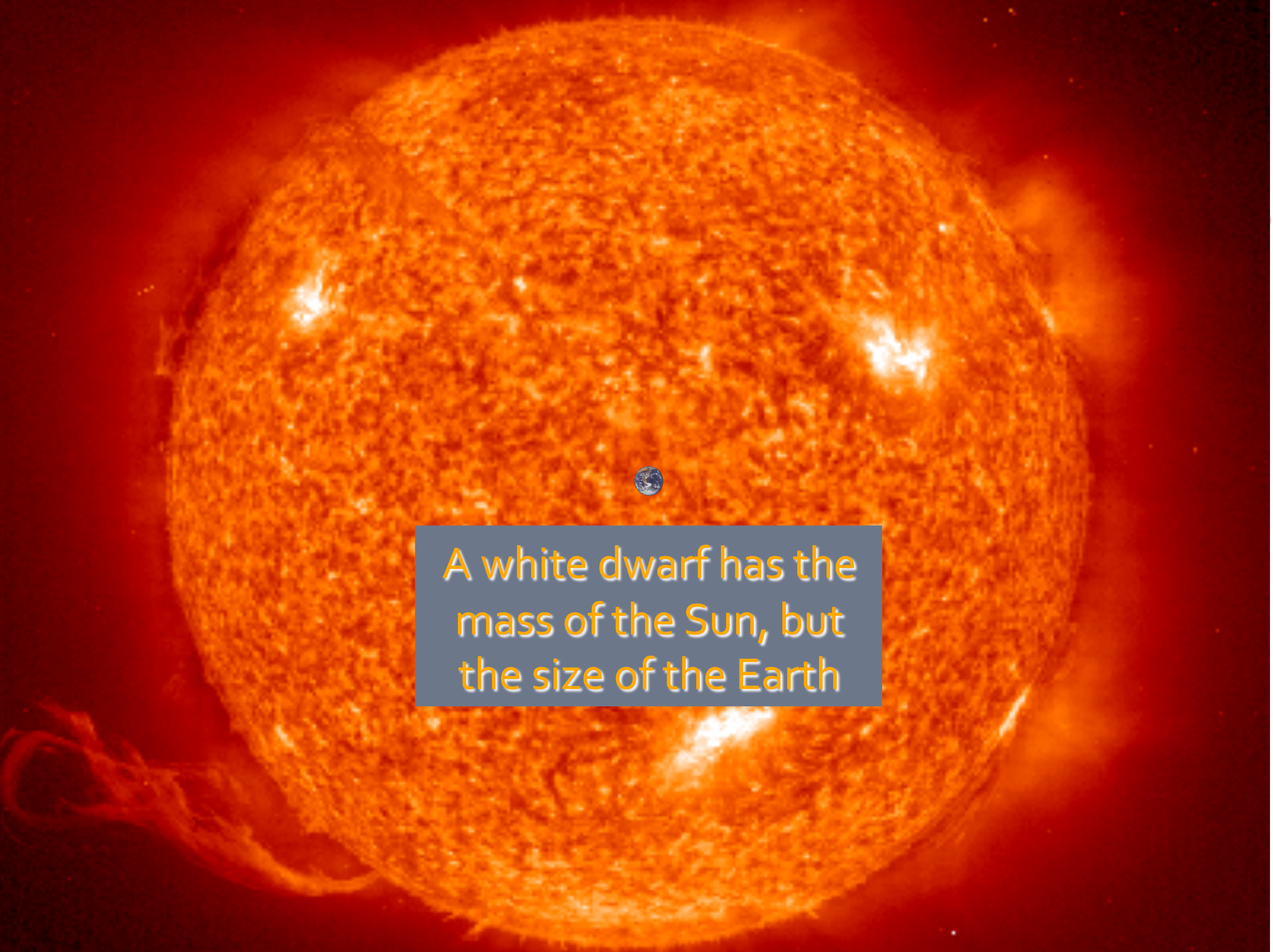
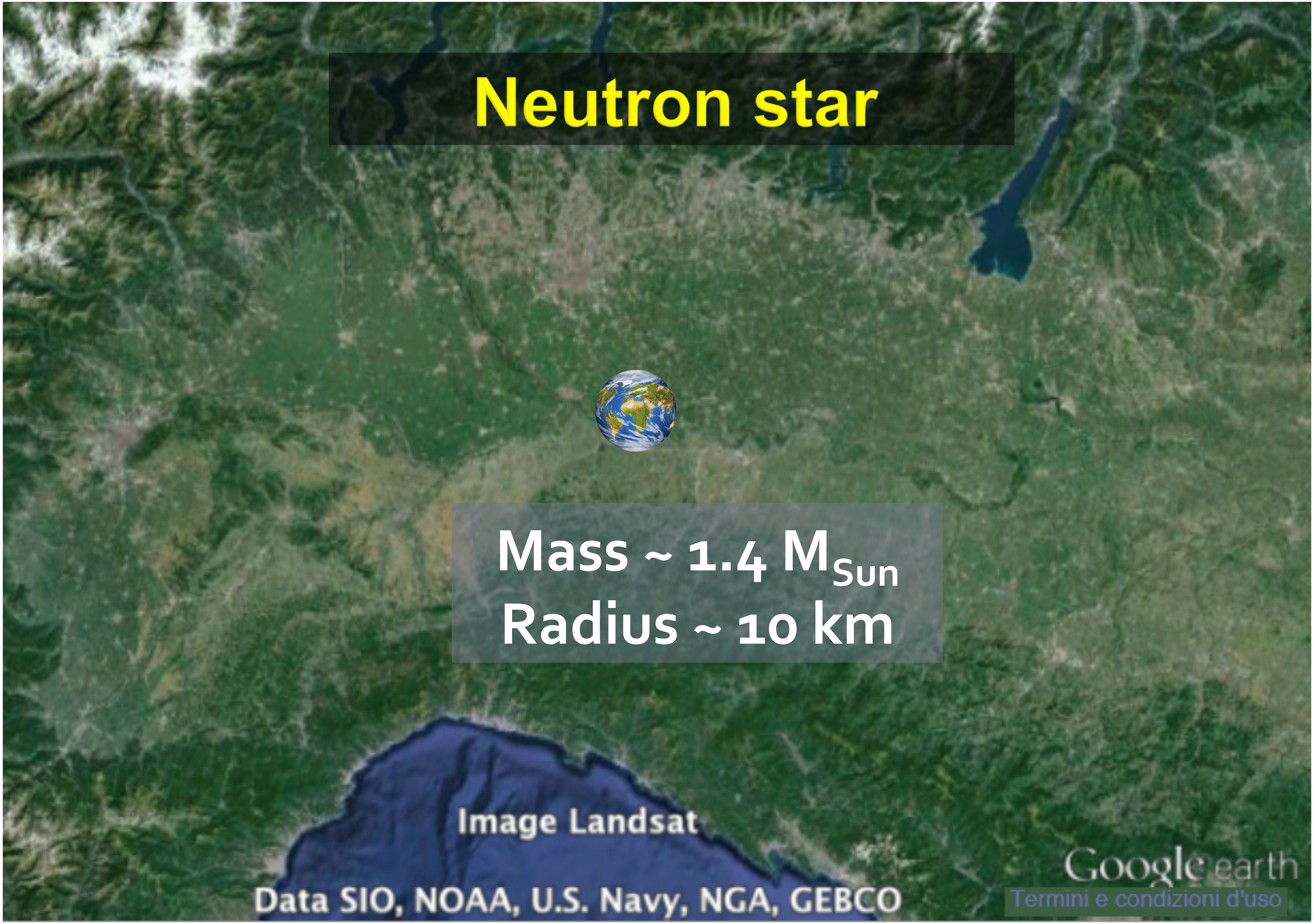

A white dwarf has the
mass of the Sun, but
the size of the Earth


Neutron star
Mass ~ 1.4 M_{Sun}
Radius ~ 10 km
Image Landsat
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
Google earth
Termini e condizioni d'uso

How compact are compact objects?

Escape velocity:

$$\frac{v^2}{R} = \frac{GM}{R^2} \Rightarrow v_e = \sqrt{\frac{GM}{R}}$$

Black Hole when $v_e \approx c$

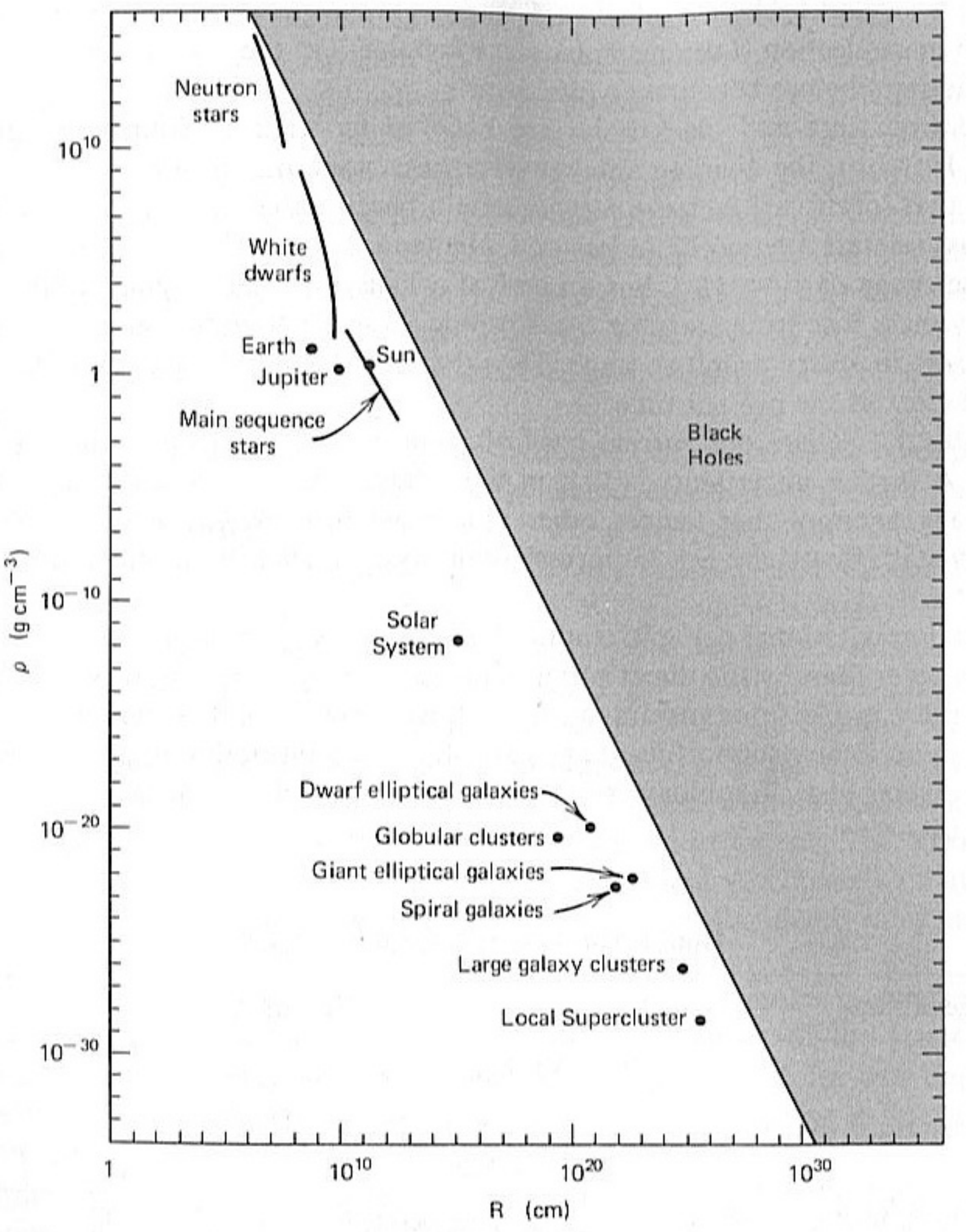
Strong gravity when

$$E_G \approx mc^2$$

$$\Rightarrow \frac{GM}{Rc^2} \approx 1$$

Compactness = 1 for BH

Sun? WD? NS?



Degeneracy Pressure

- Two particles cannot occupy the same space with the same **momentum** (energy).

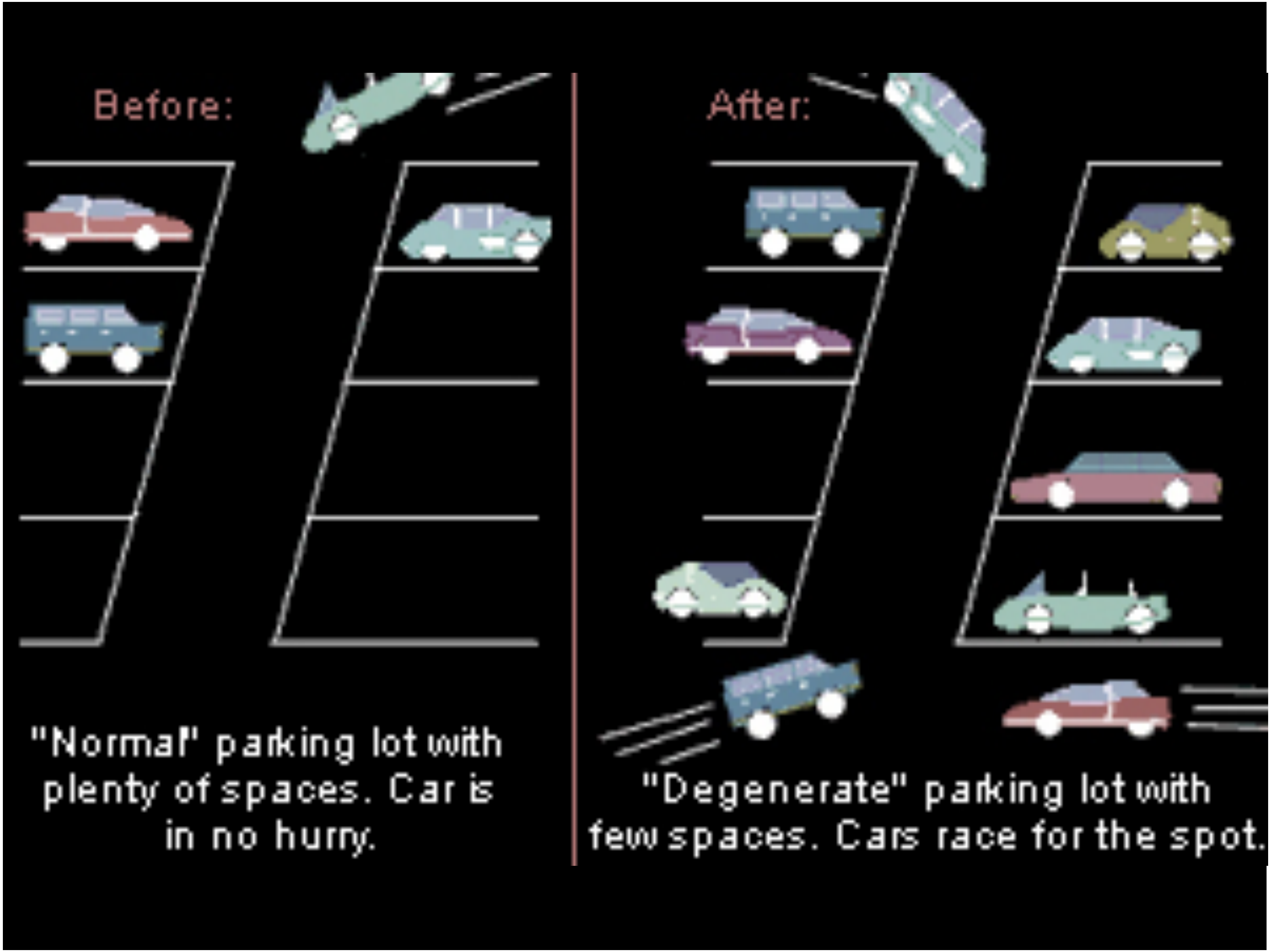
- For **very dense solids**, electrons cannot be in their ground states, they become very energetic ⇒ approaching the speed of light.

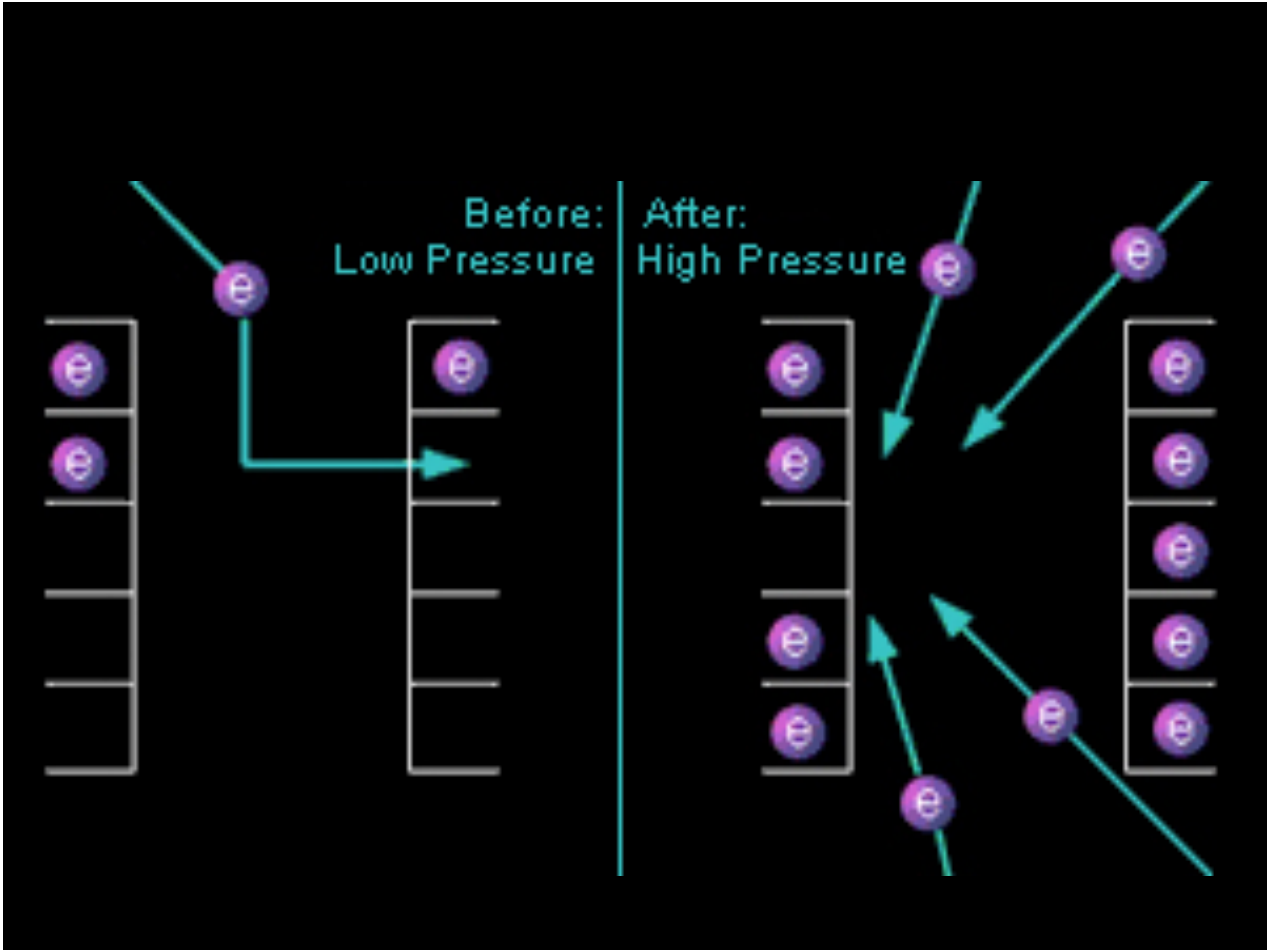
- ***Pressure** holding up star no longer depends on* ***temperature:***

$$P \propto \rho^{\gamma}$$

$\gamma=5/3$ for **non-relativistic** degenerate gas

$\gamma=4/3$ for **relativistic** degenerate gas


Before:
After:
"Normal" parking lot with plenty of spaces. Car is in no hurry.
"Degenerate" parking lot with few spaces. Cars race for the spot.


Before:
Low Pressure
After:
High Pressure
e

The Chandrasekhar's limit:

General argument by **Landau** (1932) on limiting mass for a degenerate gas of **electrons** (WDs) or **neutrons** (NSs)

N fermions in star of radius $R \Rightarrow n \sim N/R^3$

Volume per fermion $\sim 1/n$ (**Pauli exclusion principle**) and momentum $\sim \hbar n^{1/3}$ (**Heisenberg principle**)

Fermi energy of fermionic gas in relativistic regime:

$$E_F = p_F c \sim \hbar n^{1/3} c \sim \hbar c N^{1/3}/R$$

Gravitational energy per fermion:

$$E_G \sim -GMm_B/R \quad (M = Nm_B, \text{ most of the mass in baryons})$$

Equilibrium at a minimum of the total energy function:

$$E = E_F + E_G = \hbar c N^{1/3}/R - GNm_B^2/R$$

$$E(N) = E_F + E_G = \hbar c N^{1/3}/R - GNm_B^2/R$$

For arbitrary large N, E is always negative ⇒ if R decreases, E continues to decrease ⇒ collapse continues indefinitely ⇒ M_{max}

For small N, first term dominates ($E > 0$) ⇒ minimum at $E(N)=0$

$$N_{max} \sim (\hbar c/Gm_B^2)^{3/2} \sim 2 \times 10^{57} \Rightarrow M_{max} \sim N_{max}\, m_B \sim 1.7\ M_\odot$$

From this simplified calculation, same M_{max} for WDs and NSs.

Equilibrium radius: $E_F \sim mc^2$ in the relativistic regime and m is the mass of electrons or neutrons, giving WD and NS radius, respectively

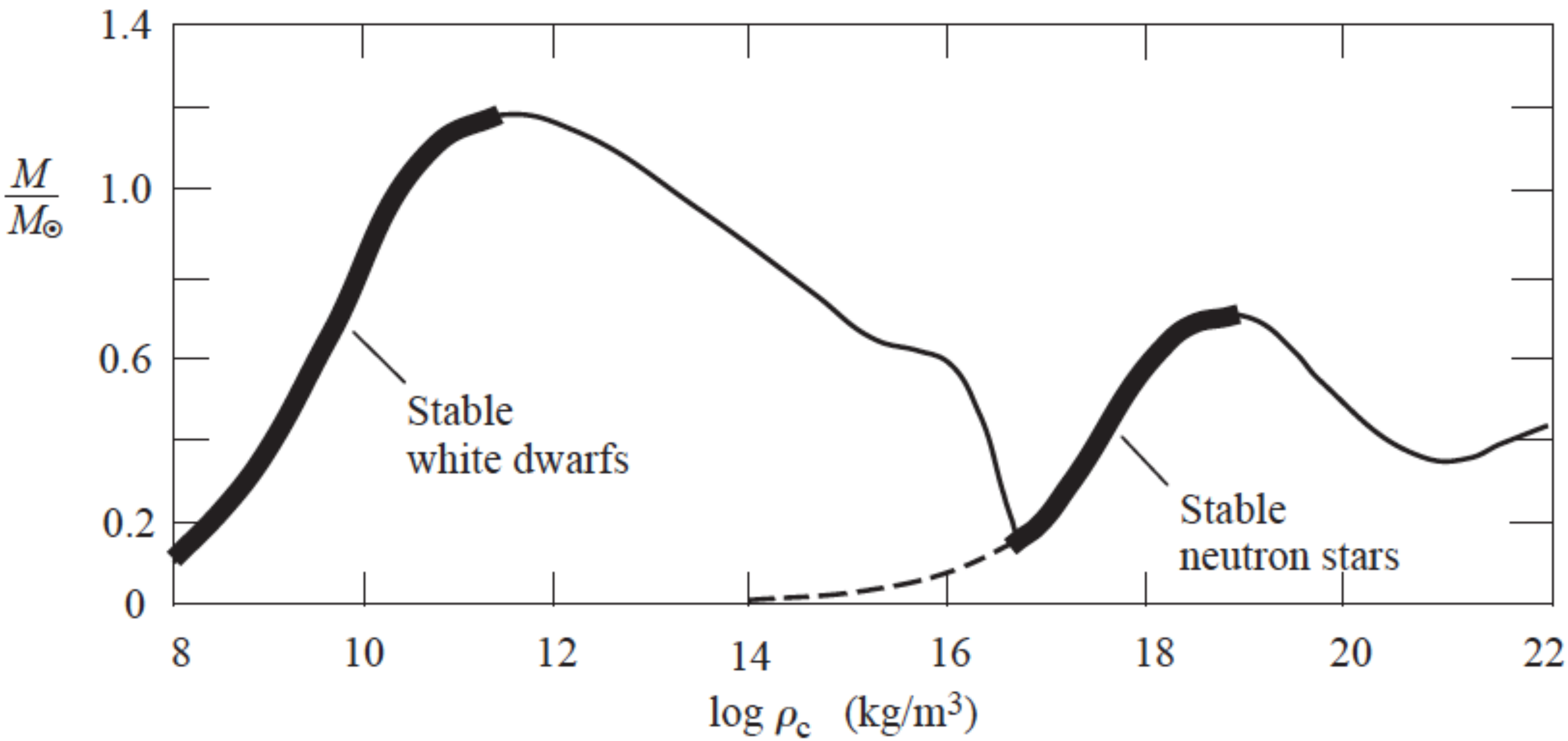
$$E_F \sim \hbar c N^{1/3}/R \sim mc^2 \quad R \sim \hbar/mc(N_{max})^{1/3} \sim \hbar/mc\ (\hbar c/Gm_B^2)^{1/2}$$

$R_{WD} \sim 5 \times 10^8$ cm for $m=m_e$; $R_{NS} \sim 3 \times 10^5$ cm for $m=m_n$

NS radii m_n/m_e times smaller than WD radii

Stable WDs and NSs

HW (1958) and OV (1939) equations of state, *ignoring nuclear forces*.

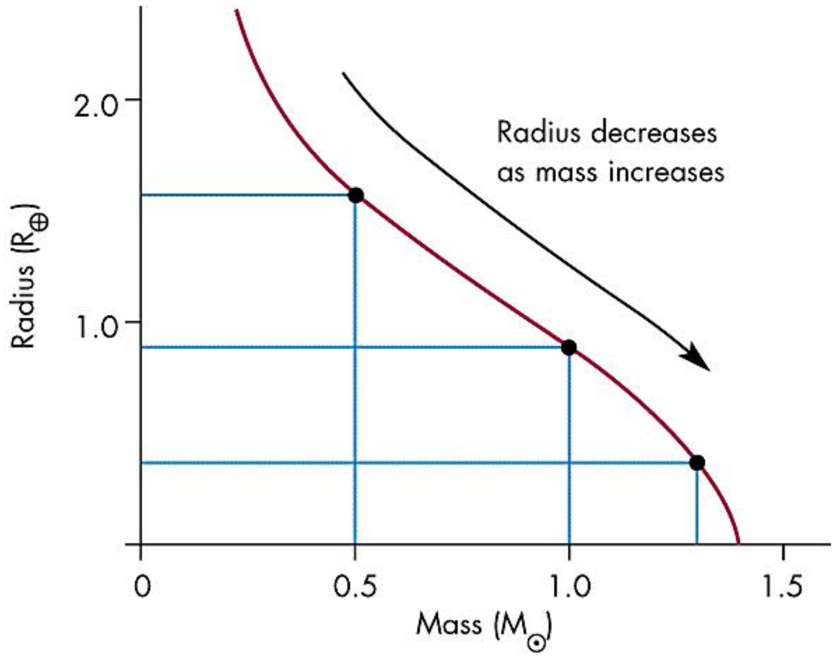


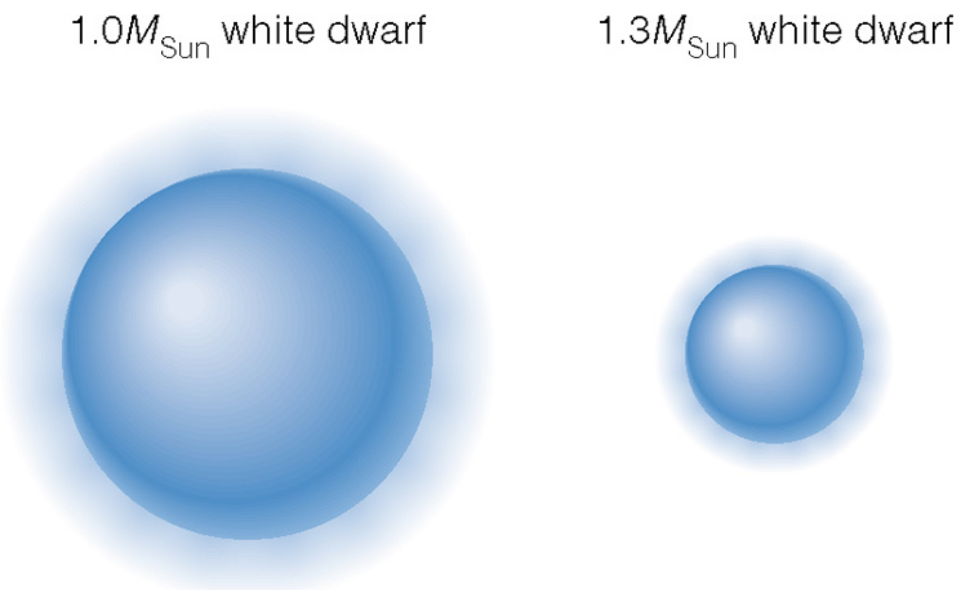
Stability only if mass increase implies larger density

⇒larger pressure to contrast gravity ($P \propto \rho^\gamma$)

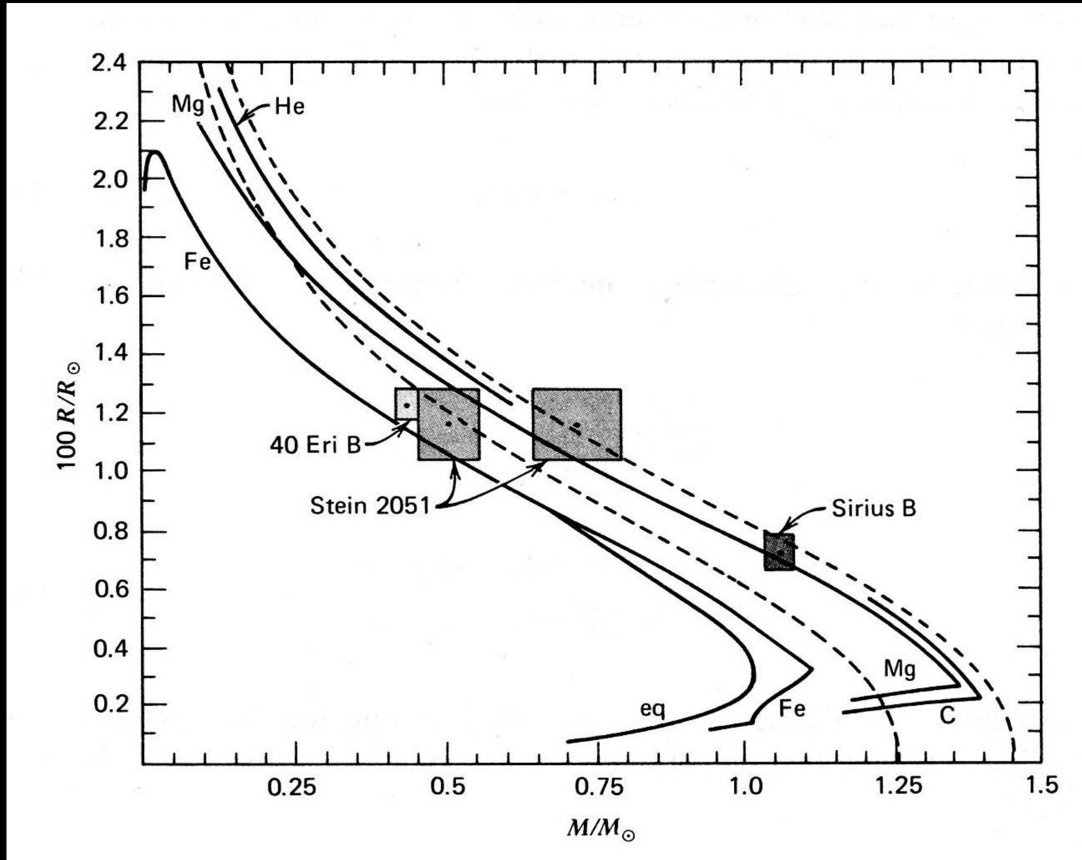
White Dwarfs

- The more mass the star has, the *smaller* the star becomes!
  - increased gravity makes the star denser
  - greater density increases degeneracy pressure to balance gravity





White dwarfs: mass-radius relation



Chandrasekhar's model (dashed line) agrees quite well with better models based on equations of state (with a dominating element, different fermions, particle interactions and electrostatic corrections).

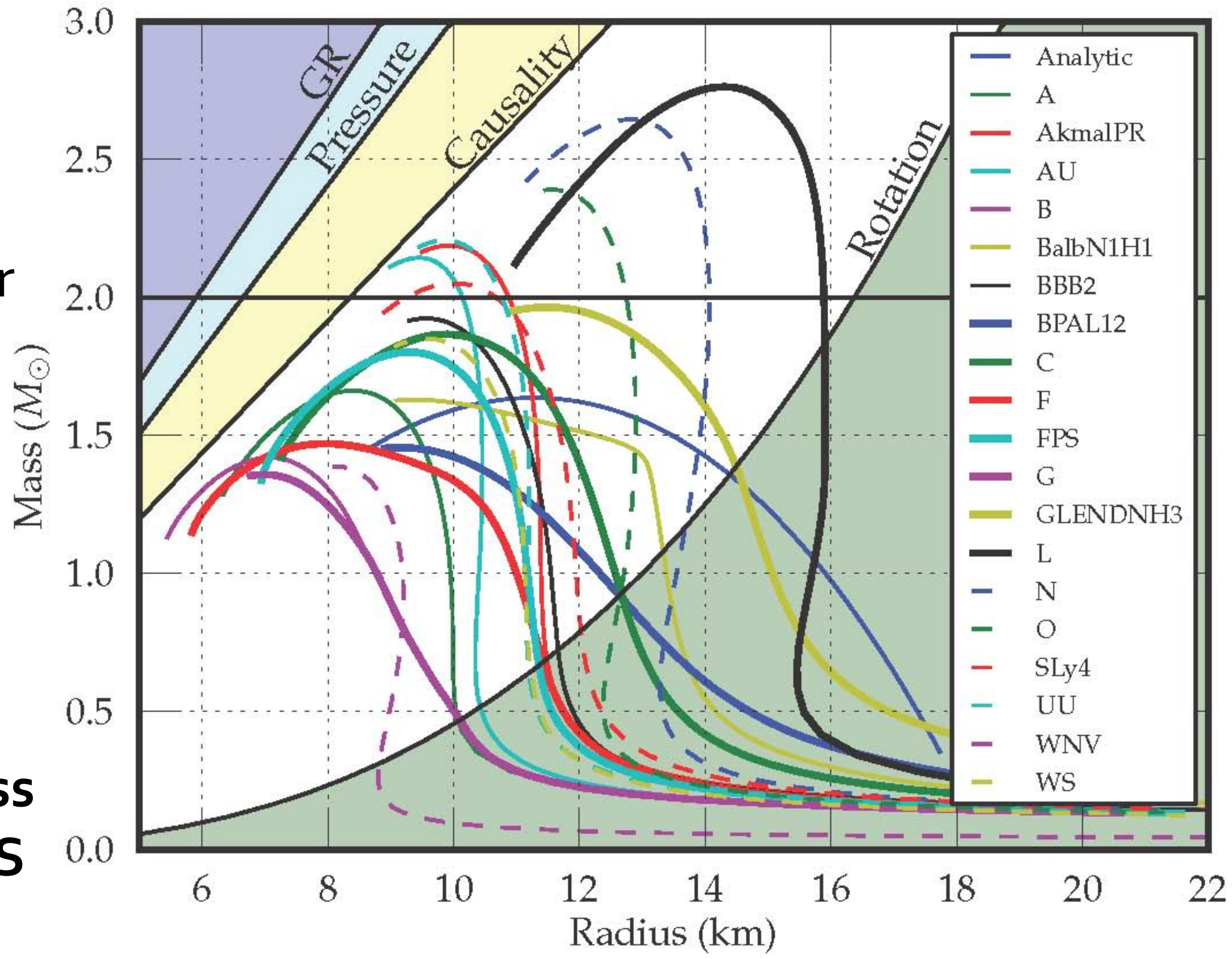
Maximum mass varies in the range 1-1.45 $M_\odot$

Neutron star: mass-radius relation

To determine NS Equation of State (**EoS**) we need to know the behavior of matter at supranuclear density and use General Relativity

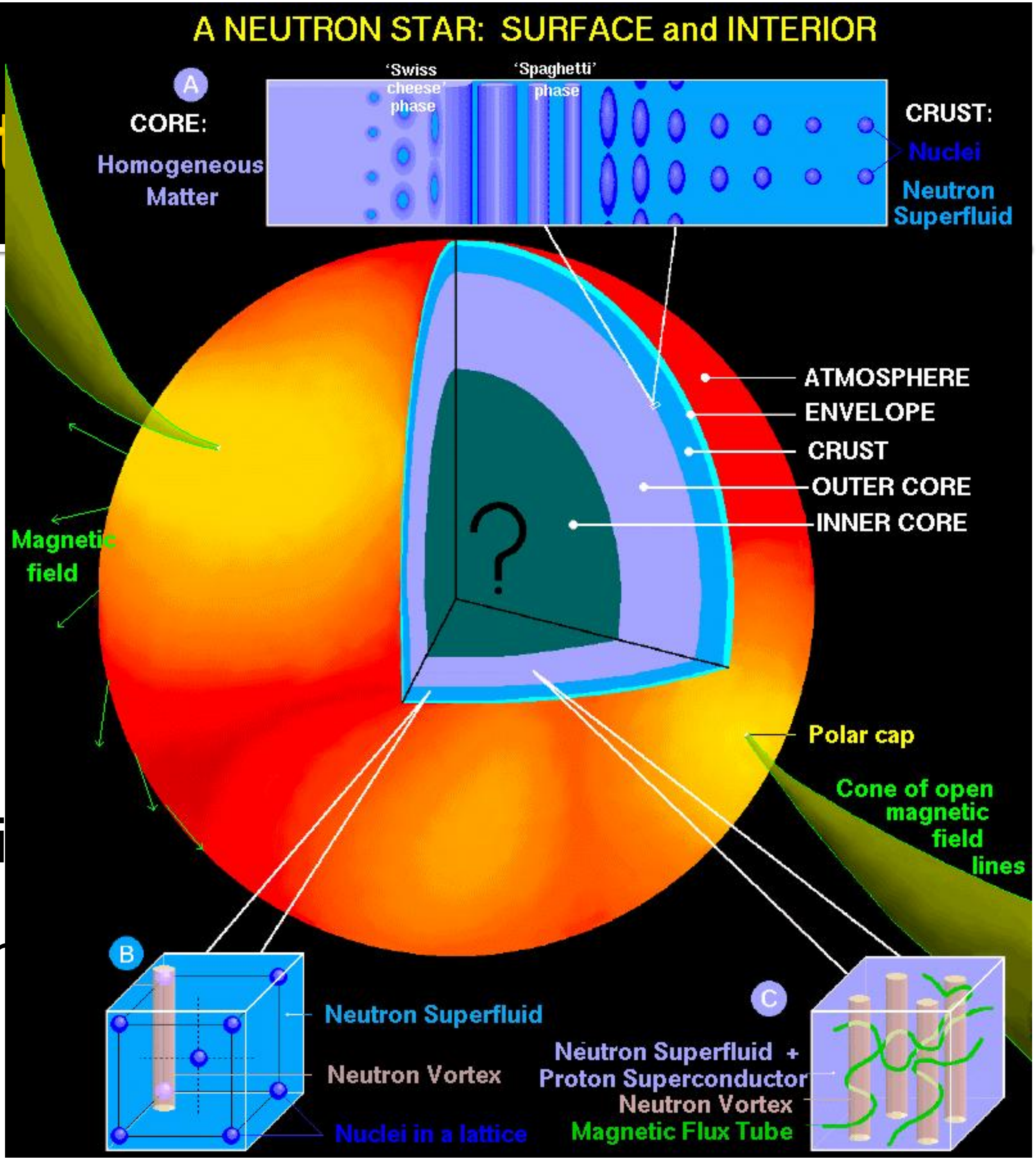
$$\left(\frac{GM_{NS}}{R_{NS}c^2} \approx 0.1\right)$$

Maximum NS mass <3 $M_\odot$ for any EoS

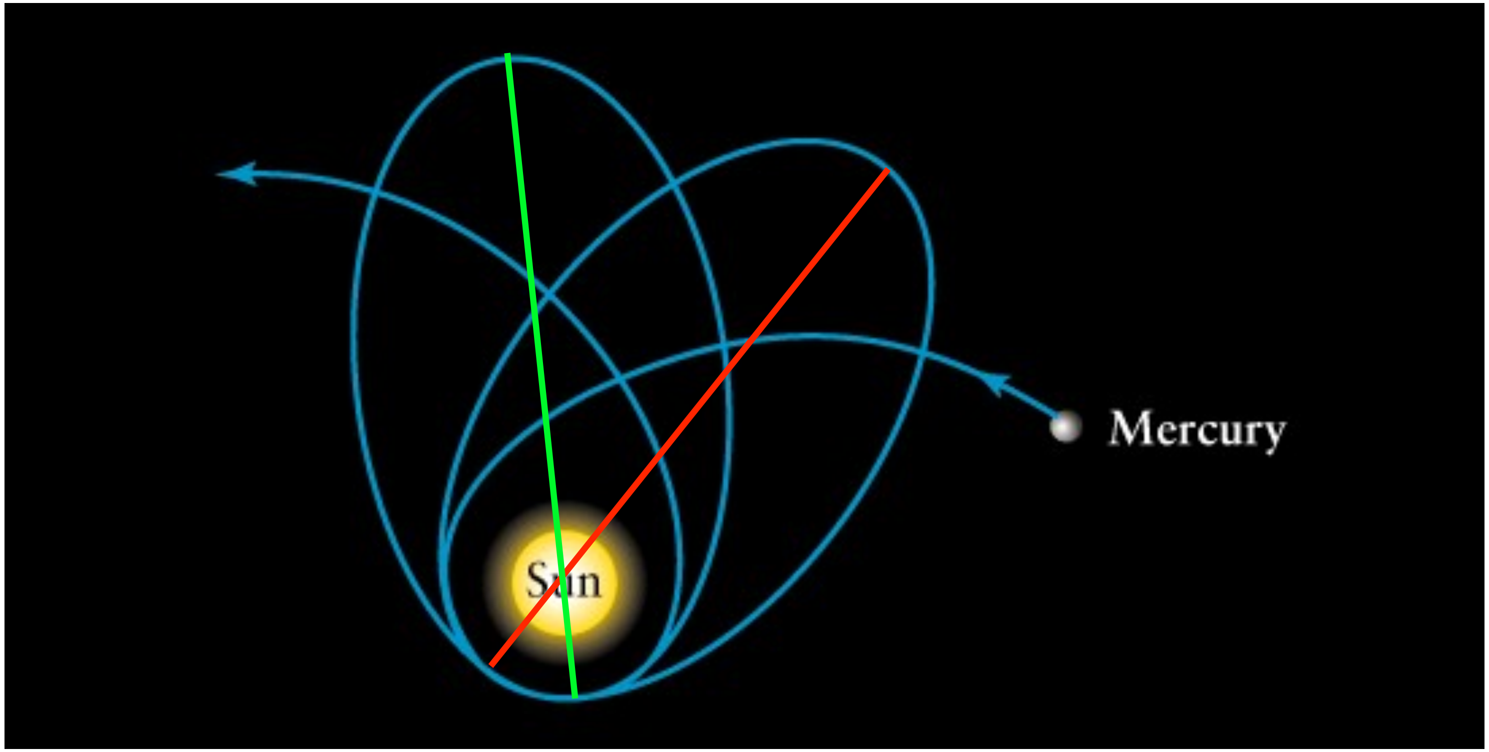


Neutron st

- Atmosphere:
- Crust: Fe
- Neutron drip:
- Superfluidity
- Nuclear densi
- Core: quark m



GR: Mercury orbit precession



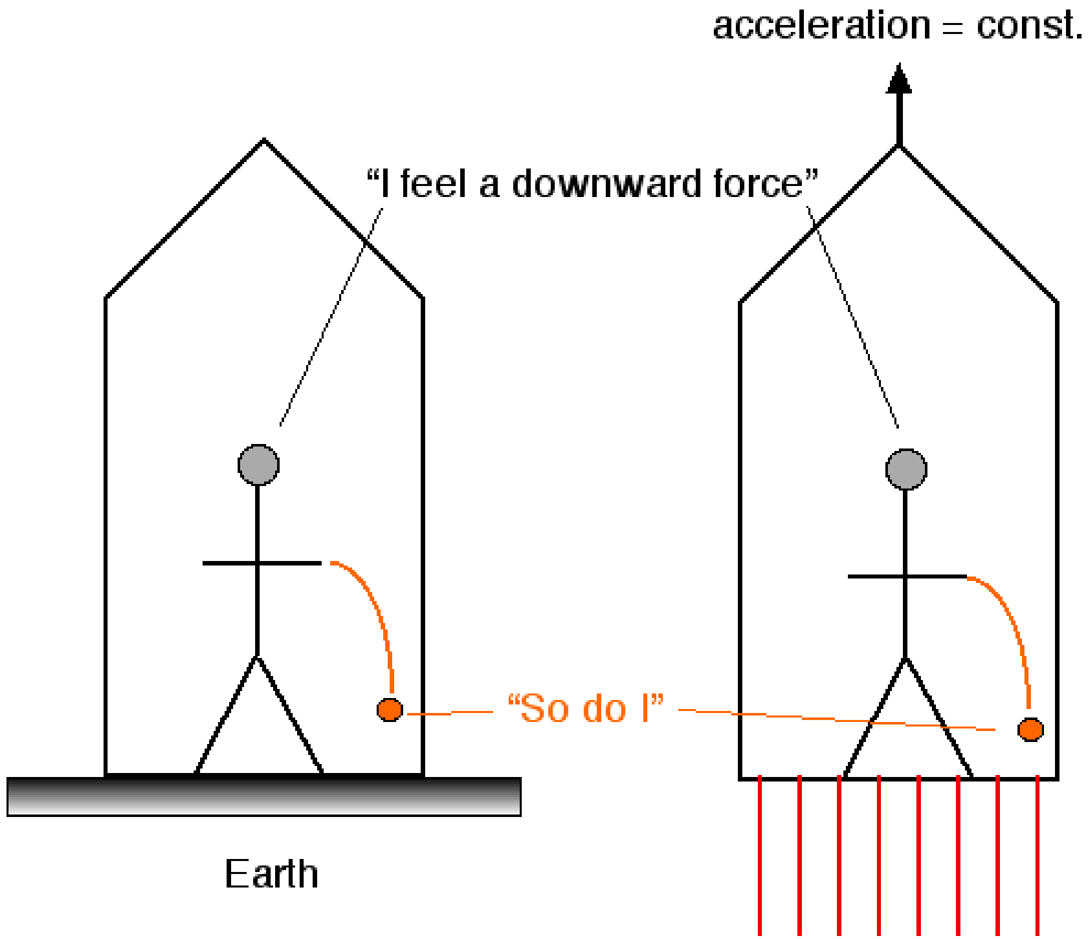
Newtonian Gravity Predicts: 5557.62 arcsec/century

Observed Value: 5600.73 arcsec/century

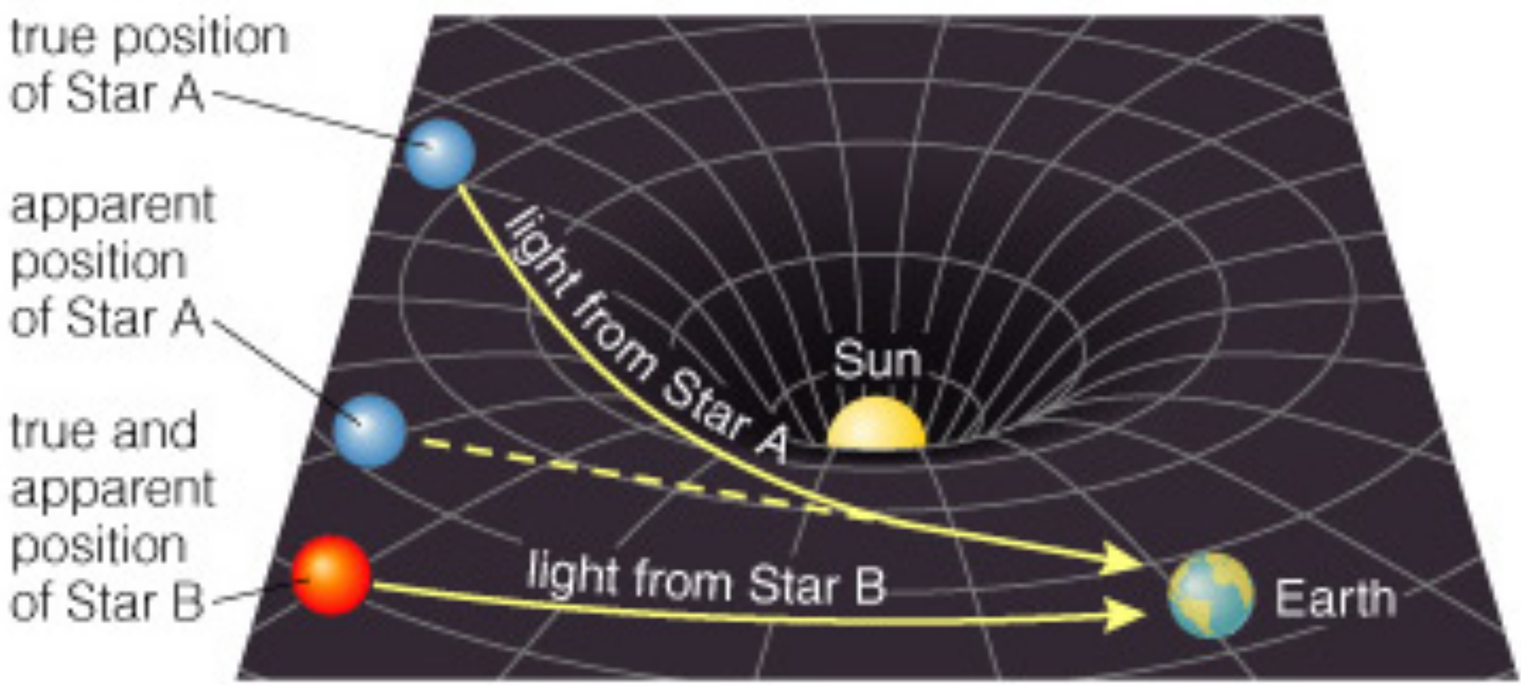
Difference: 43.11 ± 0.45 arcsec/century too fast!!

GR: The Equivalence Principle

The force of gravity is indistinguishable from the force due to accelerated motion.



GR: Deflection of Starlight

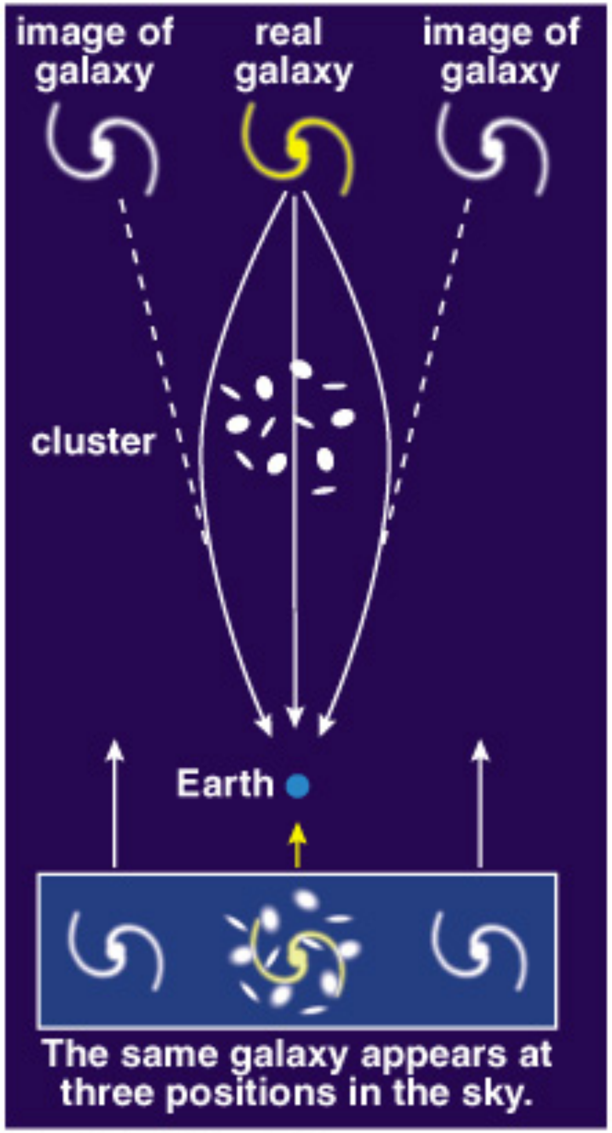


How can we measure this effect?

GR: Deflection of Starlight

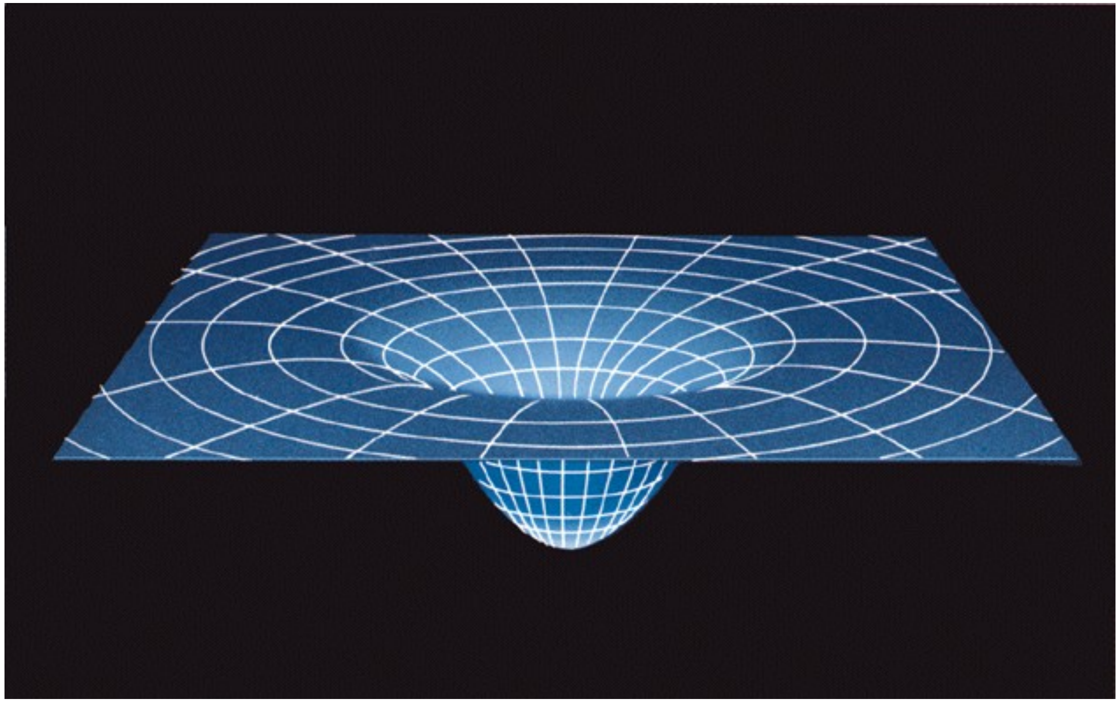
Observation by Eddington during Solar eclipse in 1919

GR: Gravitational Lensing



Distant galaxies lensed/warped in appearance by close galaxy masses (mainly Dark Matter)

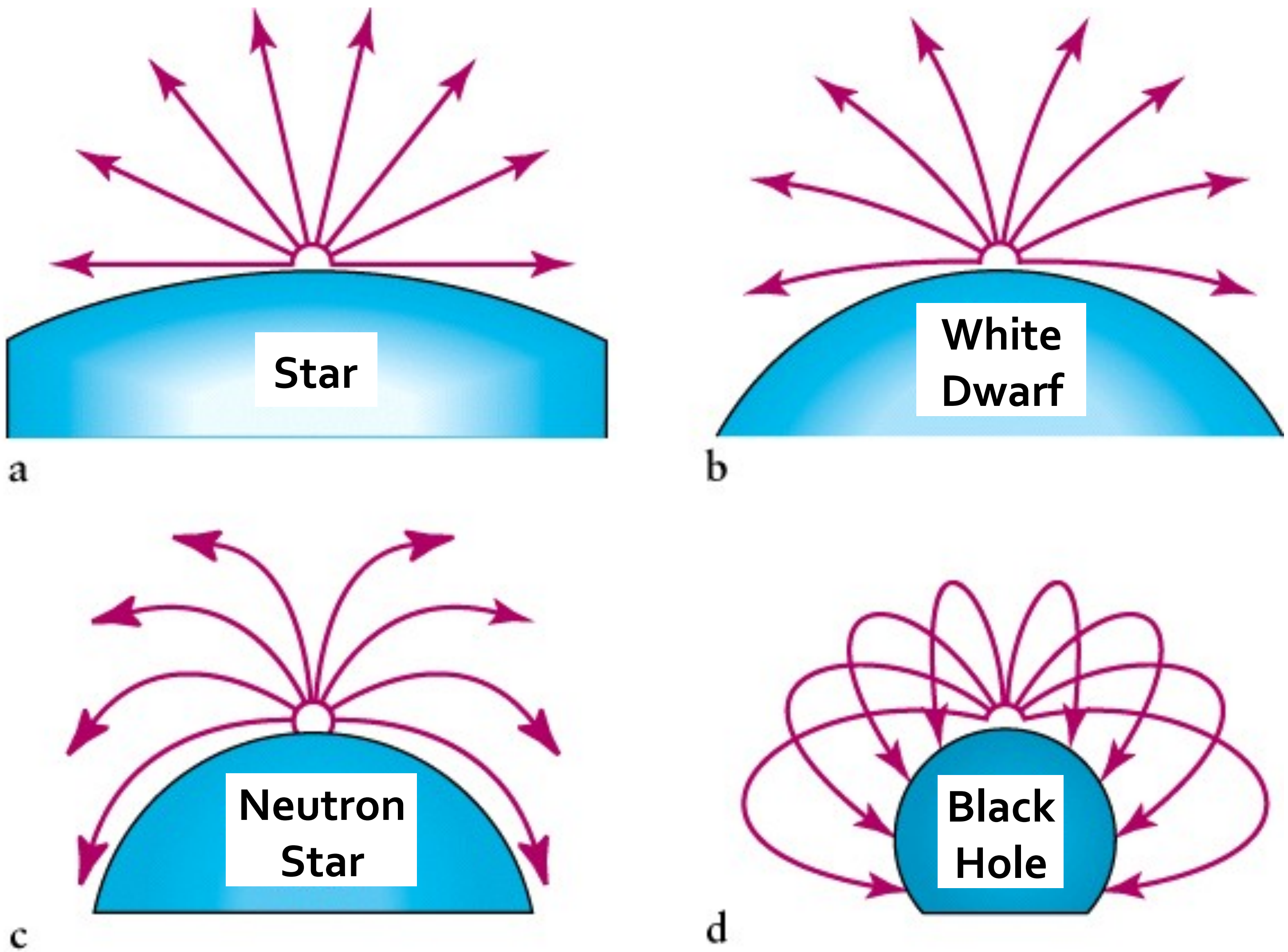
GR: Light travels along “straight” lines in a curved “space-time”



If this were a soccer field, how would a soccer ball “roll” on it?

Light behaves similarly traveling through curved 3D space

Black Holes

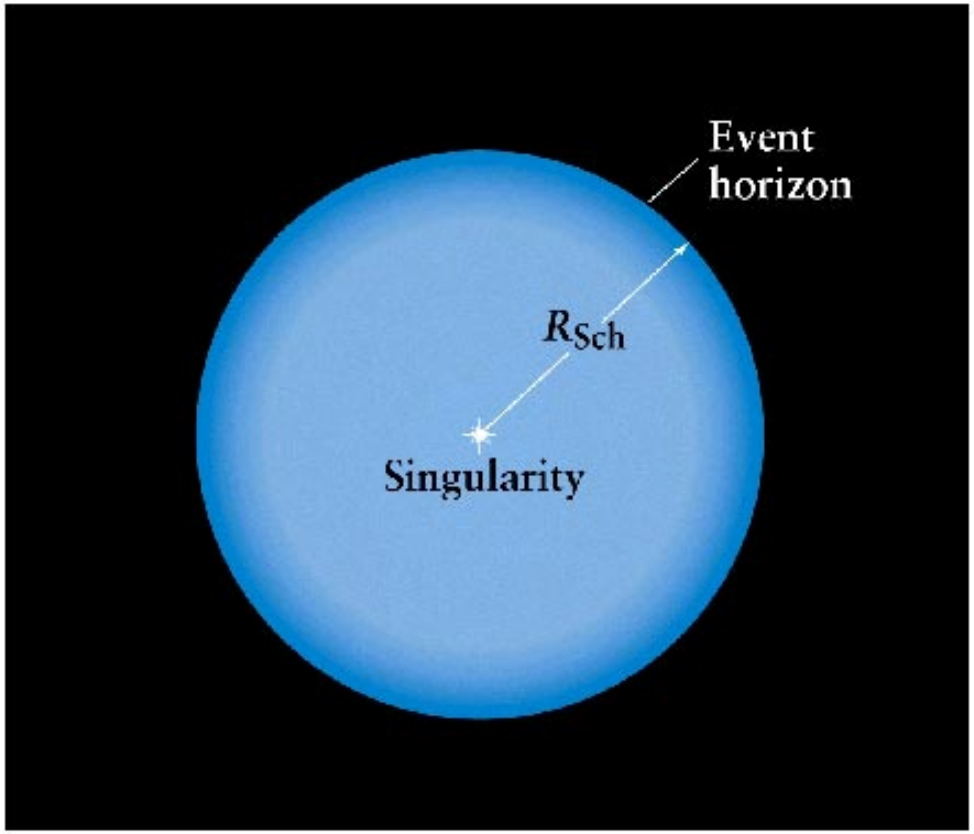


A nonrotating black hole has only a "center" and a "surface"

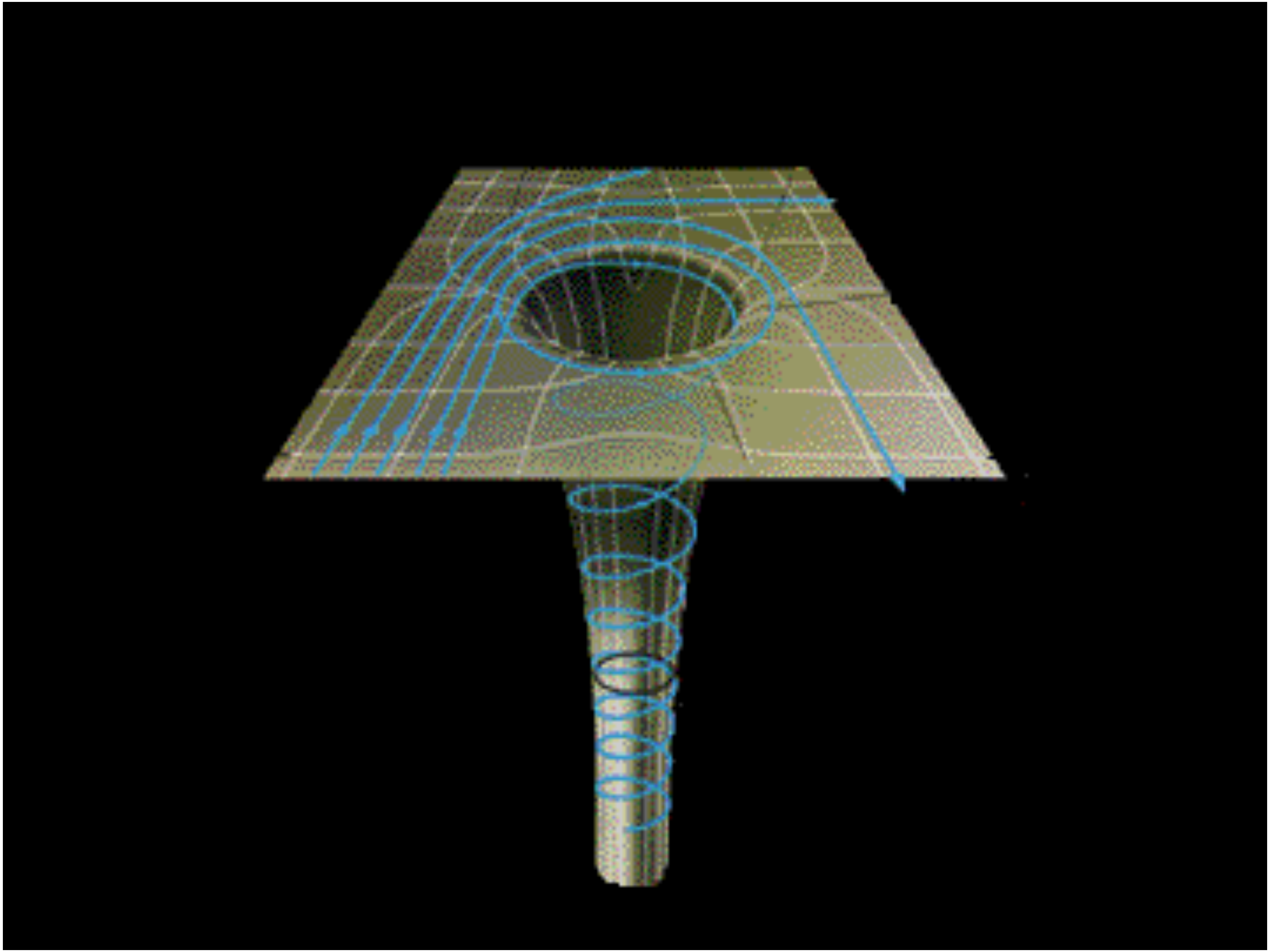
- The ***event horizon*** is the sphere from which light cannot escape
- The distance between the BH and its event horizon is the ***Schwarzschild radius***:

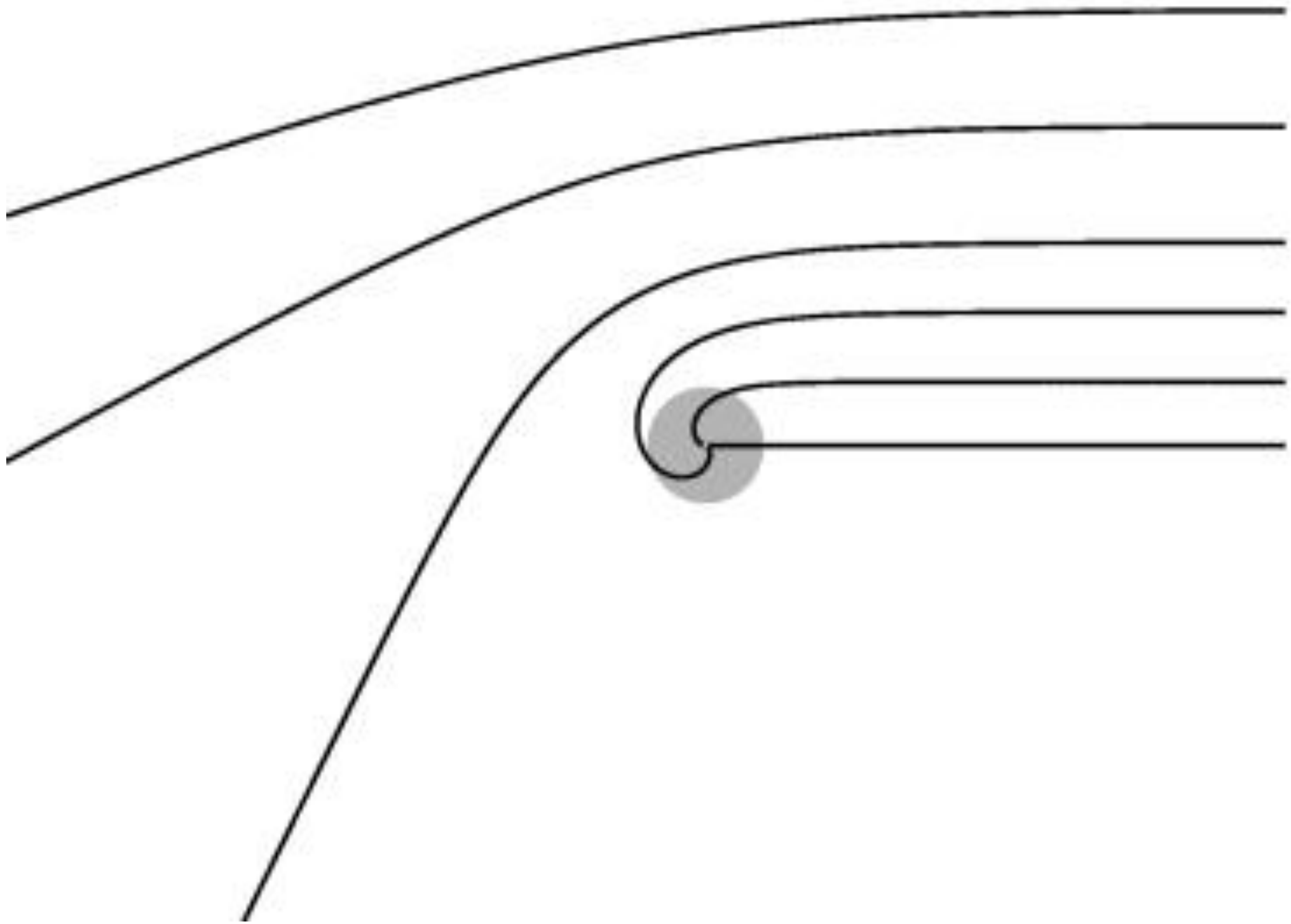
$$R_S = \frac{2GM}{c^2} \approx 3\frac{M}{M_{Sun}}\ km$$

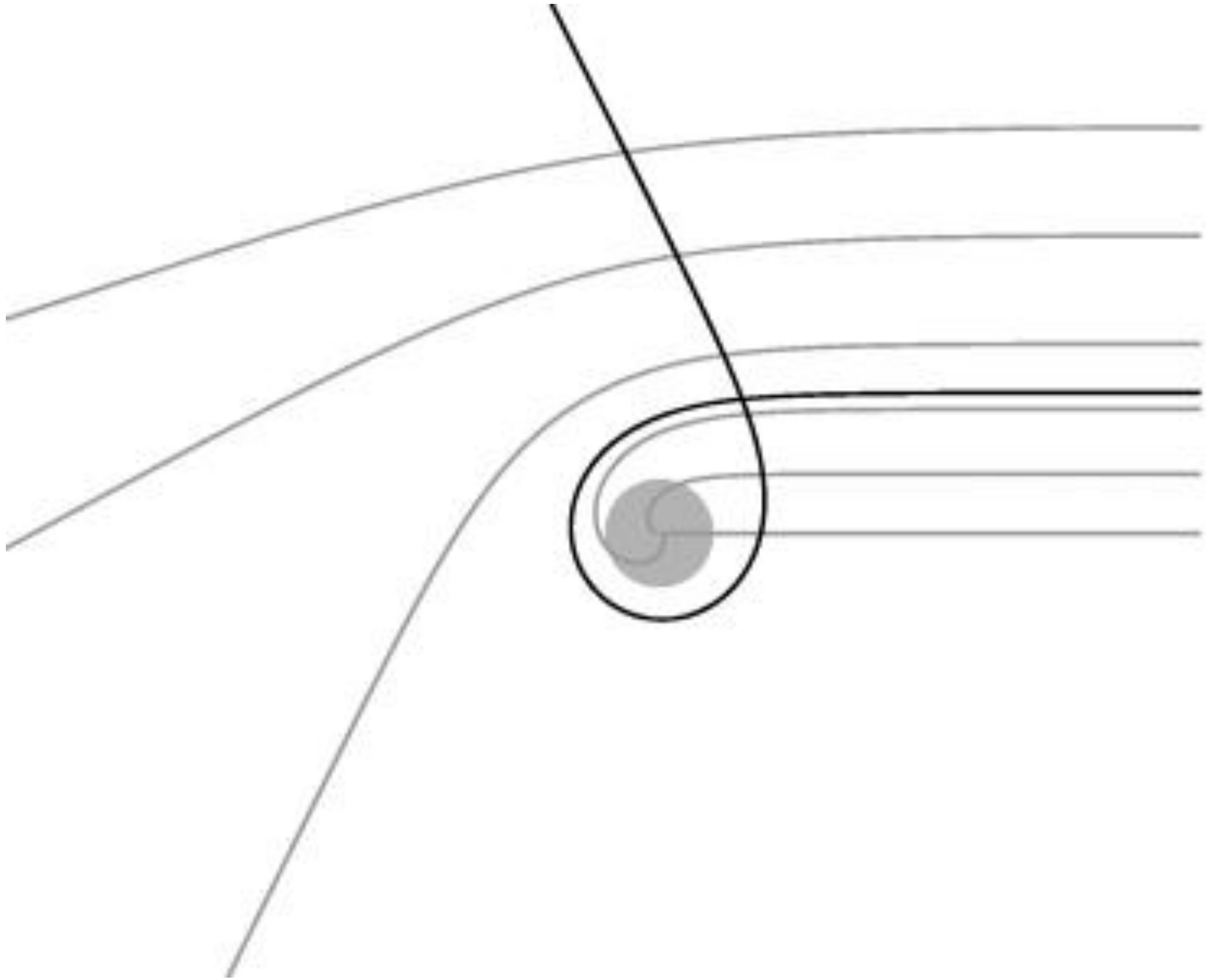
- The center of the BH is a point of infinite density and zero volume, called a ***singularity***

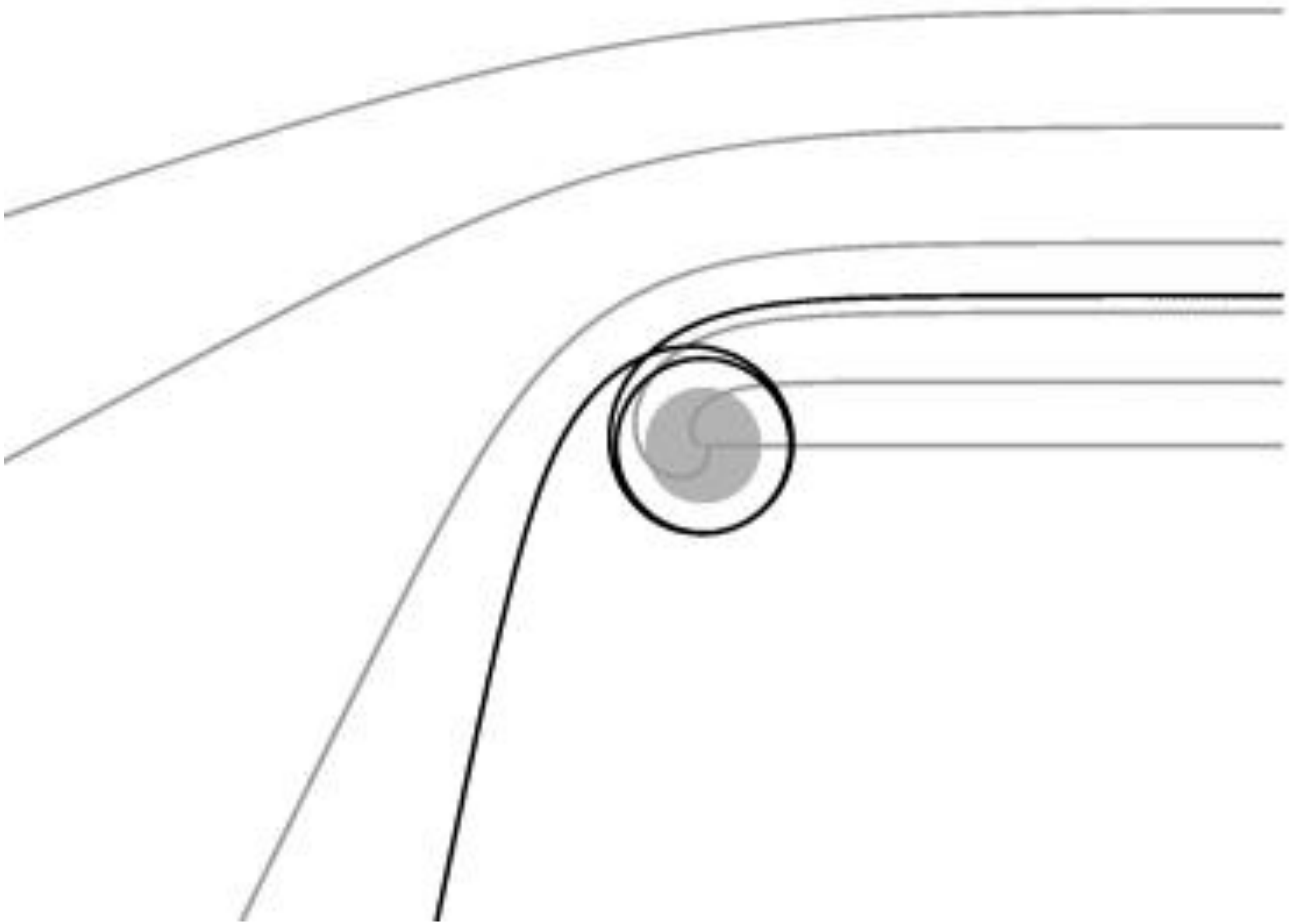


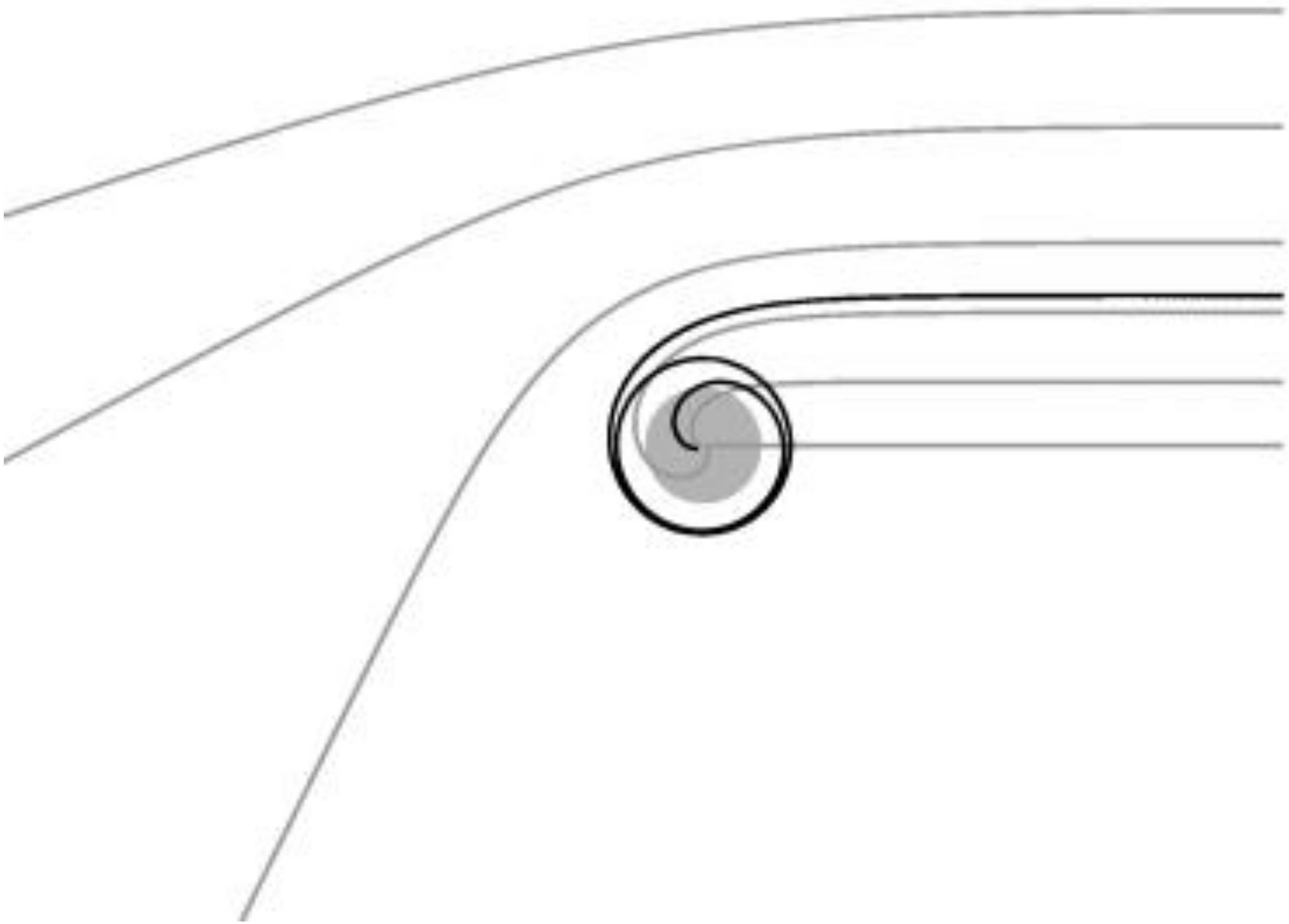
- Every object in the universe has a Schwarzschild radius, but they become a BH only if their mass is contained within R_S.

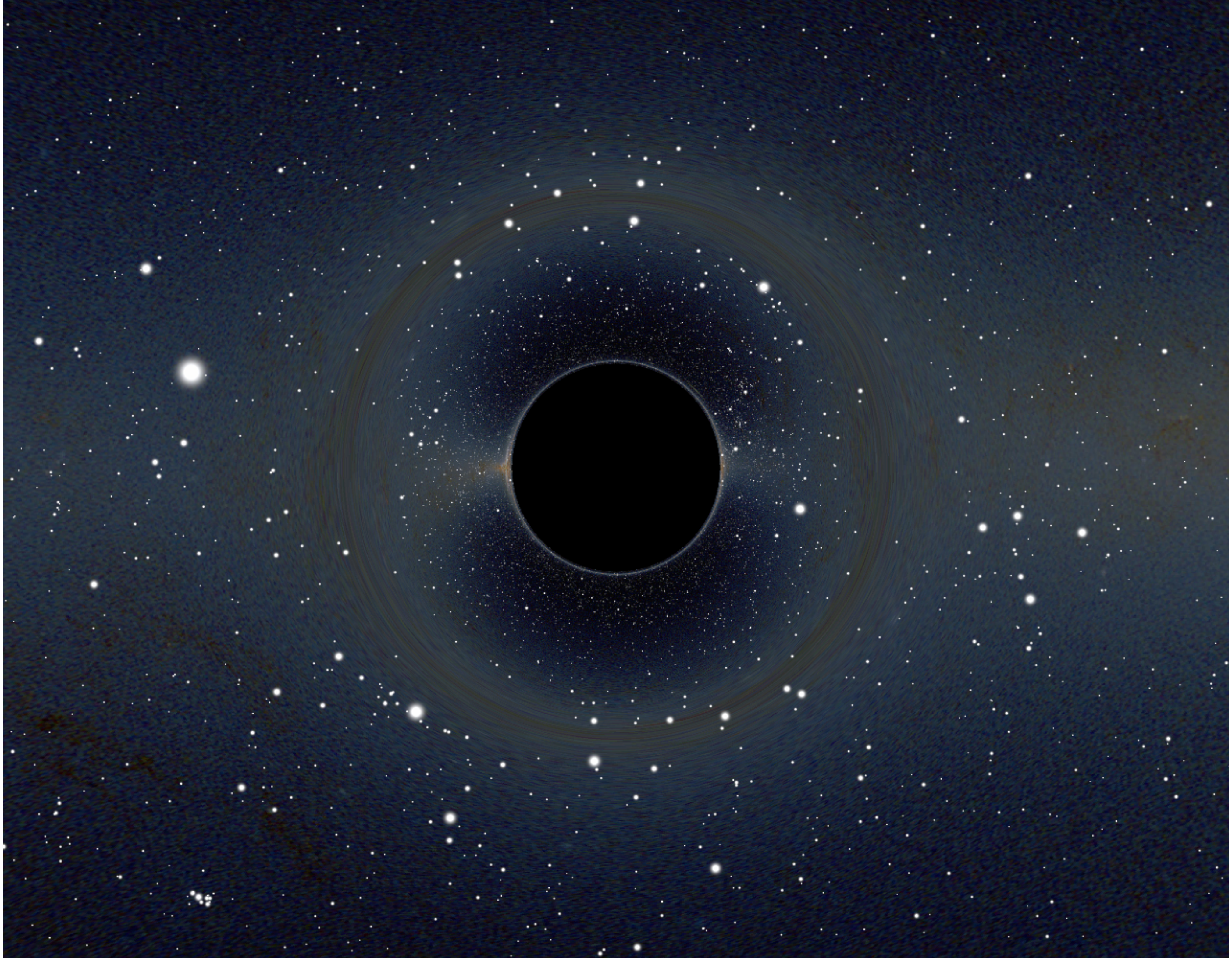


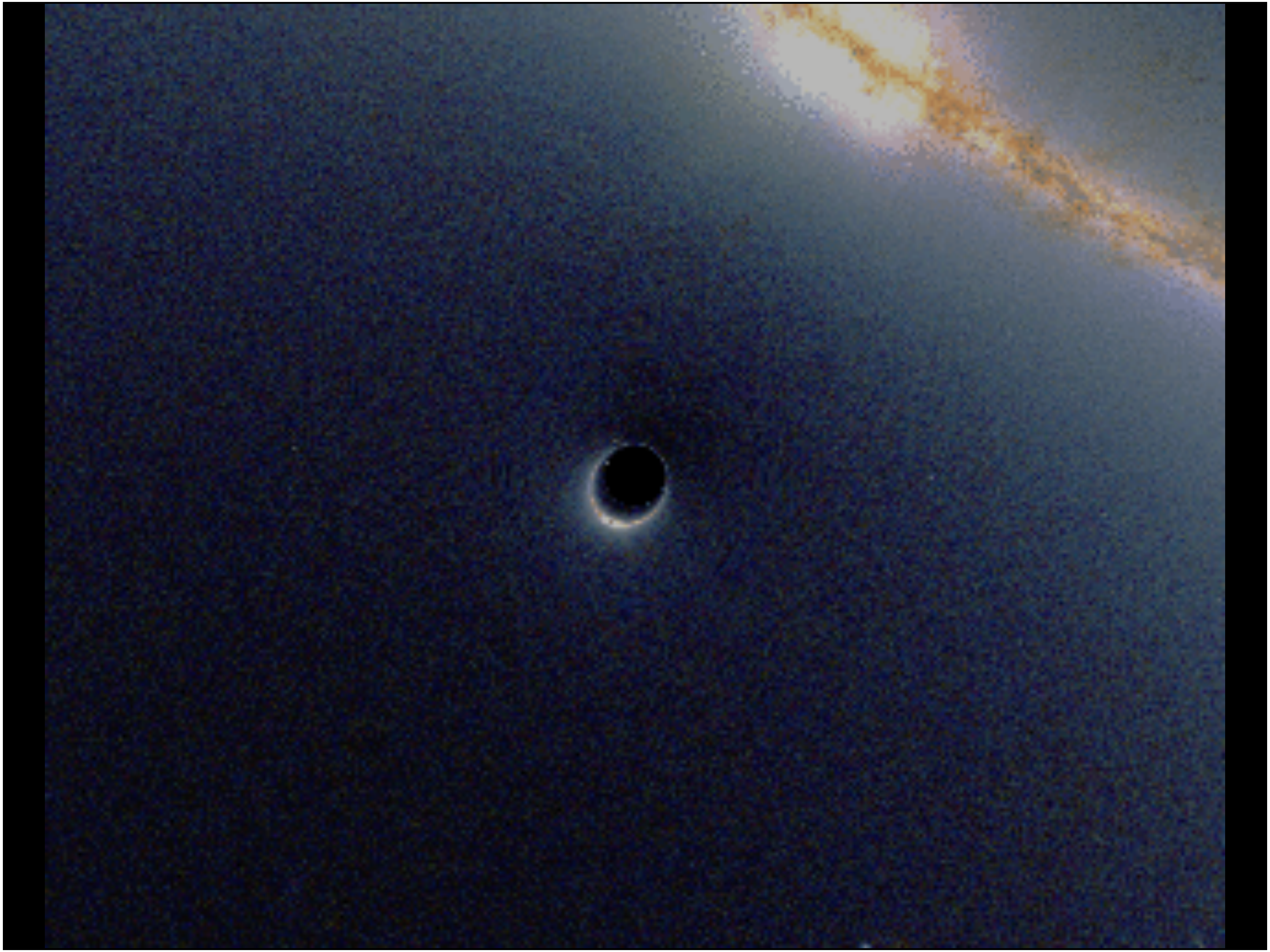






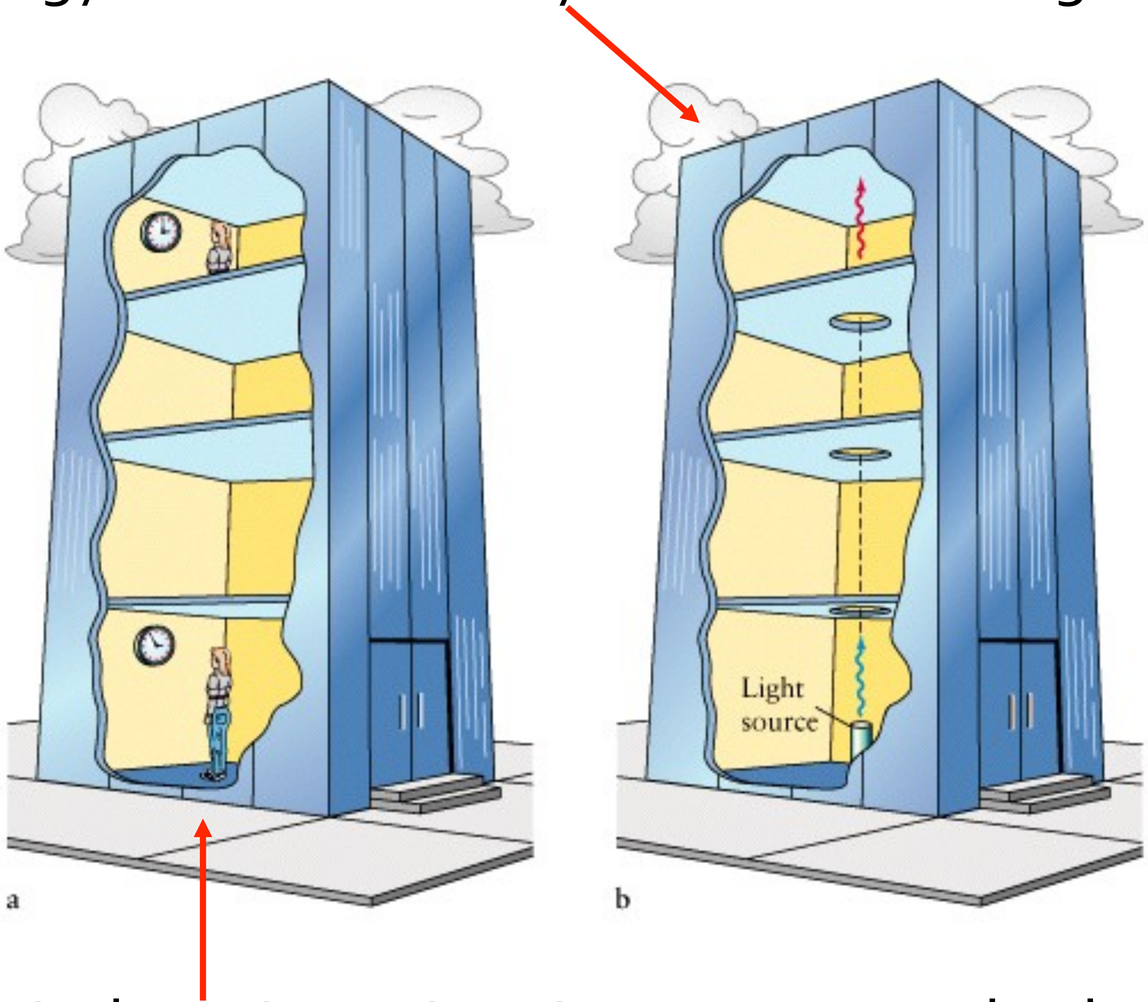






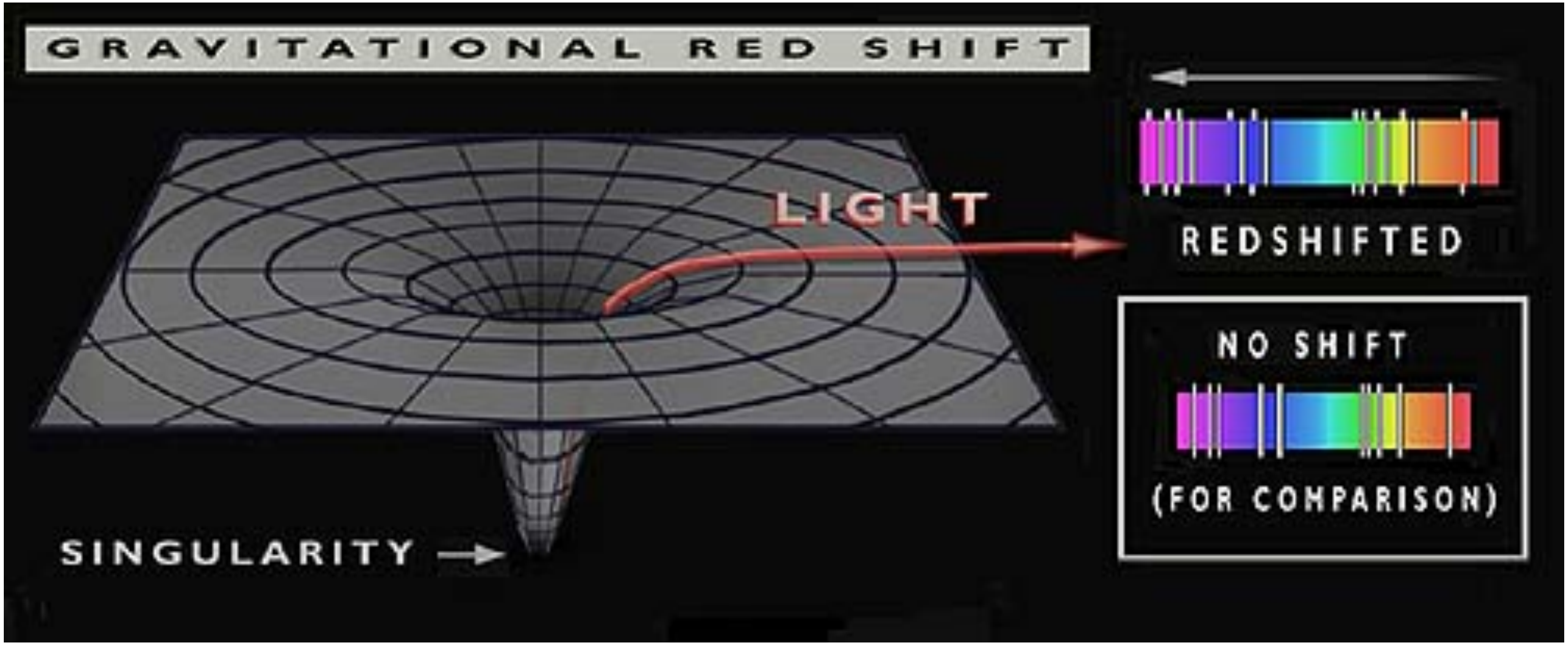
GR: Gravitational Redshift

Light loses energy as it travels away from a source of gravity



Equivalent viewpoint: time runs more slowly the closer you are to a source of gravity!

Gravitational redshift

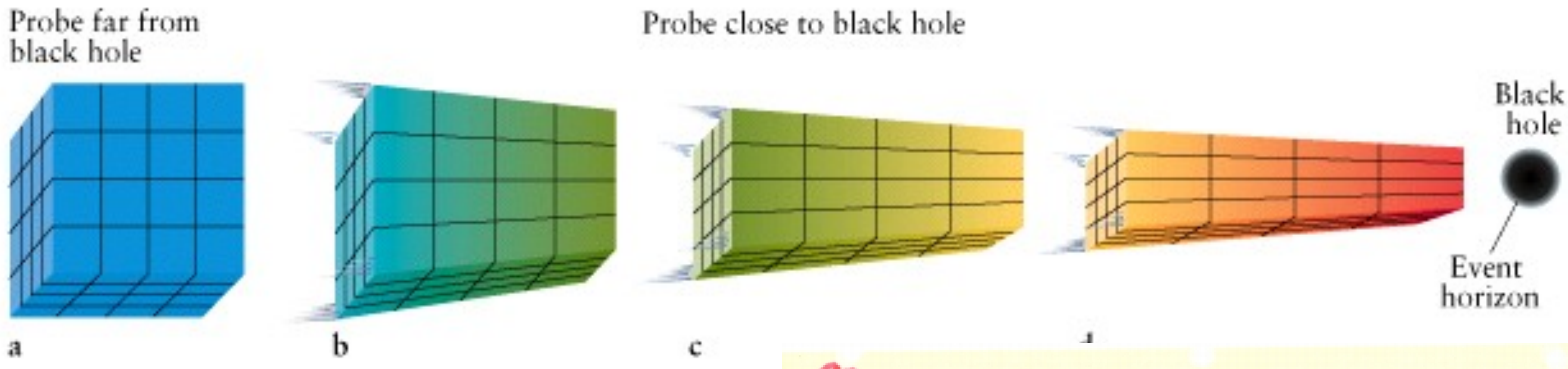


$$\frac{\nu}{\nu_r} = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} \Rightarrow \nu \rightarrow 0 \text{ for } r \rightarrow R_S$$

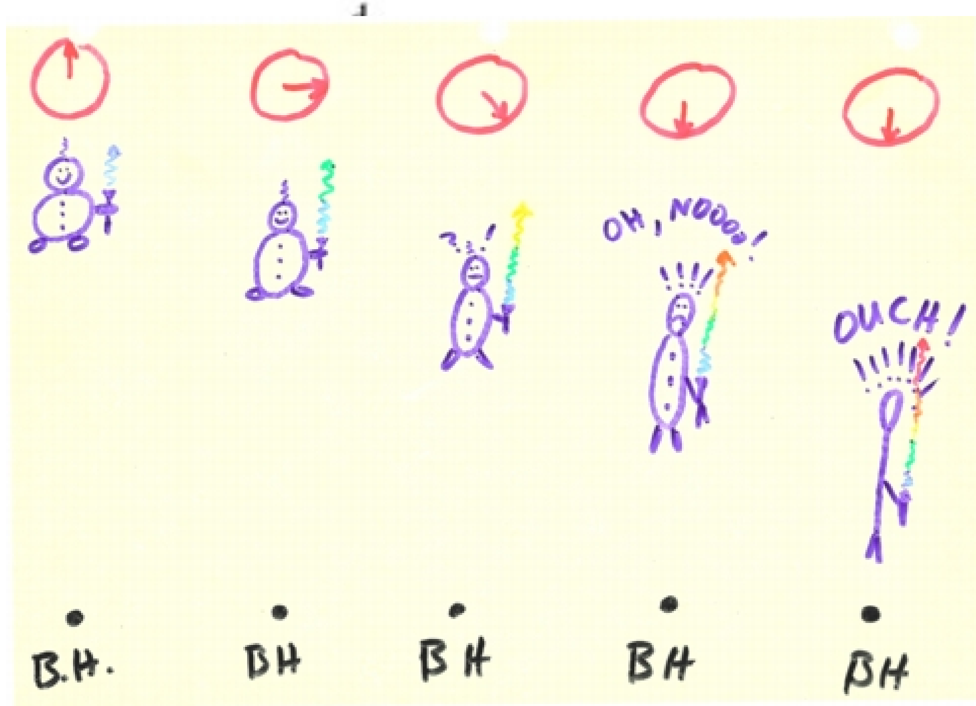
Falling into a black hole

- With a sufficiently ***large*** black hole, a **freely falling observer** would pass right through the event horizon in a finite time, **would not feel the event horizon**.
- A **distant observer** watching the freely falling observer would never see him/her fall through the event horizon (takes an **infinite time**).
- Signals sent from the freely falling observer would be **time dilated** and **redshifted**.

Falling into a black hole



Falling into smaller BH, the freely falling observer would be ripped apart by **tidal effects**.



Falling into a black hole

- Once **inside the event horizon**, **no communication** with the universe outside the event horizon is possible.
- But **incoming** signals from external world can enter.
- A black hole of mass M has exactly the same gravitational field as an ordinary mass M **at large distances**.

No-hair theorem

Three parameters completely describe the structure of a BH

- Mass ($\boldsymbol{M}$)
  - As measured by the black hole's effect on orbiting bodies, such as another star
- Total electric charge ($\boldsymbol{Q}$)
  - As measured by the strength of the electric force (Q = 0)
- Spin = angular momentum (a_*)
  - How fast the black hole is spinning ($a_* < 1$)