

Cosmic Rays interactions with interstellar medium

Special relativity

$$V = (x, y, z, ct)$$

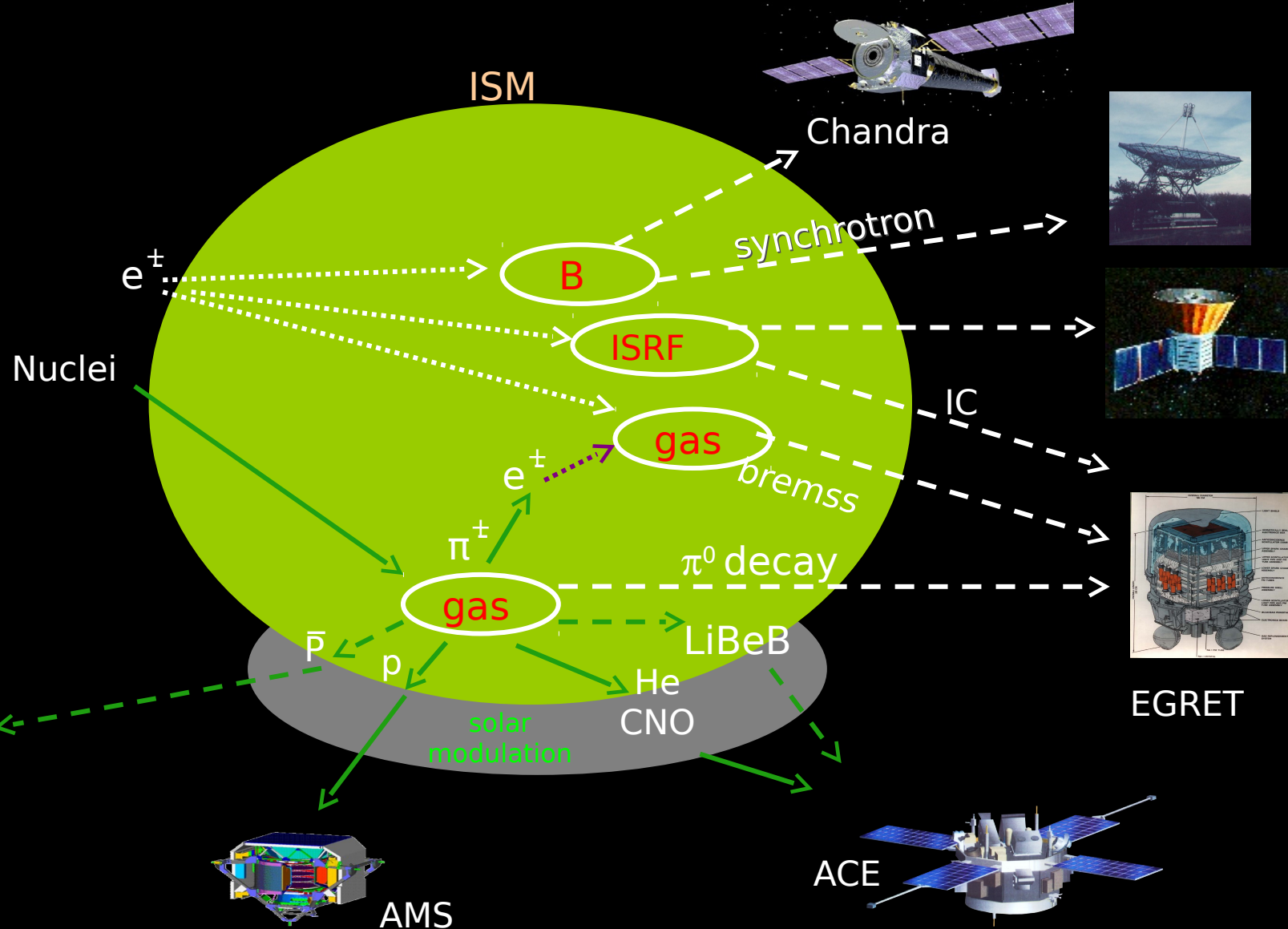
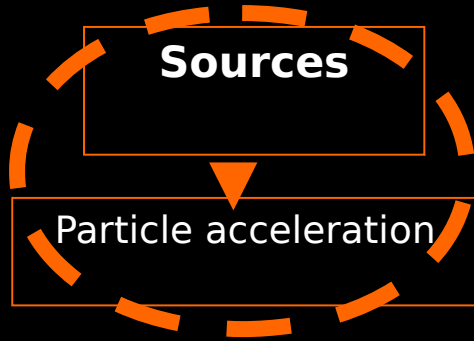
$$V^2 = V^T \eta V = x^2 + y^2 + z^2 - c^2 t^2 = r^2 - c^2 t^2$$

$$\eta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

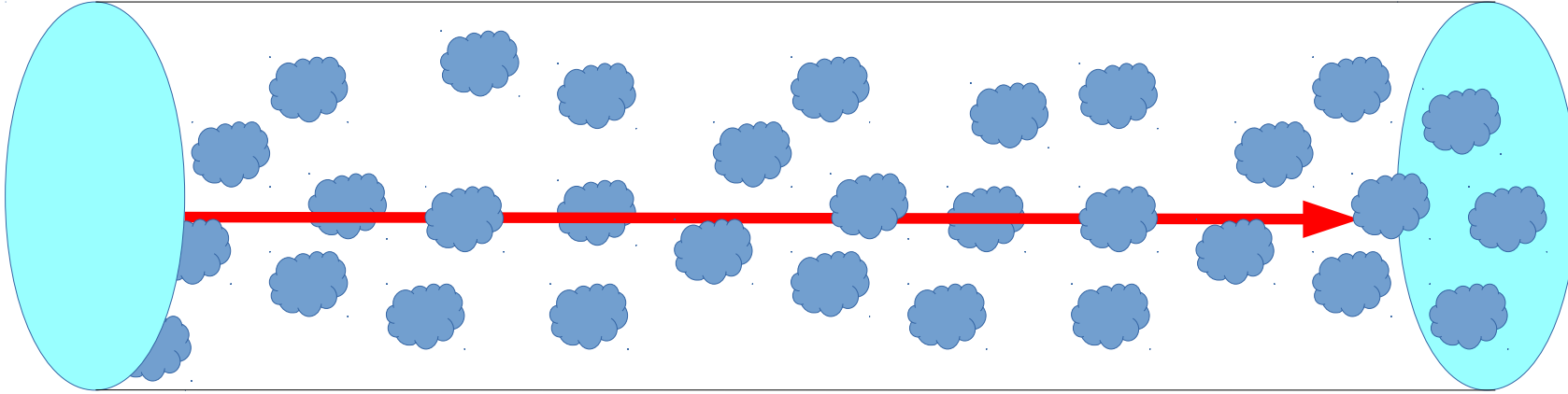
$$P = (p_x, p_y, p_z, \frac{E}{c})$$

$$P^2 = p_x^2 + p_y^2 + p_z^2 - \frac{E^2}{c^2} = p^2 - \frac{E^2}{c^2}$$

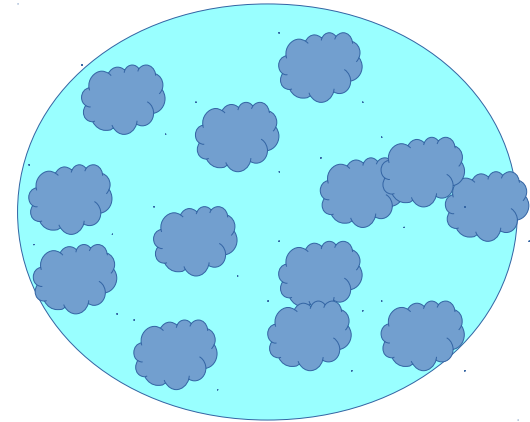
The multi-wavelength ISM



Interaction Rate



$$P = \frac{\sigma N_t}{S} = c \sigma n_t$$



Emissivity and E. losses

$$P(E_p) = c \sigma(E_p) n_t \quad \left[\frac{\text{interactions}}{s \cdot \text{particle}} \right]$$

$$\epsilon_{ic}(E_\gamma) = \int P(E_p) n_p(E_p) dE_p = \int c \sigma(E_p, E_\gamma) n_t n_p(E_p) dE_p \quad \left[\frac{ph}{cm^3 s} \right]$$

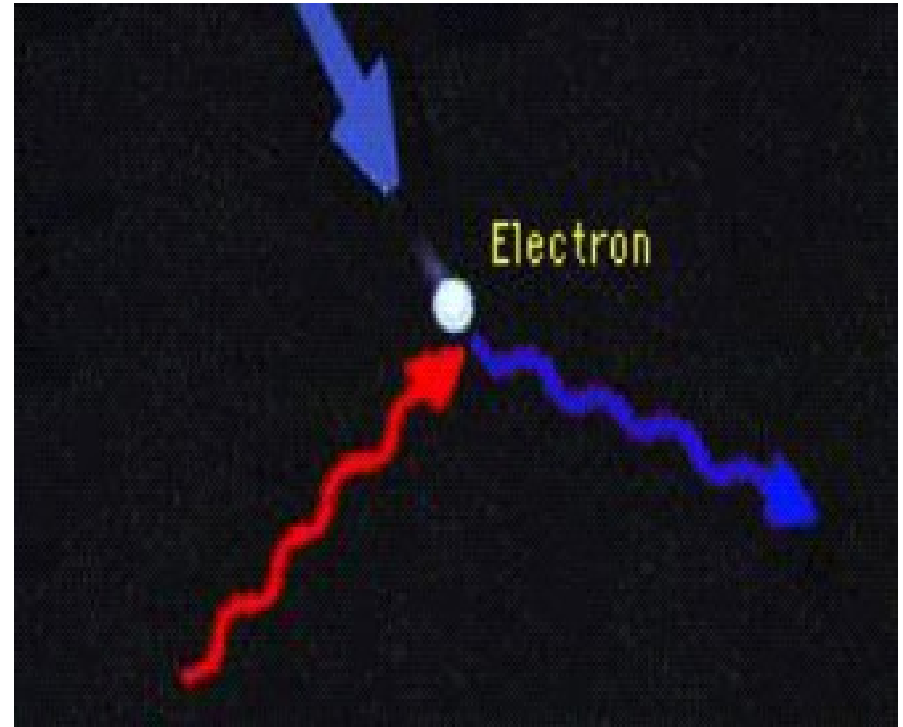
$$\frac{dE_p}{dt} = P(E_p) \delta(E_p) = c \sigma(E_p) n_t \delta(E_p) \quad \left[\frac{erg}{s} \right]$$

Inverse Compton

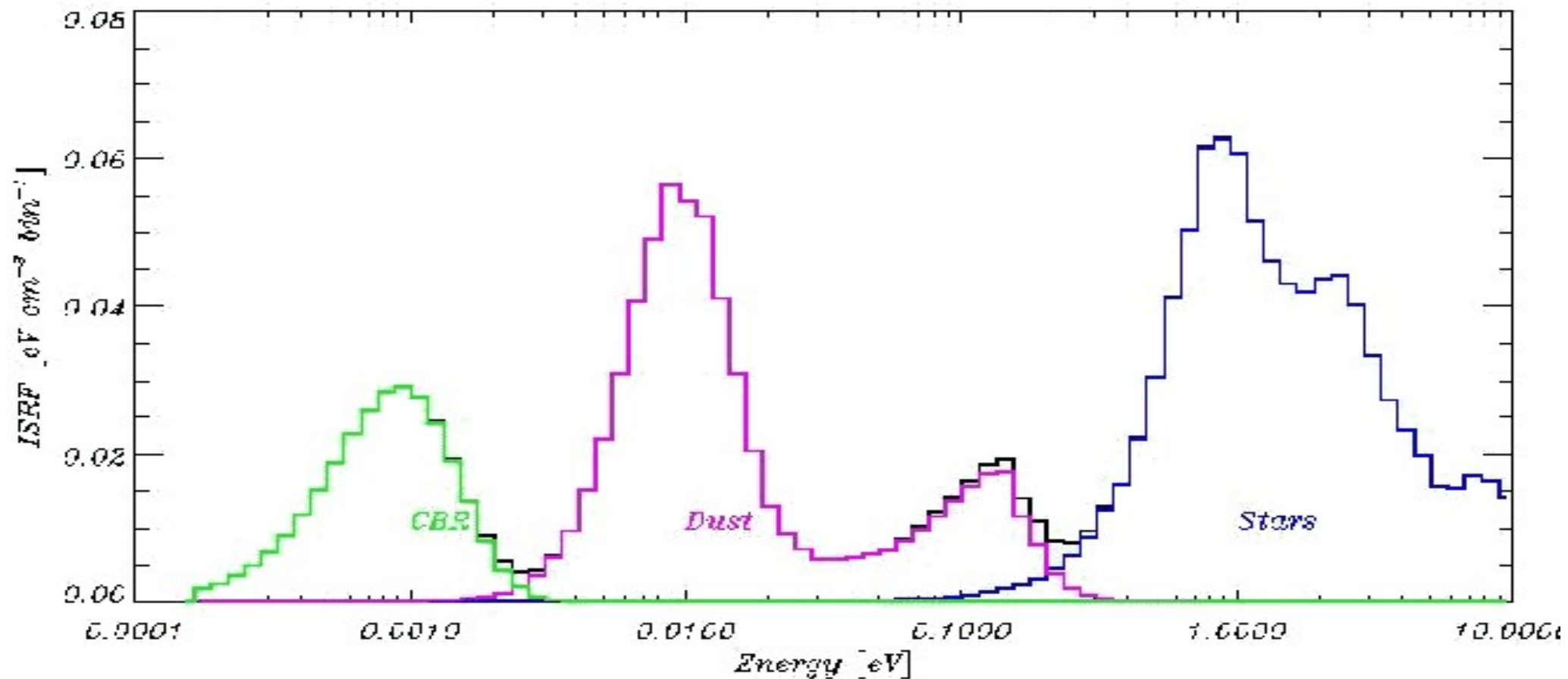
$$E_\gamma = \frac{4}{3}\gamma^2\beta^2 E_{ph} \xrightarrow{\gamma \gg 1} \frac{4}{3}\gamma^2 E_{ph}$$

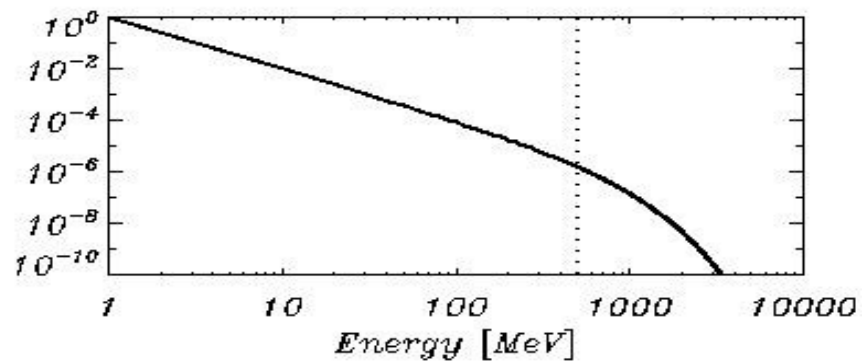
$$\epsilon_{ic}(E_\gamma)dE_\gamma = 4\pi\sigma_t n_{ph} I(E_e)dE_e$$

$$\frac{dE_p}{dt} = cn_{ph}\sigma_t E_\gamma = \frac{4}{3}cU_{ph}\sigma_t\gamma^2$$

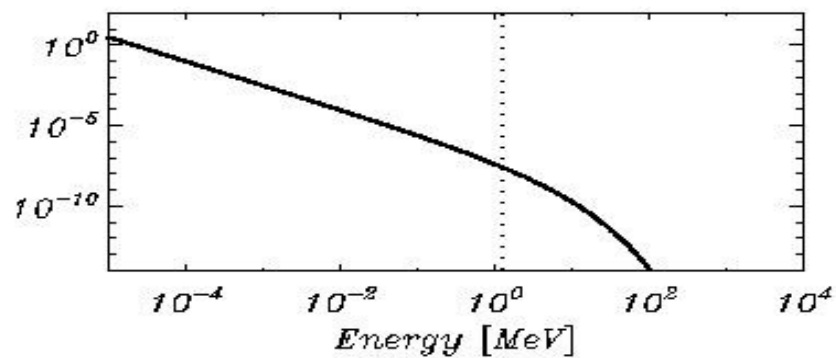


The Interstellar Radiation Field

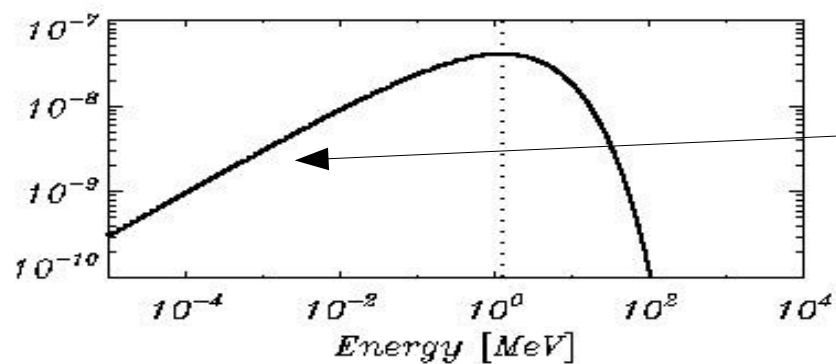




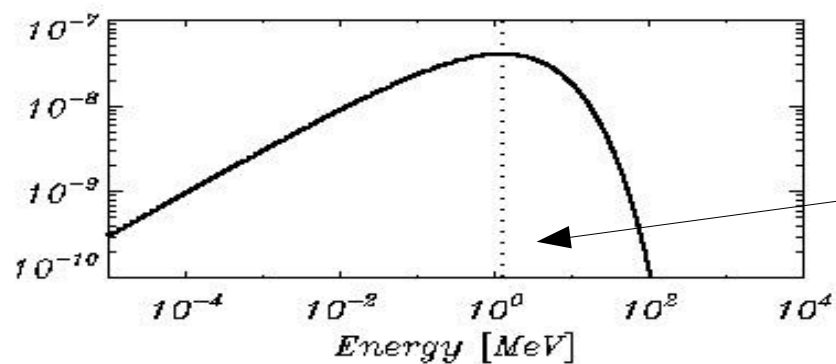
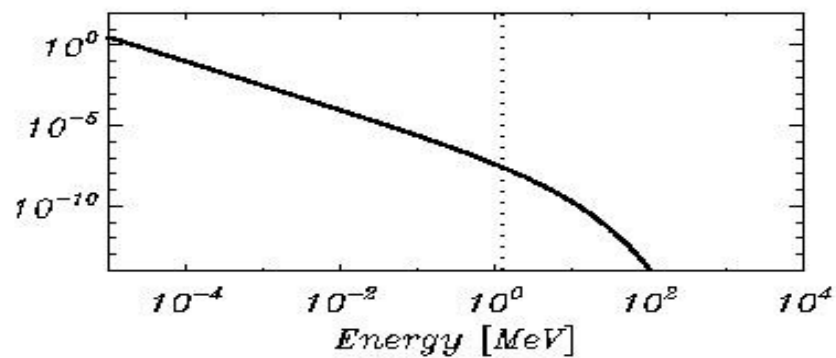
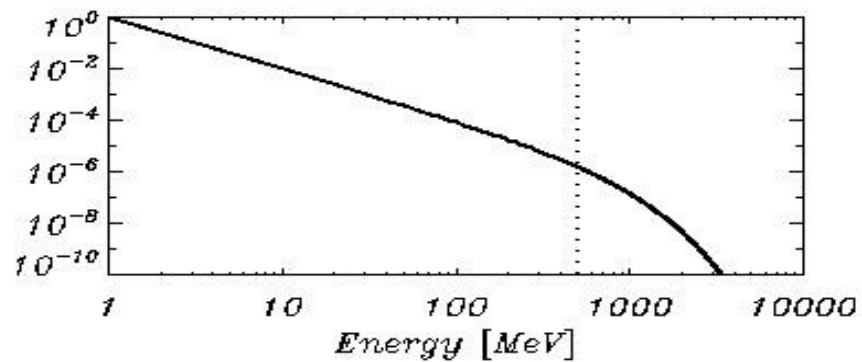
$$I_e \sim E^{-a}$$



$$I_\gamma \sim E^{-(a+1)/2}$$

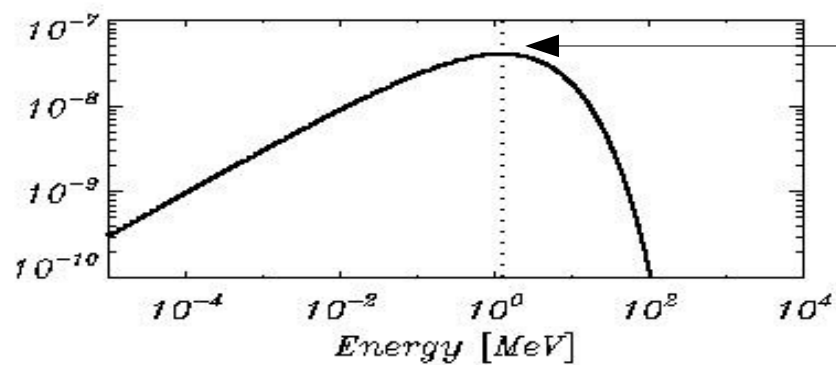
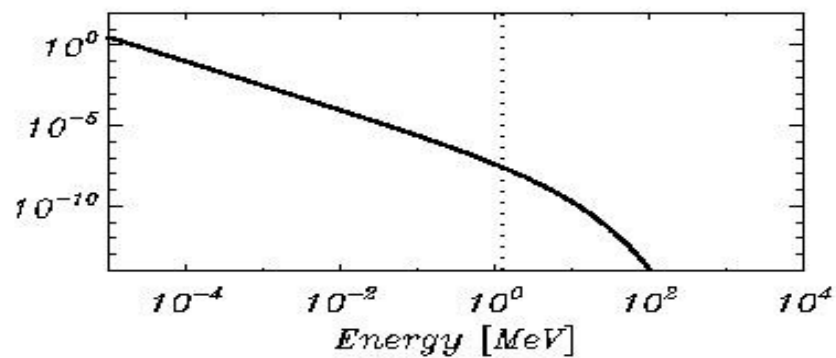
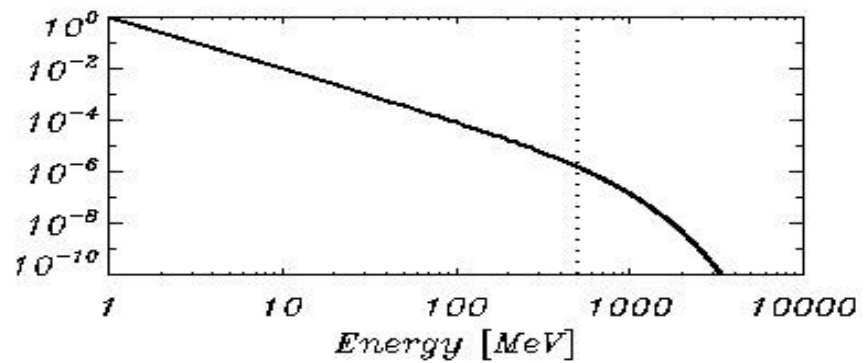


$$I_\gamma E^2 \sim E^{(3-a)/2}$$



Cut off !

$$\nu_{ic} = \frac{4}{3} \gamma^2 \nu_{ph}$$

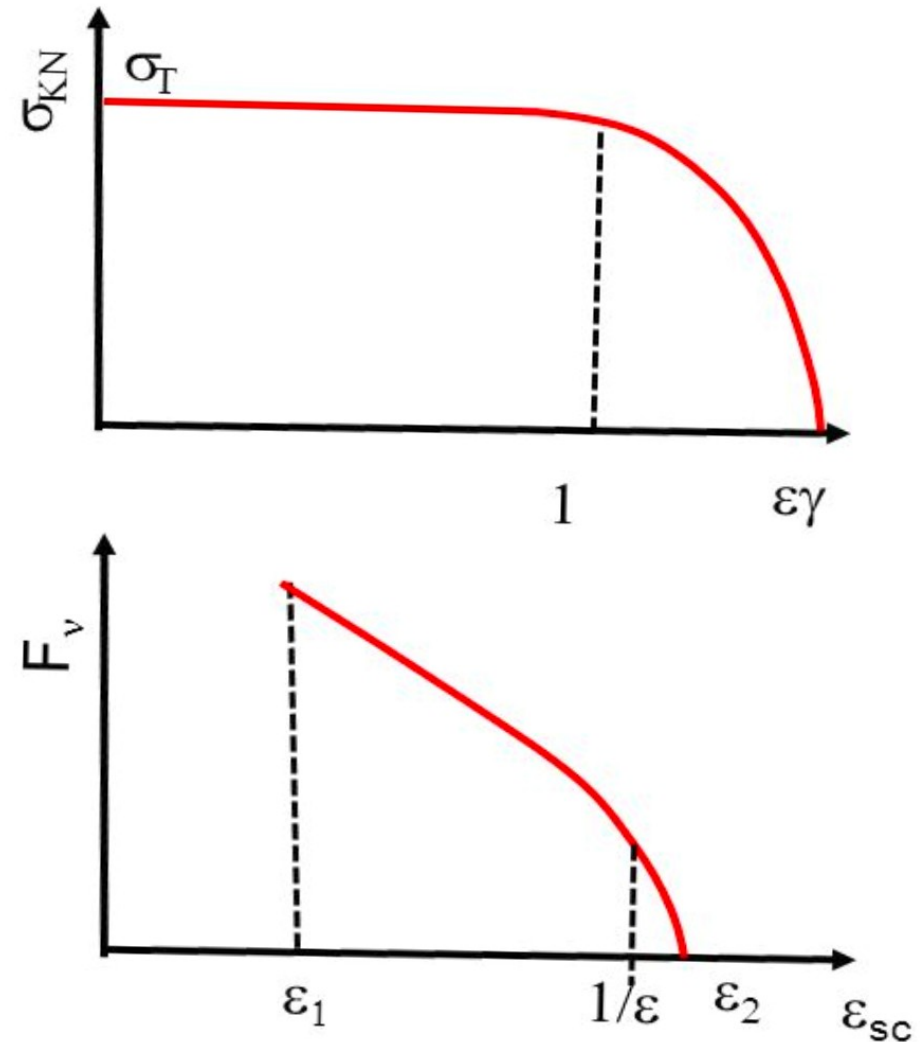


$\sim U$

Klein–Nishina cross section

$$\gamma E_{ph} \sim m_e c^2$$

$$\sigma_{KN} = \frac{\pi^2 r_e^2}{h\nu} \left(\ln 2h\nu + \frac{1}{2} \right).$$



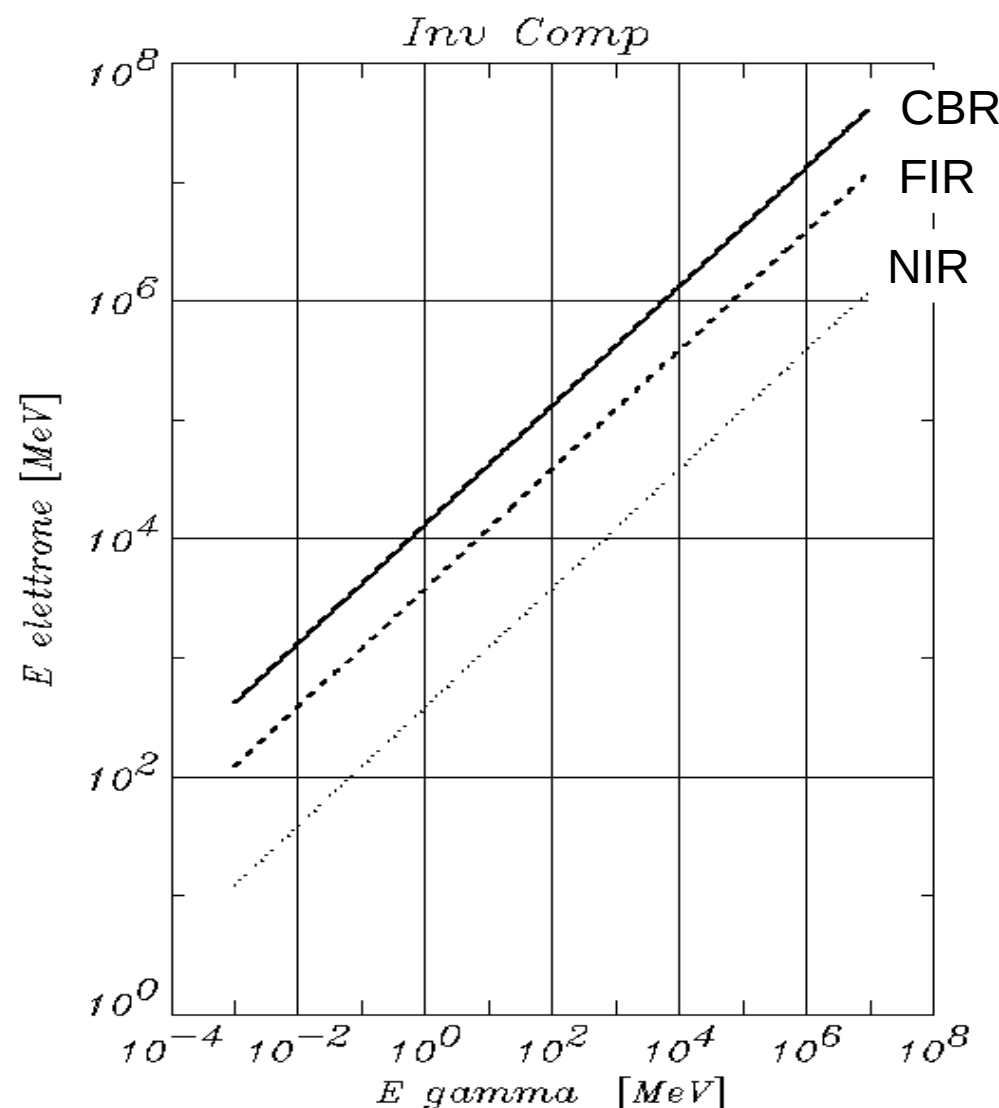
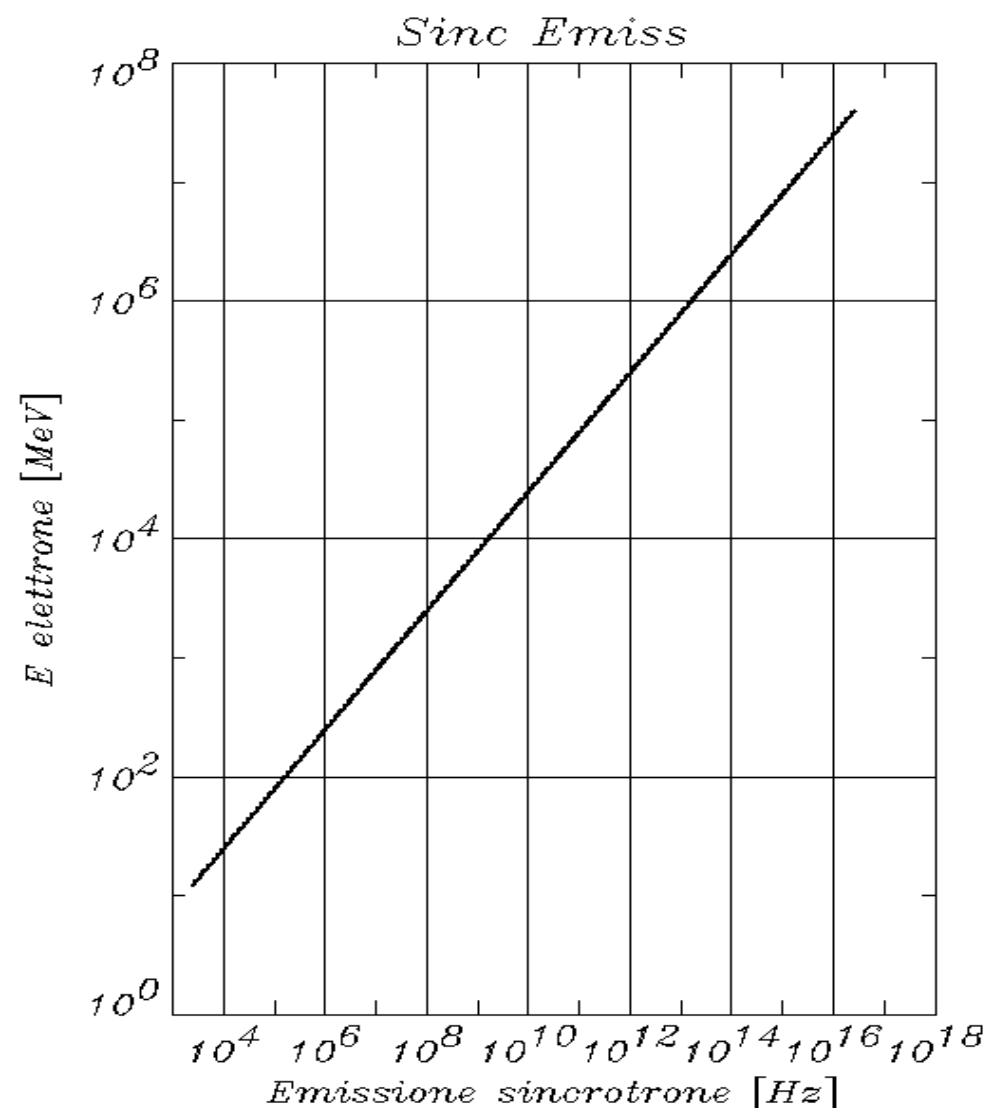
Synchrotron

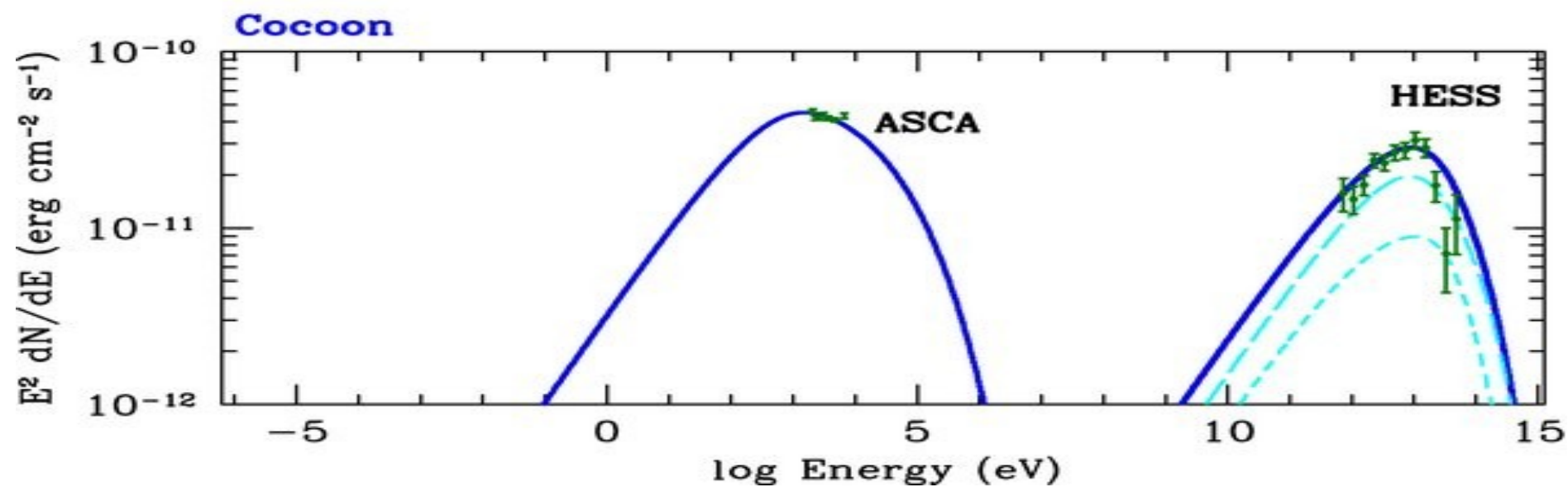
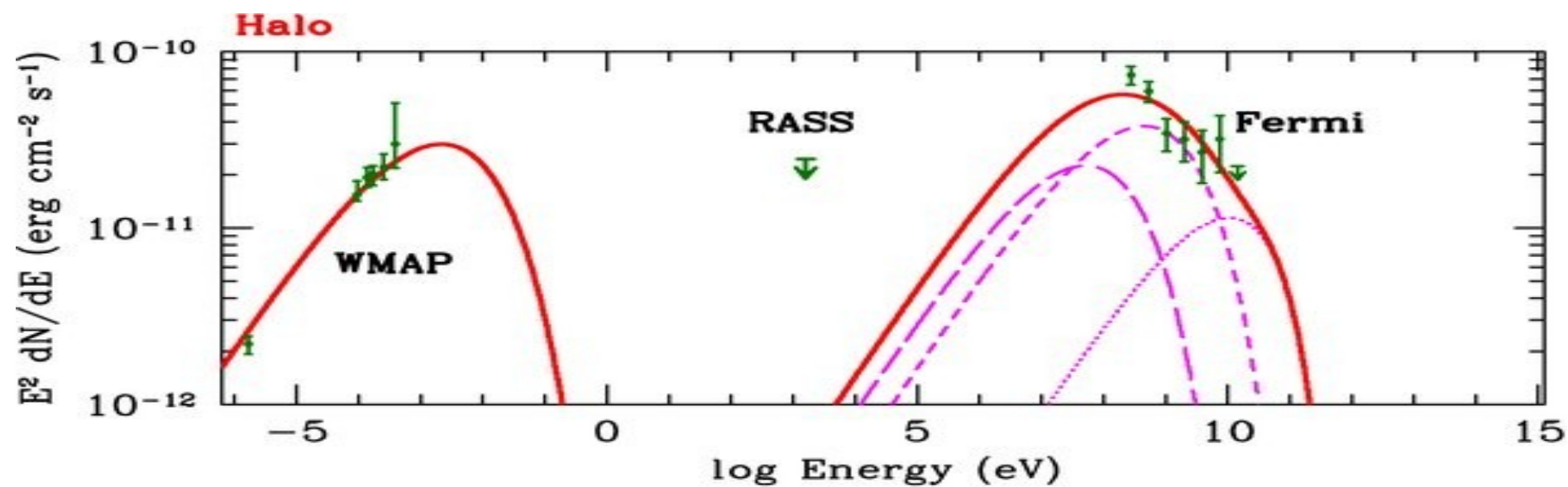
$$\nu_{syn} = \frac{4}{3} \gamma^2 \nu_{gyr}$$

$$\nu_{gyr} = \frac{e B}{2 \pi m_e c}$$

$$\frac{dE}{dt} = \frac{4}{3} \sigma_T U_B c \gamma^2$$

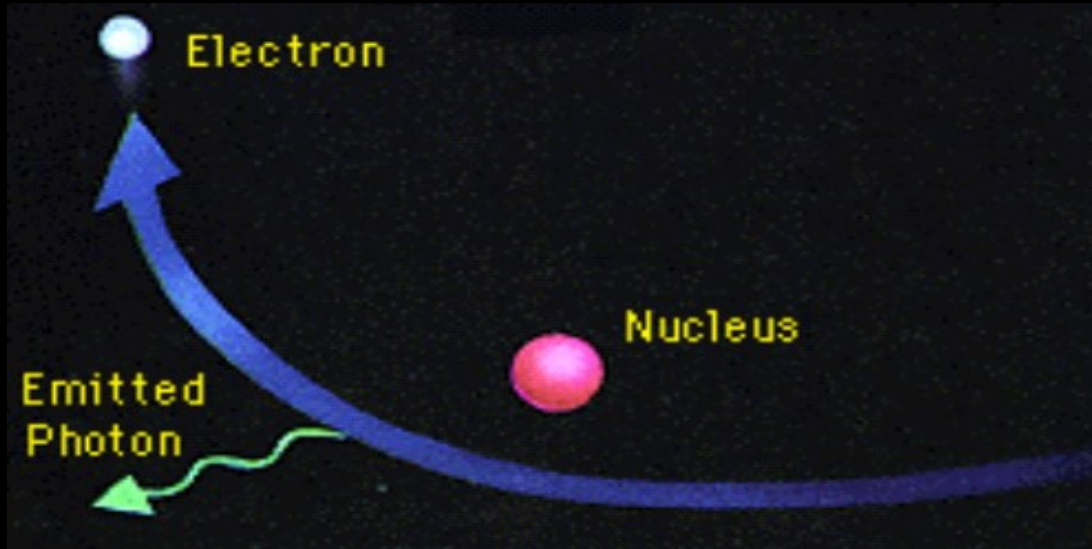
Inverse Compton emissivity





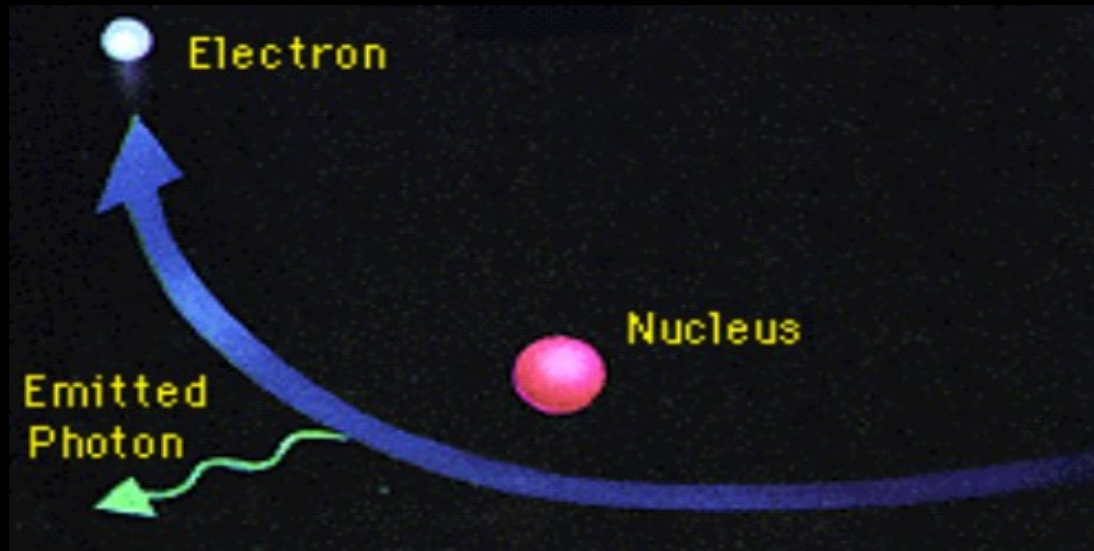
Bremsstrahlung

CR electrons + ISM nuclei $\rightarrow$ γ rays



Bremsstrahlung

CR electrons + ISM nuclei $\rightarrow \gamma$ rays

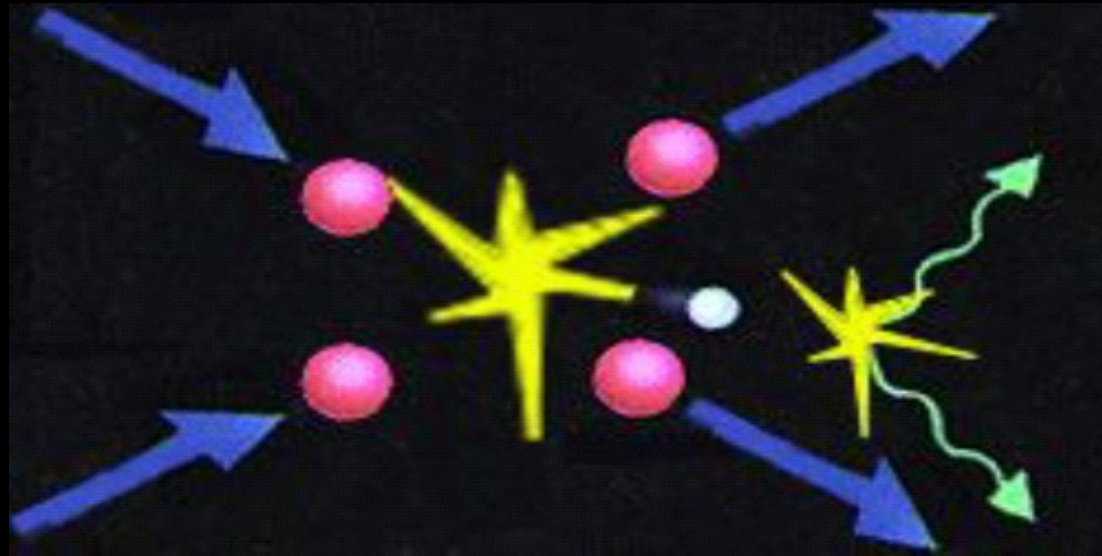


$$-\frac{dE}{dt} = 4c\alpha r_e^2 N G_a E$$

$$N_e(E) = k E^{-a}$$

$$I(\hbar\omega) = 4Z^2 c\alpha r_e^2 N G_a k \frac{(\hbar\omega)^{-a}}{a-1}$$

Emission processes: Neutral π decay



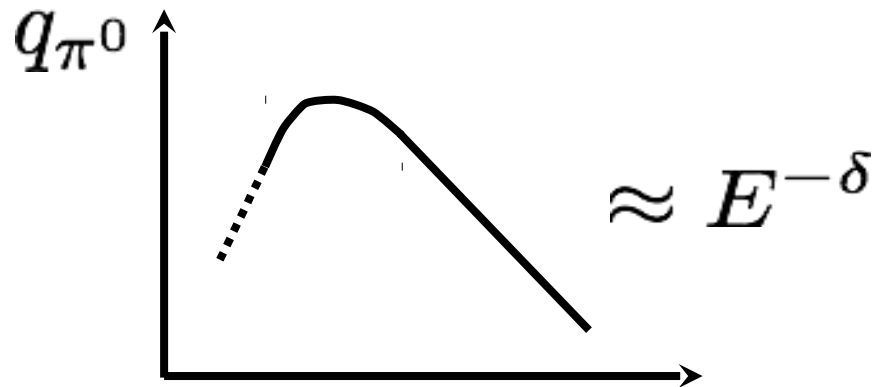
CR protons + ISM nuclei $\rightarrow \pi^0 \rightarrow \gamma$ rays

p-p interactions

We assume a power law spectrum for CRs: $N_p(E_p) \propto E_p^{-\delta}$

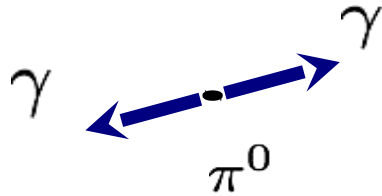
Fraction of proton kinetic energy transferred to pion
(from data):

$$f_{\pi^0} \approx 0.17$$



p-p interactions

Pion rest frame:



$$E_{\gamma}^* = \frac{m_{\pi^0}}{2}$$

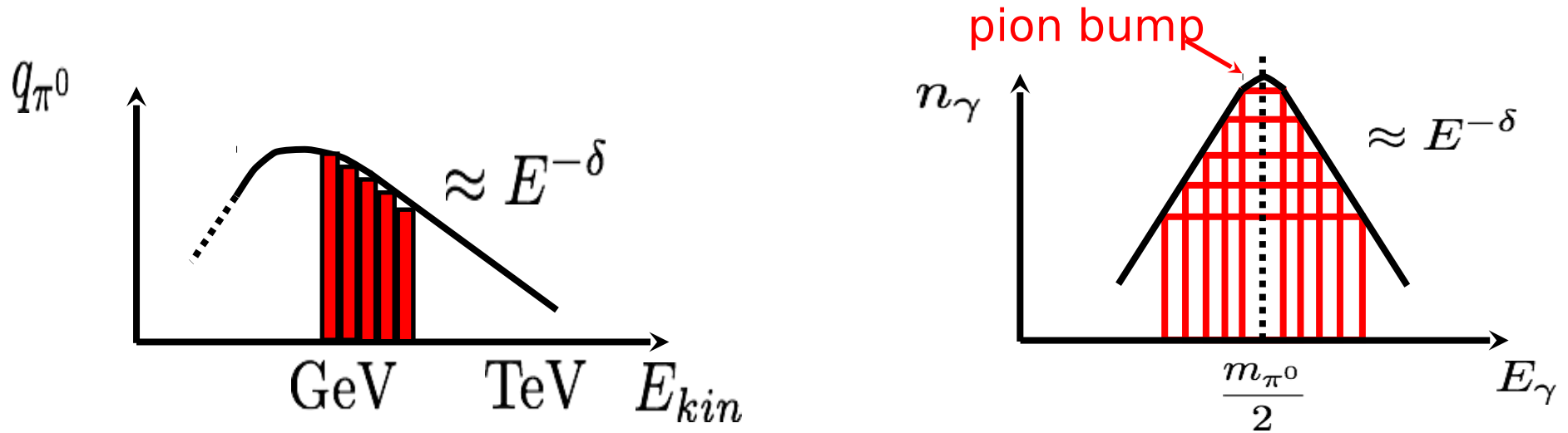
Lab frame:

$$E_{\gamma} = \gamma (E_{\gamma}^* + v p_{\gamma}^* \cos \theta^*)$$

max and min energies -> $\cos \theta^* = \pm 1$

$$\frac{m_{\pi^0}}{2} \sqrt{\frac{1 - \beta}{1 + \beta}} \leq E_{\gamma} \leq \frac{m_{\pi^0}}{2} \sqrt{\frac{1 + \beta}{1 - \beta}}$$

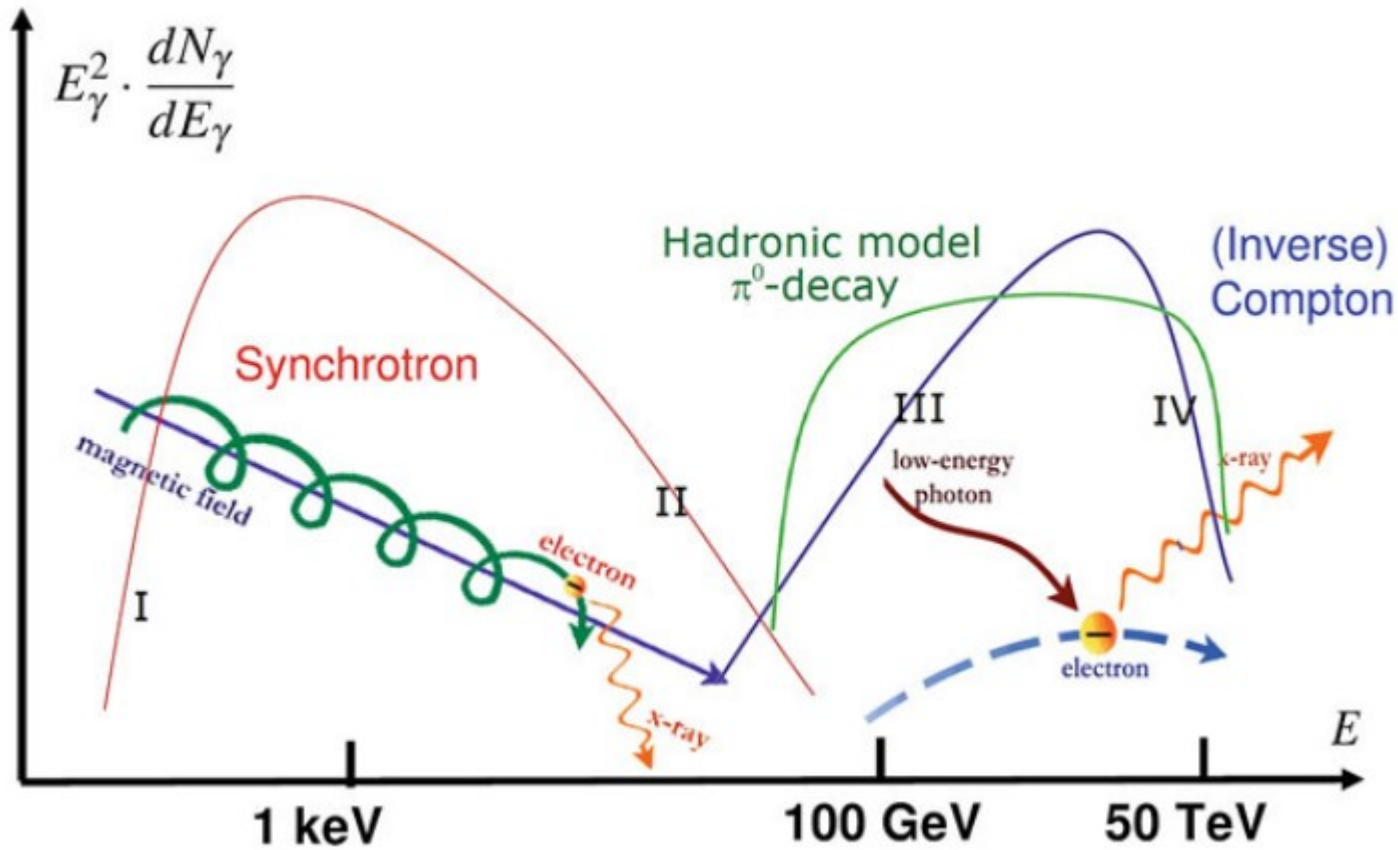
p-p interactions



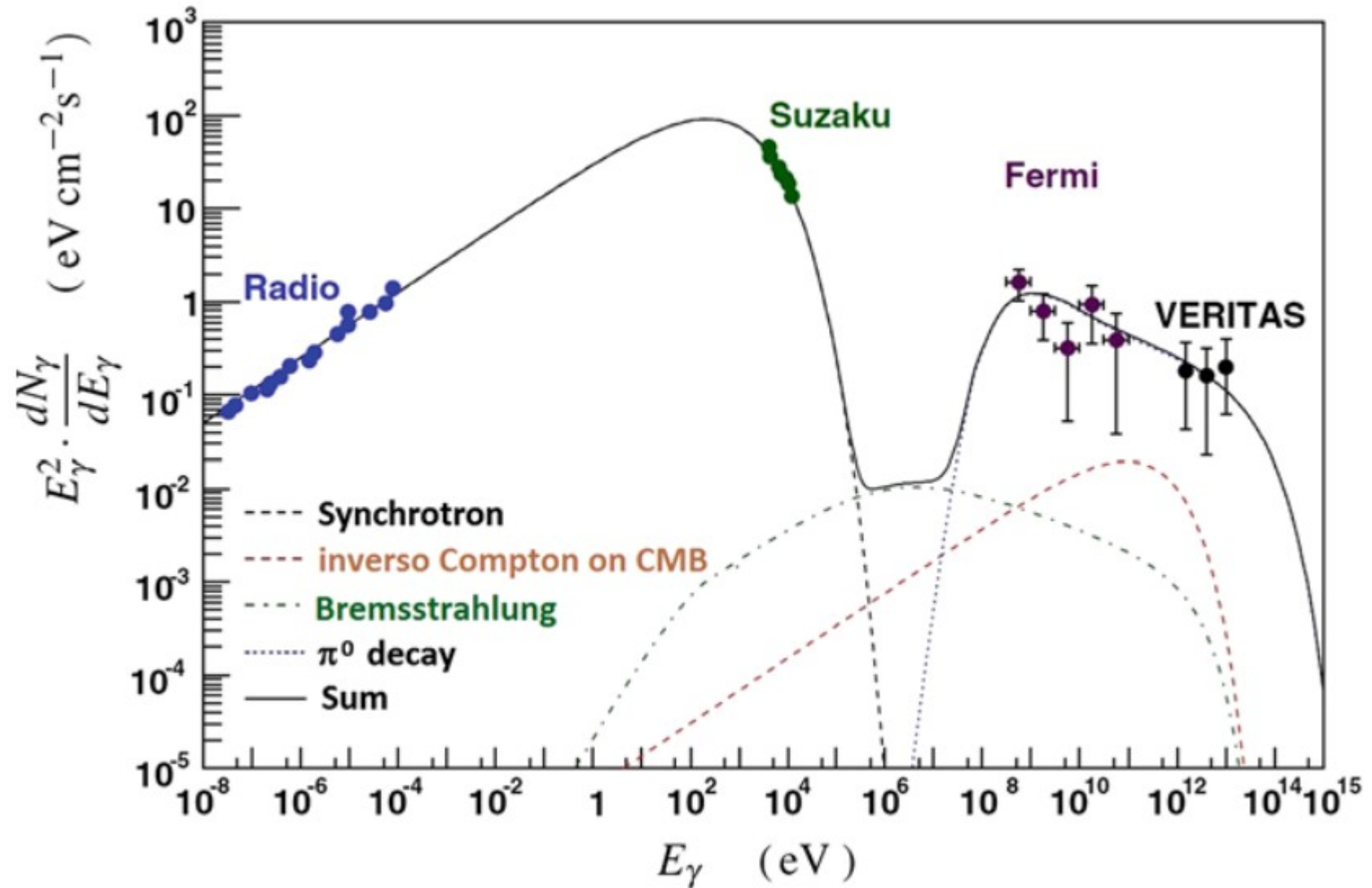
the gamma ray spectrum is symmetric (in log-log) with respect to: $\frac{m_{\pi^0}}{2} \sim 70 \text{ MeV}$

at high energy the spectrum mimic the CR spectrum, with (roughly): $E_{\gamma} \approx \frac{E_{CR}}{10}$

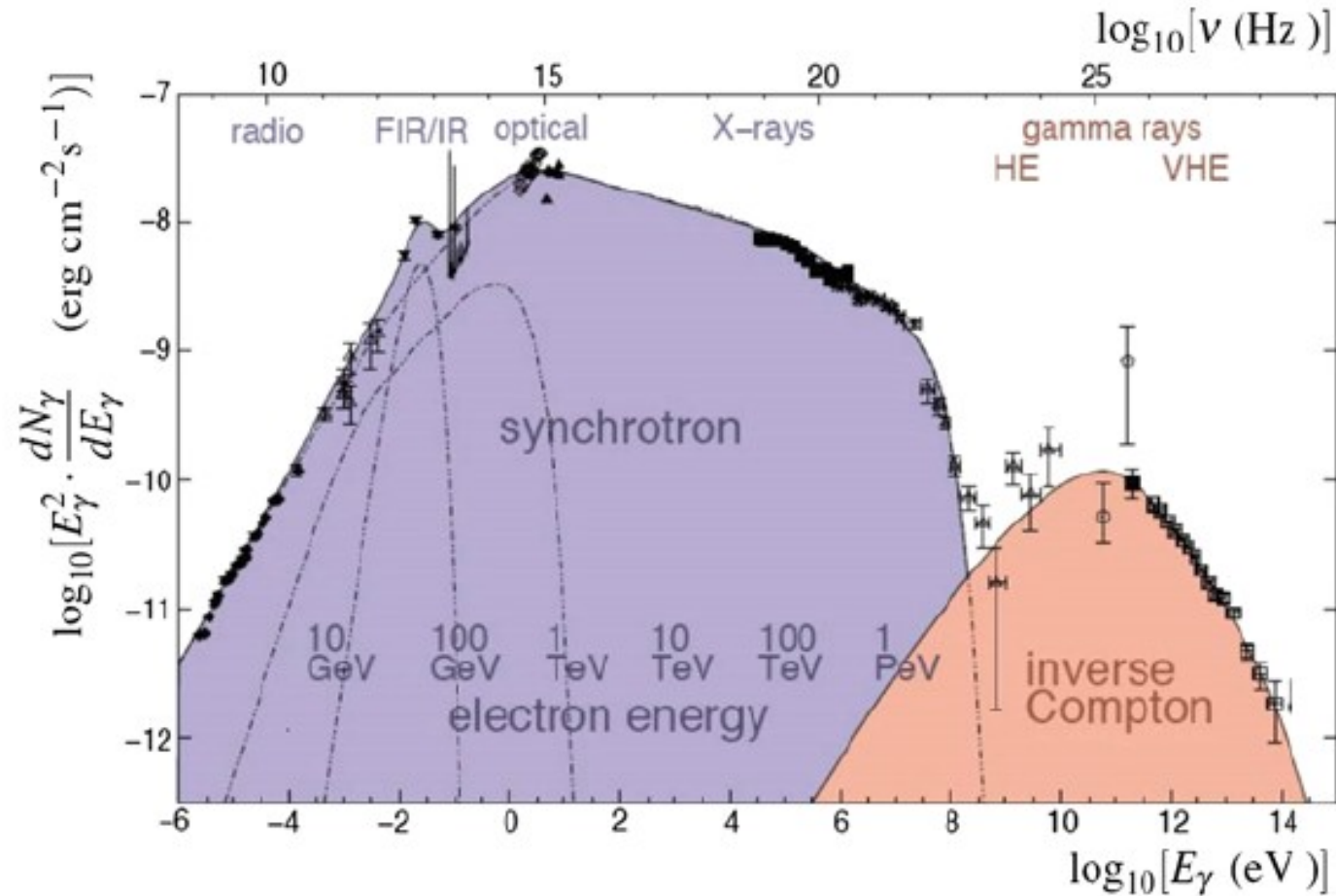
SED Modeling



SED of the Tycho's SNR



Crab SED



SED of the AGN Mrk 421

