Problem 9.22. (a) If the average luminosity (10^{39} W) of a quasar (Table 1) arises because the presumed black hole at its center accretes a star of $\sim 1\ M_\odot$ every now and then, about how often should it have one of these "star" meals? Assume that about 10% of the rest-mass energy mc^2 of the eaten star is converted to the radiation we observe. (b) Find (classically) an expression for the distance from a central mass M to which a mass m must fall from infinity, such that the loss of potential energy of m equals its entire rest mass energy mc^2. (It is impossible to extract more energy than this.) Twice your answer will be the famed Schwarzschild radius, the event horizon for a non-rotating black hole. What is the Schwarzschild radius for a quasar mass of $10^8\ M_\odot$? What is it for the sun ($1\ M_\odot$)? (c) Write in terms of $\dot{m}$ an expression for the maximum luminosity one could conceivably obtain from accretion at the rate of $\dot{m} = \mathrm{d}m/\mathrm{d}t$ (kg/s). Discussion topics. Why is a large quasar mass required to support a large quasar luminosity? (Hint: look up *Eddington luminosity*.) What is the meaning of the Schwarzschild radius? How might it affect the luminosities from accretion onto a black hole. [Ans. ~ 7 months; ~1 AU, ~1 km; —]

Problem 12.31. The principle of relativistic time dilation states that the mean life at rest, $\tau = 2.2\ \mu s$, of a fast moving muon will be extended to τ' according to

$$\tau' = \gamma\tau \qquad \gamma = \frac{E}{mc^2} \tag{12.17}$$

where E is the total energy (rest energy + kinetic energy), and $mc^2 = 106$ MeV is the muon rest energy. What is the kinetic energy E_k of a muon (in GeV) that will just make it to sea level at the end of its (extended) mean life? Assume the muon is created at a depth of 1000 kg/m^2 in the atmosphere, or about 18 km above sea level. You may approximate its speed of travel as about equal to the speed of light c and neglect any energy loss due to ionization during passage through the atmosphere. To what extent are these assumptions warranted? [Ans. ~3 GeV]

Problem 12.33. A fairly large EAS due to an incident gamma ray reaches its maximum size of 1×10^7 electrons, positrons, and gamma rays as it reaches sea level (atmospheric depth 10 300 kg/m^2). Consider all of the created particles to still be present at the maximum. What approximately is the mean interaction length in the atmosphere, in kg/m^2, for either the pair production or the bremsstrahlung process; assume they are equal. This is known as the "radiation length" in air. Hint:

Problem 12.32. A relativistic muon passes vertically through you while you are standing up. (a) About how much energy is dissipated by it inside your body, in units of MeV? The energy deposition rate per kg/m^2 is about the same as in the atmosphere; see text. (b) Roughly, how much energy is deposited in your brain by cosmic ray muons in one second? (See text for muon flux.) How many molecules (mostly water) of your brain are likely to suffer an atomic ionization in that one second? (It takes 13 eV to ionize either an H or an O atom.) By what age might you expect to have all the molecules in your brain ionized, if recombination is neglected? [Ans. ~200 MeV; 10^{12} yr]