sun**

- Formation of absorption features can also be understood in terms of the temperature of the local source function decreasing towards the line centre



# Limb Darkening





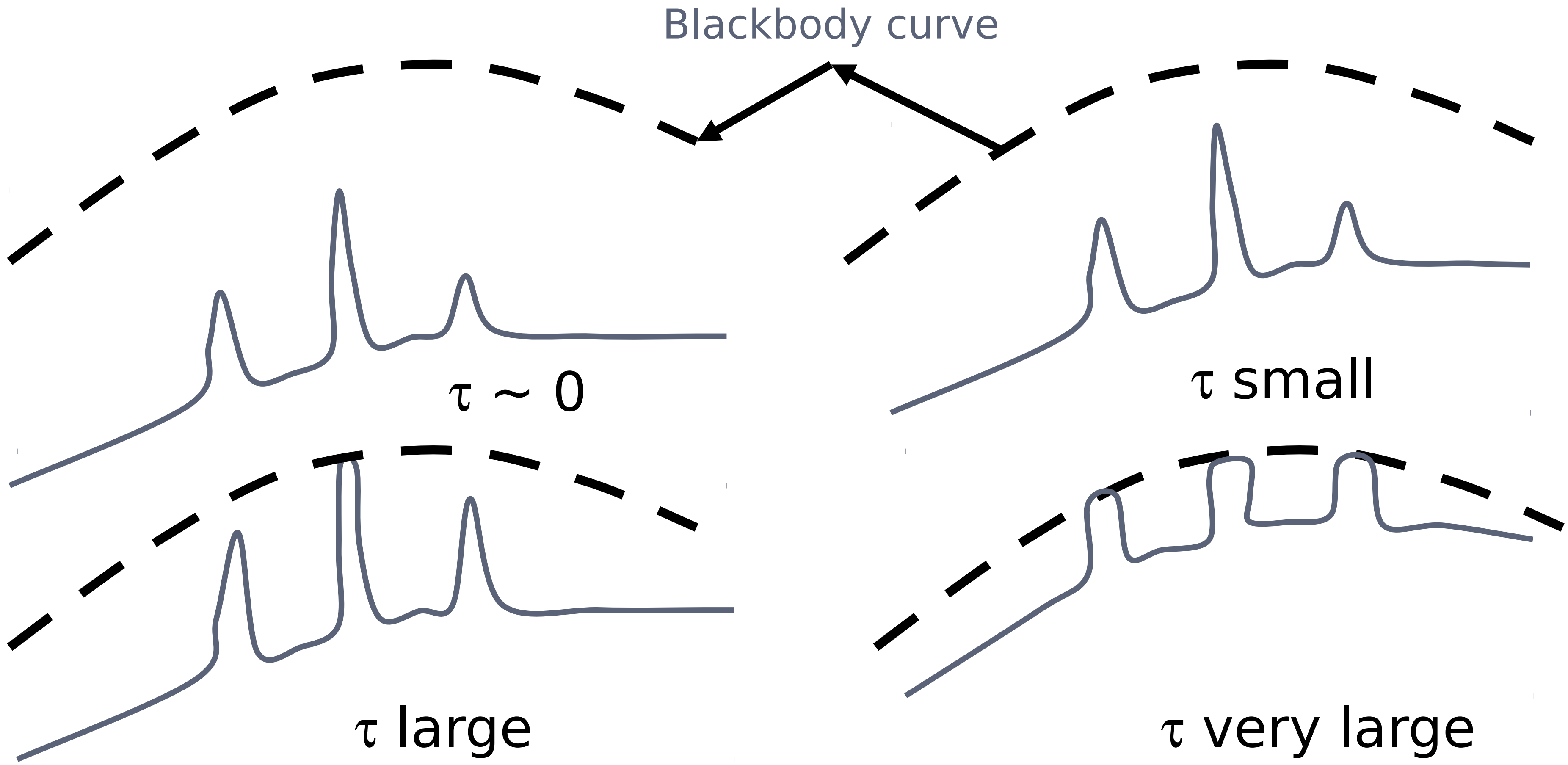
*The Sun □ redder at the edges, also dimmer at the edges…*

# *Thermalisation*

- Consider a uniform slab of gas of thickness L and temperature T that radiates like a blackbody, with an absorption coefficient $\alpha_\nu$ which is small everywhere except at a strong line of frequency $\nu_0$

- Compare the emitted intensity in the line relative to the neighbouring continuum for different limiting optical thicknesses of the slab

# Approach to thermalisation




**At a given temperature, BB has the largest luminosity ⇒ maximum efficiency**

# Emission or absorption?

Spherical BB with $T_c$ surrounded by shell with $T_s$.

Emission or absorption at $\nu_0$ if $\alpha_{\nu 1} << \alpha_{\nu 0}$?

**1.** $T_c > T_s \Rightarrow B_\nu(T_c) > B_\nu(T_s)$

$$I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + S_\nu\,(1 - e^{-\tau_\nu})$$

**Case A:**

$\alpha_{\nu 1}$ small $\Rightarrow I_{\nu 1} \approx B_{\nu 1}(T_c)$

$I_{\nu 0}(0) > S_{\nu 0}(T_s) = B_{\nu 0}(T_s)$

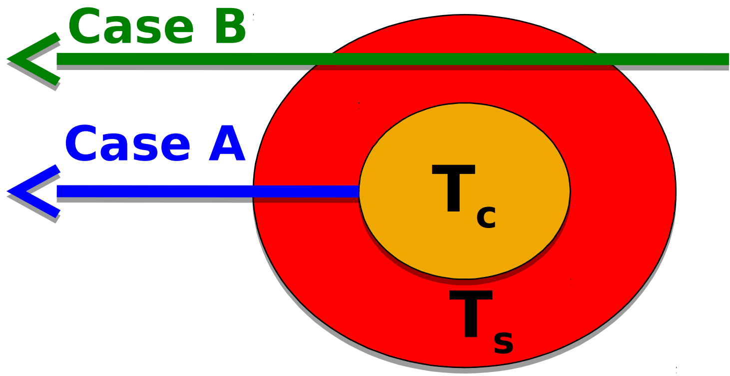
$\Rightarrow I_{\nu 0} = S_{\nu 0} + (I_{\nu 0}(0) - S_{\nu 0})\, e^{-\tau_{\nu 0}} > S_{\nu 0}$

$dI_{\nu 0}/d\tau_{\nu 0} = S_{\nu 0} - I_{\nu 0} < 0 \Rightarrow$ absorption

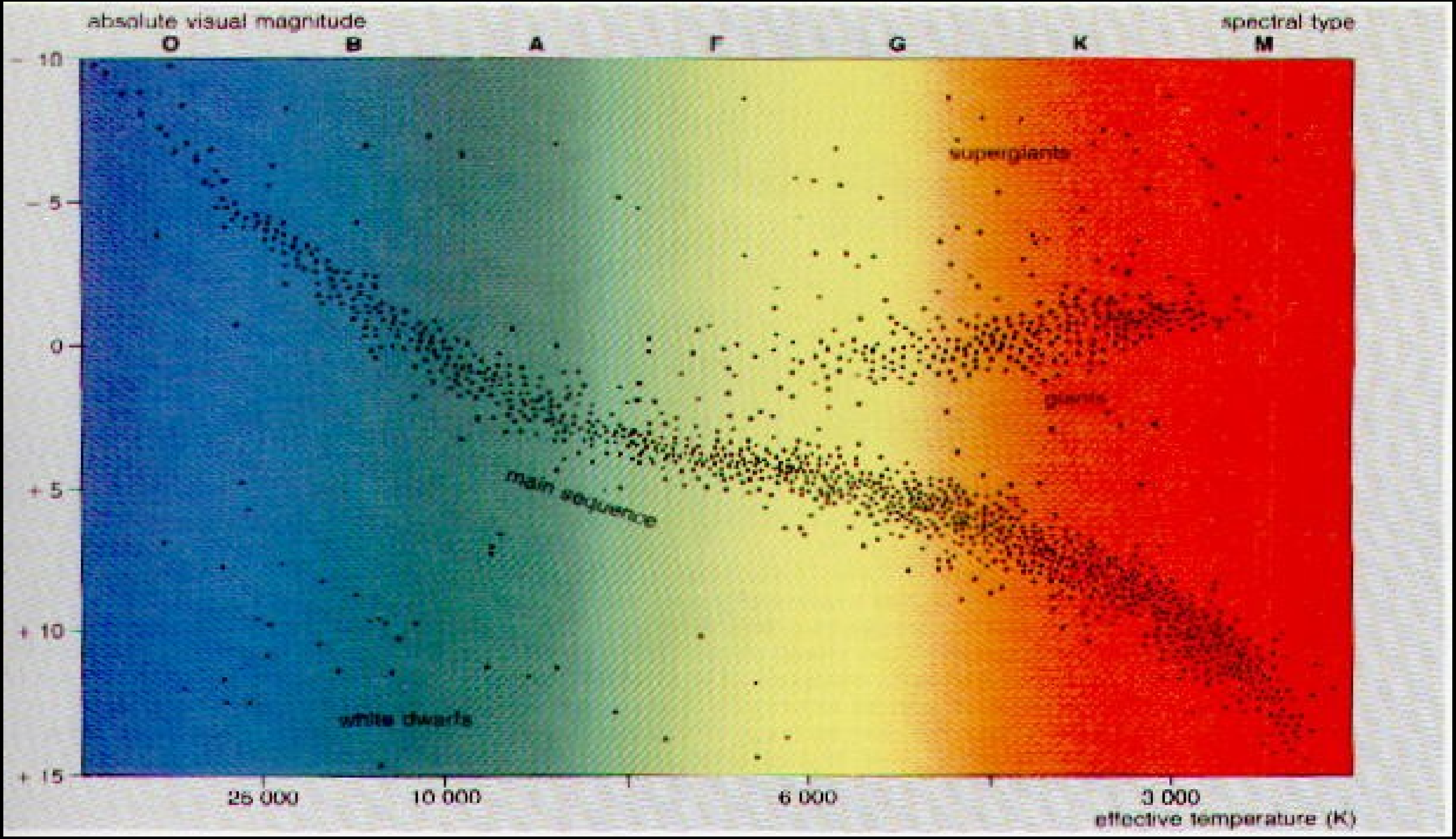
**Case B:**

$I_\nu(0) = 0$

$I_{\nu 0} < S_{\nu 0} = B_{\nu 0}(T_s) \Rightarrow dI_{\nu 0}/d\tau_{\nu 0} = S_{\nu 0} - I_{\nu 0} > 0 \Rightarrow$ emission



# Hertzsprung-Russel Diagram

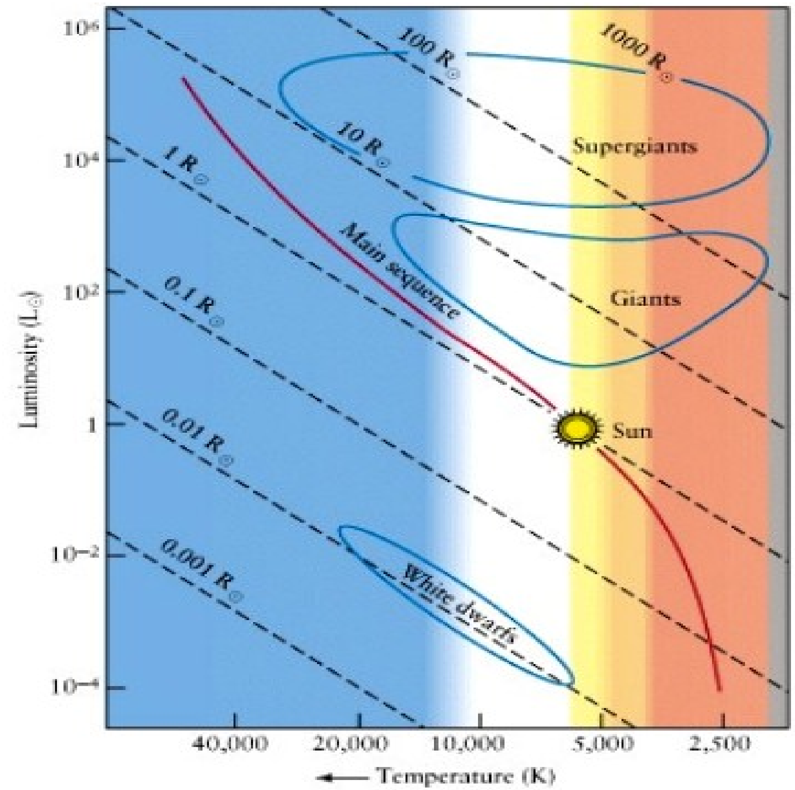


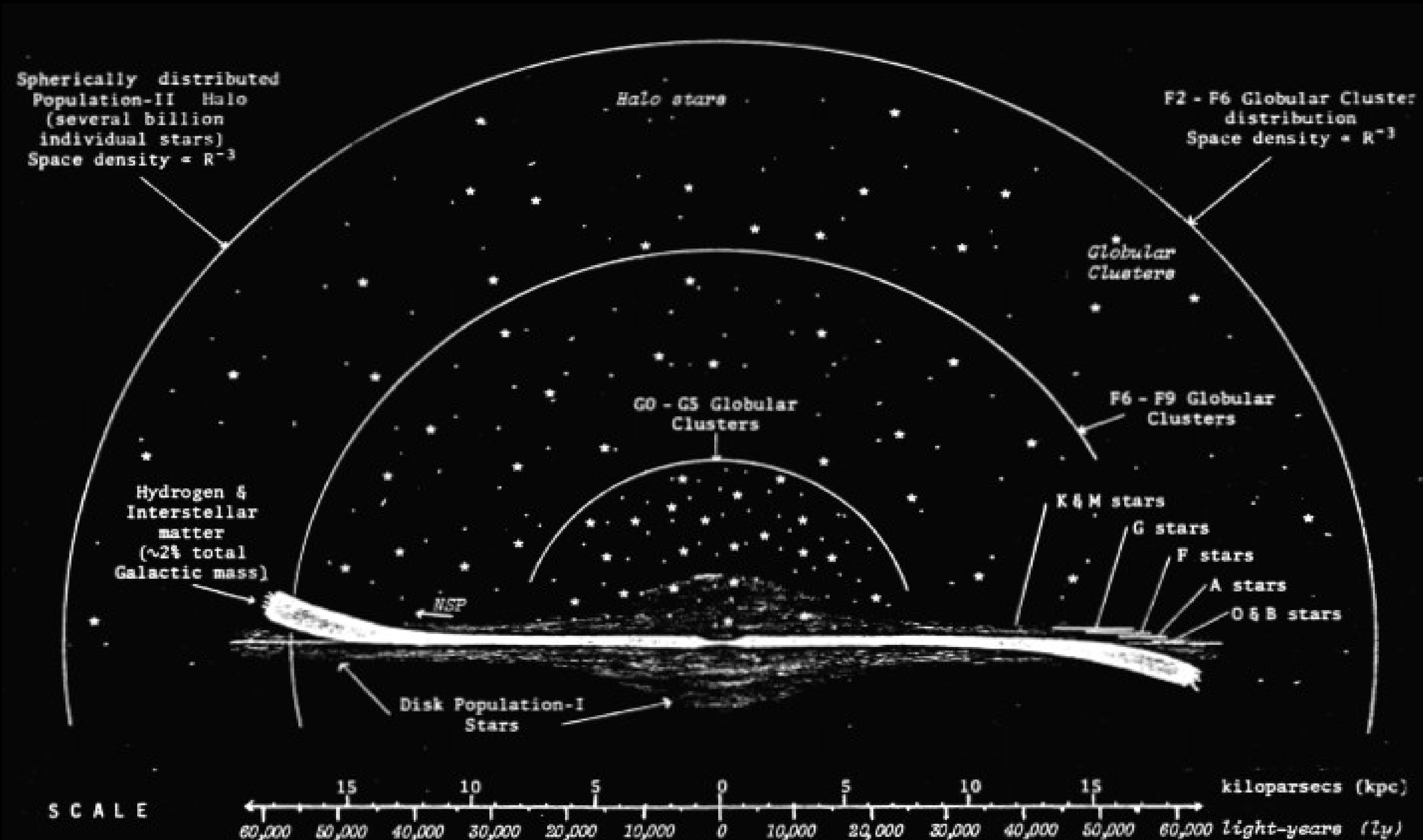
$$L \propto T^4$$

Why the Main Sequence is not a straight line?

$$L = 4\pi \boldsymbol{R}^2 \sigma T^4$$

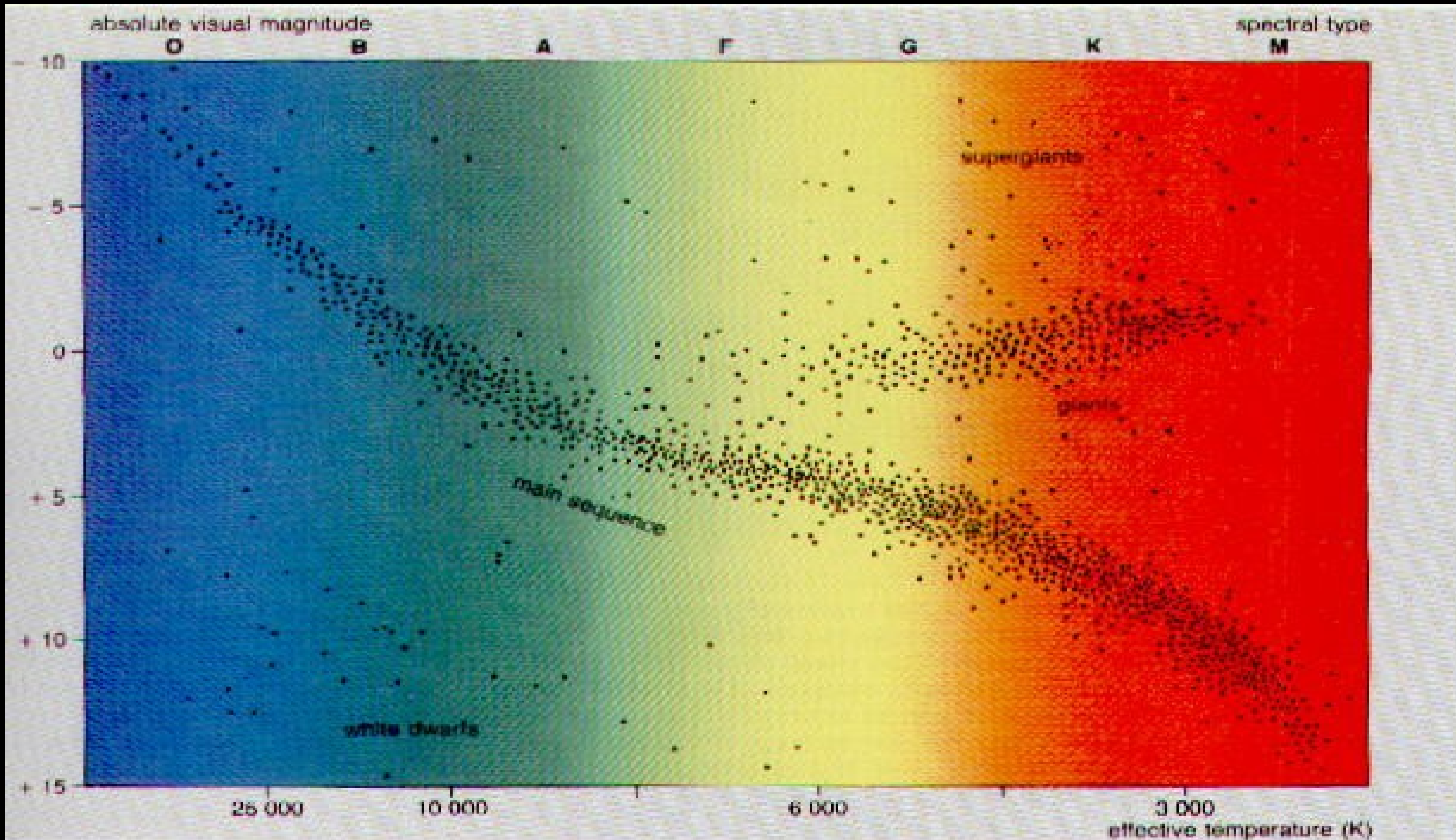
defines lines of **constant radius**

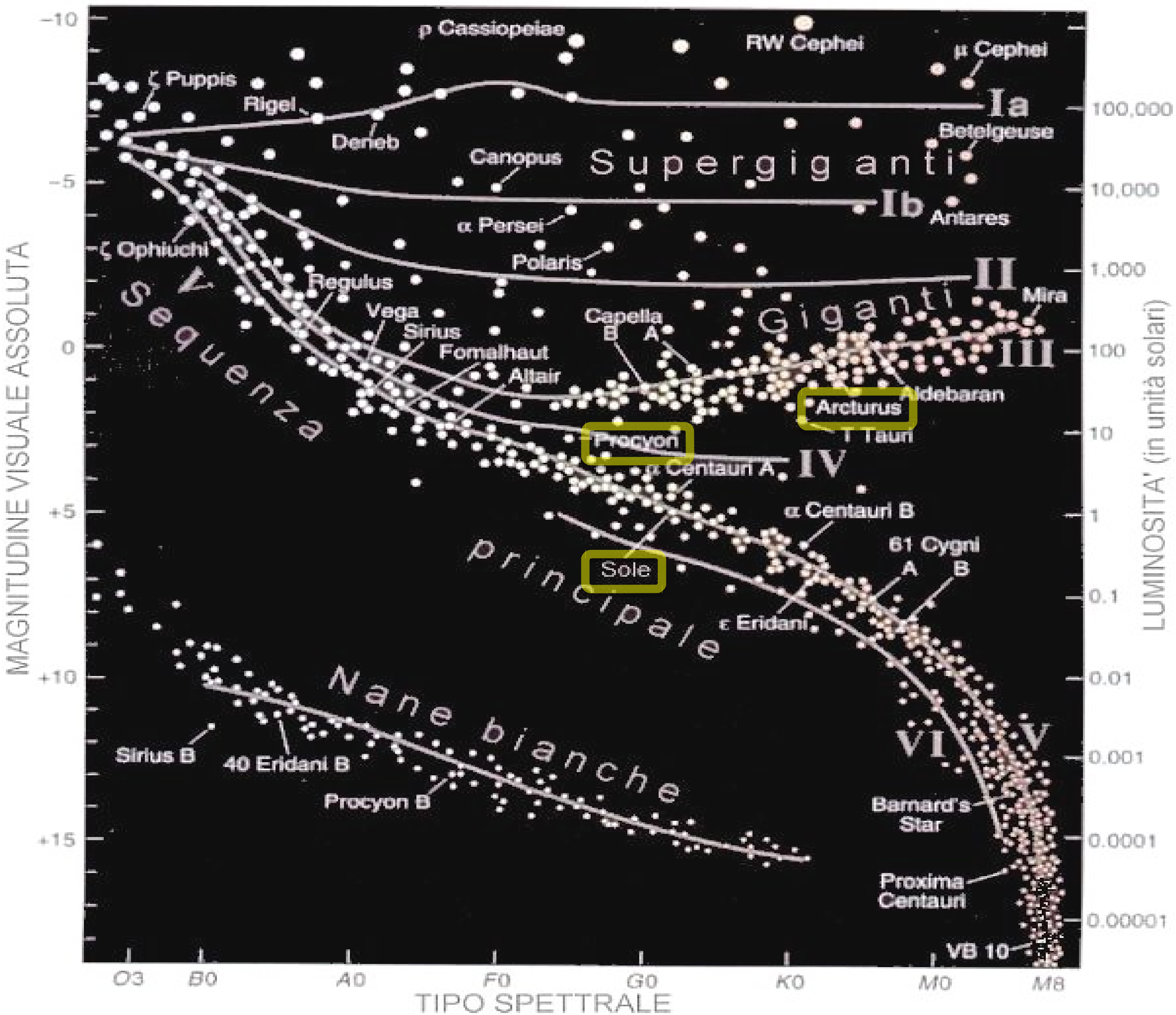




Spherically distributed Population-II Halo (several billion individual stars) Space density ∝ $R^{-3}$
Halo stars
F2 - F6 Globular Cluste: distribution Space density ∝ $R^{-3}$
Globular Clusters
F6 - F9 Globular Clusters
G0 - G5 Globular Clusters
Hydrogen & Interstellar matter (~2% total Galactic mass)
NSP
K & M stars
G stars
F stars
A stars
O & B stars
Disk Population-I Stars
SCALE
15 10 5 0 5 10 15 kiloparsecs (kpc)
60,000 50,000 40,000 30,000 20,000 10,000 0 10,000 20,000 30,000 40,000 50,000 60,000 light-years (ly)

# Hertzsprung-Russel Diagram




ρ Cassiopeiae
RW Cephei
μ Cephei
ζ Puppis
Rigel
Deneb
Ia
Betelgeuse
Canopus
Supergiganti
Ib
Antares
α Persei
Polaris
II
ζ Ophiuchi
Regulus
Mira
Vega
Sirius
Fomalhaut
Altair
Capella
B A
Giganti
III
Aldebaran
Arcturus
Procyon
γ Tauri
α Centauri A
IV
α Centauri B
Sequenza principale
V
Sole
61 Cygni
A B
ε Eridani
Nane bianche
VI
Sirius B
40 Eridani B
Procyon B
Barnard's Star
Proxima Centauri
VB 10
-10
-5
0
+5
+10
+15
100,000
10,000
1,000
100
10
1
0.1
0.01
0.001
0.0001
0.00001
O3
B0
A0
F0
G0
K0
M0
M8
TIPO SPETTRALE
MAGNITUDINE VISUALE ASSOLUTA
LUMINOSITA' (in unità solari)

# Hydrostatic Thermostat

Nuclear fusion reactions are **temperature** sensitive:

- Higher Core Temperature = More Fusion

**BUT**

- More fusion makes the core hotter
- Hotter core leads to even more fusion

Why don't stars **explode** like

Hydrogen Bombs?

If the reactions run **too fast**:

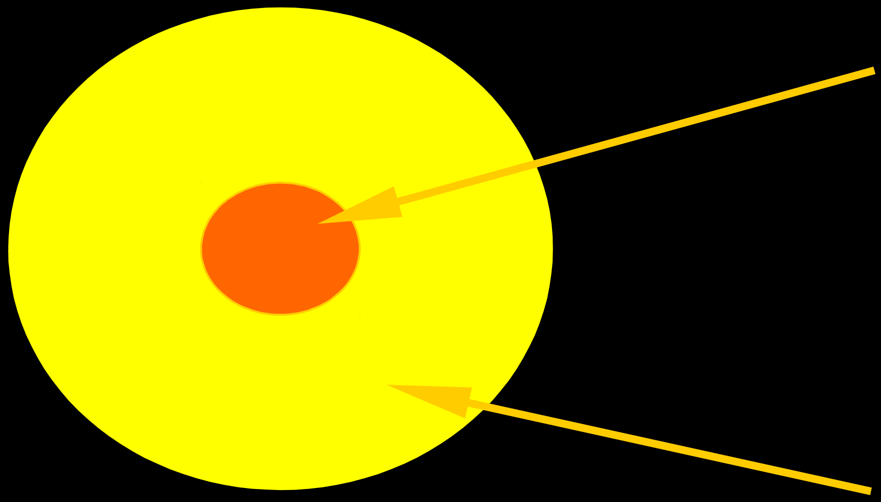
- The core heats up ⇒ higher Pressure (P)
- Higher P ⇒ expansion
- Expansion cools core, slowing the rate of fusion

If the reactions run **too slow**:

- The core cools ⇒ lower P
- Lower P ⇒ contraction
- Contraction heats core, increasing the fusion rate

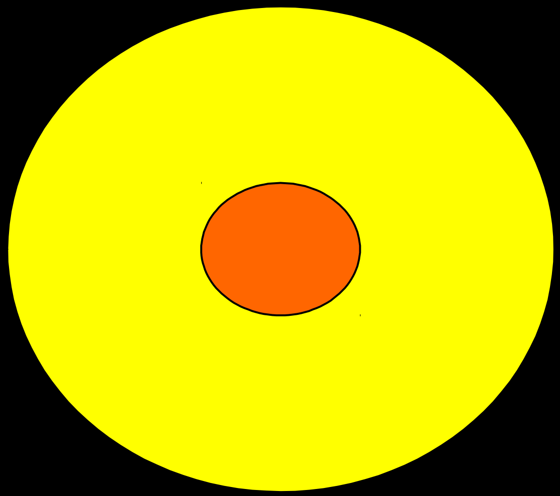
Result is like a **thermostat**

# Sun’s Structure

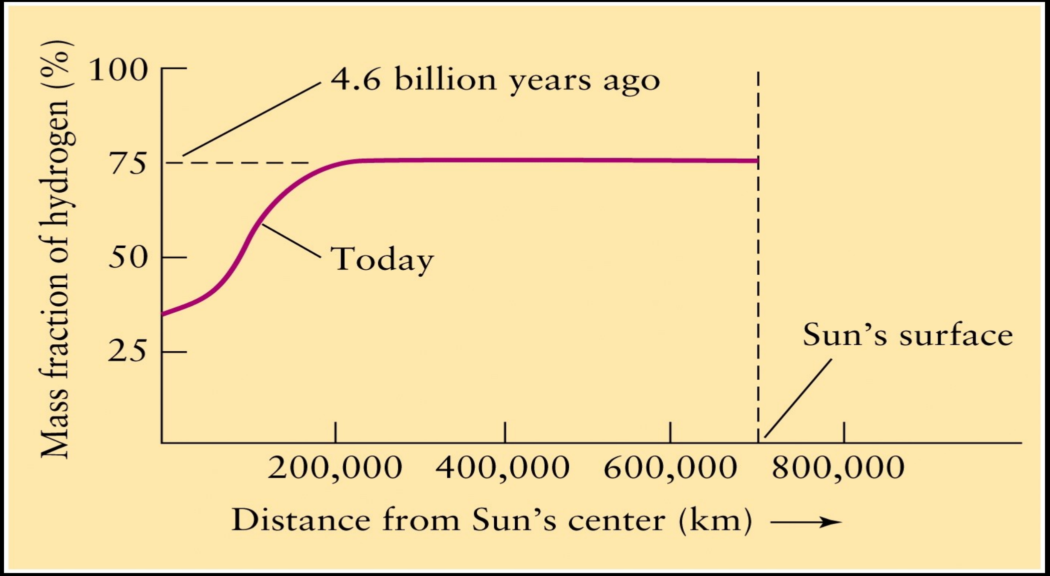


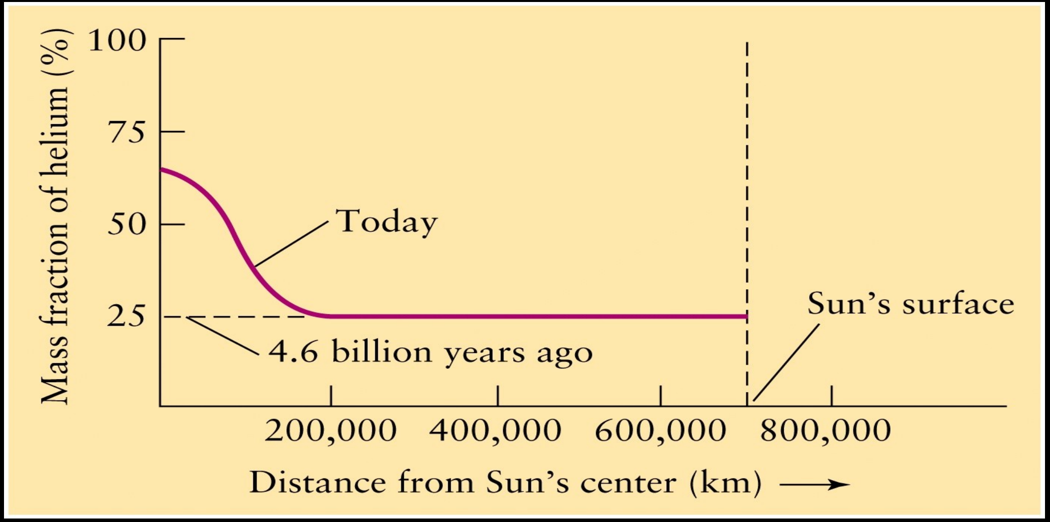
- Core
  - Where nuclear fusion occurs
- Envelope
  - Supplies gravity to keep core hot and dense

# Main Sequence Evolution



- Core starts with same fraction of hydrogen as whole star
- Fusion changes H → He
- Core gradually shrinks and Sun gets hotter and more luminous

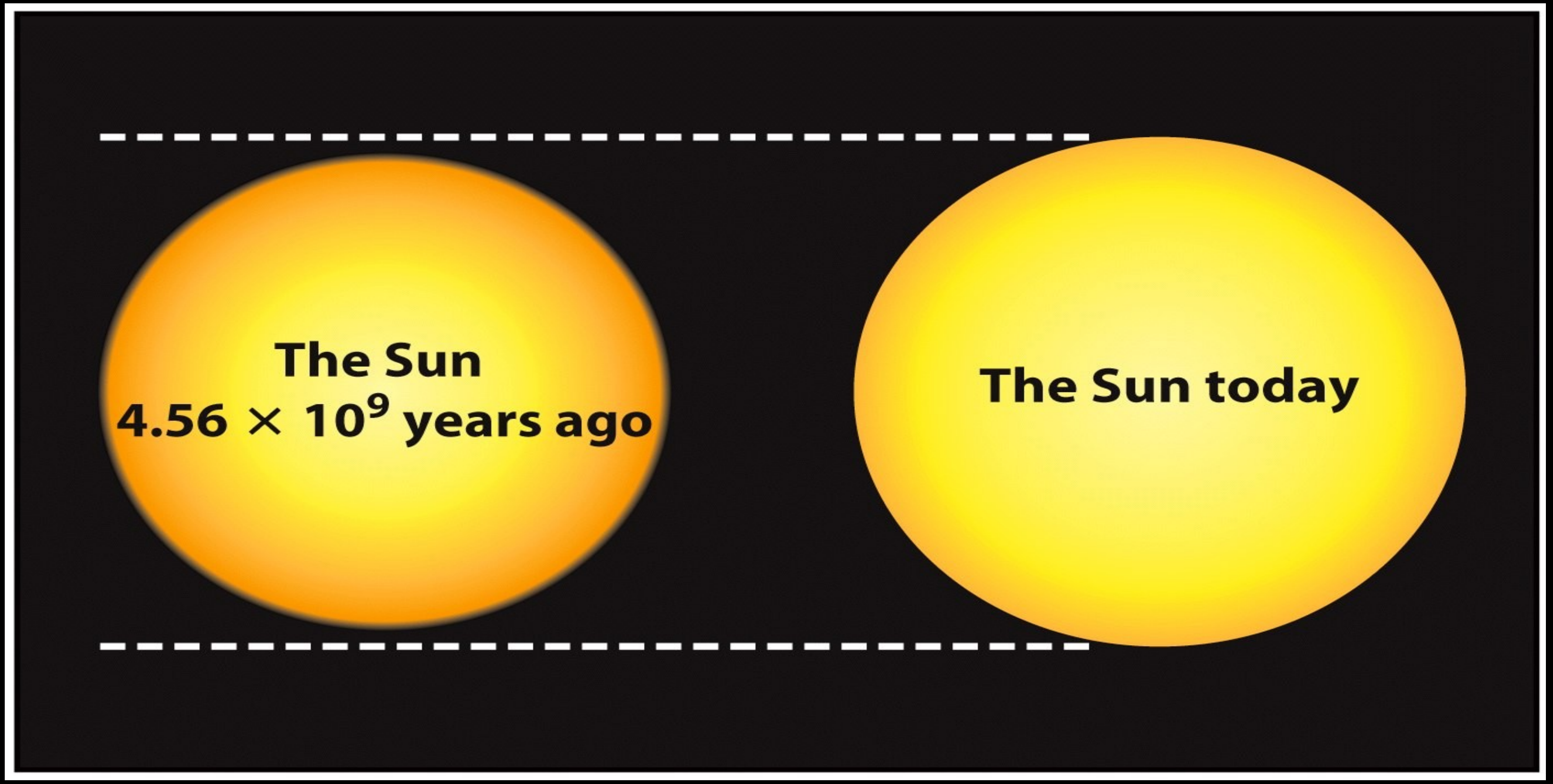



Since reaching the Main Sequence, H has been depleted in the core, while He has been built up there

***We do not see this on the surface!***

# Gradual change in size of Sun



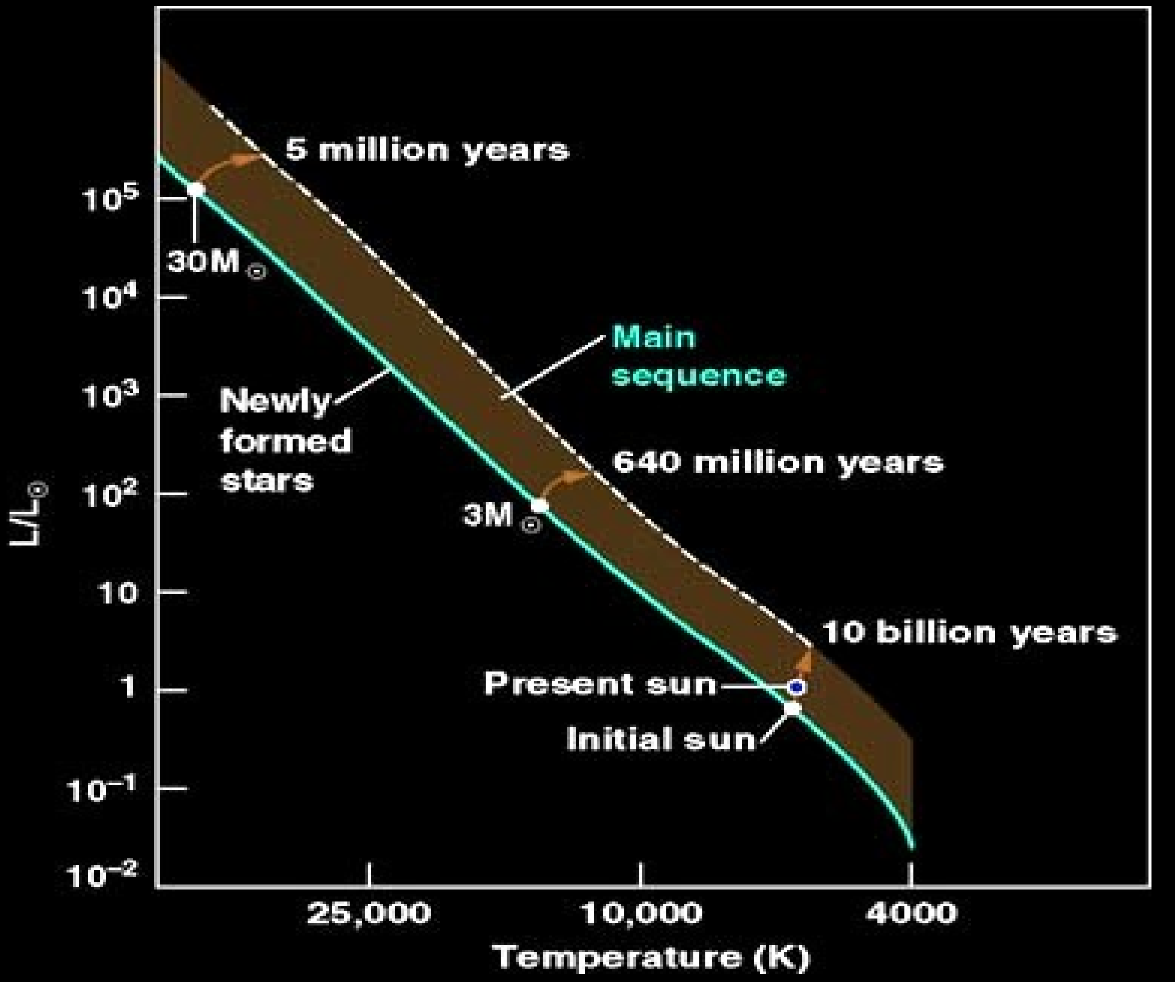

Now 6% larger, 5% hotter ⇒ 40% brighter

# Main Sequence Evolution

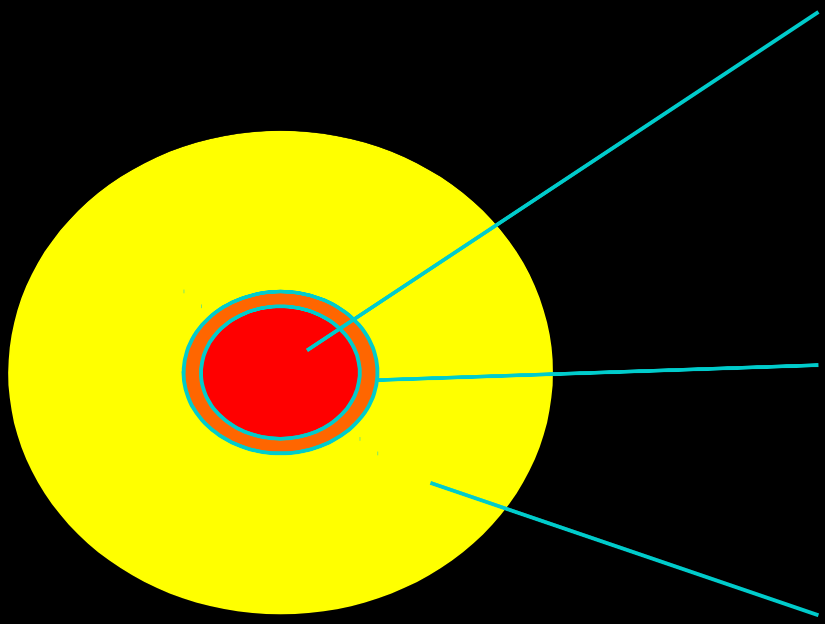
When stars initiate H burning in their cores, they are located on the ***zero-age main sequence*** (ZAMS).

As they age, they evolve slowly away from the ZAMS.

Most stars, regardless of their mass, spend roughly 90% of their total lifetimes as main sequence stars.

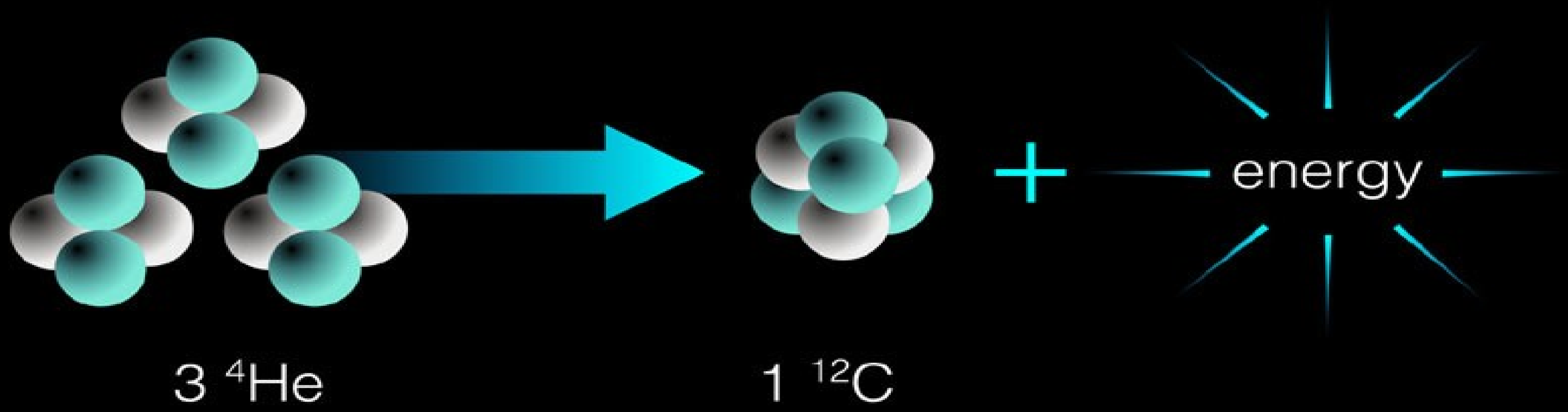


# Red Giant Phase



- He core
  - No nuclear fusion
  - Gravitational contraction produces energy
- H layer
  - Nuclear fusion
- Envelope
  - Expands because of increased energy production
  - Cools because of increased surface area

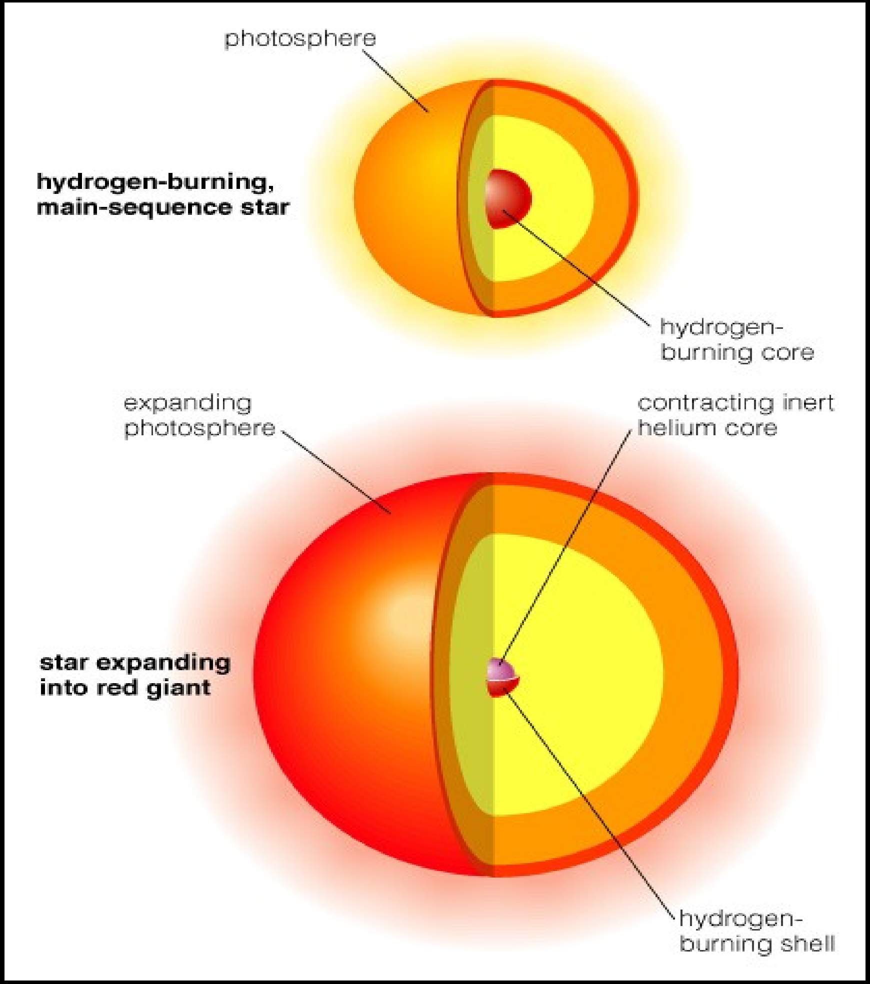
# Helium fusion




Helium fusion does not begin right away because it requires higher temperatures than hydrogen fusion—larger charge leads to greater repulsion

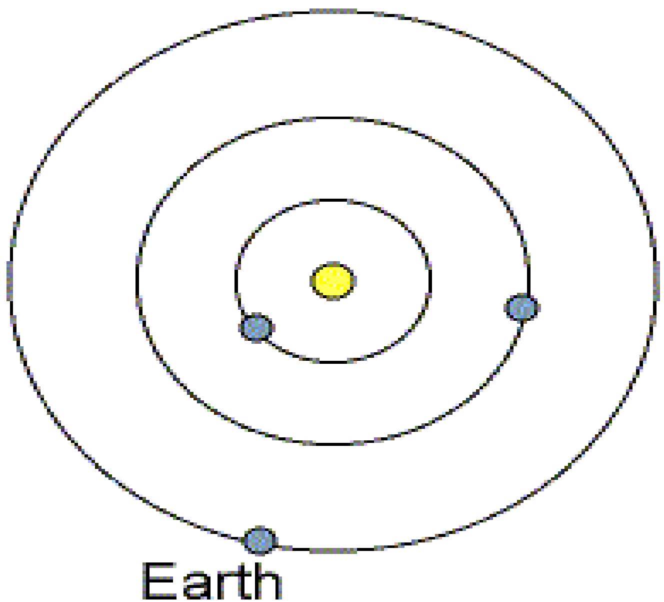
Fusion of two helium nuclei doesn't work, so helium fusion must combine three He nuclei to make carbon

# Broken Thermostat

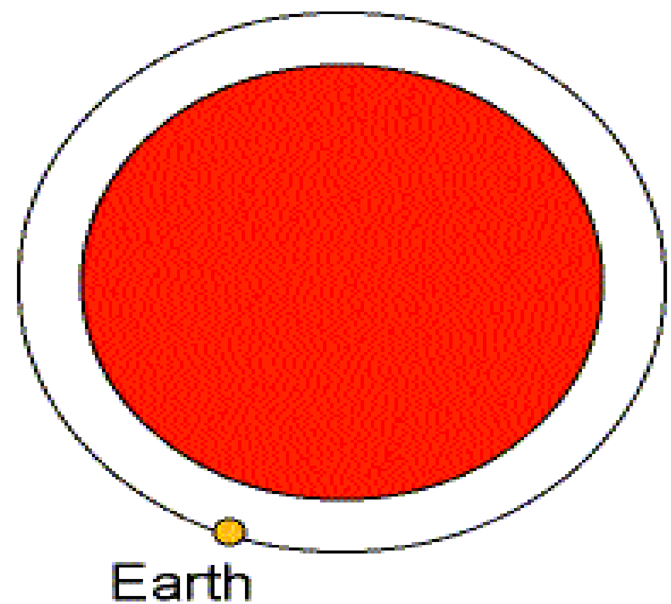



- As the core contracts, H begins fusing to He in a shell around the **degenerate** core
- Luminosity increases because the core thermostat is broken (no nuclear reactions) ⇒ the increasing fusion rate in the shell does not stop the core from contracting

# Sun's Red Giant Phase

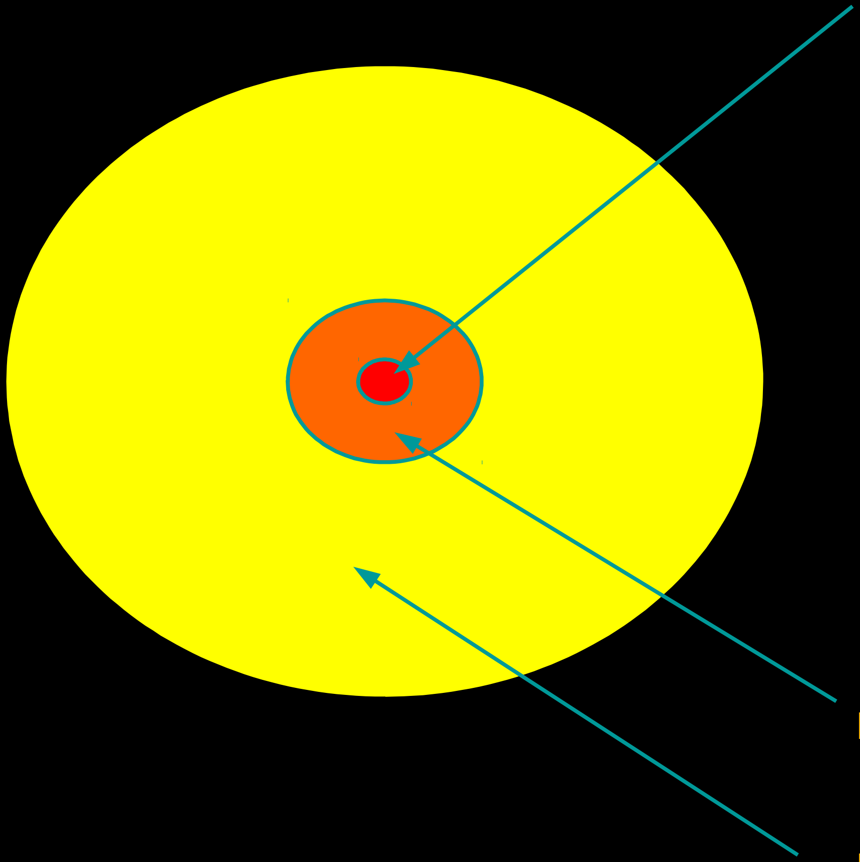



Now: hot core + warm surface; small size.



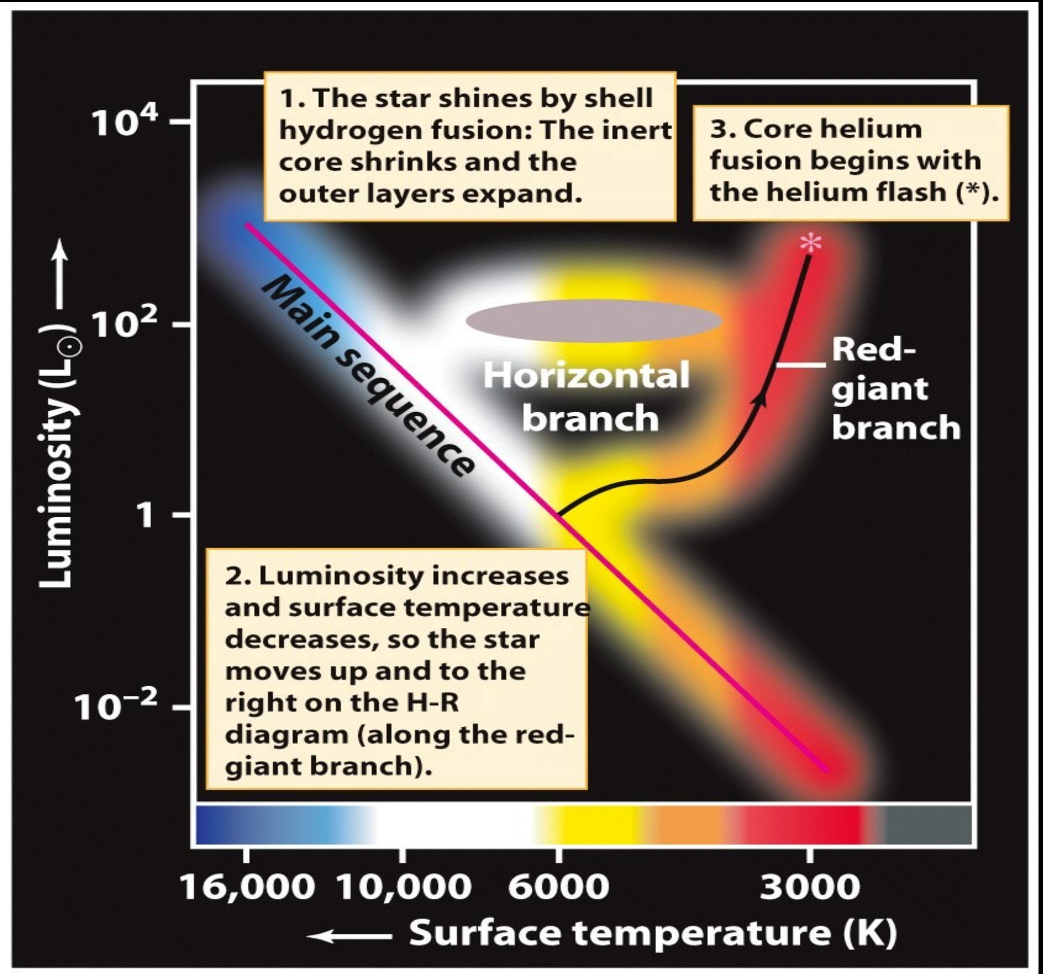

Future: very hot core + cool surface. Large size

# Helium Flash



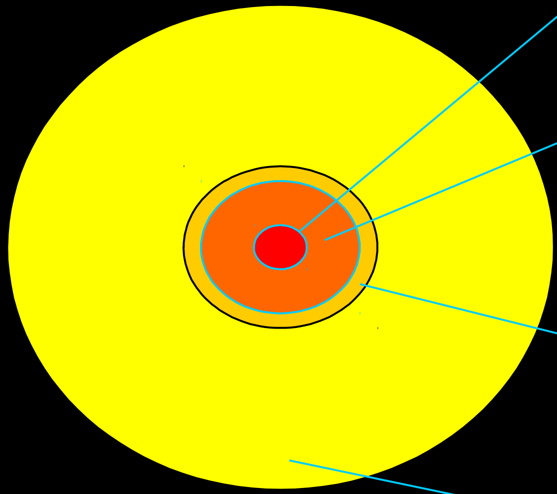
- He core
  - Eventually the core gets hot enough to fuse Helium into Carbon.
  - This causes the temperature to increase rapidly to 300 million K and there's a sudden flash when a large part of the Helium gets burned all at once.
  - We don't see this flash because it's buried inside the star.
- H layer
- Envelope

# Movement on HR diagram



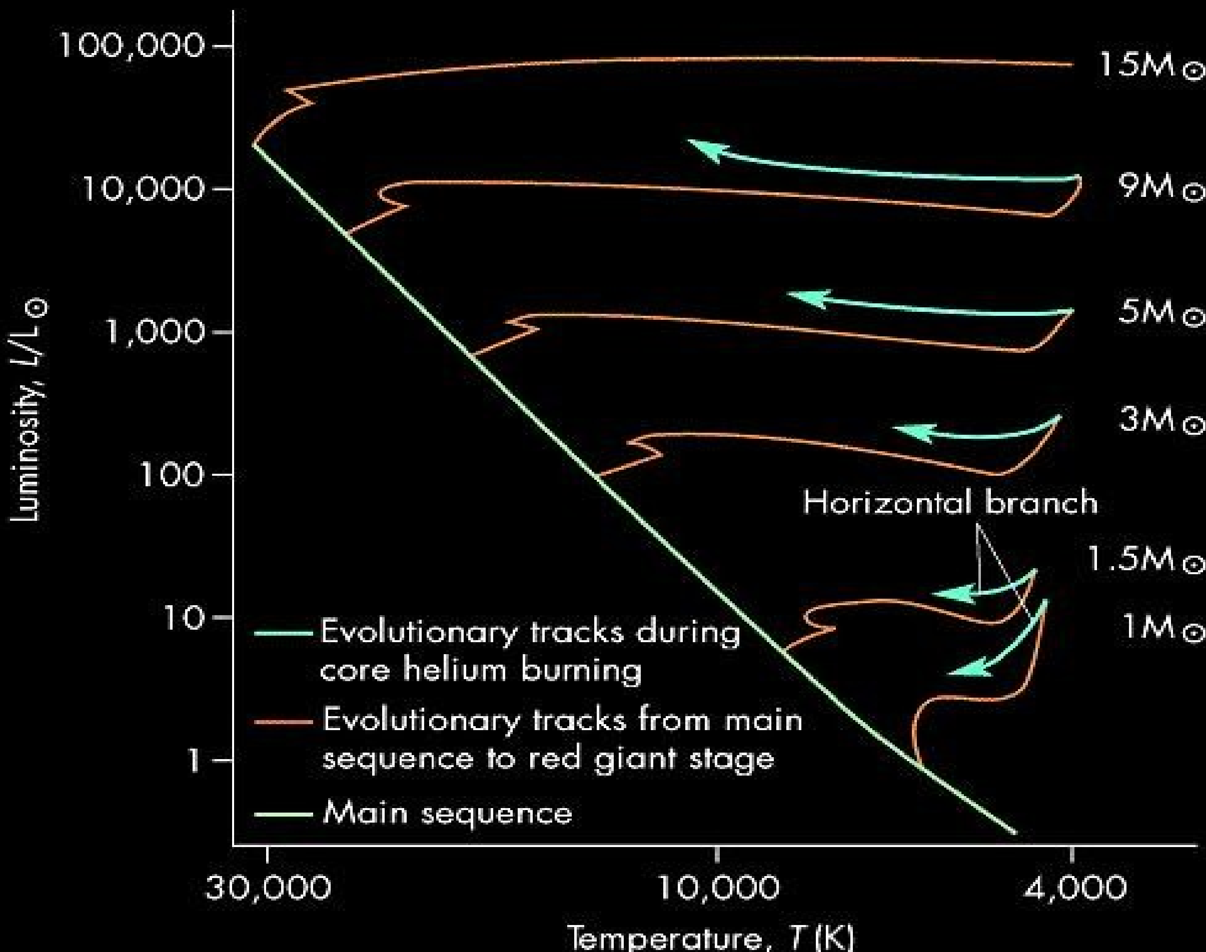

Before the helium flash: A red-giant star

# Red Giant after Helium Ignition



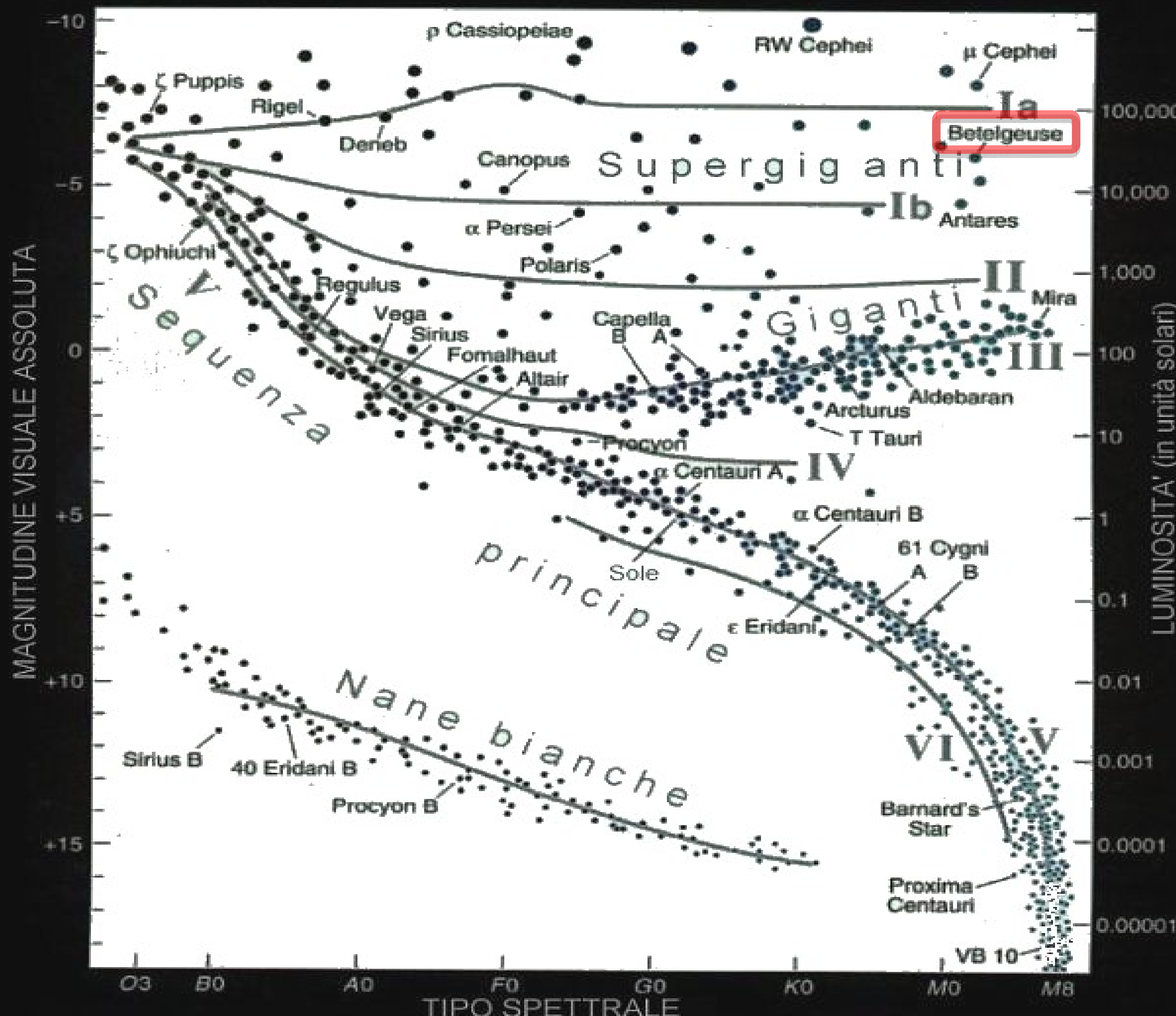
- He burning core
  - Fusion burns He into C, O
- He rich core
  - No fusion
- H burning shell
  - Fusion burns H into He
- Envelope

# Sun moves onto Horizontal Branch




Sun burns He into Carbon and Oxygen in the core

Sun becomes hotter and smaller (L~constant): **Horizontal Branch**



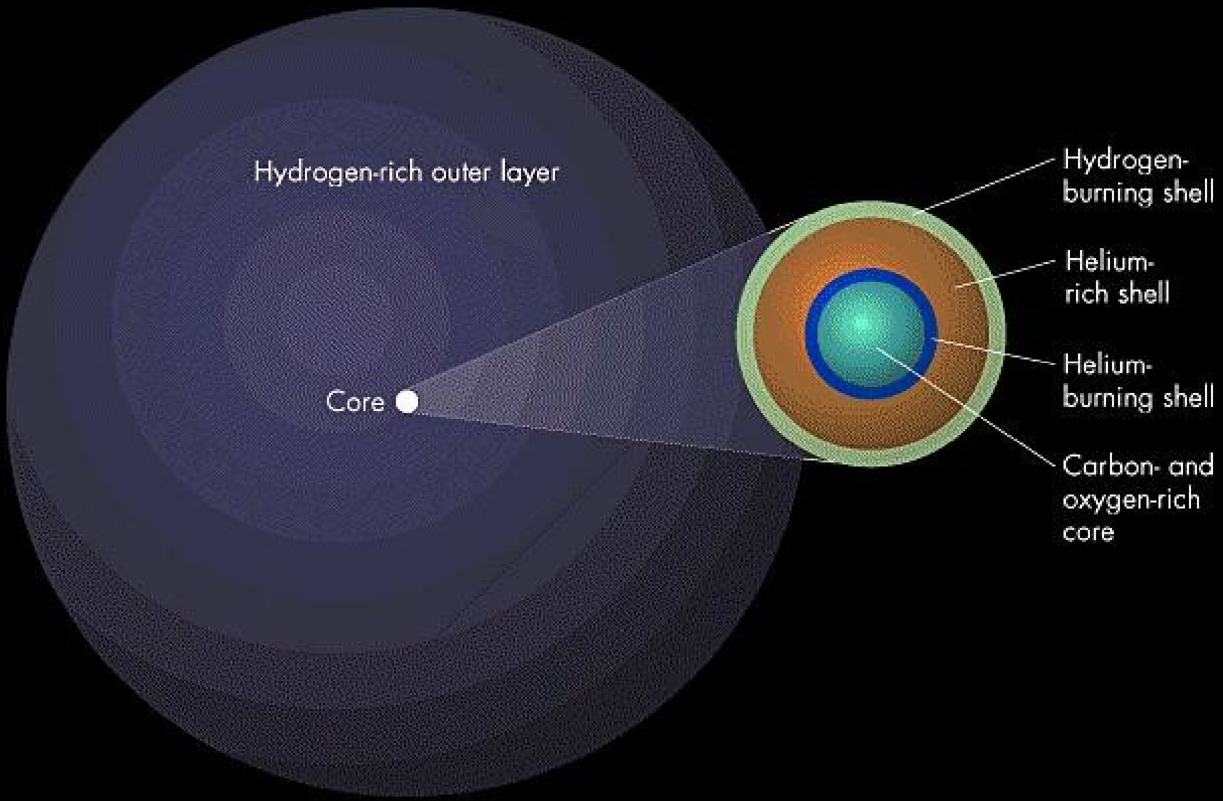
ρ Cassiopeiae
RW Cephei
μ Cephei
ζ Puppis
Rigel
Deneb
Canopus
Ia
Betelgeuse
Supergiganti
Ib
Antares
α Persei
ζ Ophiuchi
Polaris
II
Regulus
Mira
Vega
Sirius
Capella
B A
Giganti
III
Fomalhaut
Altair
Aldebaran
Arcturus
T Tauri
Procyon
IV
α Centauri A
α Centauri B
61 Cygni
A B
Sole
ε Eridani
V
Sequenza principale
Nane bianche
VI
V
Sirius B
40 Eridani B
Procyon B
Barnard's Star
Proxima Centauri
VB 10
MAGNITUDINE VISUALE ASSOLUTA
-10
-5
0
+5
+10
+15
LUMINOSITA' (in unità solari)
100,000
10,000
1,000
100
10
1
0.1
0.01
0.001
0.0001
0.00001
O3
B0
A0
F0
G0
K0
M0
M8
TIPO SPETTRALE

# Helium burning in the core stops

H burning is continuous

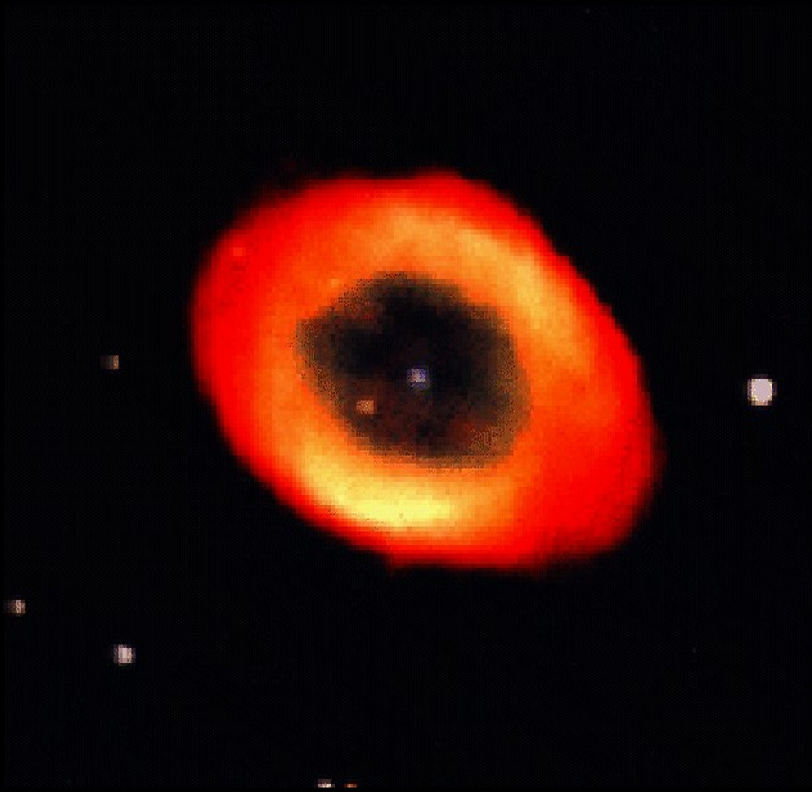
He burning happens in "thermal pulses"

Core is degenerate

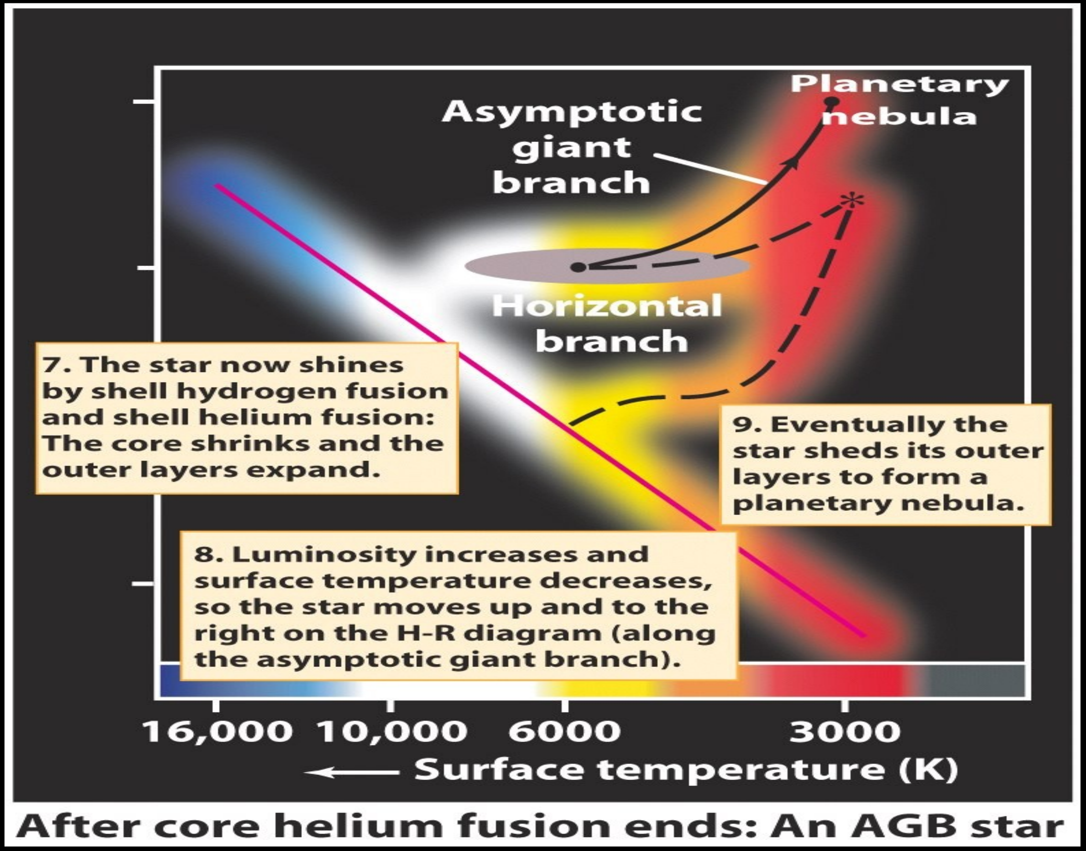


# Sun loses mass via winds

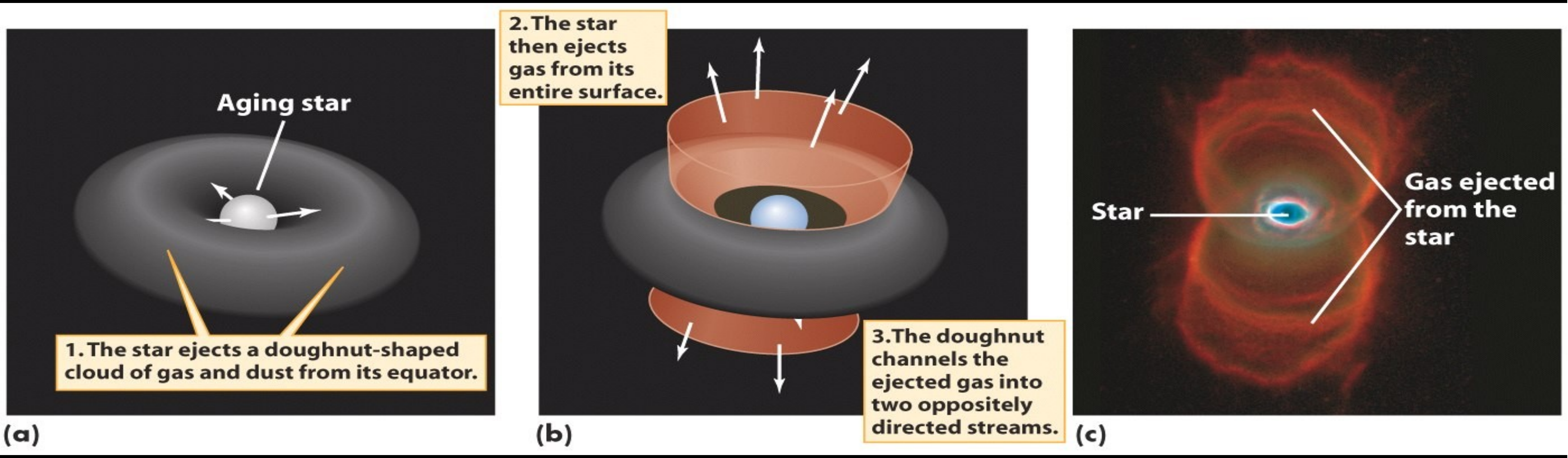
- Creates a "planetary nebula"
- Leaves behind core of carbon and oxygen surrounded by thin shell of hydrogen
- Hydrogen continues to burn



# Sun moves onto Asymptotic Giant Branch (AGB)



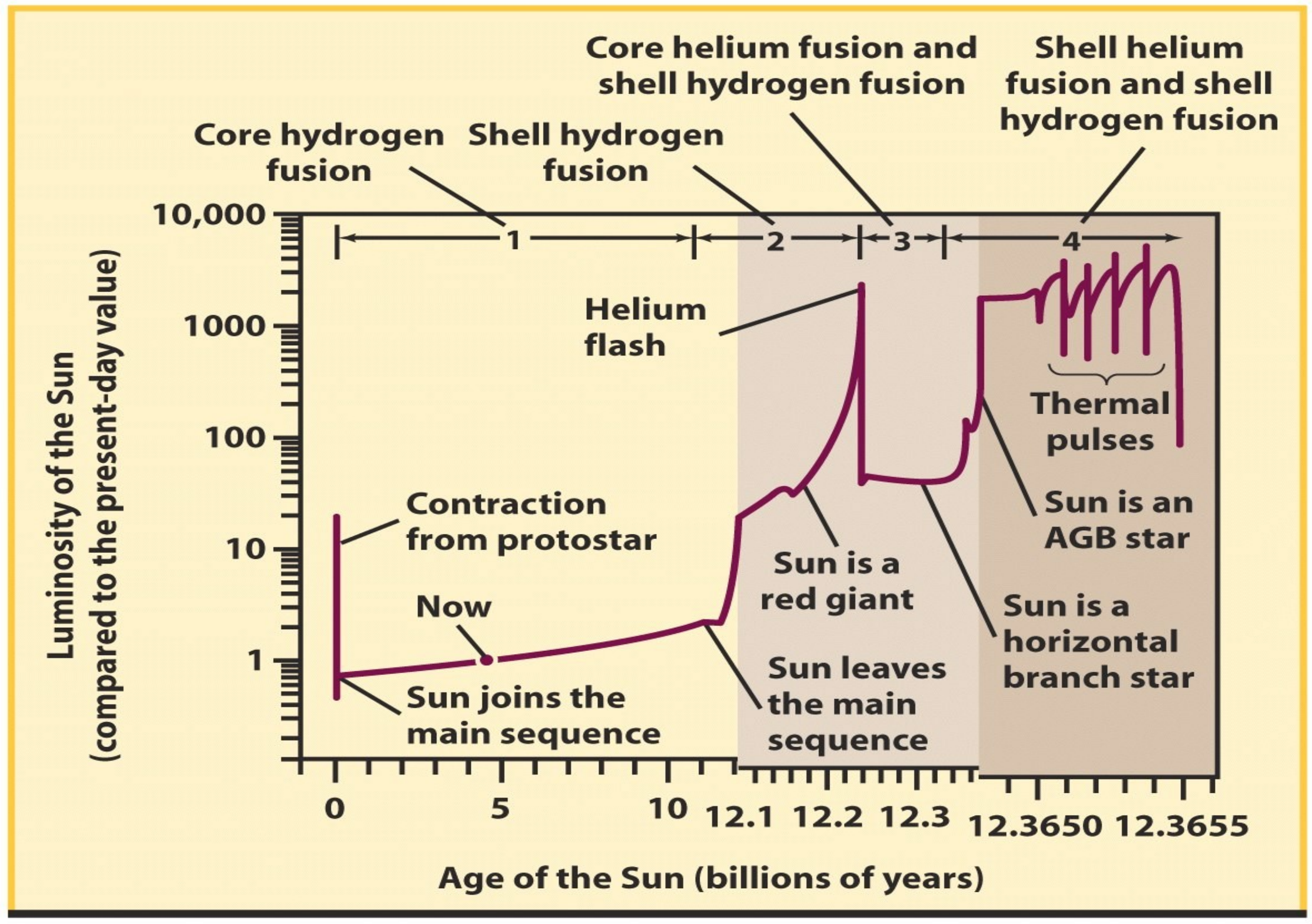
# Bipolar planetary nebulae



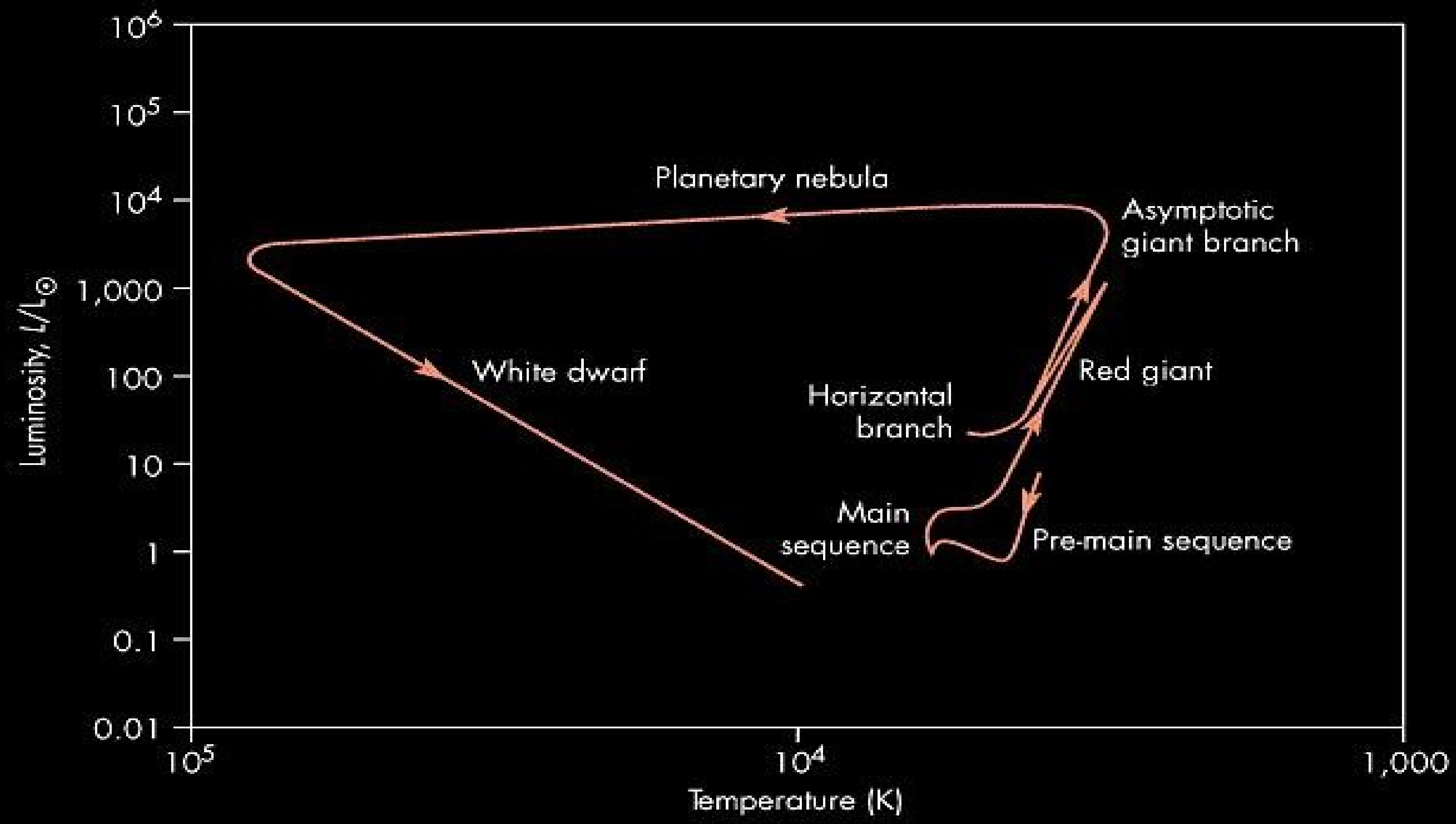
# White dwarf

- Star burns up rest of hydrogen
- Nothing remains but degenerate core of Oxygen and Carbon
- “White dwarf” cools but does not contract because core is degenerate
- No energy from fusion, no energy from gravitational contraction
- White dwarf slowly fades away…

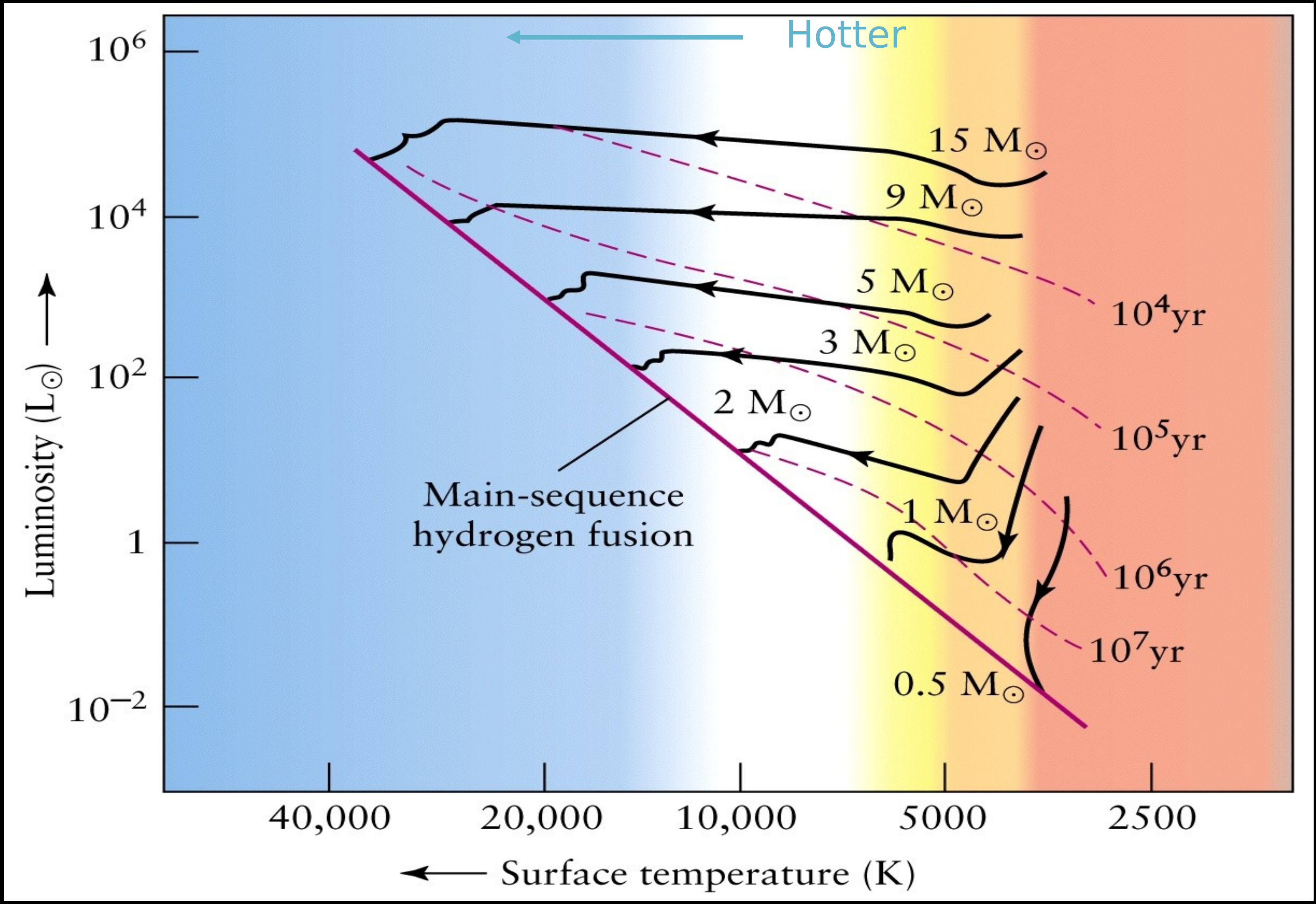
# Time line for Sun’s evolution



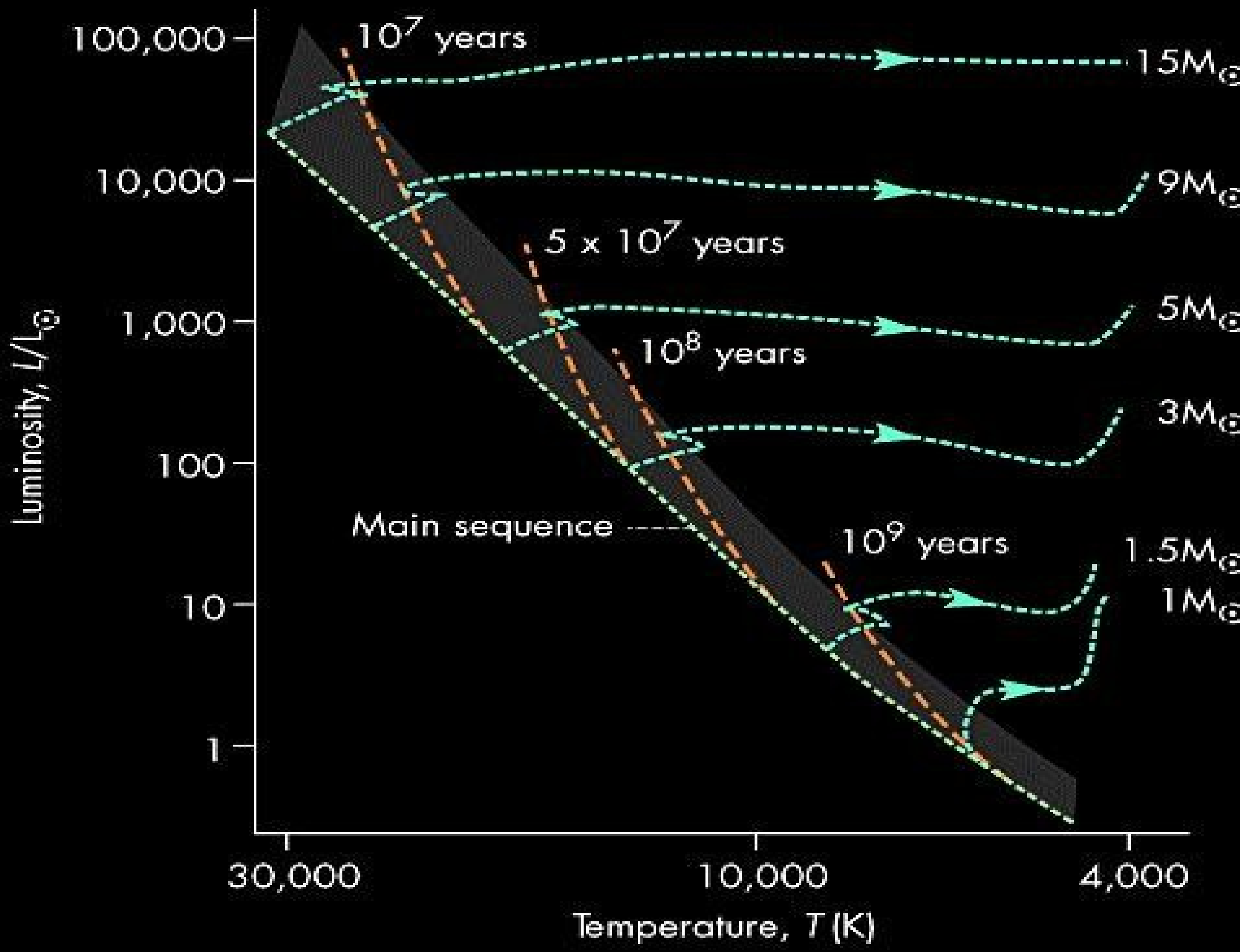
# Evolution on HR diagram



# Higher mass protostars contract faster



# Higher mass stars spend less time on the main sequence



## Table 21-1 Main-Sequence Lifetimes

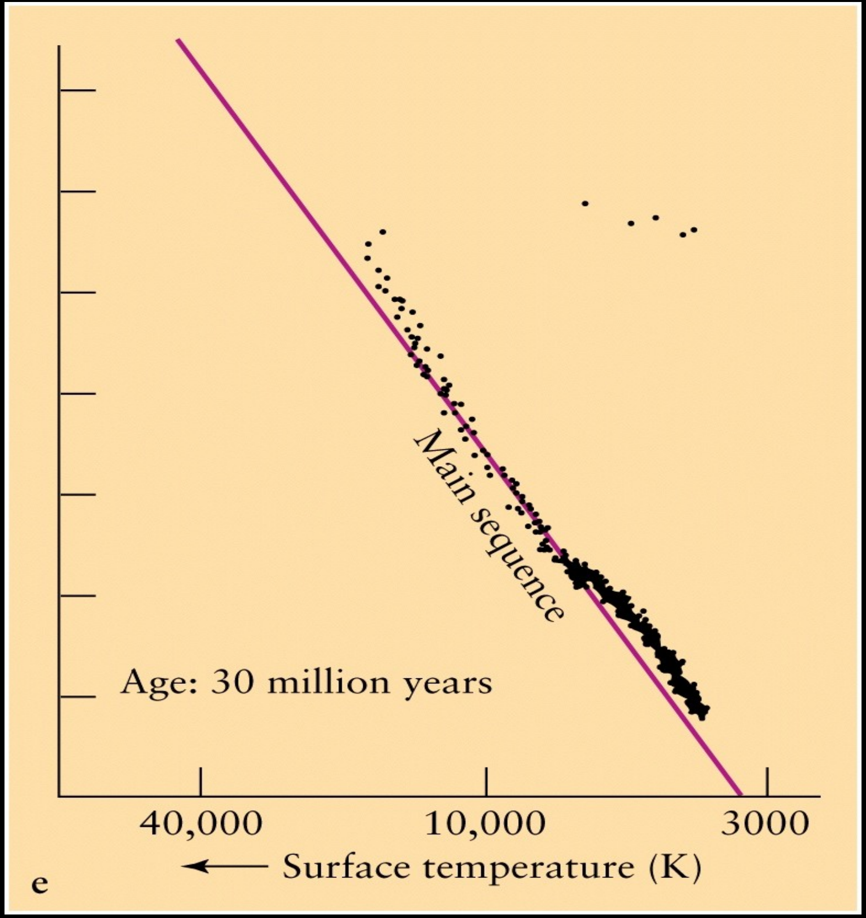
| Mass ($M_\odot$) | Surface temperature (K) | Spectral class | Luminosity ($L_\odot$) | Main-sequence lifetime ($10^6$ years) |
|---|---|---|---|---|
| 25 | 35,000 | O | 80,000 | 3 |
| 15 | 30,000 | B | 10,000 | 15 |
| 3 | 11,000 | A | 60 | 500 |
| 1.5 | 7000 | F | 5 | 3000 |
| 1.0 | 6000 | G | 1 | 10,000 |
| 0.75 | 5000 | K | 0.5 | 15,000 |
| 0.50 | 4000 | M | 0.03 | 200,000 |

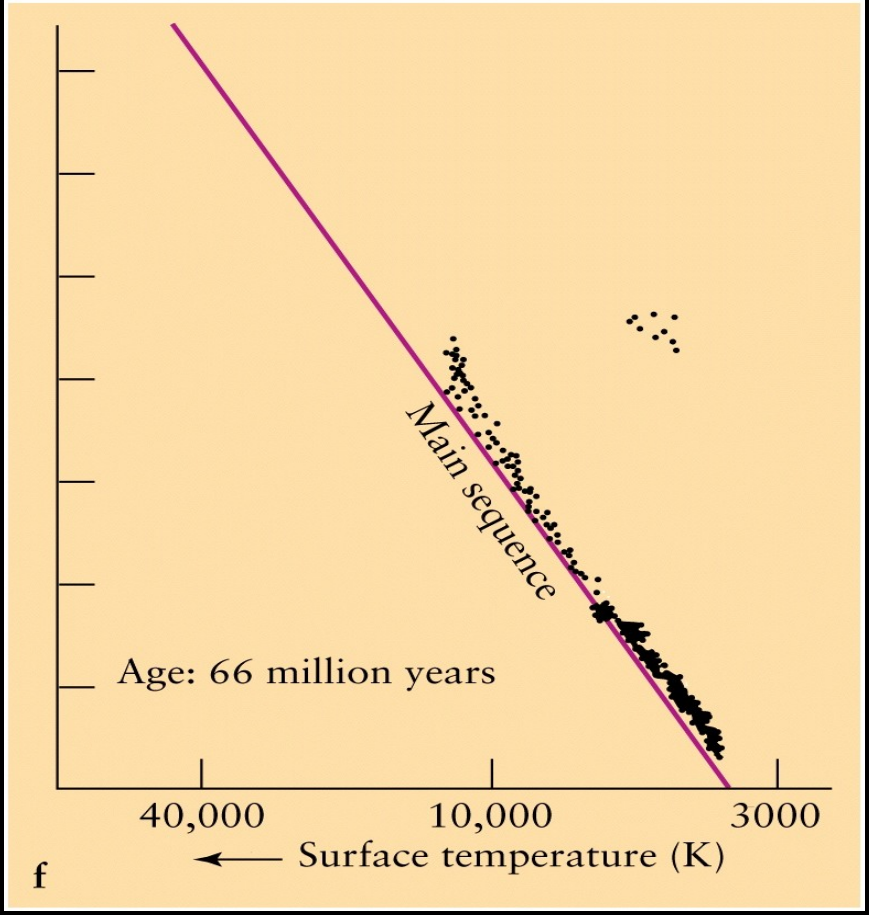
# Determining the age of a star cluster

- Imagine we have a cluster of stars that were all formed at the same time, but have a variety of different masses

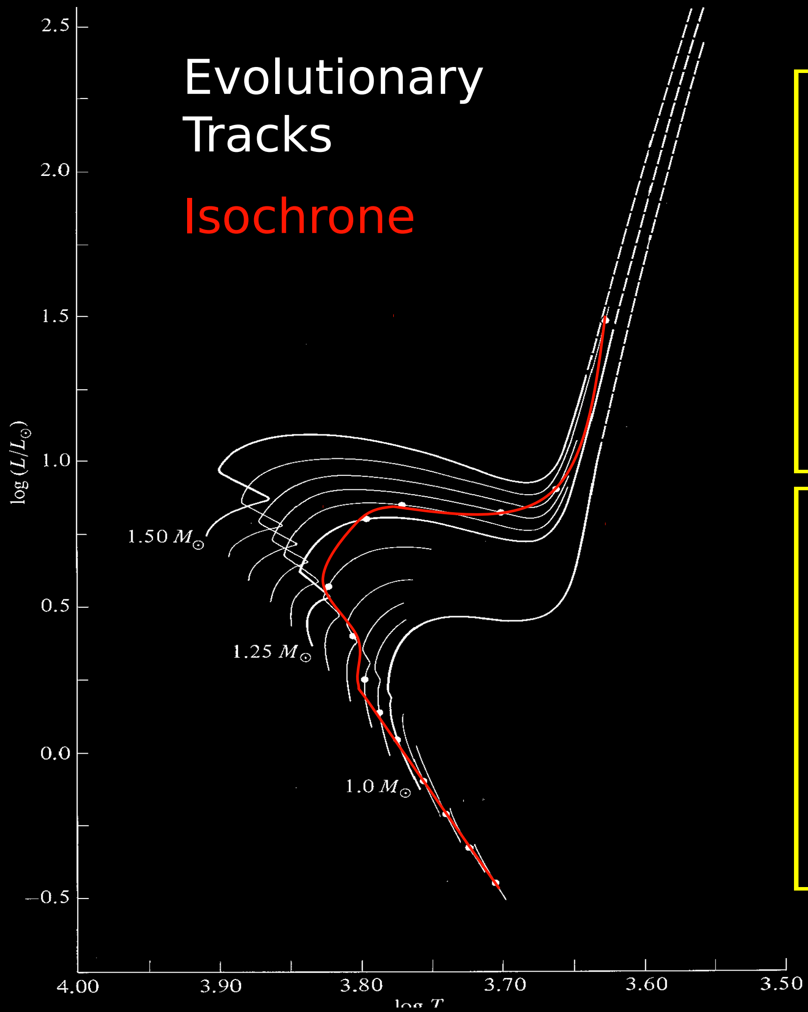
- Using what we know about stellar evolution is there a way to determine the age of the star cluster?

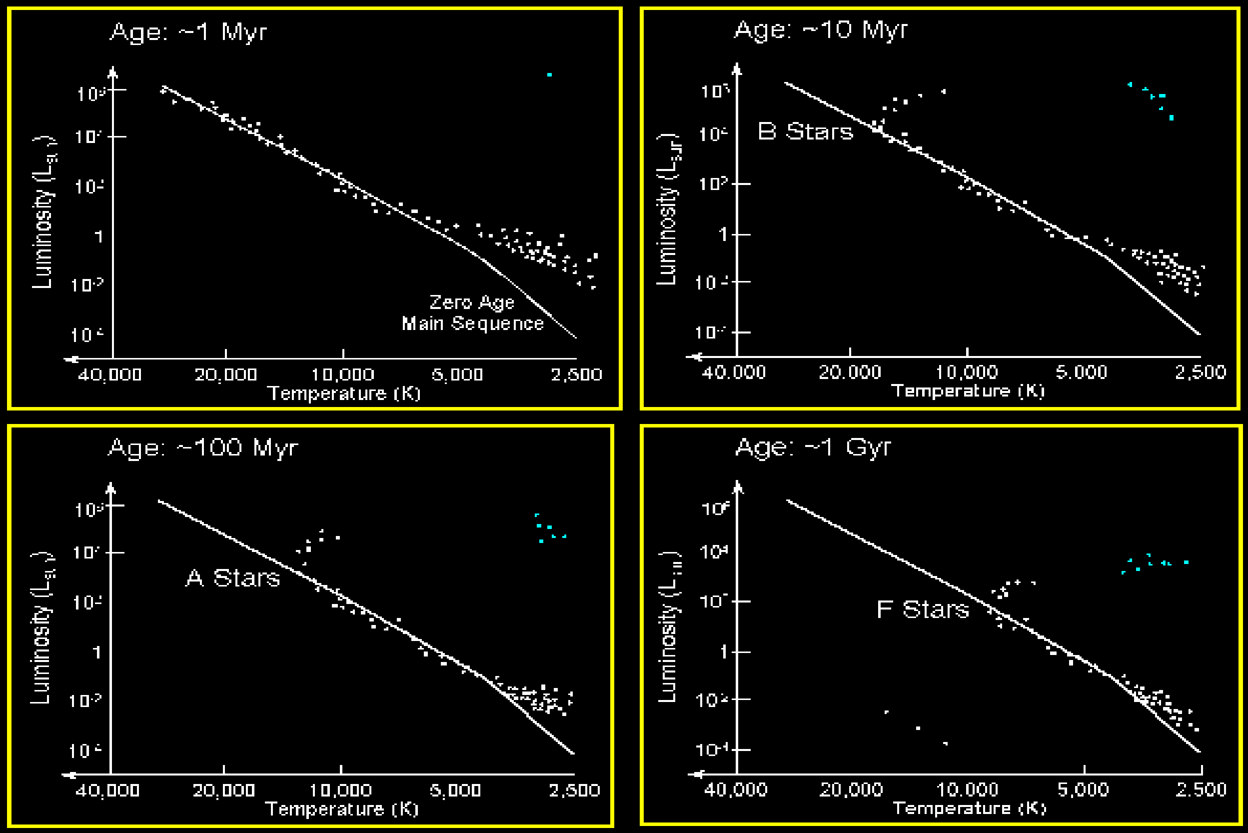
# *For a group of stars formed at the same approximate time, the more luminous ones evolve faster.*



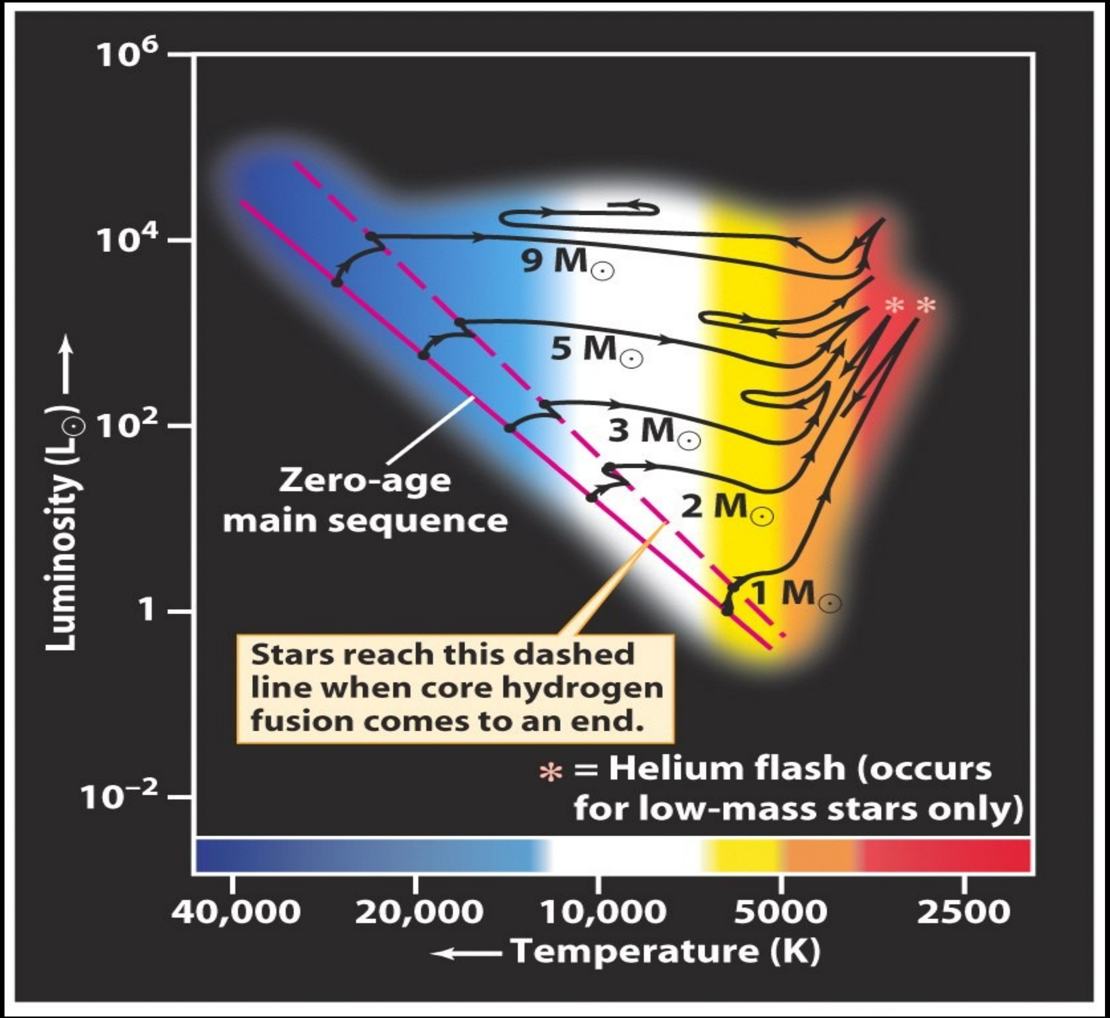



# Cluster age and turn-off point

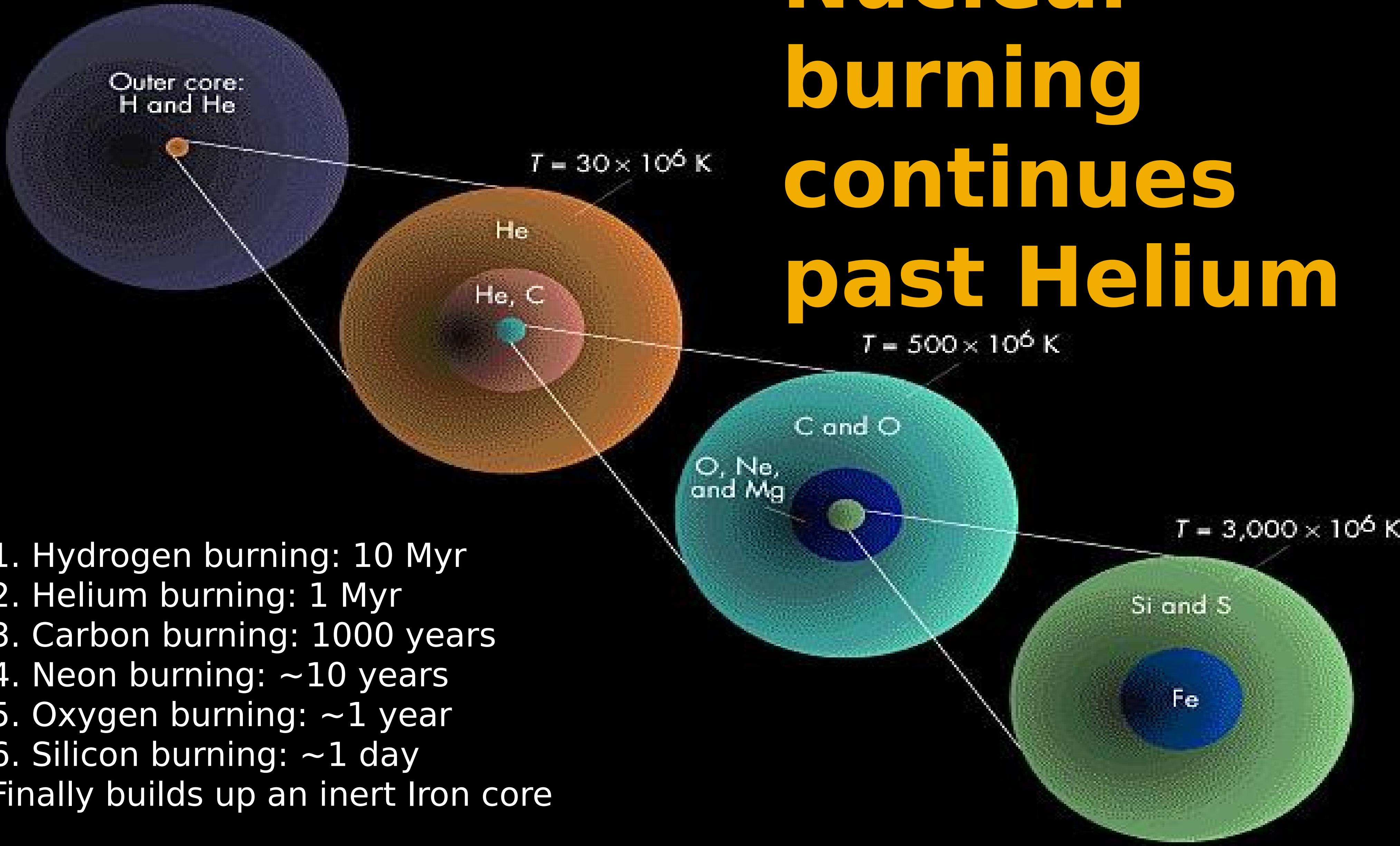


# Higher mass stars do not have helium flash



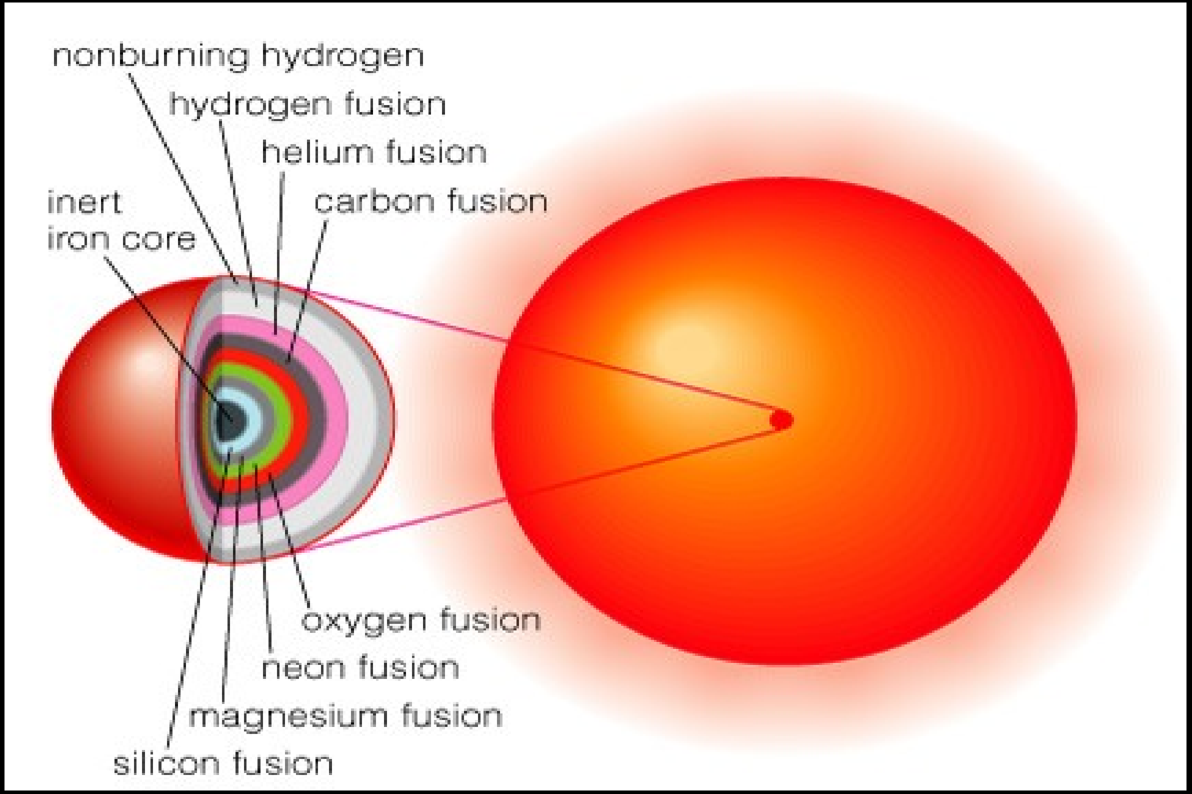
# Nuclear burning continues past Helium



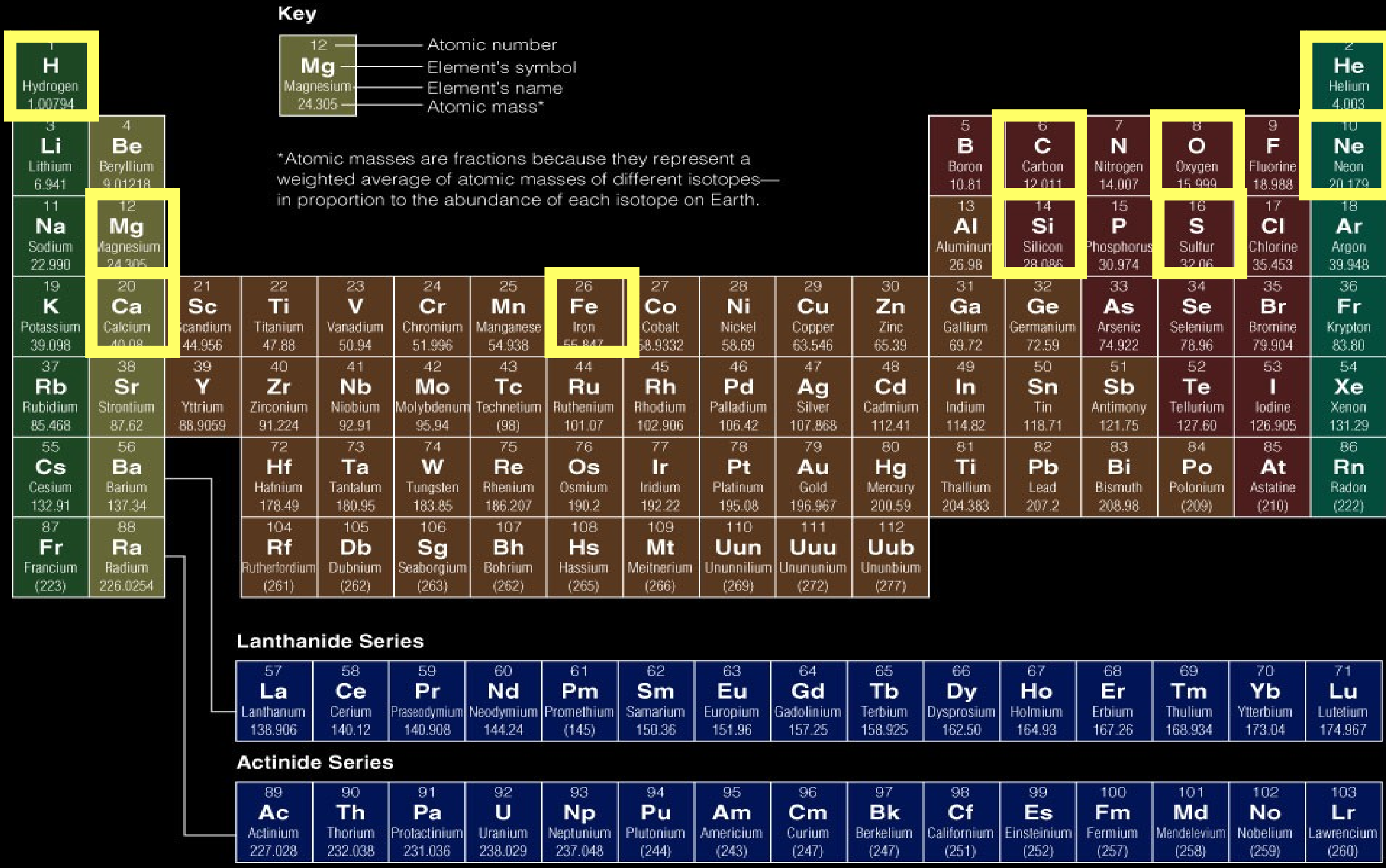

1. Hydrogen burning: 10 Myr
2. Helium burning: 1 Myr
3. Carbon burning: 1000 years
4. Neon burning: ~10 years
5. Oxygen burning: ~1 year
6. Silicon burning: ~1 day

Finally builds up an inert Iron core

# Multiple Shell Burning

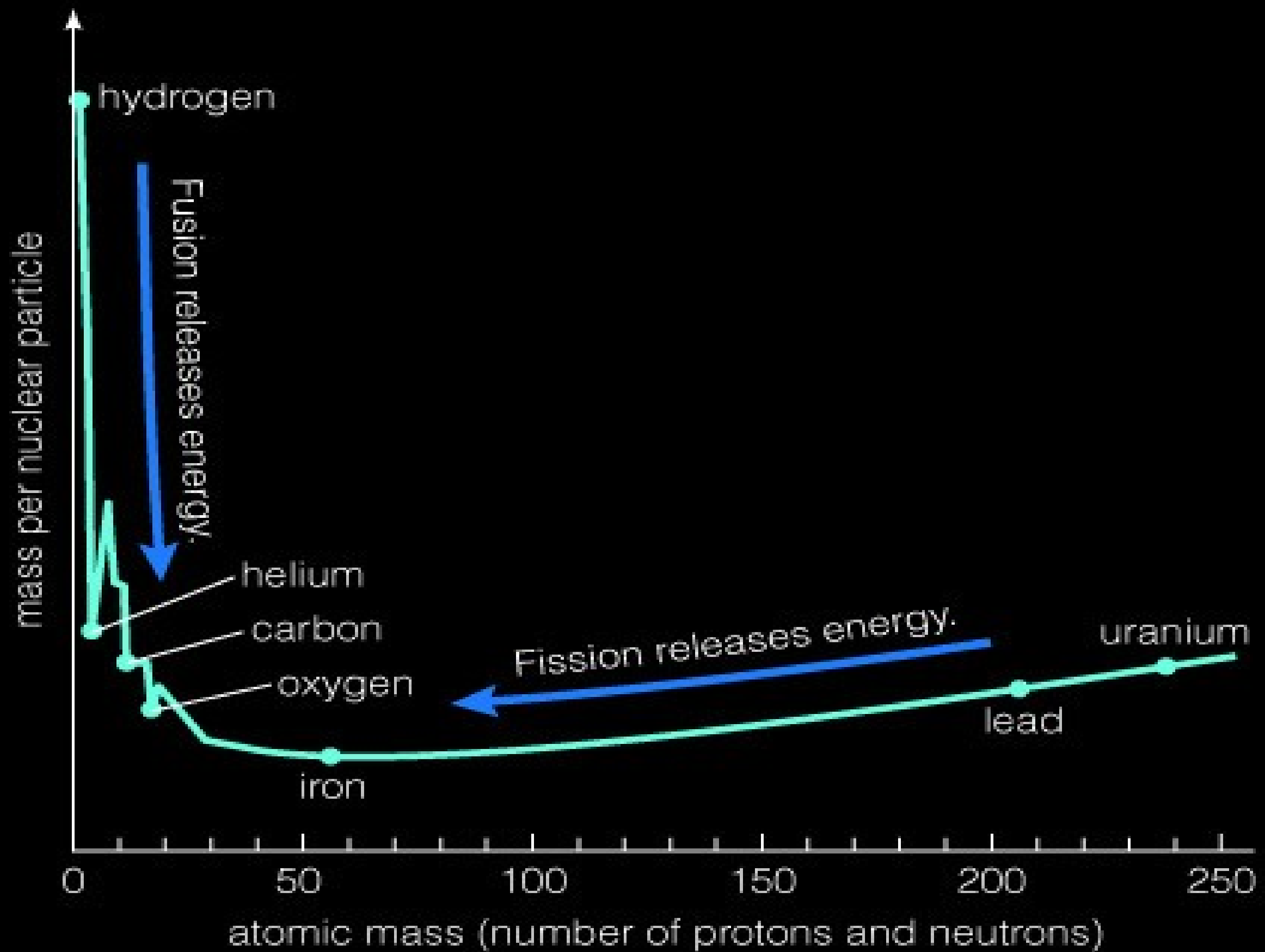



- Advanced nuclear burning proceeds in a series of nested shells



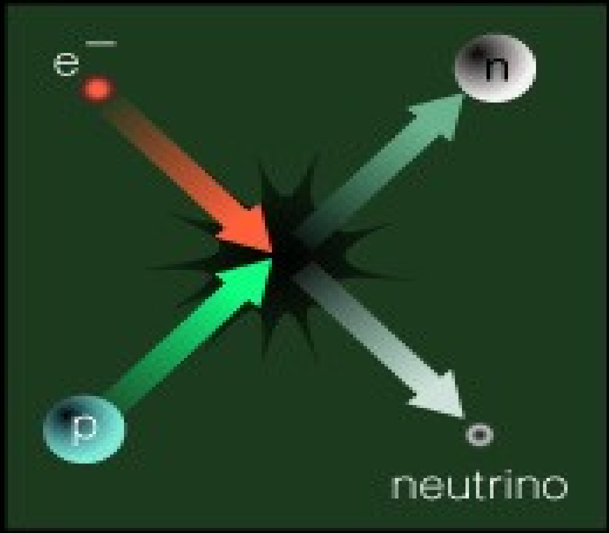

Advanced reactions in stars make elements like Si, S, Ca, Fe

# Why does fusion stop at Iron?



# Core collapse

- Iron core is degenerate and grows until it is too heavy to support itself
- Core collapses and iron nuclei are converted into neutrons with the emission of neutrinos
- Core collapse stops, neutron star is formed
- Rest of the star bounces off the new neutron star (also pushed outwards by the neutrinos)



# Supernova explosion