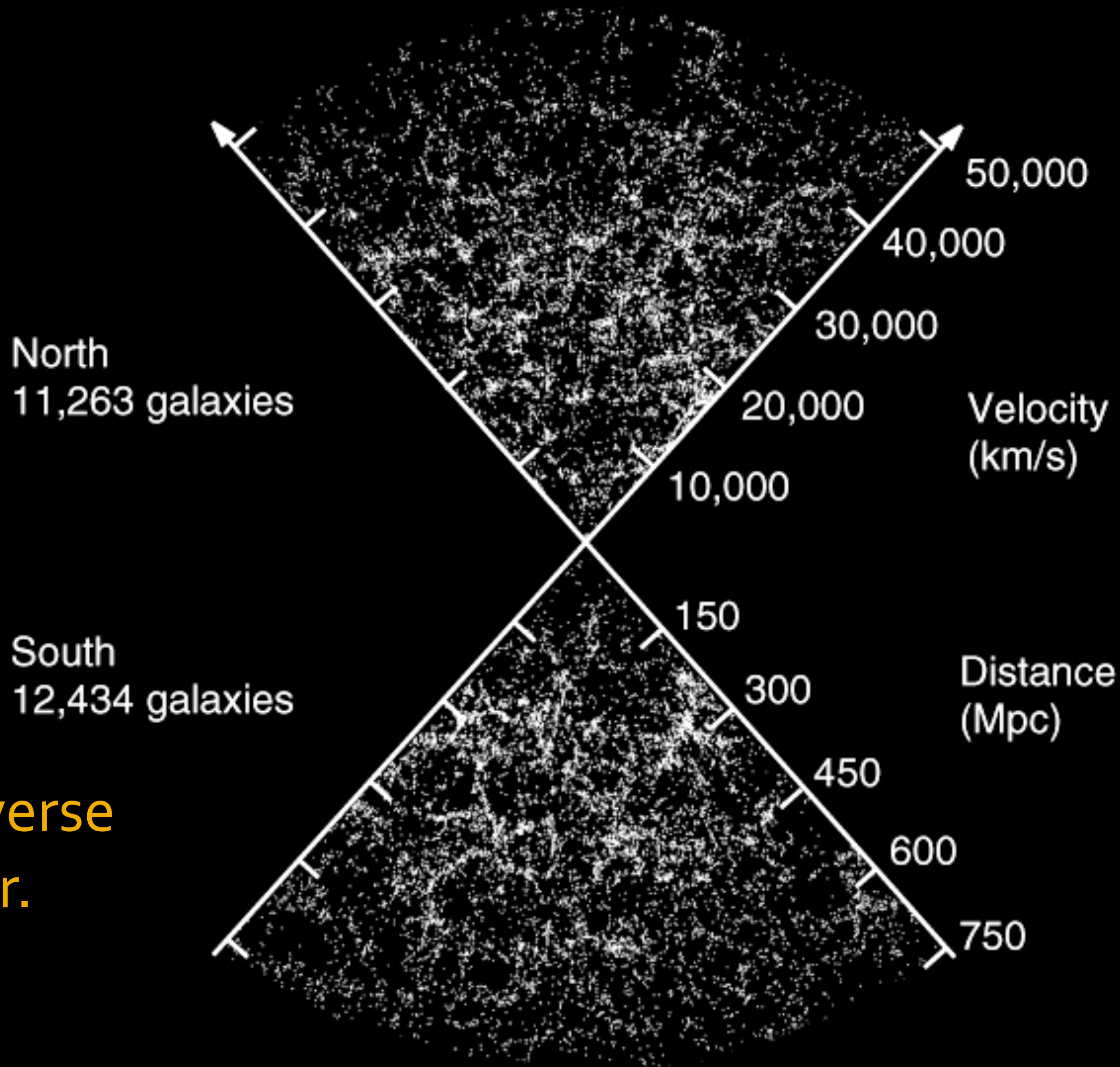
Cosmology

- What is the Universe made of?
- How big is it?
- How old is it?
- How did it form?
- What will happen to it?

The Cosmological Principle : the universe is homogeneous and isotropic on sufficiently large scales

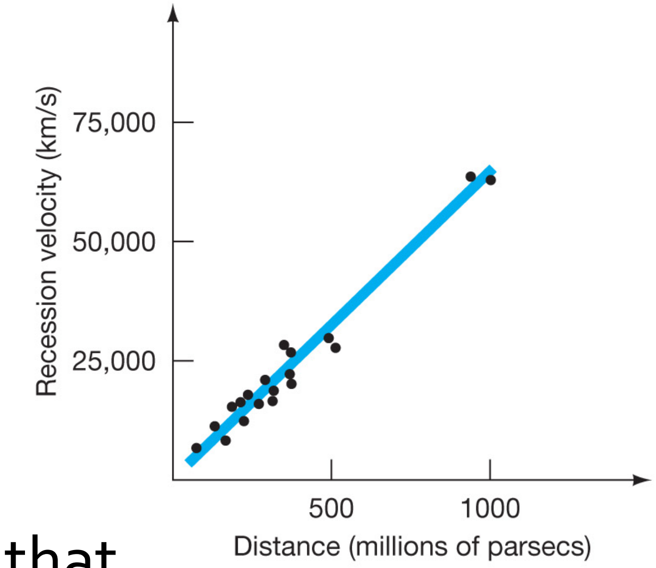
The universe looks pretty much like this everywhere: "walls" and "voids" are present but no larger structures are seen....



It follows that the Universe has no "edge" or center.

Universal expansion: Hubble's law

$$\textit{recession velocity} = H_o \times \textit{distance}$$



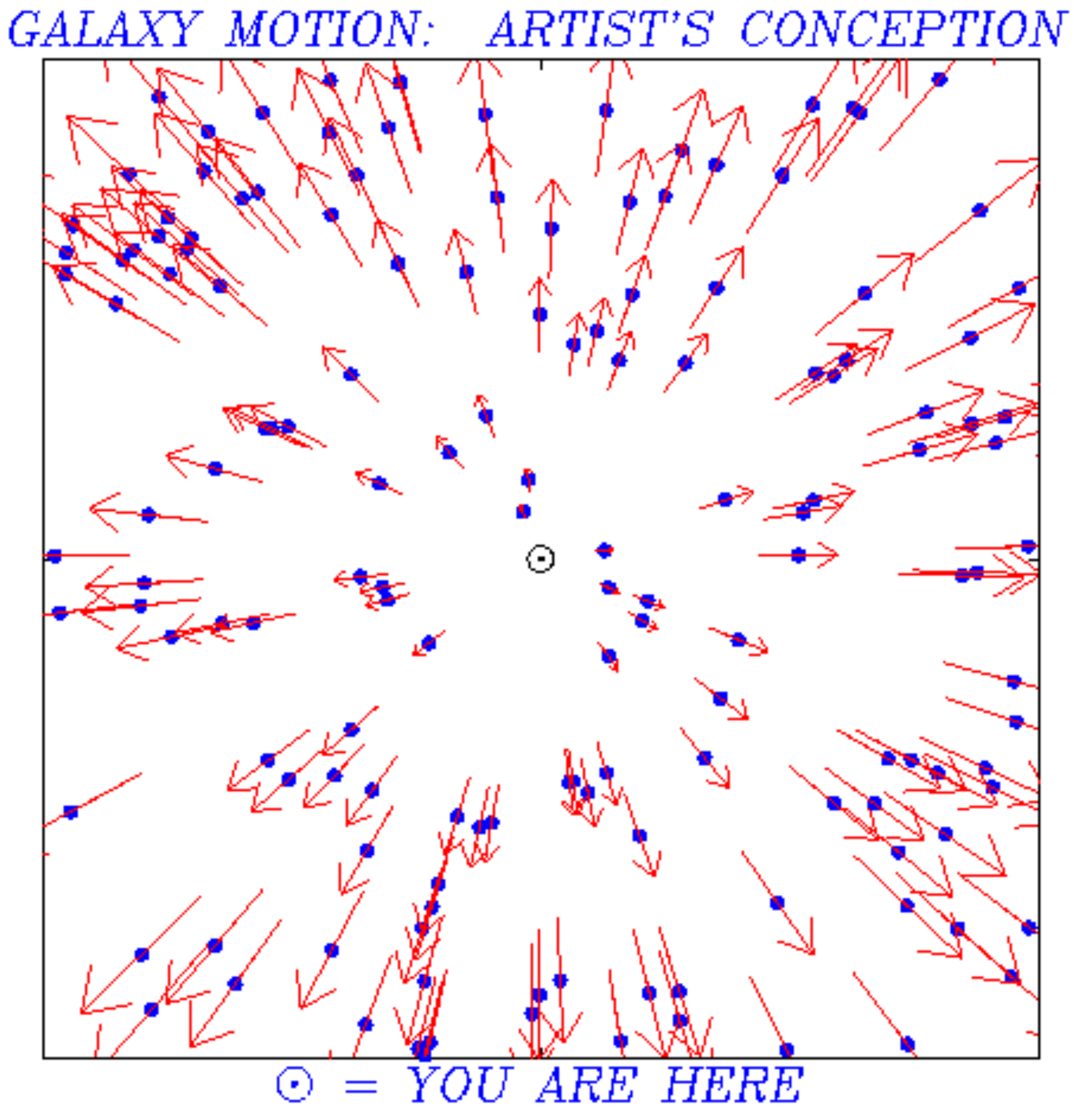
- The cosmological principle does not imply that the Universe is *constant* at all times (this was once thought to be the case - **Steady State Universe**).

- Universal **expansion** points to a beginning of the Universe and implies that the Universe is changing over time

What does Hubble's Law mean?

In a homogenous Universe, what does the farther away the faster they move away mean?

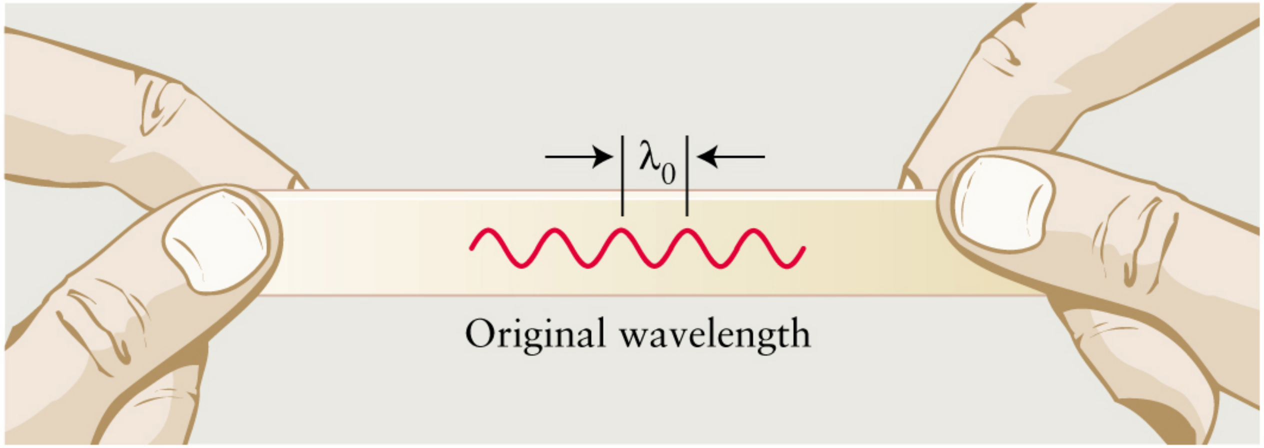
Are we at the **center of the Universe**?!



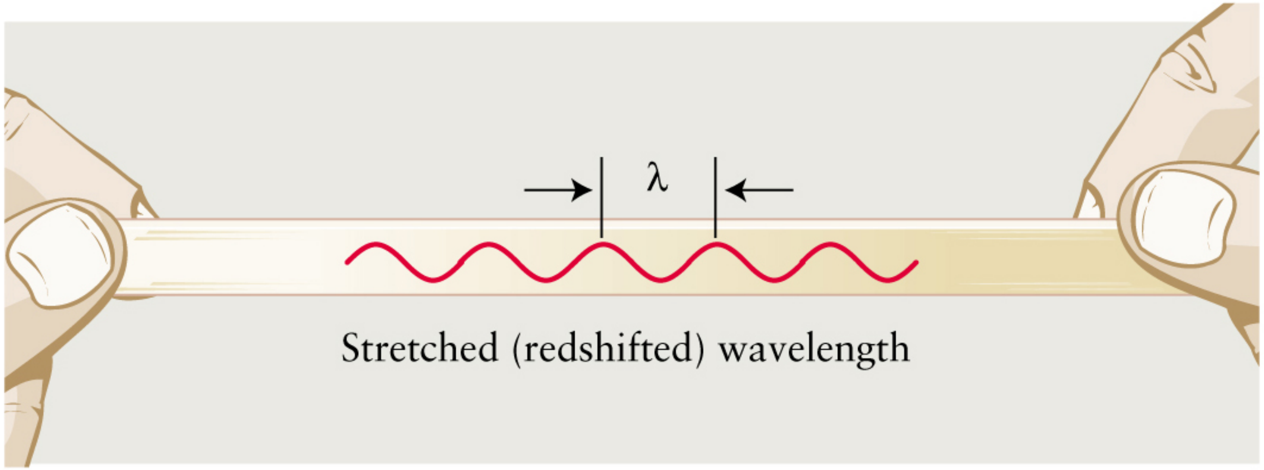
Expanding Cake Analogy

Just as all the chocolate chips move apart as the cake rises, all the galaxies recede from each other as the universe expands.

Cosmological Redshift



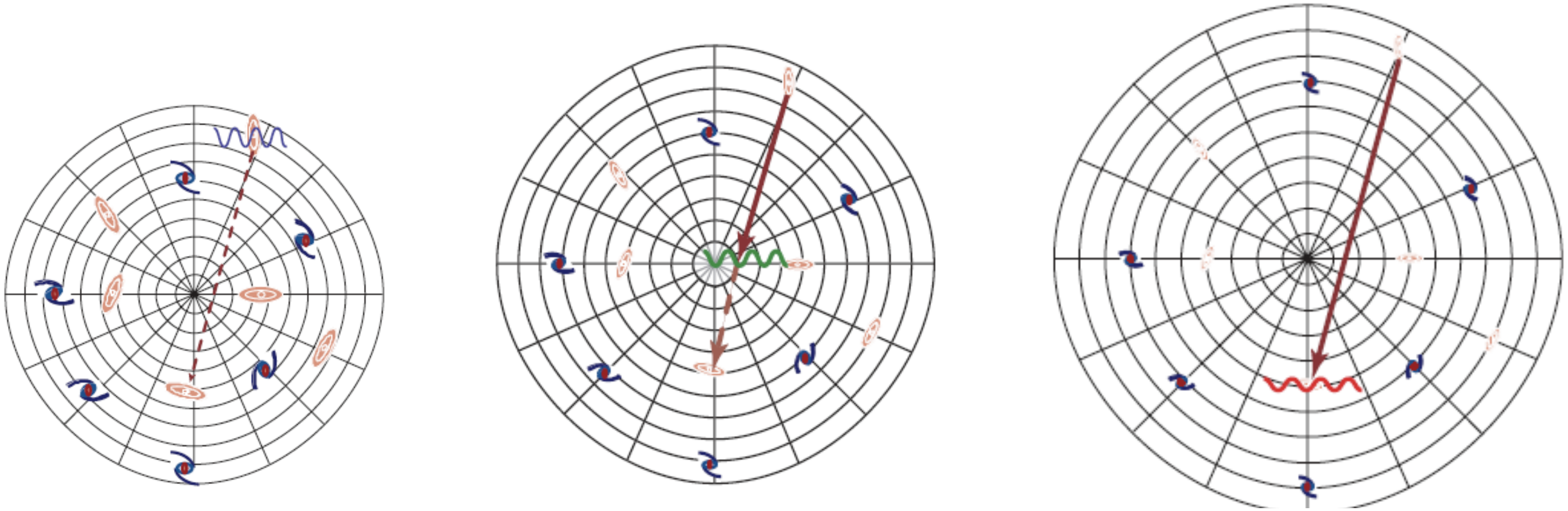
a A wave drawn on a rubber band …



b … increases in wavelength as the rubber band is stretched.

Cosmological Redshift

As Universe expands, λ expands proportionally to the scale factor **R(t)** = $r(t)/r(t_o)$.



If radiation is emitted at wavelength λ_1 at epoch t_1, and detected at wavelength λ_2 at epoch t_2, then

$$\frac{\lambda_2}{\lambda_1} = \frac{R(t_2)}{R(t_1)}$$

If we let t_1 be some arbitrary time, and t_2 be the reference epoch t_0 (for which $R = 1$), this becomes

$$\frac{\lambda_0}{\lambda} = \frac{1}{R(t)}$$

We define the *redshift*, z, to be

$$z = \frac{\Delta\lambda}{\lambda}$$

where $\Delta\lambda = \lambda_0 - \lambda$, so $\lambda_0/\lambda = 1 + z$. This gives

$$1 + z = \frac{1}{R(t)}$$

Remember, since the radiation is emitted before the reference epoch, $R(t) < 1$, so $z > 0$.

Since redshift arises due to expanding wavelength of all photons traveling through an expanding Universe, it is called a **cosmological redshift.**

As a consequence, we don't normally convert a z to a **distance**, since we need to assume a particular model for how R(t) has evolved.

We often need to study events (objects) spread out in space and time so we must compute distances between 2 events in **4D spacetime**

Space-time interval in **spherical coordinates**

$$(ds)^2 = (cdt)^2 - R^2(ct)[(dr)^2/(1-\kappa r^2) + r^2[(d\theta)^2+\sin^2\theta(d\phi)^2]]$$

Robertson-Walker metric

where R is the scale factor of the Universe and r, θ, ϕ are the usual spherical coordinates of objects (like galaxies). These are the *comoving coordinates* of a point in space. If the expansion of the universe is homogeneous and isotropic, comoving coordinates are constant with time.

The **photons** from distant galaxies follow *null geodesics* (where **ds=0**).

The universe probably originated in an explosion called the Big Bang

- In the 1940s, based on Hubble's Law, George **Gamow** proposed that the universe began in a colossal explosion
- In the 1950s, the term **BIG BANG** was coined by an unconvinced Sir Fred Hoyle
- In the 1990s, there was an international competition to **rename** the BIG BANG with a more appropriate name, but no new name was selected

BIG BANG is a relatively simple idea

- If the universe is expanding, it must have been **smaller in the past**
- If it was smaller in the past, then something must have made it **begin** to expand
- This “event” is called the **BIG BANG**
- The **age of the universe** is simply the separation distance of the most distant galaxies divided by their recessional velocities

The Age of the Universe

Hubble: $\mathbf{v = H\,D}$
$\boldsymbol{D}$**:** distance between 2 particles (galaxies)

If **constant** speed,
distance = speed x time

$$\mathbf{D = v\,t}$$

Put it all together:

$$\mathbf{v = H\,D = H\,(v\,t) = H\,t\,v}$$

so: $\boldsymbol{H\,t = 1}$

$$\boldsymbol{t=1/H}$$

“expansion age” of Universe

The Age of the Universe

Other methods to date the Universe:

- **Radioactivity** in Rocks

  uranium decays to lead

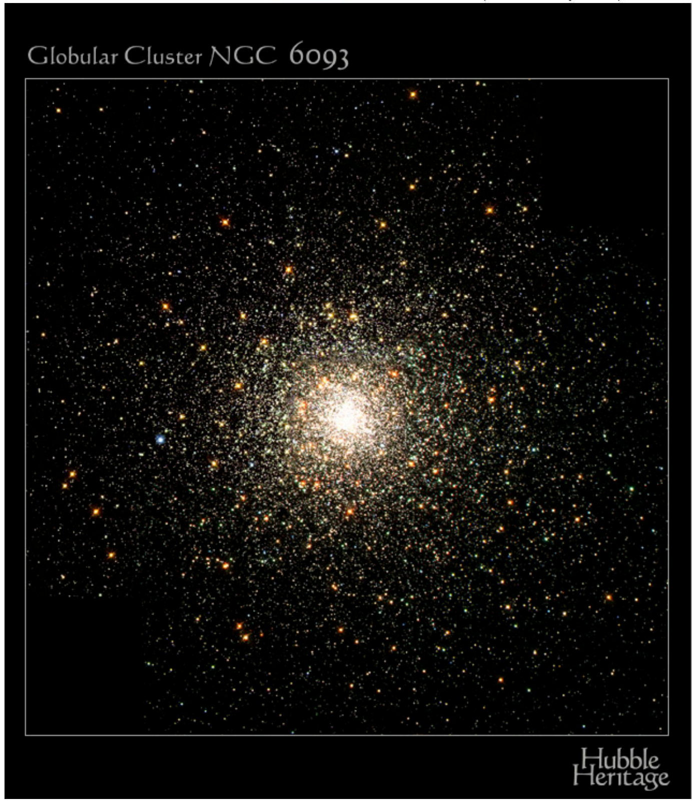
  decay is "clock": tells time since uranium made in star

  age > 10 billion yrs

- **Globular clusters**

  oldest stars

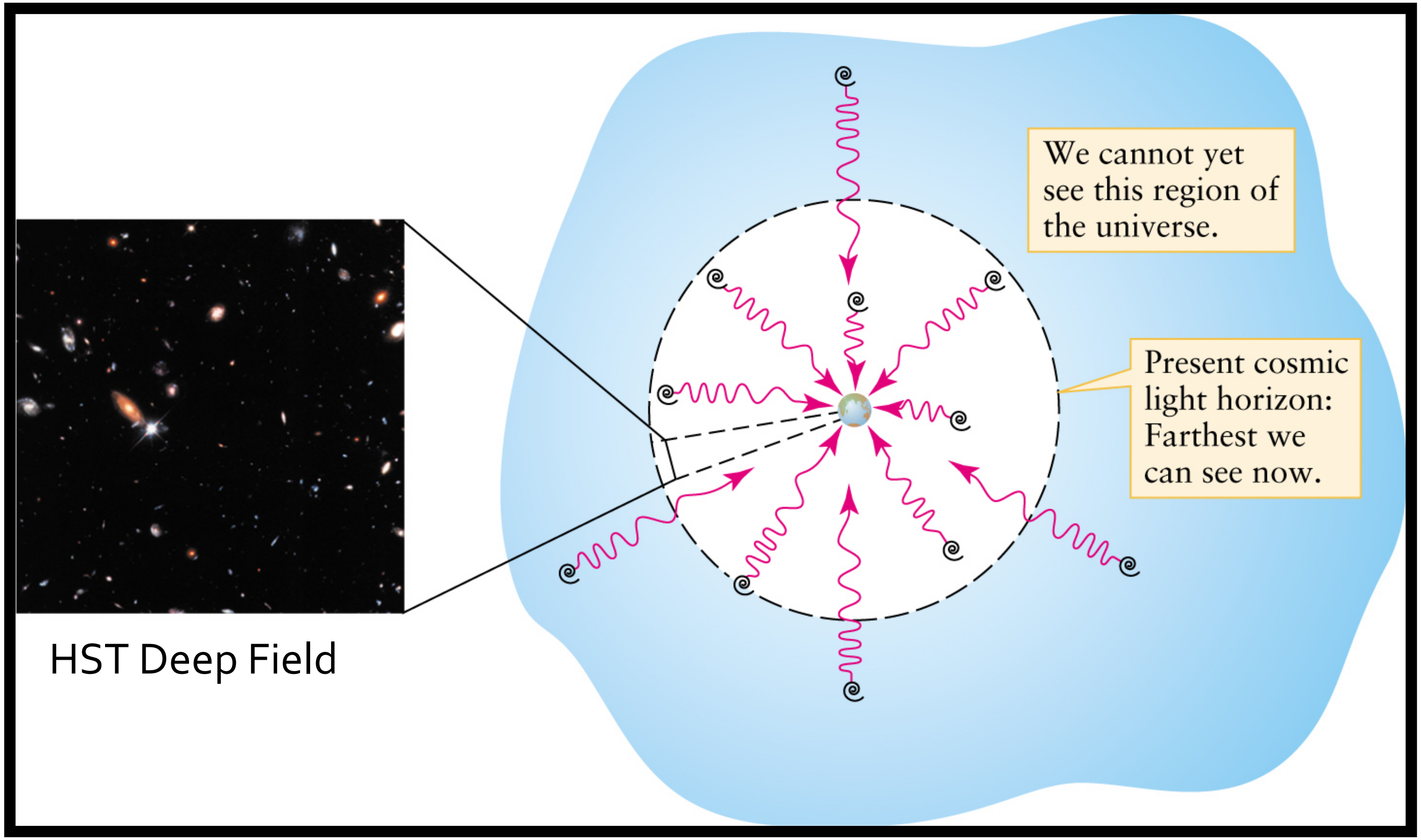
  age about 13 billion years



Best estimate (Planck, from CMB):

- Age ***t* = 13.82 billion years**

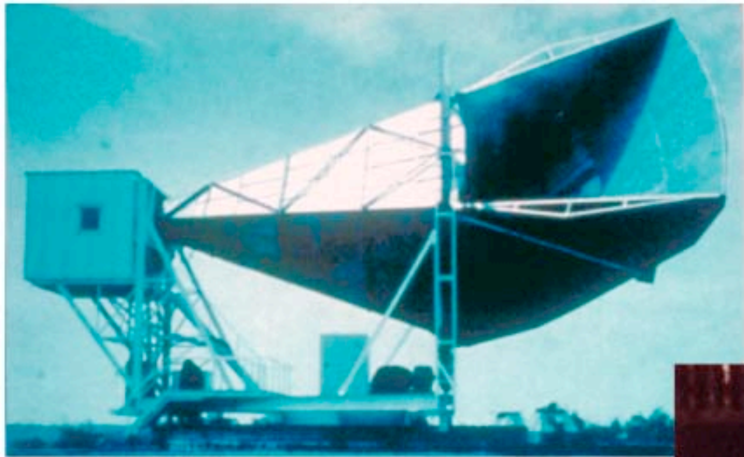
Observable Universe



The cosmic light **horizon** today is about 13.8 billion light-years away in all directions. The **farthest galaxies** we can see are as they were within a few hundred million years after the Big Bang.

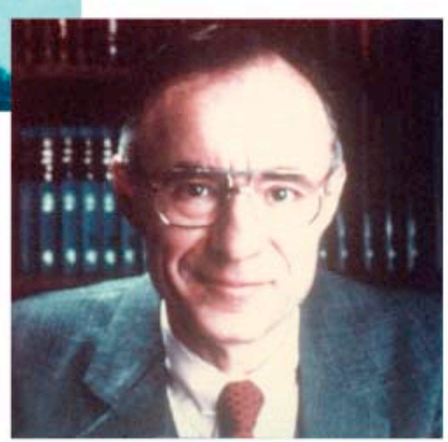
The Early Universe was *HOT*

- If the early Universe was so hot, we should be able to see the **blackbody** radiation, **redshifted**
- It is redshifted down to the **microwaves**. Called the Cosmic Microwave Background (**CMB**).
- First detected by Robert **Wilson** and Arno **Penzias** in 1965 (1978 Nobel prize).



Microwave Receiver

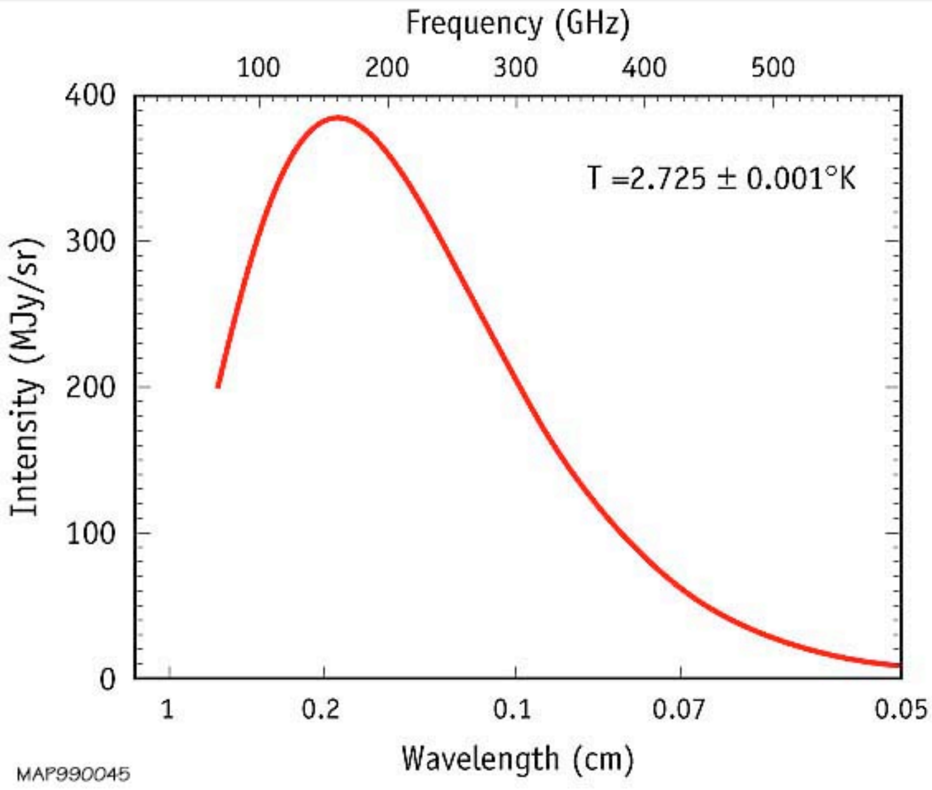
Robert Wilson



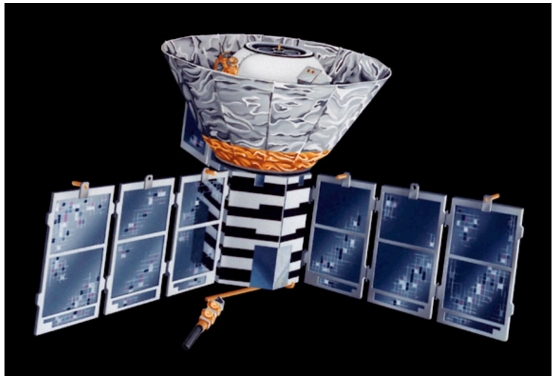
Arno Penzias

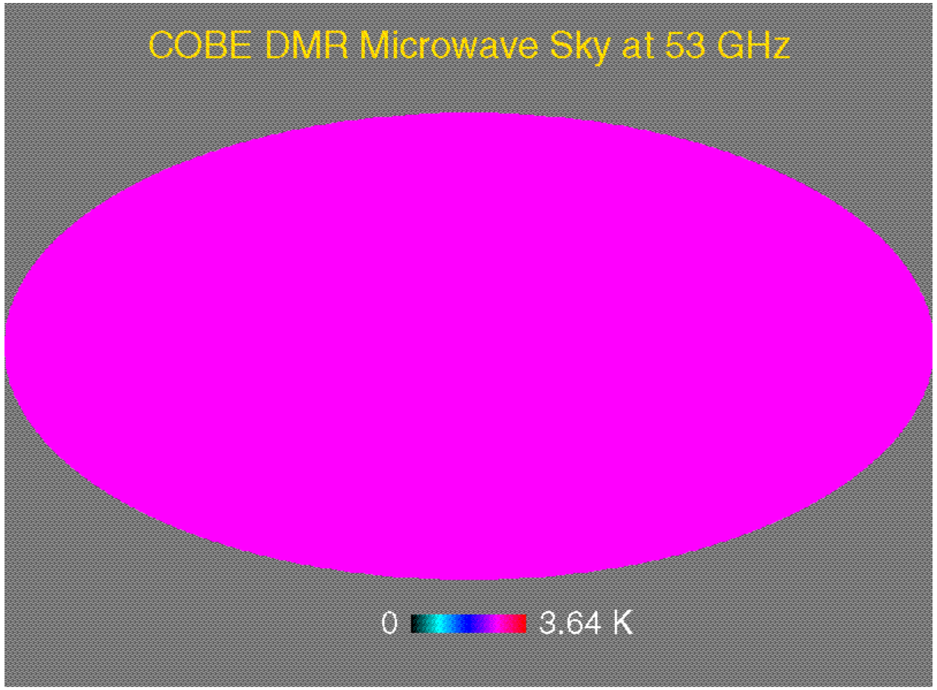
MAP990045

CMB: a perfect blackbody?



Cosmic Background Explorer (**COBE**) satellite (launched in 1989)

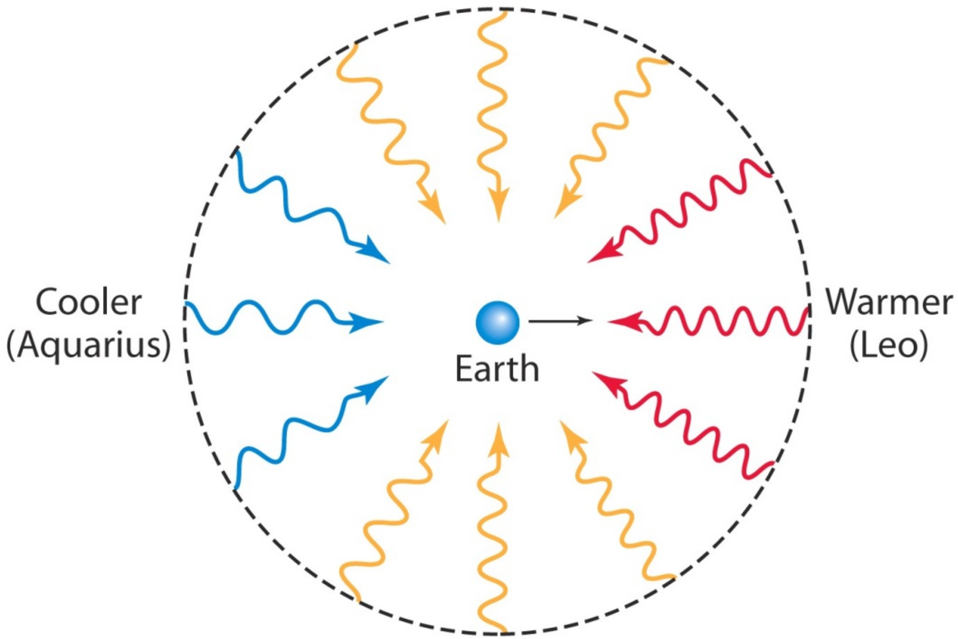


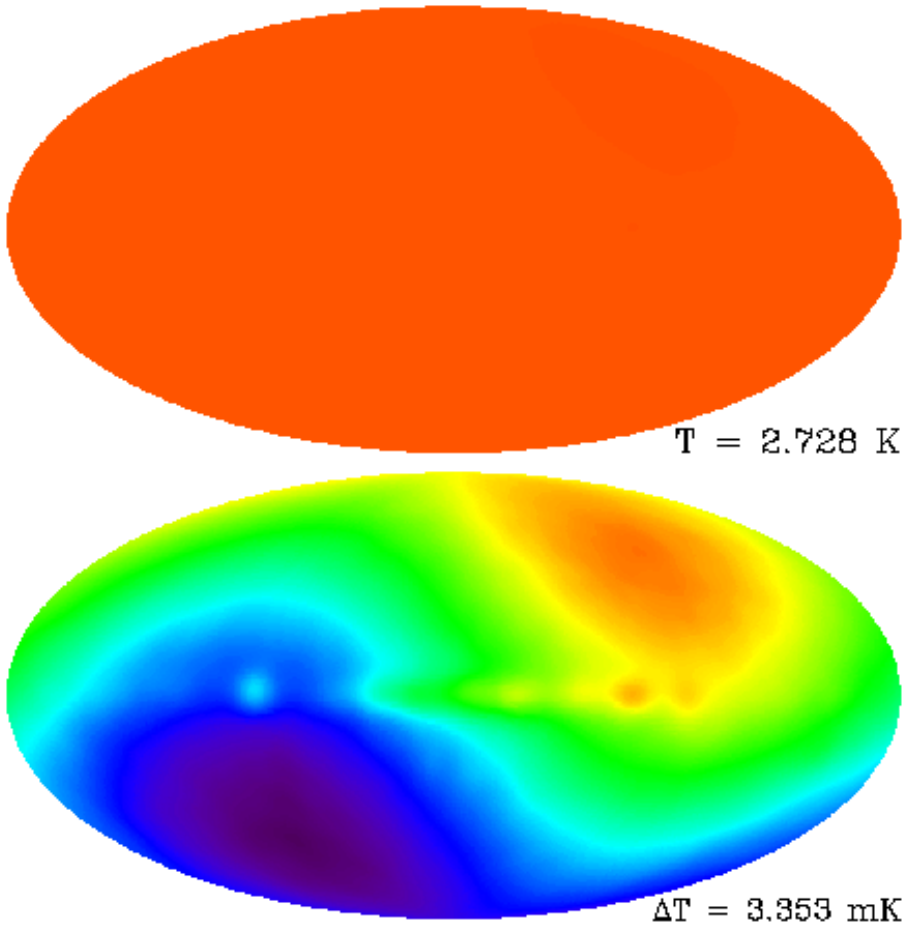


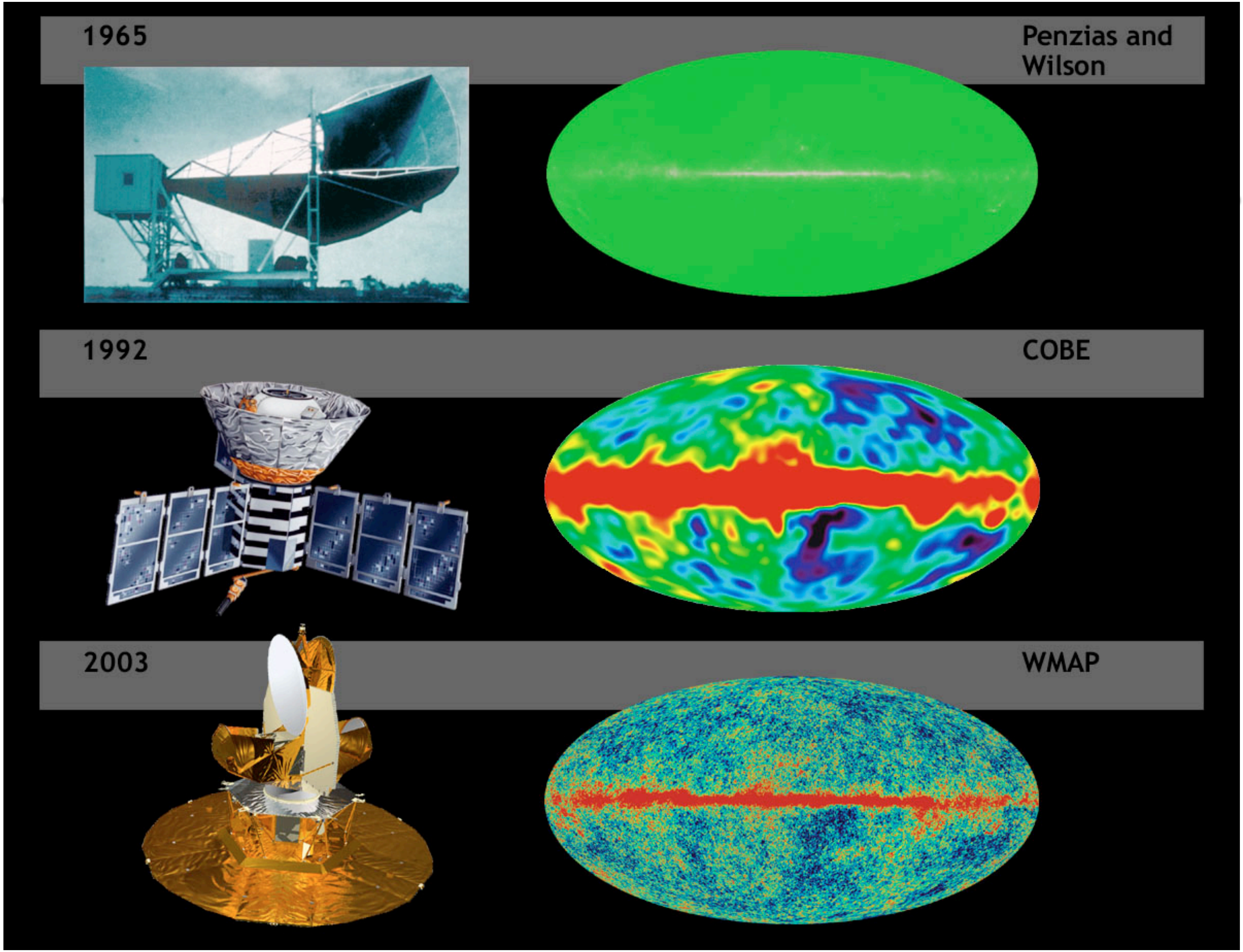
Is the spectrum perfectly Planckian and the temperature identical in every direction?

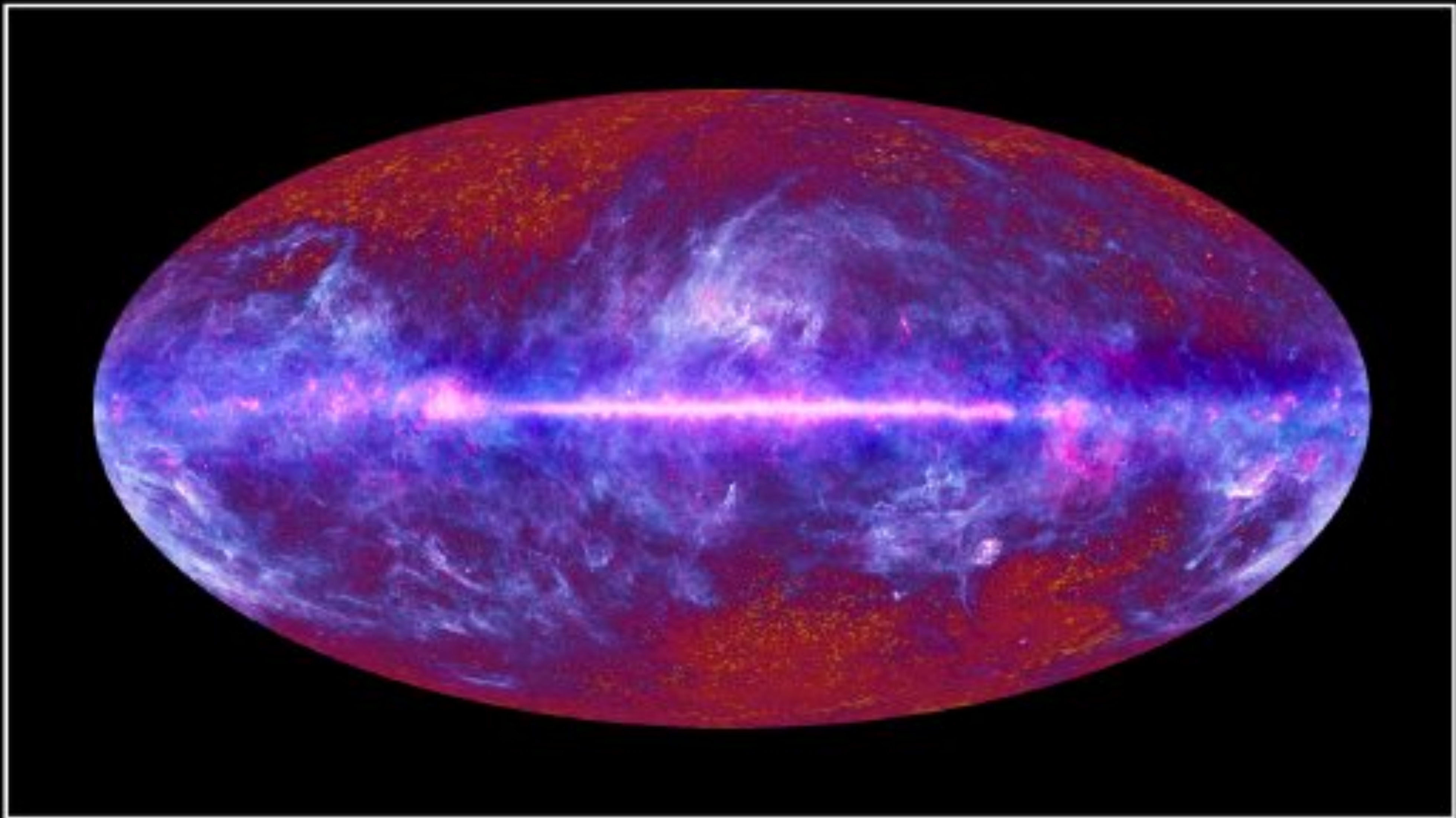
Small Anisotropy

- Small scale variation, due to our movement with respect to the background.
- We are moving at ~600 km/s

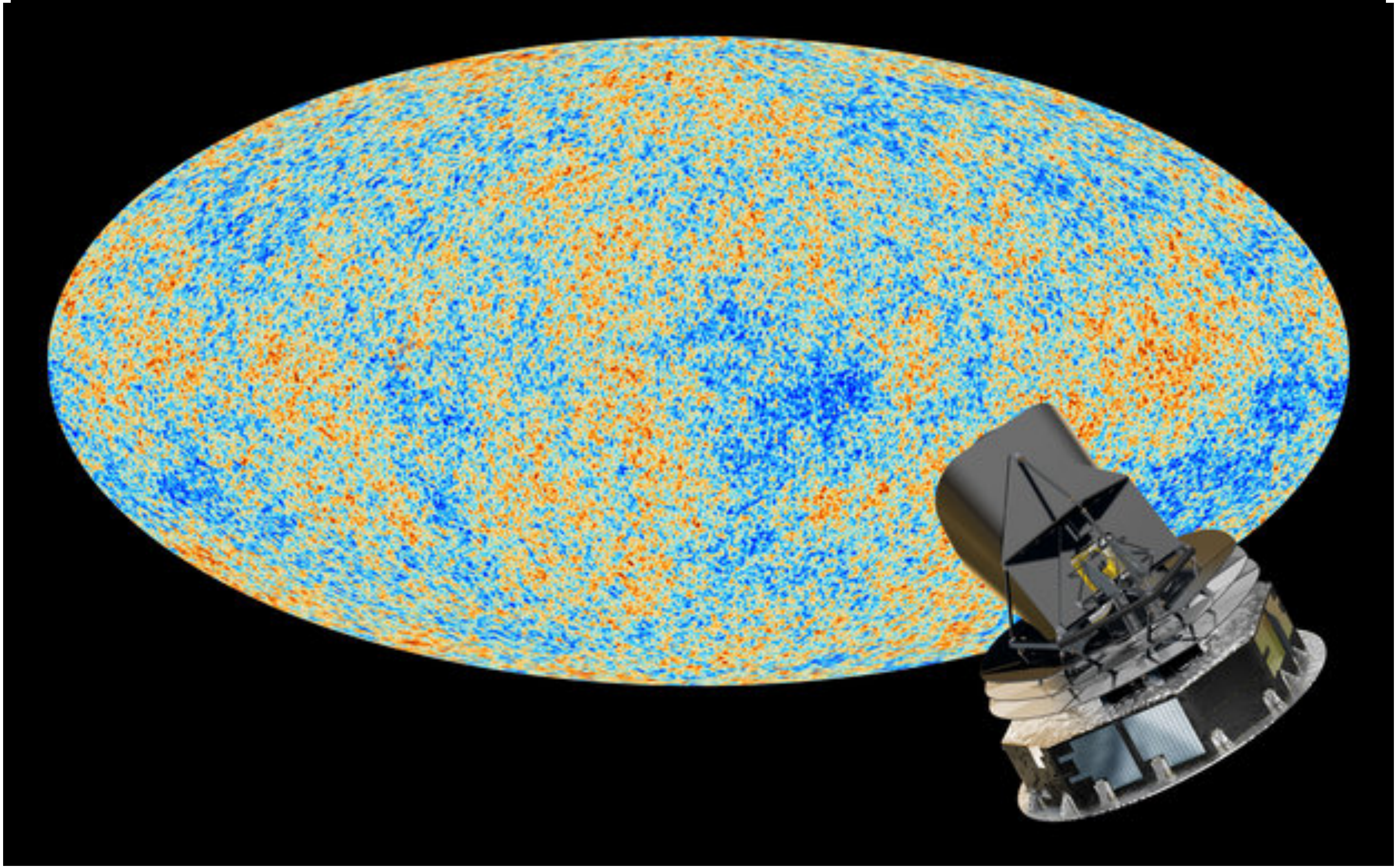




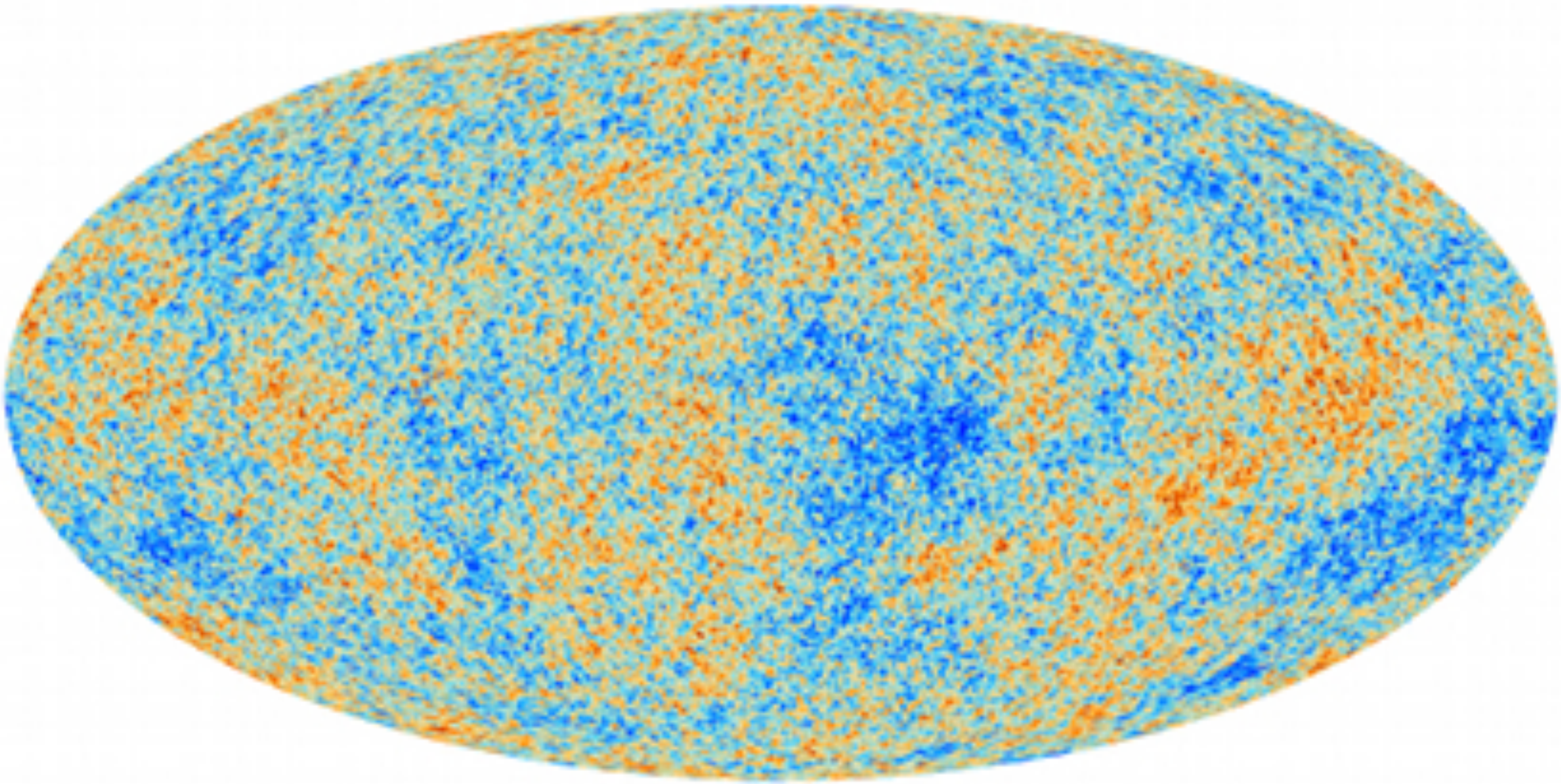

1965
Penzias and Wilson
1992
COBE
2003
WMAP

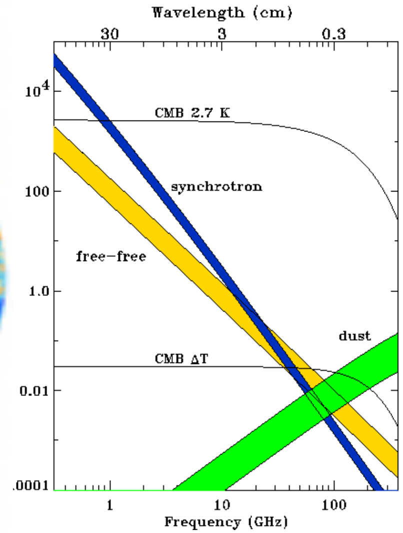


The Planck one-year all-sky survey

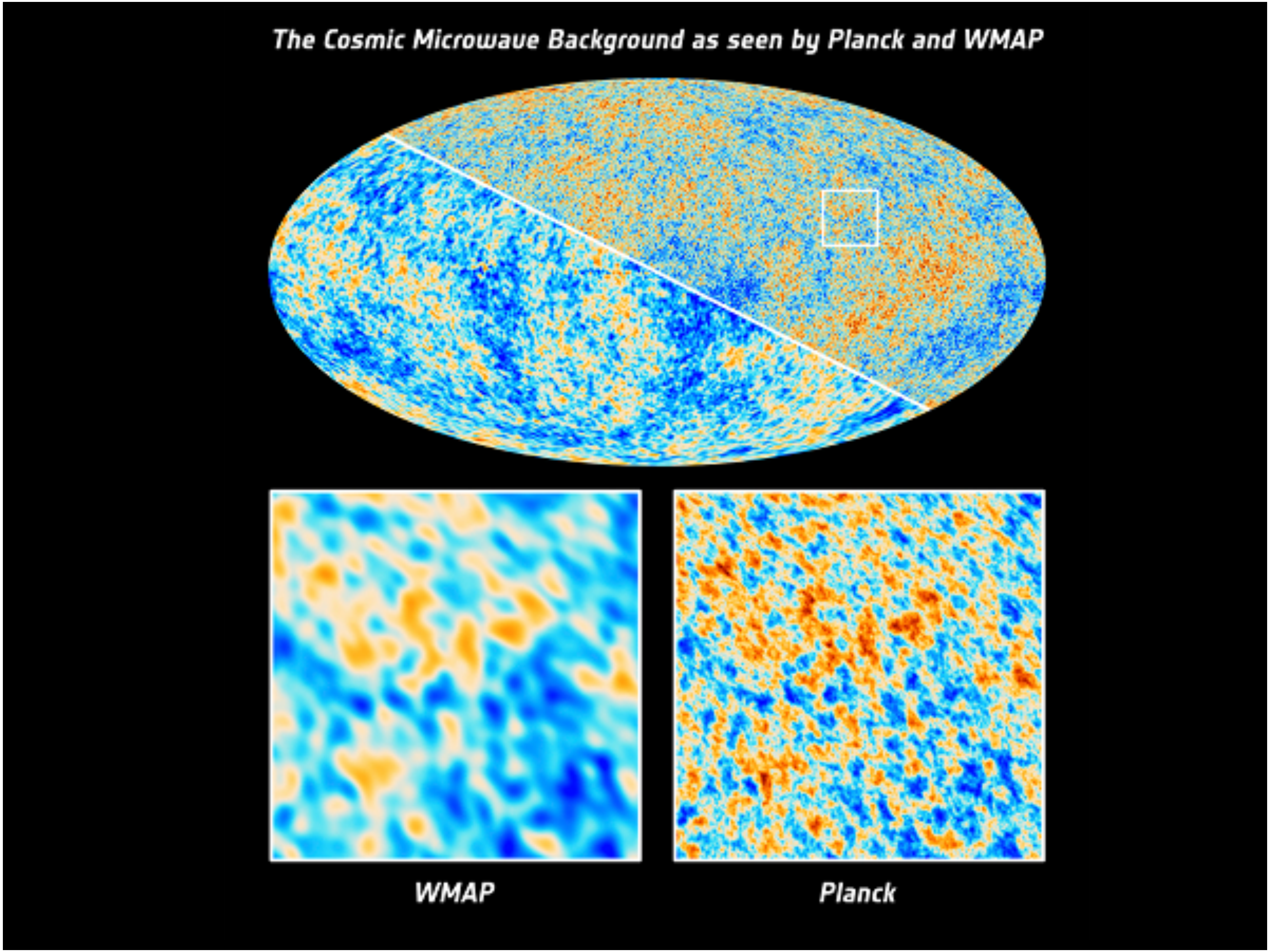


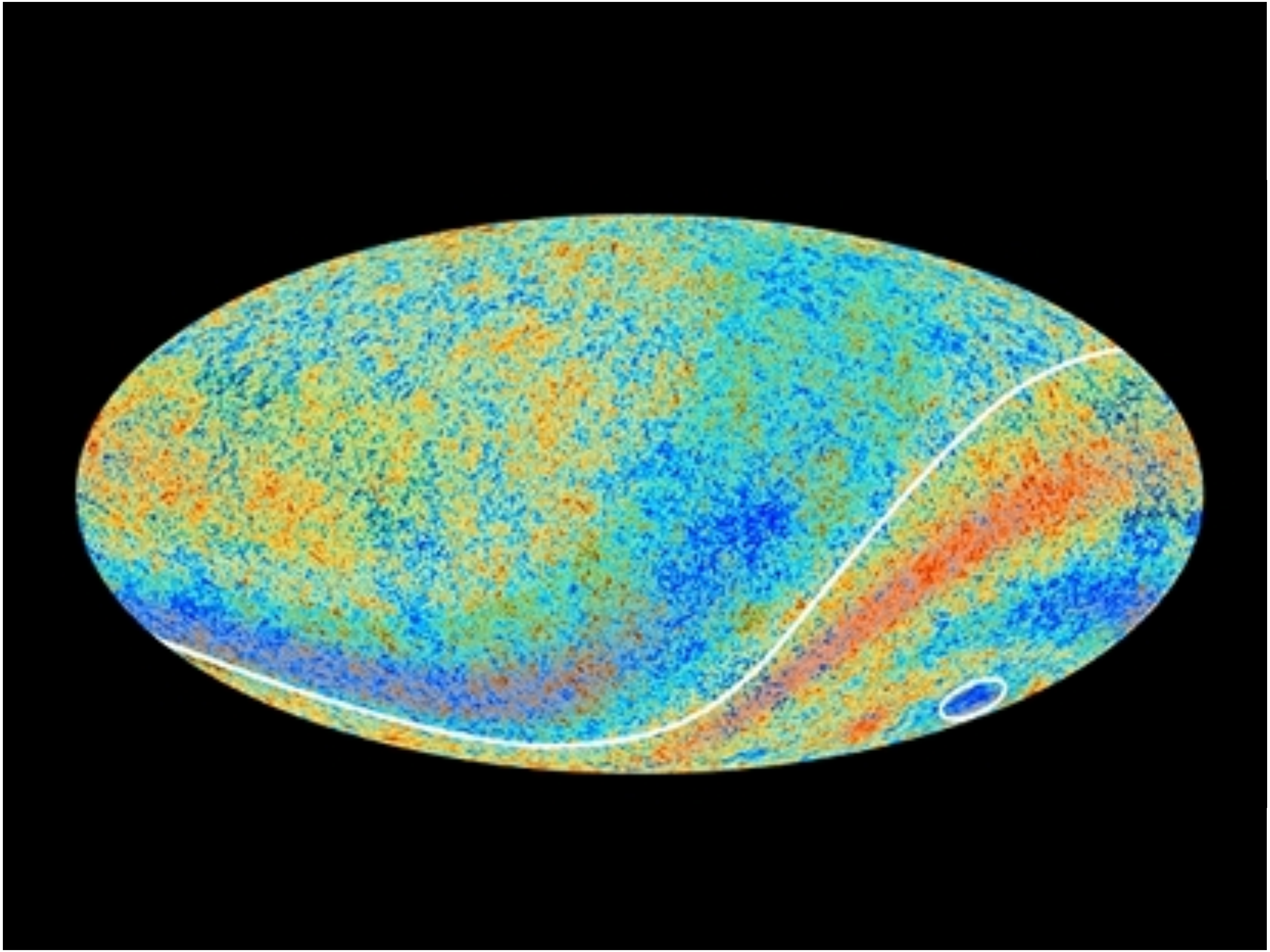
The Seeds of Galaxies





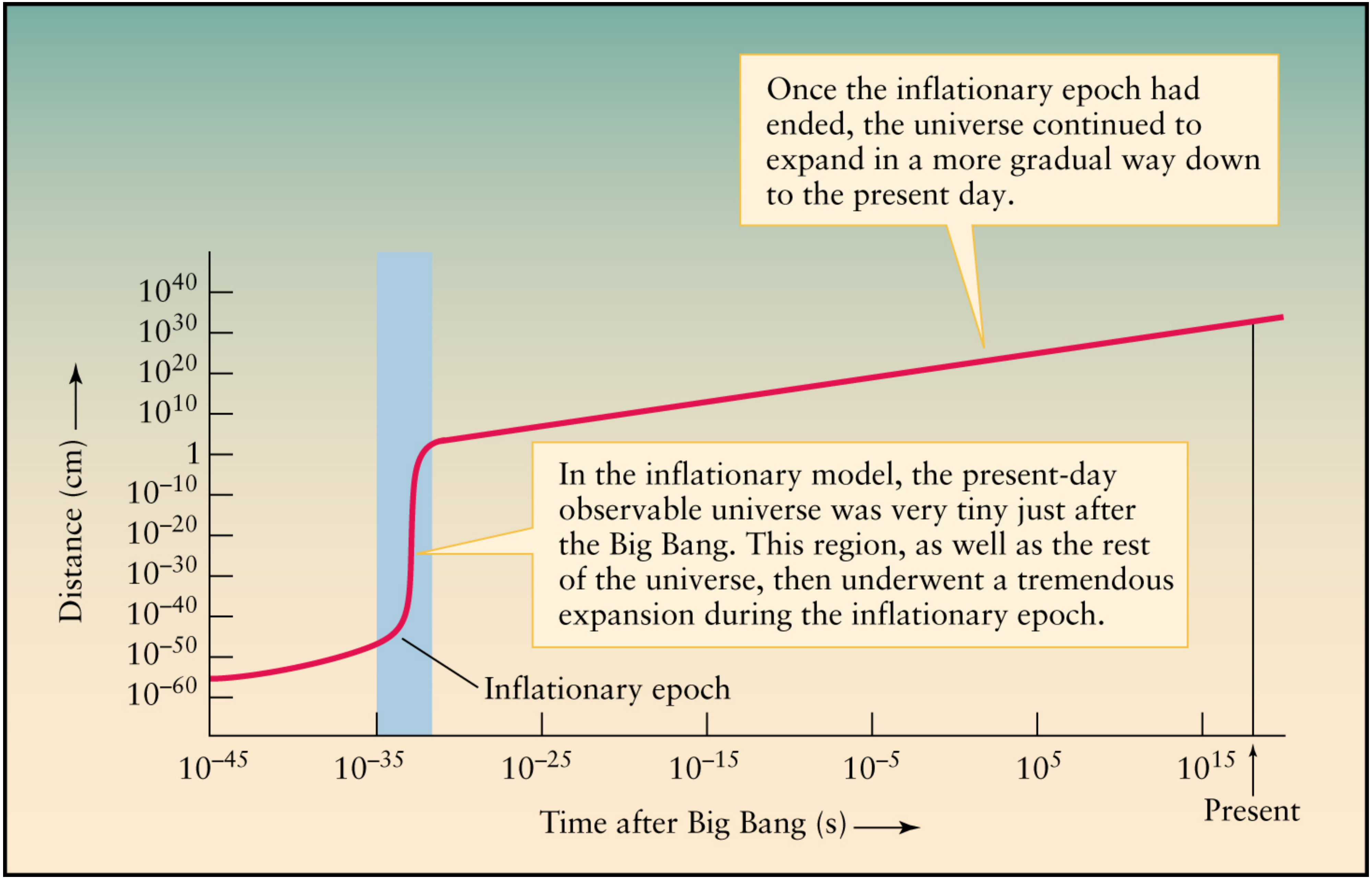
These small (< 1/100,000) perturbations are the **fluctuations** that caused the **large scale structures** we see today. Discovered by COBE (2006 Nobel Prize to George **Smoot** and John **Mather**)


The Cosmic Microwave Background as seen by Planck and WMAP
WMAP
Planck

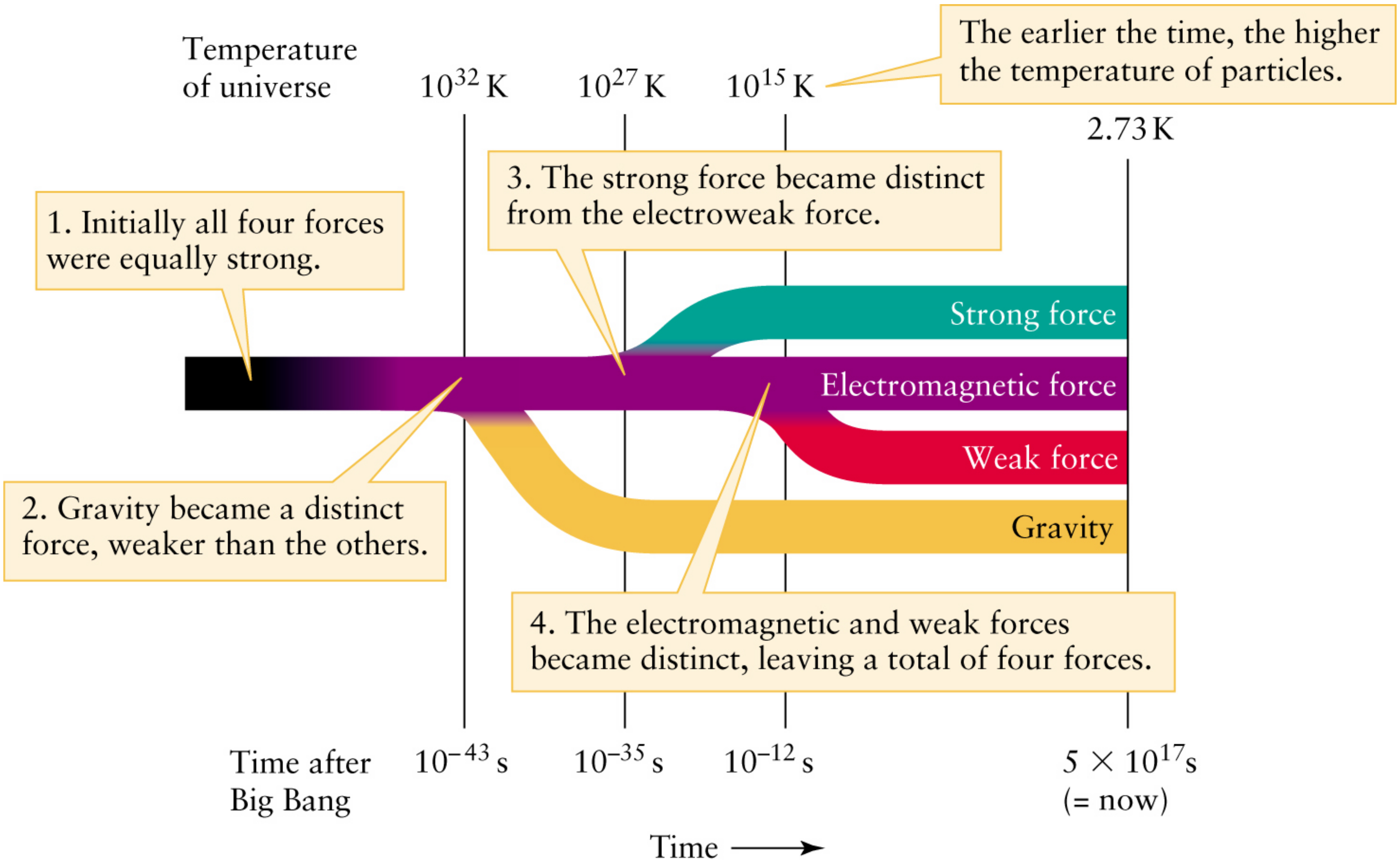


A period of vigorous inflation followed the Big Bang

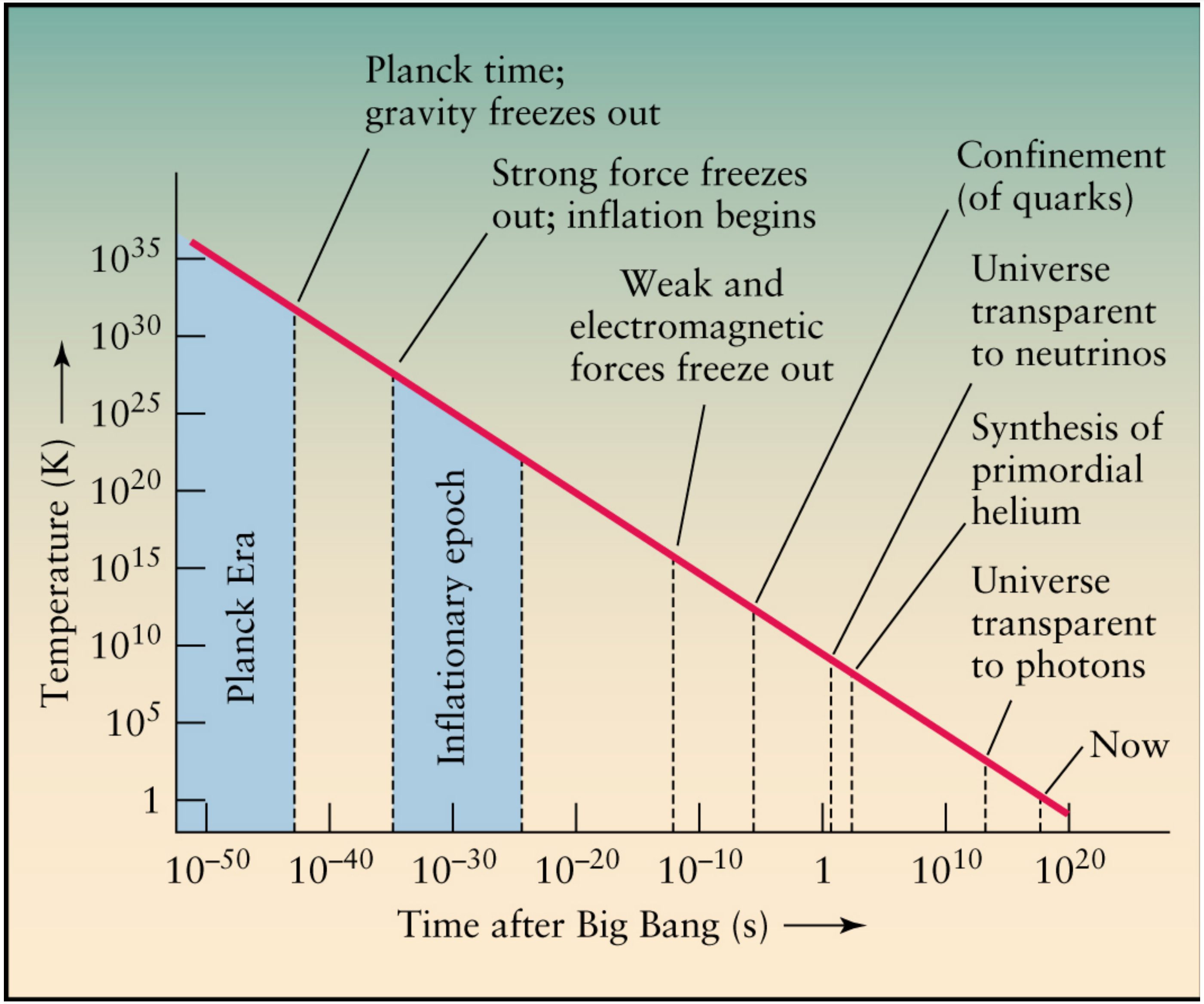
- PROBLEM: If the universe is at least 26 billion light years across (13 billion in each direction), how could both sides have exactly the **same temperature** if they couldn't "communicate"?
- SOLUTION: During the first fraction of second, the universe underwent a rapid expansion, called the **inflationary epoch**, in which it became many times larger than its original size.


Once the inflationary epoch had ended, the universe continued to expand in a more gradual way down to the present day.
In the inflationary model, the present-day observable universe was very tiny just after the Big Bang. This region, as well as the rest of the universe, then underwent a tremendous expansion during the inflationary epoch.
Inflationary epoch
Distance (cm)
10^{40}
10^{30}
10^{20}
10^{10}
1
10^{-10}
10^{-20}
10^{-30}
10^{-40}
10^{-50}
10^{-60}
10^{-45}
10^{-35}
10^{-25}
10^{-15}
10^{-5}
10^{5}
10^{15}
Present
Time after Big Bang (s)

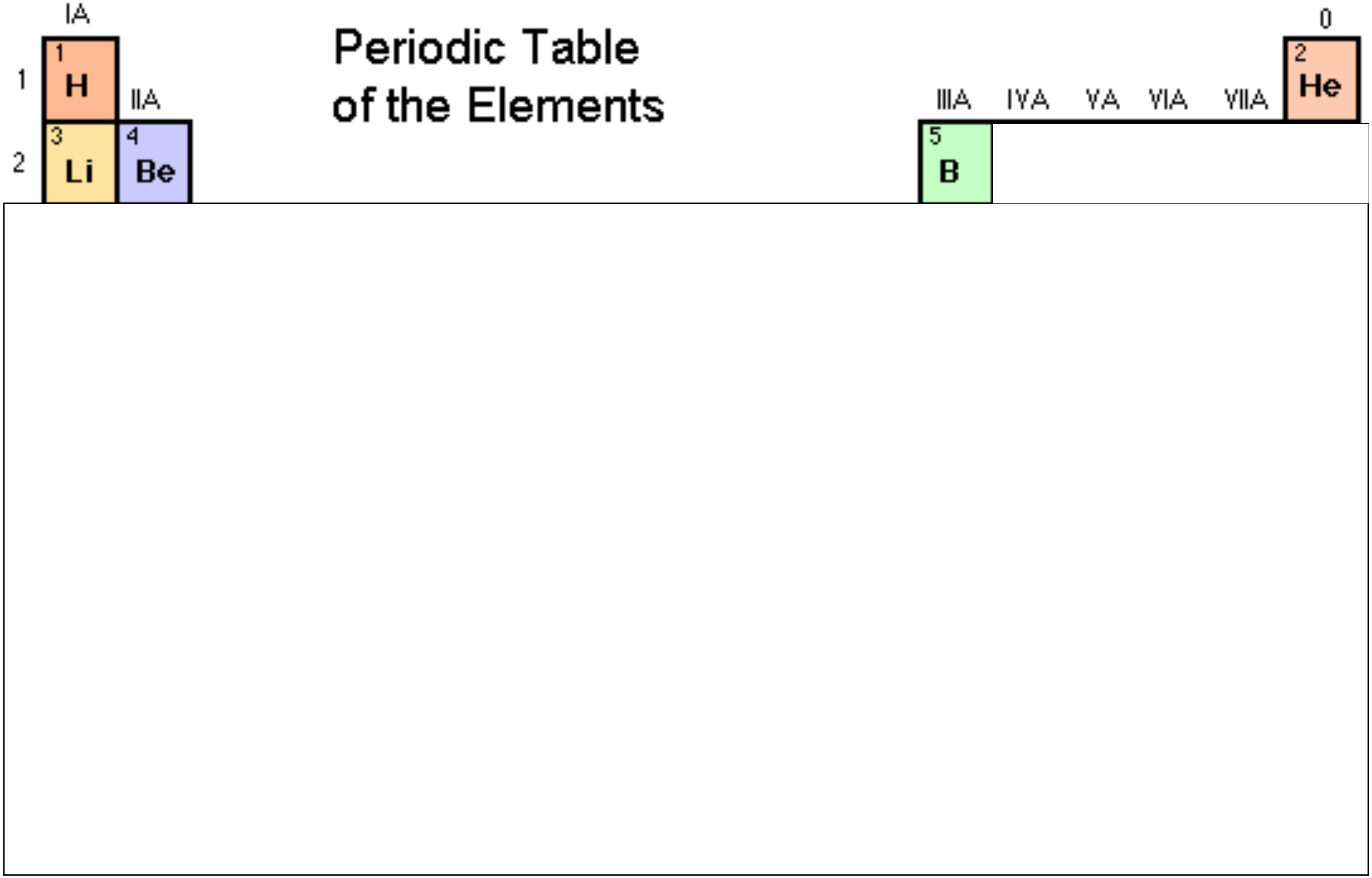
Unification of the Four Forces



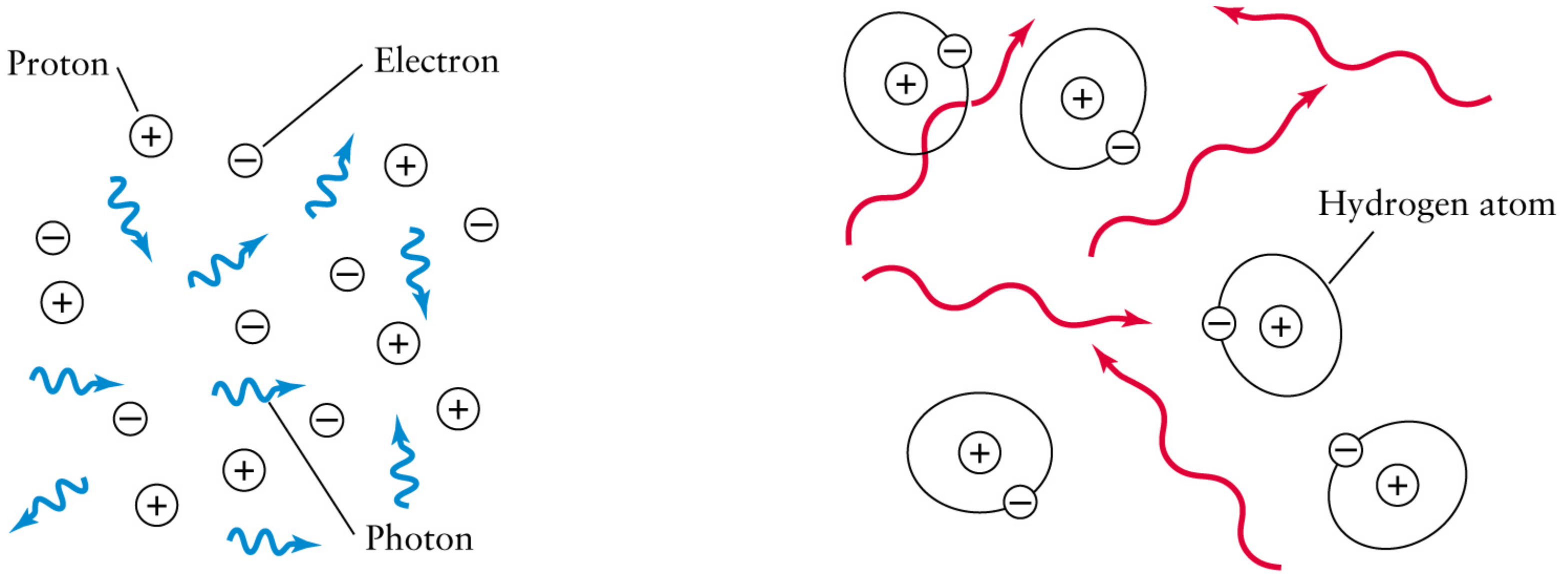
Early History of the Universe



Cosmological Nucleosynthesis



Era of Recombination



a Before recombination:
- Temperatures were so high that electrons and protons could not combine to form hydrogen atoms.
- The universe was opaque: Photons underwent frequent collisions with electrons.
- Matter and radiation were at the same temperature.

b After recombination:
- Temperatures became low enough for hydrogen atoms to form.
- The universe became transparent: Collisions between photons and atoms became infrequent.
- Matter and radiation were no longer at the same temperature.

When (temperature, redshift) did recombination occur? ***T~3000 k; z~1000***

Friedmann Equations

General relativistic approach: Insert metric into Einstein equation to obtain differential equation for $R(t)$:

Einstein equation:

$$\underbrace{R_{\mu\nu} - \frac{1}{2}\mathscr{R} g_{\mu\nu}}_{G_{\mu\nu}} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{8.34}$$

where

$g_{\mu\nu}$**:** Metric tensor ($\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$)

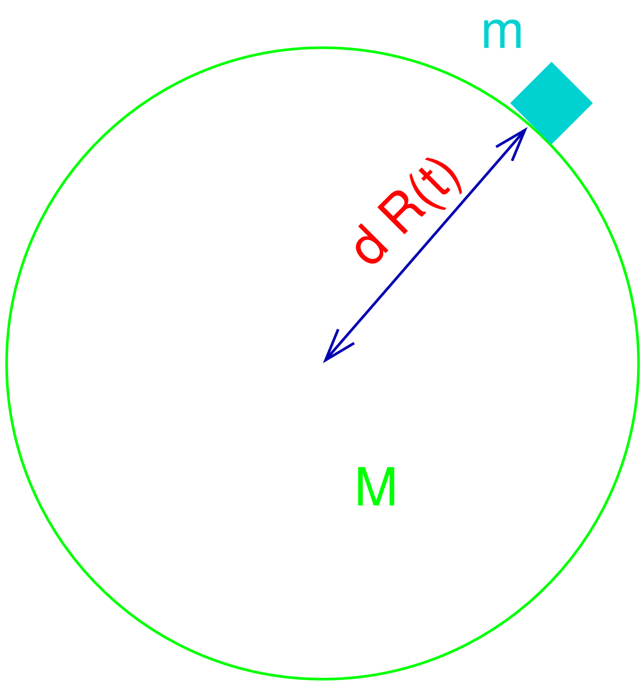
- $R_{\mu\nu}$**:** Ricci tensor (function of $g_{\mu\nu}$)
- $\mathscr{R}$**:** Ricci scalar (function of $g_{\mu\nu}$)
- $G_{\mu\nu}$**:** Einstein tensor (function of $g_{\mu\nu}$)

$T_{\mu\nu}$**:** Stress-energy tensor, describing curvature of space due to fields present (matter, radiation,...)

Λ**:** Cosmological constant

$\Longrightarrow$ Messy, but doable

Friedmann Equations



Here, Newtonian derivation of Friedmann equations: Dynamics of a mass element on the surface of sphere of density $\rho(t)$ and comoving radius d, i.e., proper radius $d \cdot R(t)$ (McCrea, 1937)

Mass of sphere:

$$M = \frac{4\pi}{3}(d\,R)^3\rho(t) = \frac{4\pi}{3}d^3\rho_0 \quad \text{where} \quad \rho(t) = \frac{\rho_0}{R(t)^3} \tag{8.35}$$

Force on mass element:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\big(d\,R(t)\big) = -\frac{GMm}{(dR(t))^2} = -\frac{4\pi G}{3}\frac{d\rho_0}{R^2(t)}\,m \tag{8.36}$$

Canceling $m \cdot d$ gives momentum equation:

$$\ddot{R}(t) = -\frac{4\pi G}{3}\frac{\rho_0}{R(t)^2} = -\frac{4\pi G}{3}\rho(t)R(t) \tag{8.37}$$

Multiplying Eq. (8.37) with $\dot{R}$ and integrating yields the energy equation:

$$\frac{1}{2}\dot{R}(t)^2 = +\frac{4\pi G}{3}\frac{\rho_0}{R(t)} + \text{const.} = +\frac{4\pi G}{3}\rho(t)R^2(t) + \text{const.} \tag{8.38}$$

where the constant can only be obtained from GR.

Friedmann Equations

Problems with the Newtonian derivation:

1. Cloud is implicitly assumed to have $r_{\text{cloud}} < \infty$

   (for $r_{\text{cloud}} \to \infty$ the force is undefined)

   $\Longrightarrow$ violates cosmological principle.

2. Particles move *through* space

   $\Longrightarrow$ $v > c$ possible

   $\Longrightarrow$ violates SRT.

Why do we get correct result?

GRT $\longrightarrow$ Newton for small scales and mass densities

Since universe is isotropic: scale invariance on Mpc scales

$\Longrightarrow$ Newton sufficient (classical limit of GR).

(In fact, point 1 above *does* hold in GR: Birkhoff's theorem).

Friedmann Equations

The exact GR derivation of Friedmanns equation gives:

$$\ddot{R} = -\frac{4\pi G}{3} R \left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right]$$
$$\dot{R}^2 = +\frac{8\pi G\rho}{3} R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right] \tag{8.39}$$

Notes:

1. For $\Lambda = 0$: Eq. (8.39) $\longrightarrow$ Eq. (8.38).
2. k determines the curvature of space (and is *not* an integer here!).
3. The density, ρ, includes the contribution of all different kinds of energy (remember mass-energy equivalence!).
4. There is energy associated with the vacuum, parameterized by the parameter Λ.

In Eq. 8.49, it will be shown that the Hubble parameter can be expressed as $H(t) = \frac{\dot{R}}{R}$.

For $\Lambda = 0$, it evolves as:

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2(t) = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} \tag{8.40}$$

The Critical Density

Solving Eq. (8.40) for k:

$$\frac{R^2}{c}\left(\frac{8\pi G}{3}\rho - H^2\right) = k \tag{8.41}$$

$\Longrightarrow$ Sign of curvature parameter k only depends on density, ρ. With

$$\rho_c = \frac{3H^2}{8\pi G} \qquad \text{and} \qquad \Omega = \frac{\rho}{\rho_c} \tag{8.42}$$

it is easy to see that:

$$\begin{aligned}
\Omega > 1 &\Longrightarrow k > 0 \Longrightarrow \text{closed universe}\\
\Omega = 1 &\Longrightarrow k = 0 \Longrightarrow \text{flat universe}\\
\Omega < 1 &\Longrightarrow k < 0 \Longrightarrow \text{open universe}
\end{aligned}$$

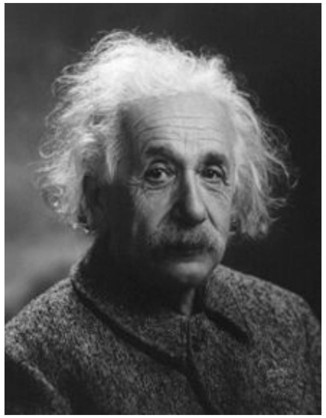
ρ_c is called the critical density.

For $\Omega \leq 1$ the universe will expand until ∞,

For $\Omega > 1$ we will see the "big crunch".

Current value of ρ_c: $\sim 1 \times 10^{-23}\,\mathrm{g\,cm^{-3}}$ ($3 \ldots 10$ H-atoms $\mathrm{m^{-3}}$).

Cosmology and General Relativity

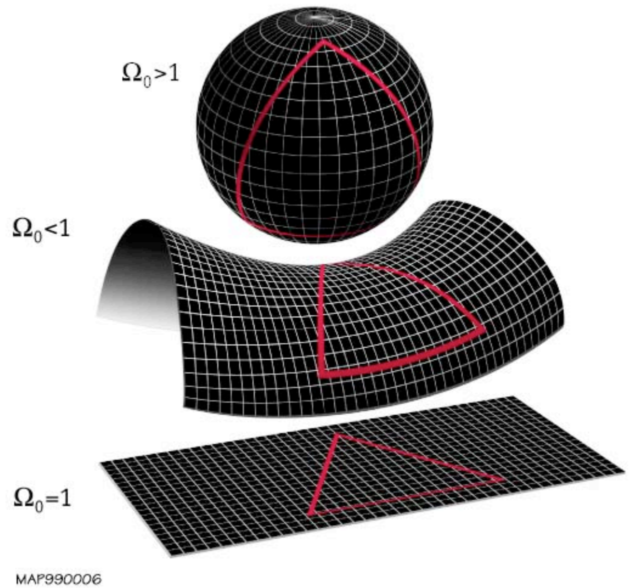


Einstein's **principle of equivalence** found that space-time is **curved** in the presence of a gravitational field. The mass density of the Universe tells us about the geometry of space-time.

Curvature of space (in 2-d) can be described in terms of angles/areas of a triangle. If the 3 angles on the triangles drawn below are α, β, γ then

$$\alpha + \beta + \gamma = 180° + (\kappa A/r_c^2)$$

where κ is the curvature constant and r_c is the radius of curvature



κ = +1 or Ω > 1, Bound/Closed Universe

κ = -1 or Ω < 1, Open Universe

κ = 0 or Ω = 1, Flat/Critical Universe

The Cosmological Constant

Einstein used **General Relativity** to improve upon Newtonian cosmology and provide a description of the Universe as a whole. He introduced the **cosmological constant Λ** into his **GR equations** since he found (as in the Newtonian case) that if the density of the Universe is non-zero, the Universe must be expanding. As this was before Hubble's work and most believed in a Steady State Universe, this constant allowed for a **non-zero density** and a **static Universe**.

Newtonian equation of motion:

$$\ddot{R} = -\frac{4\pi}{3}\frac{G\rho_o}{R^2}$$

Let **ρ = ρ – Λ/8πG** and a non-zero density is possible with $\dot{R}$=0

After Hubble's discovery, Einstein called this his "greatest blunder".

Einstein Equation and models of the Universe

Einstein's equation (analogous to Newtonian case):

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

In General Relativity, not only density, but also pressure is a source of gravity.

$$q_0 = -\frac{1}{H_0^2}\left[\frac{\Lambda c^2}{3} - 4\pi G\left(\frac{\rho_0}{3} + \frac{P_0}{3c^2}\right)\right]$$

This reduces to classical case when Λ=P=0

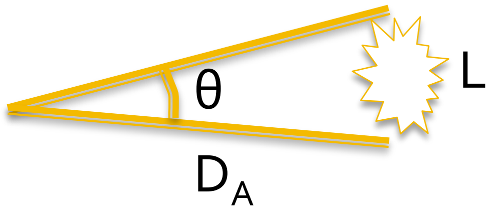
Several cosmological theories were developed upon GR principles

- de Sitter: flat Universe and positive Λ
- Friedmann: Λ=0 and zero pressure (low density)
- Lemaitre: non-zero density and Λ

Expressing Distances in an Expanding Universe

The geometry and expansion rate of the Universe effects angular sizes and distances measured.

D_A = L(size)/θ(angular size) → Angular Distance



D_L = sqrt (Luminosity/4πFlux) → Luminosity Distance

$D_L = (1+z)^2 D_A$

If Λ = 0, then

$D_L = 2c/H_o [z/(G+1)] \{1+[z/(G+1)]\}$ where $G = (1 + 2q_o z)^{1/2}$

If non-zero Λ, use a Cosmology Calculator:
http://www.astro.ucla.edu/~wright/CosmoCalc.html

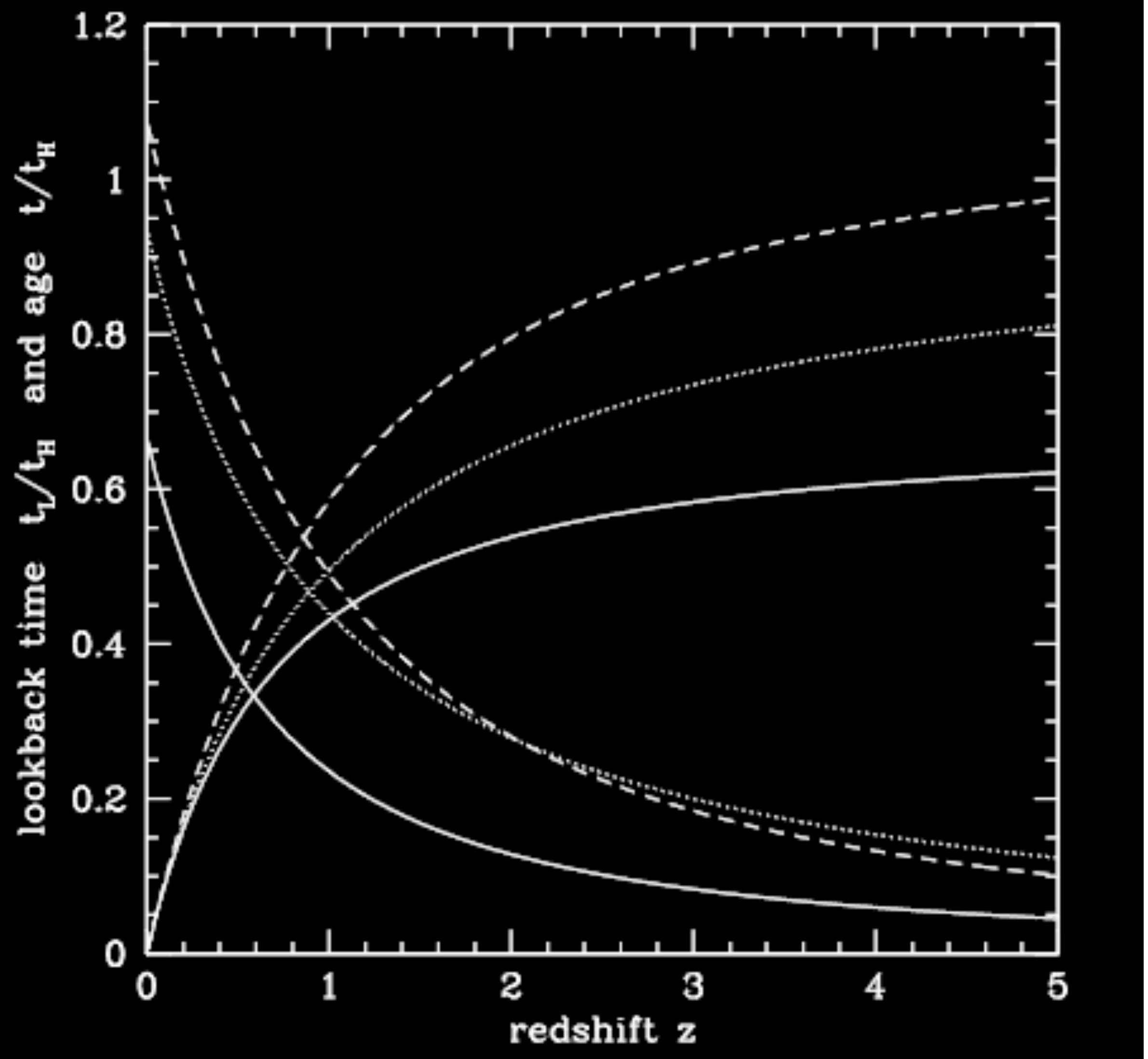


Figure 6: The dimensionless lookback time t_L/t_H and age t/t_H. Curves cross at the redshift at which the Universe is half its present age. The three curves are for the three world models, $(\Omega_M, \Omega_\Lambda) = (1,0)$, solid; $(0.05,0)$, dotted; and $(0.2,0.8)$, dashed.

Lookback time t_L is difference between age of Universe now and age t_e when photons left emitting source

flat, $\Lambda=0$ – solid
open, $\Lambda=0$ – dotted
flat, non-zero Λ - dashed

How to determine if the Universe is Open, Closed or Flat?

1) **Add up all matter in the Universe to determine density**

Luminous matter: only ~1% of ρ_{crit}

Dark matter?

Table 20.2. Mass-to-light ratios on different scales.	
Scale	M/L (solar units)
Milky Way to Sun	3
Spiral galaxy disk	10
Elliptical galaxy	30
Halo of giant elliptical	40
Rich cluster of galaxies	200
To close the universe	1200

Still a factor of ~5 short to close the Universe with "known" DM (where dynamical evidence exists)

2) **look for gravitational effects of the mass of the Universe**, such as the slowing down of the expansion rate: $\dot{H}(t_0)$

$$q_0 = -\left[\frac{\dot{H}(t_0)}{H_0^2} + 1\right]$$

3) **Measure the curvature of space-time by surveying the Universe on large scales:** galaxy counts and sizes show that the radius of curvature is comparable to or greater than the size of the observable Universe ⇒ **essentially flat!**

Why should we be so close to the boundary ($\rho_o = \rho_{crit} = 3H^2/(8\pi G)$)?

With a non-zero Λ, it is still possible to have a flat universe and low mass density

$\Omega_M = \rho_M/\rho_{crit}$

Let $\rho_{\Lambda,eff} = \Lambda/8\pi G$
then $\Omega_\Lambda = \Lambda/8\pi G\rho_{crit}$

Total density parameter for the Universe is then

$$\Omega_{tot} = \Omega_M + \Omega_\Lambda$$

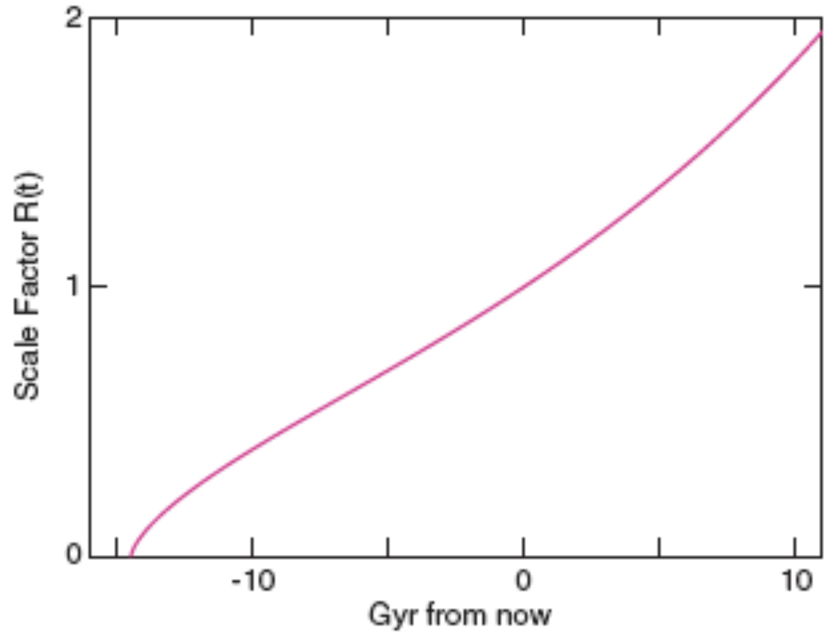


Fig 20.10. Scale factor vs. time for the cosmological model which best fits current data. The model has a Hubble constant of 65 km/s/Mpc and $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$. [© Edward L. Wright, used with permission]

Einstein equations are integrated to give R(t) by looking at limiting cases (i.e. zero cosmological constant and zero density). Each case must be evaluated at zero, + and - curvature

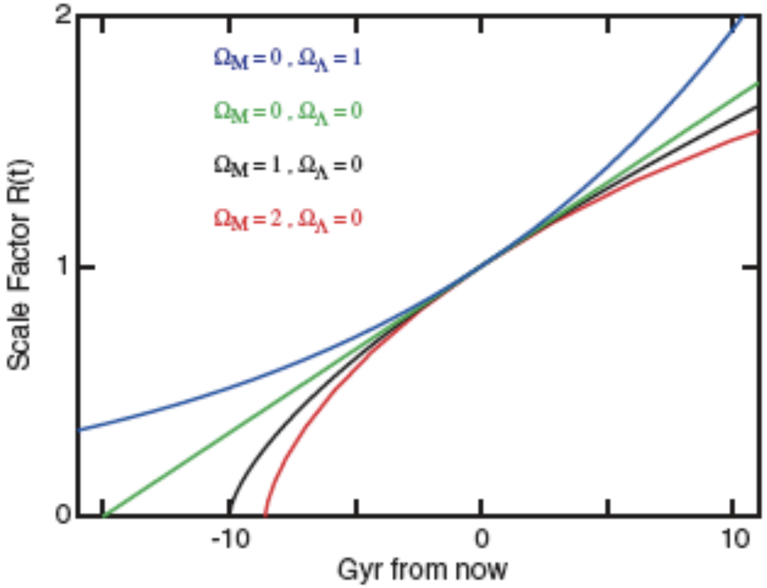
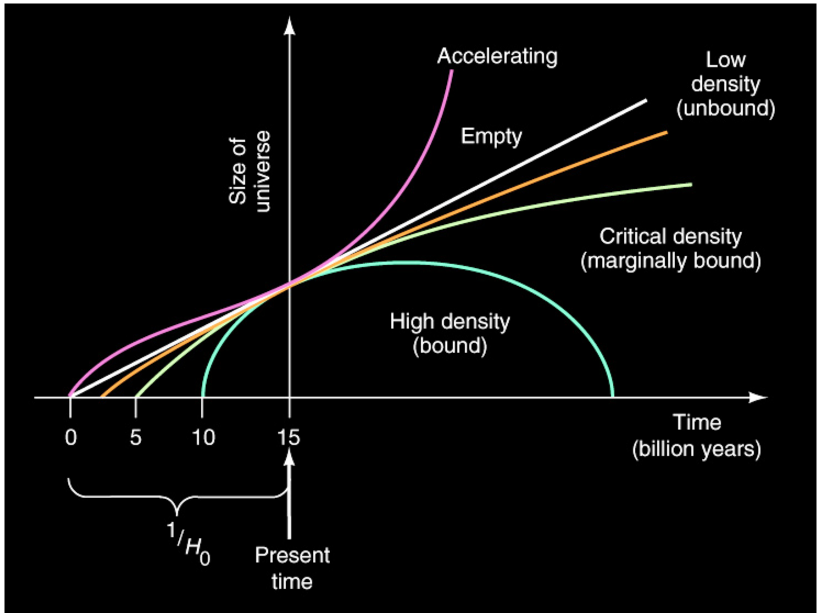


Fig 20.9. Scale factor vs. time for various cosmological models. Models are all chosen to have $R = 1$, now, and a Hubble constant = 65 km/s/Mpc (so they all have the same slope now). In terms of the density parameter (defined in equation 20.45), the models are (from top to bottom): ($\Omega_M = 0, \Omega_\Lambda = 1$), ($\Omega_M = 0, \Omega_\Lambda = 0$) ($\Omega_M = 1, \Omega_\Lambda = 0$) ($\Omega_M = 2, \Omega_\Lambda = 0$). [© Edward L. Wright, used with permission]

Table 1. Comparison of Different Cosmological Models

Name	Ω_0	Ω_Λ	t_0	q_0
Einstein-de Sitter	1	0	$\frac{2}{3}t_H$	$\frac{1}{2}$
Empty, no Λ	0	0	t_H	0
Example Open	0.3	0	$0.82t_H$	0.15
Example Closed	2	0	$0.58t_H$	1
Example Flat, Lambda	0.5	0.5	$0.84t_H$	-0.25
Concordance	.27	.73	$\approx 1.001t_H$	-0.6
Steady State	1	0	∞	-1

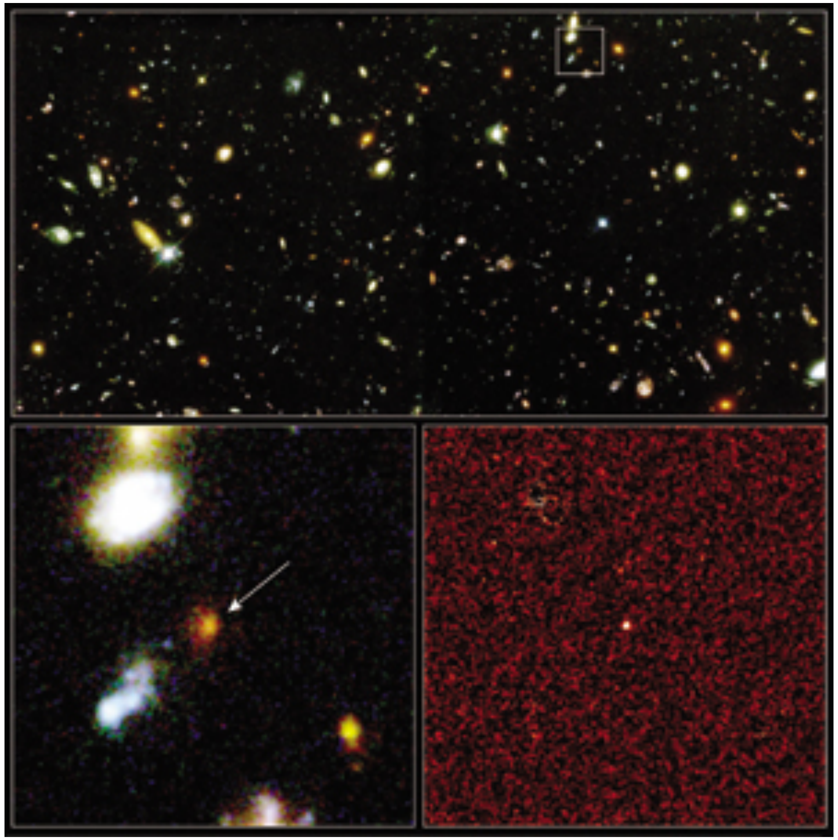


Type Ia supernovae

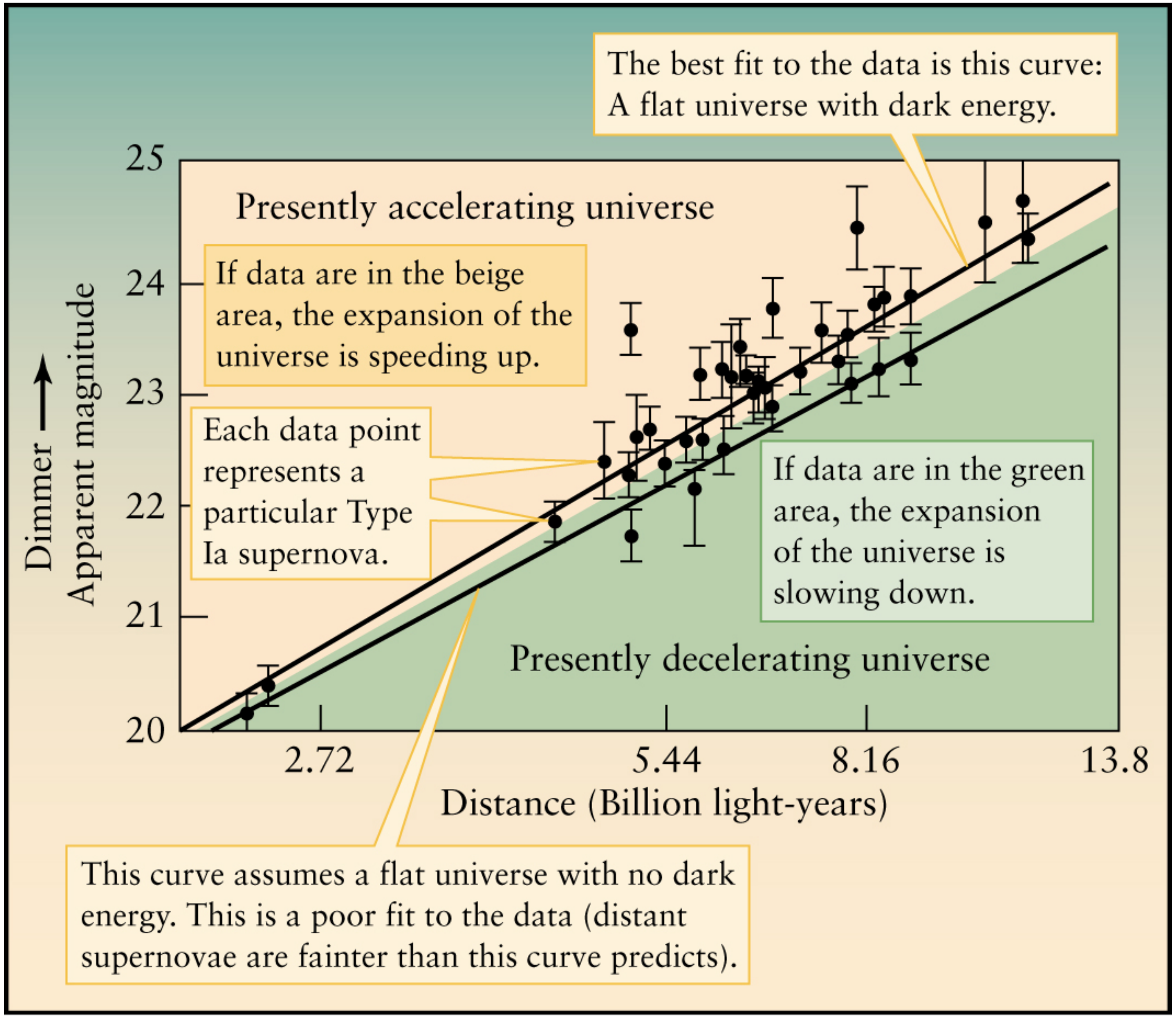
- Standard candles
- Their intrinsic luminosity is known
- Their apparent luminosity can be measured
- The ratio of the two can provide the luminosity-distance (d_L) of the supernova
- The redshift z can be measured independently from spectroscopy
- Finally, one can obtain d_L (z) or equivalently the magnitude(z) and draw a Hubble diagram

Dimmer Distant Supernova

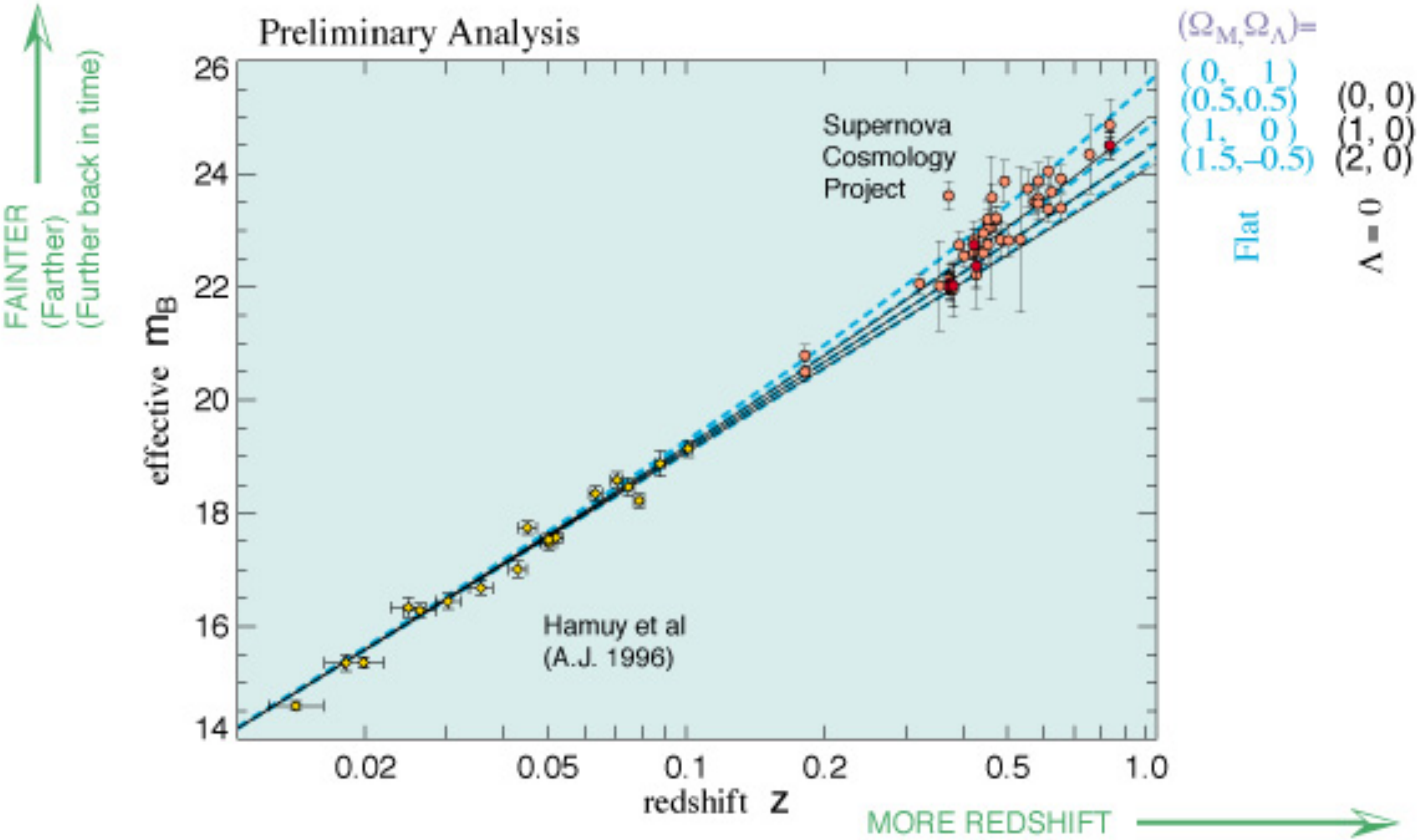
- These HST images show the galaxy in which the supernova SN 1997ff occurred.
- This supernova was dimmer than expected.
- The distance to it is greater than it would be if the universe had been continually slowing down.
- An outward force is acting over vast distances in the universe.

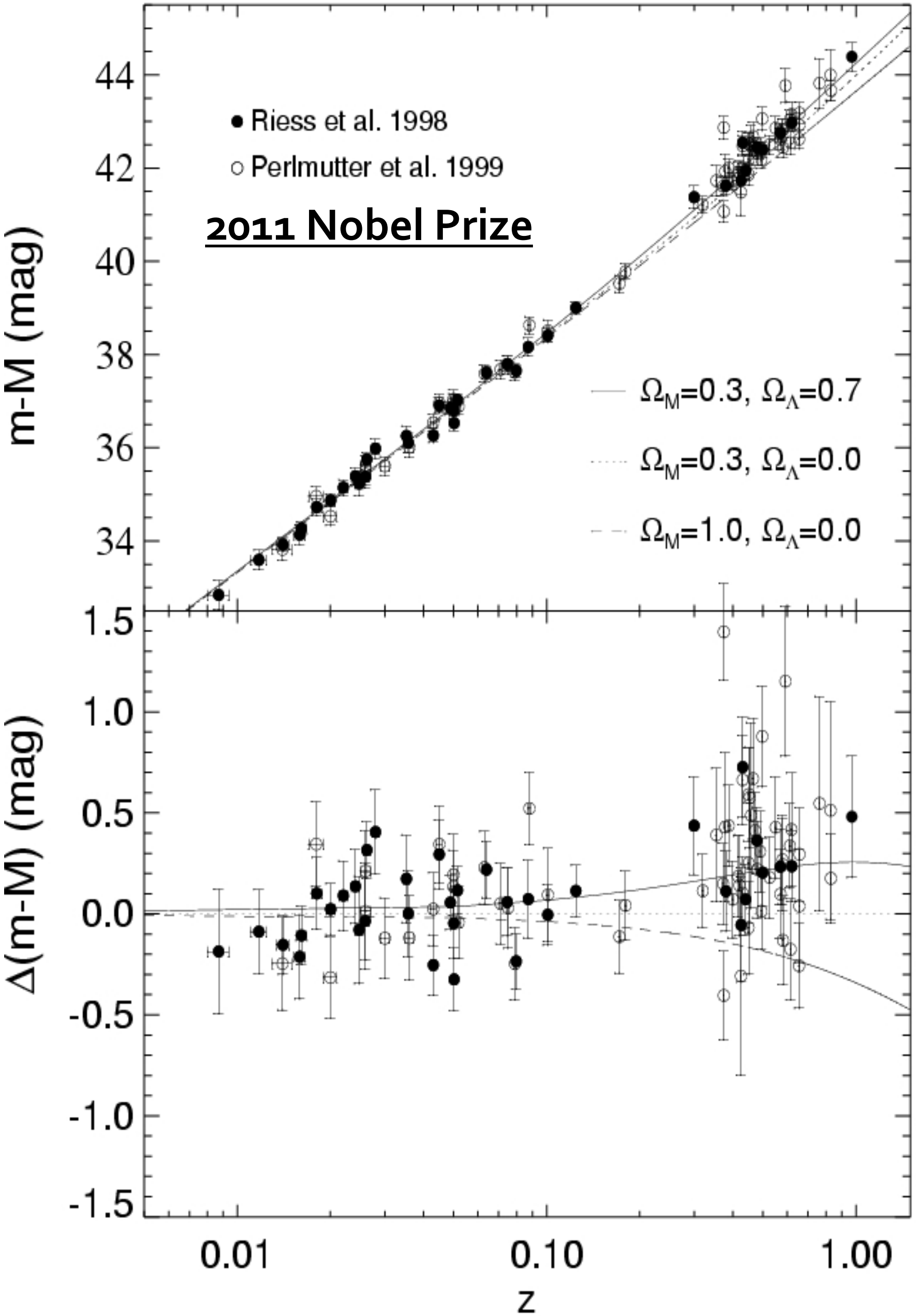


Dimmer Distant Supernova

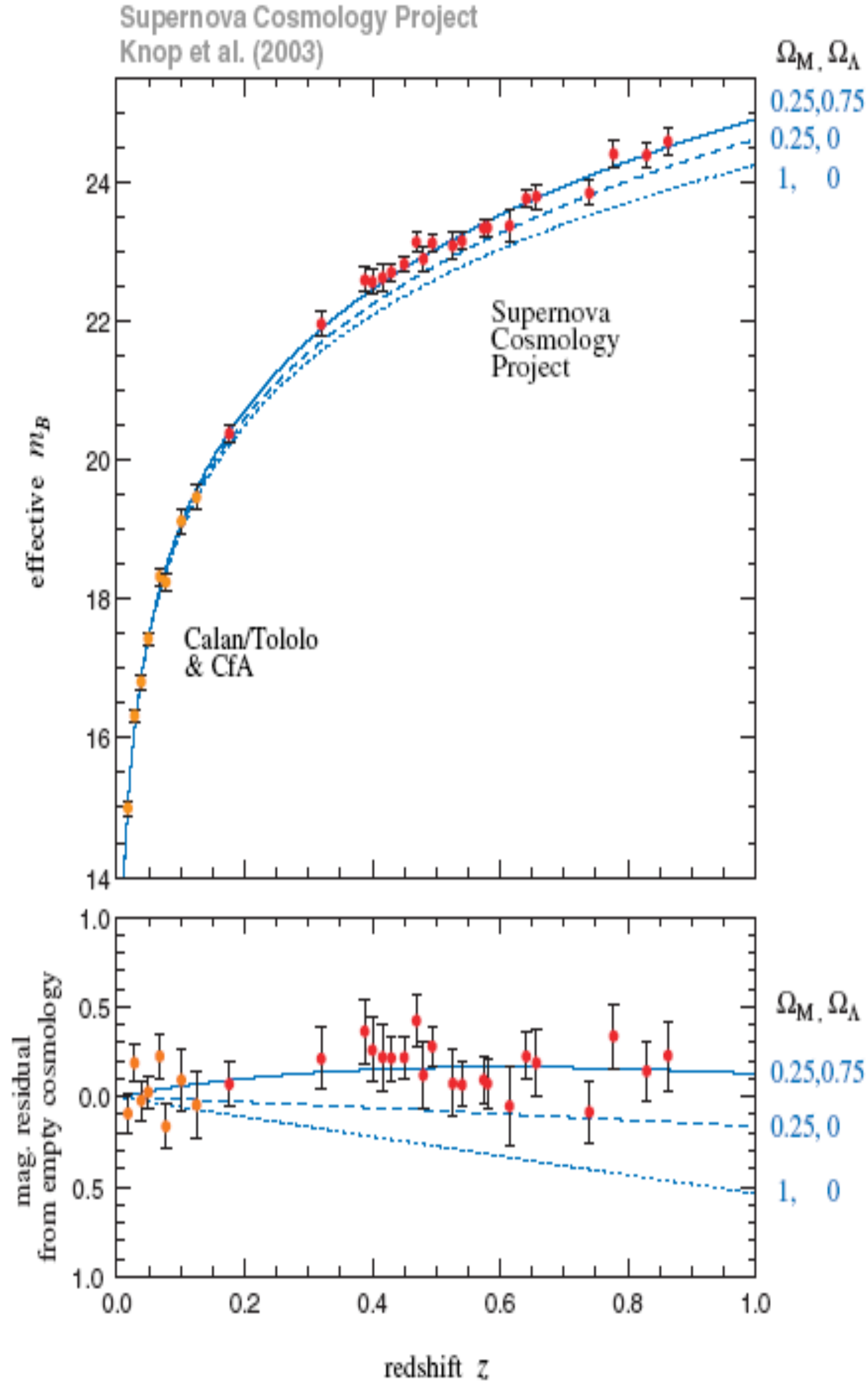


Hubble Plots



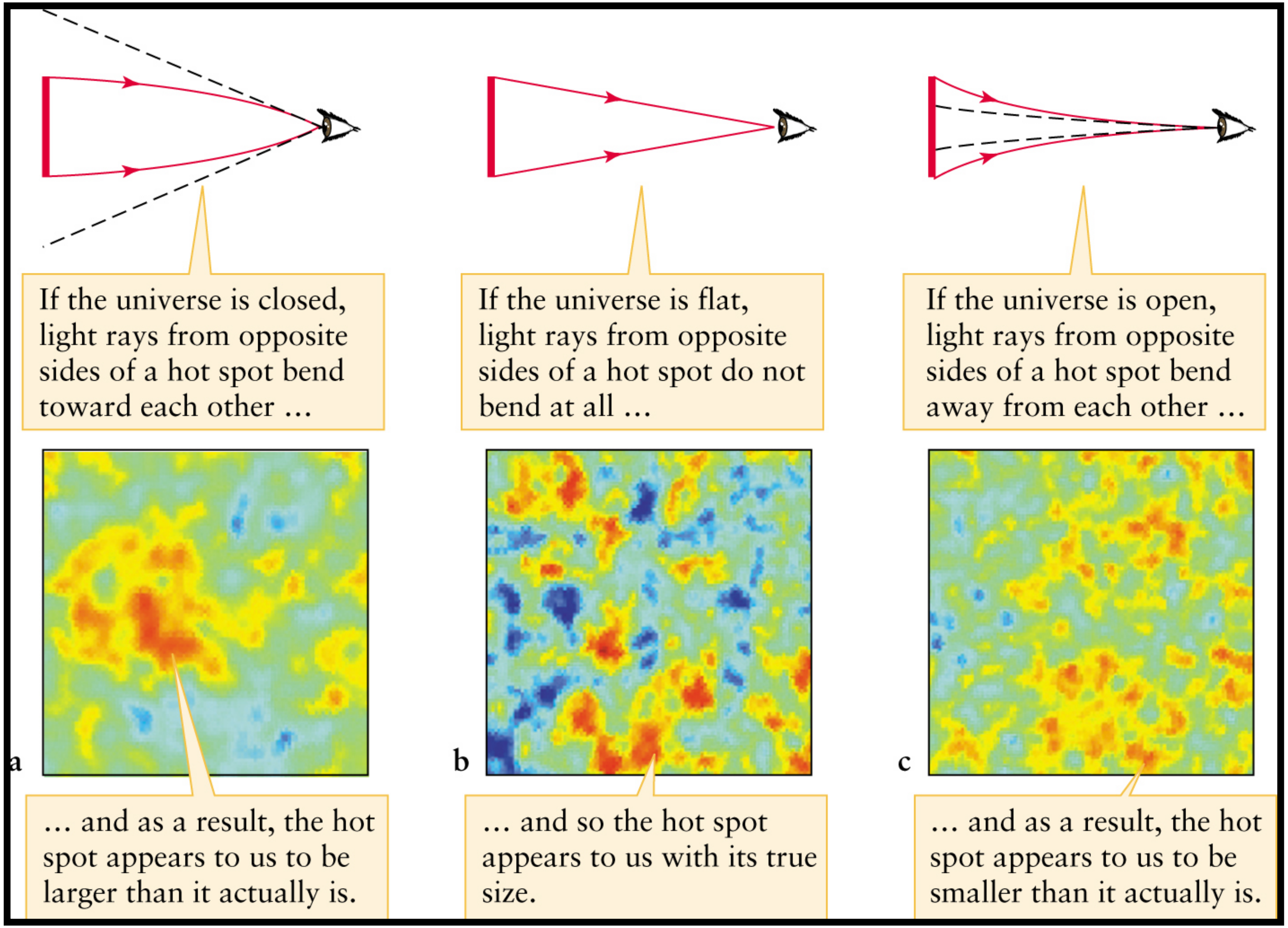


Riess et al. 1998
Perlmutter et al. 1999
2011 Nobel Prize
Ω_M=0.3, Ω_Λ=0.7
Ω_M=0.3, Ω_Λ=0.0
Ω_M=1.0, Ω_Λ=0.0
m-M (mag)
Δ(m-M) (mag)
z
0.01
0.10
1.00



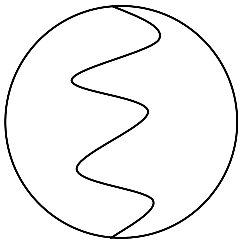
Supernova Cosmology Project
Knop et al. (2003)
Ω_M, Ω_Λ
0.25,0.75
0.25, 0
1, 0
Supernova Cosmology Project
Calan/Tololo & CfA
effective m_B
mag. residual from empty cosmology
redshift z

Cosmic Microwave Background and the Curvature of Space



Can we predict the primordial perturbations?

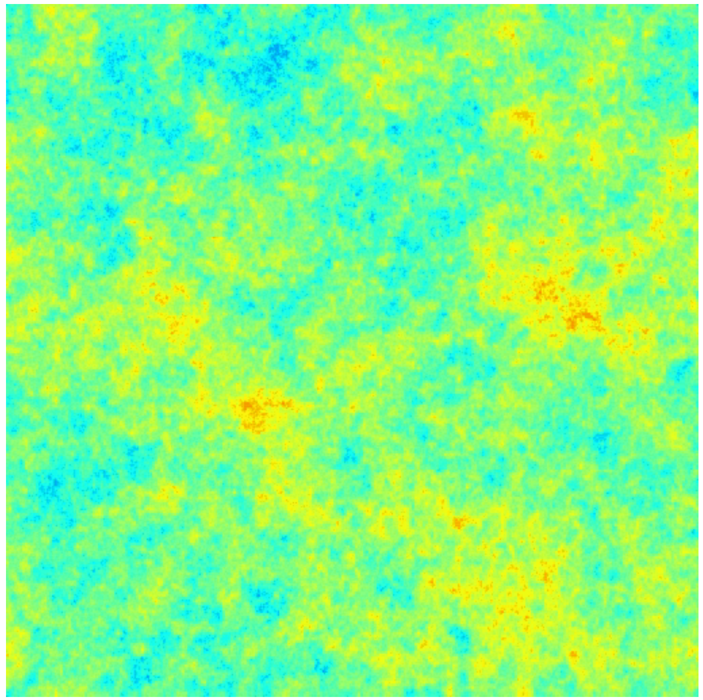
Maybe..



Quantum Mechanics

Inflation

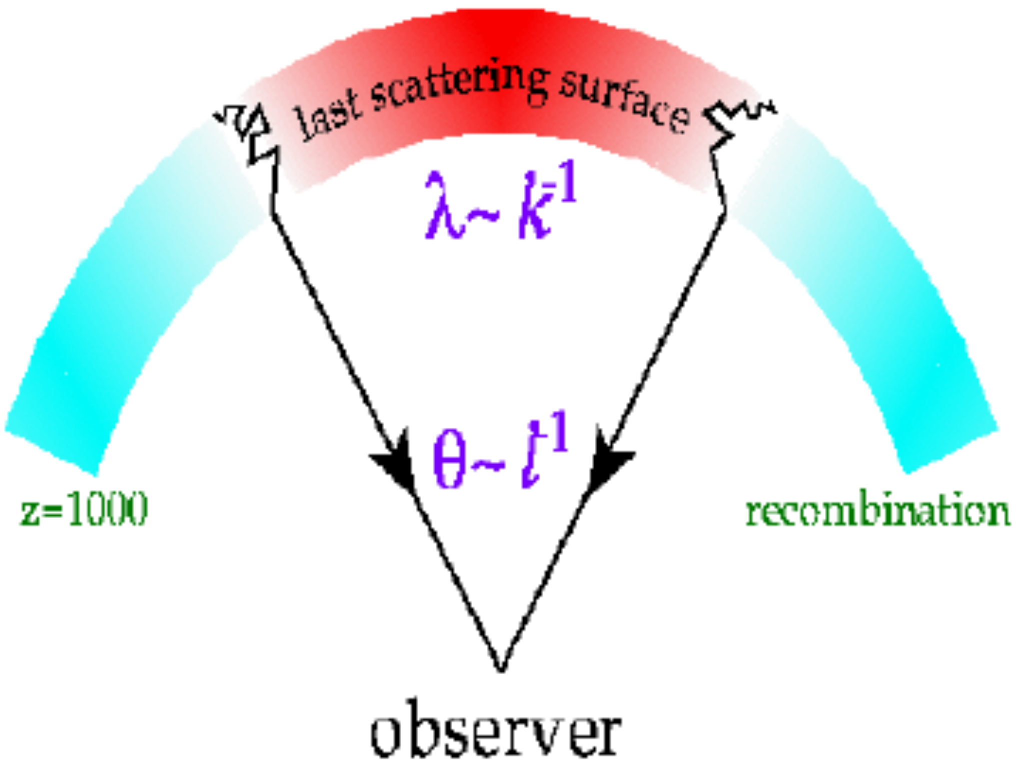
make $>10^{30}$ times bigger



After inflation

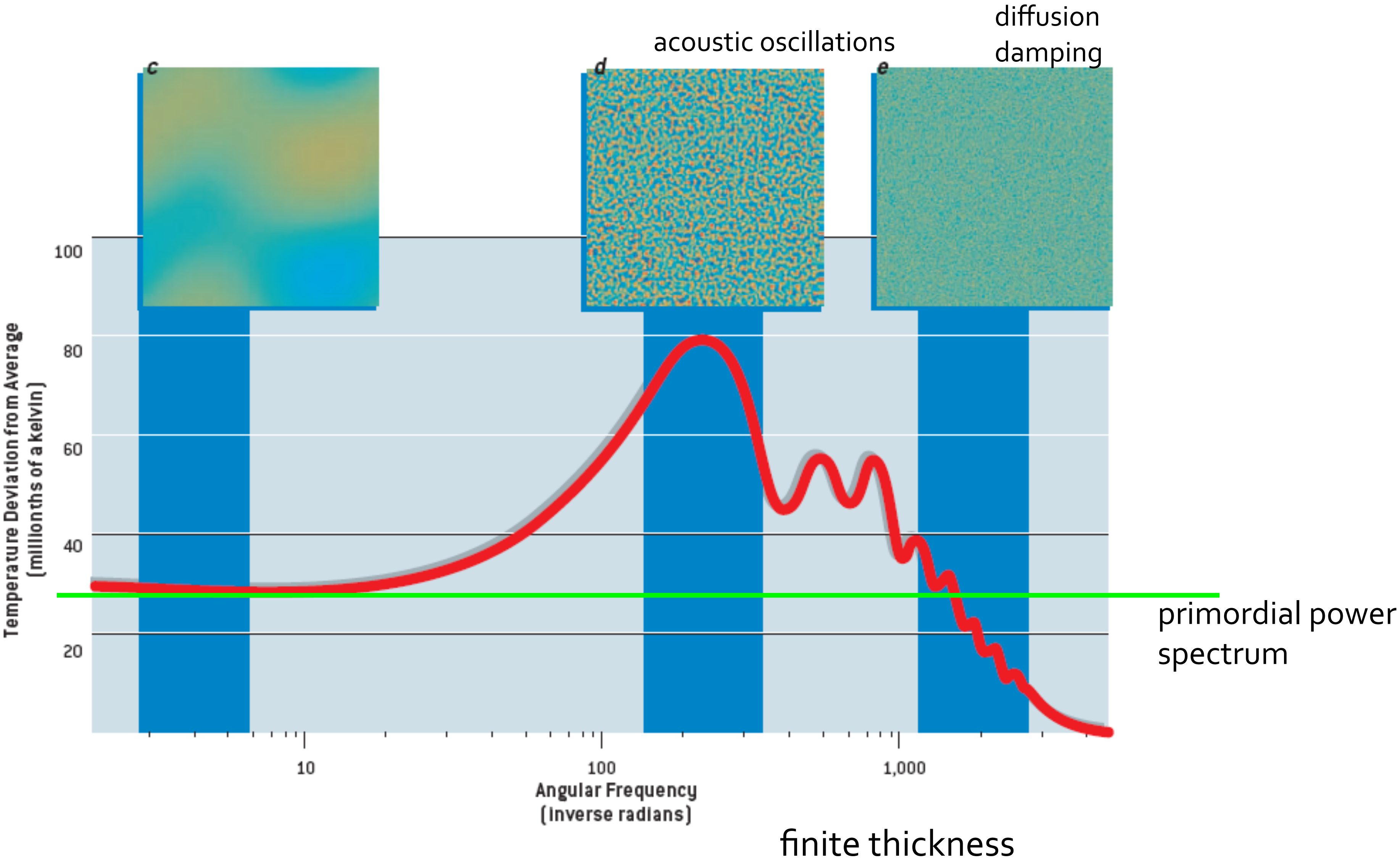
Huge size, amplitude ~ 10^{-5}

Observed ΔT as function of angle on the sky



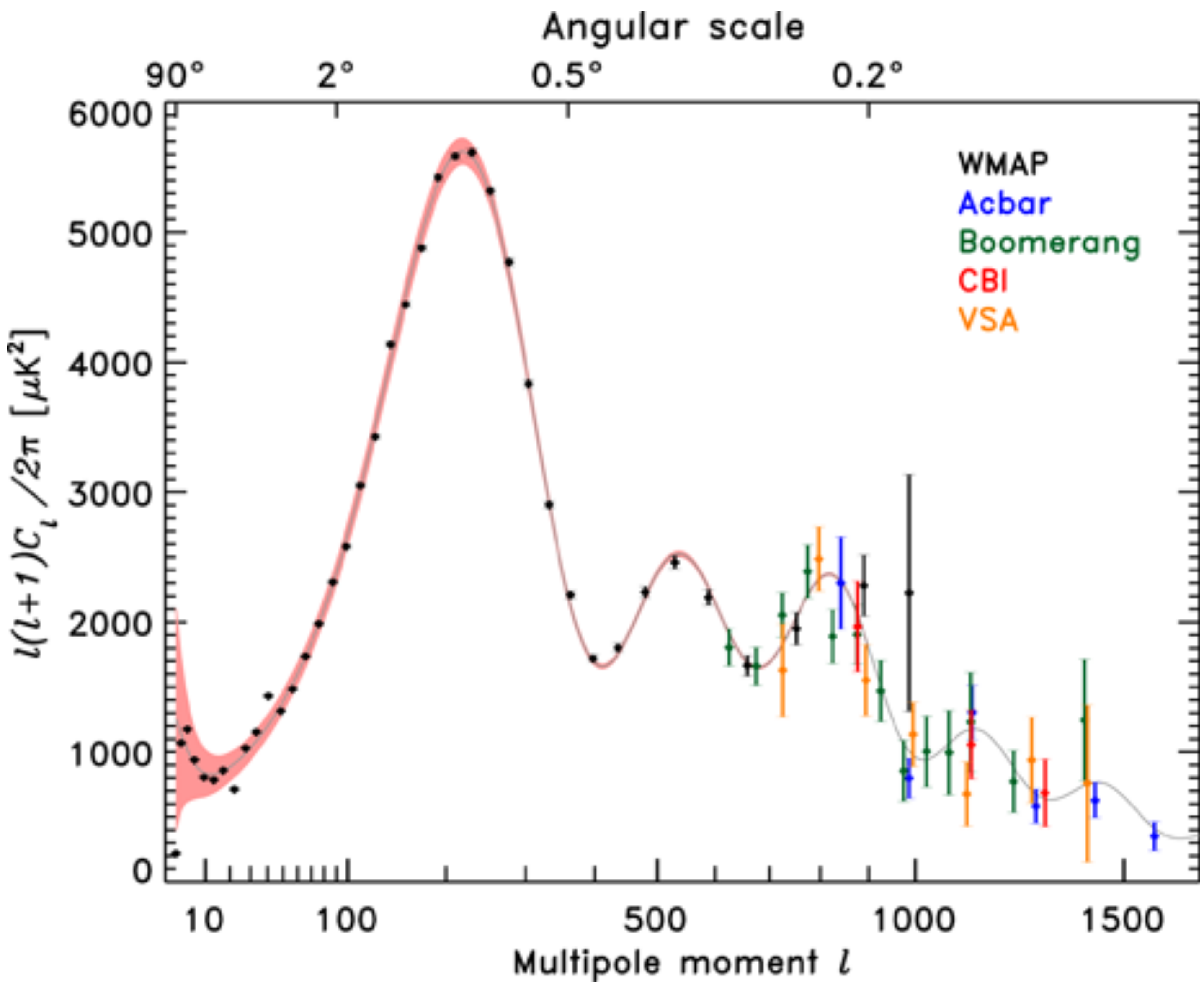
CMB temperature power spectrum

Primordial perturbations + later physics



Hu & White, Sci. Am., 290 44 (2004)

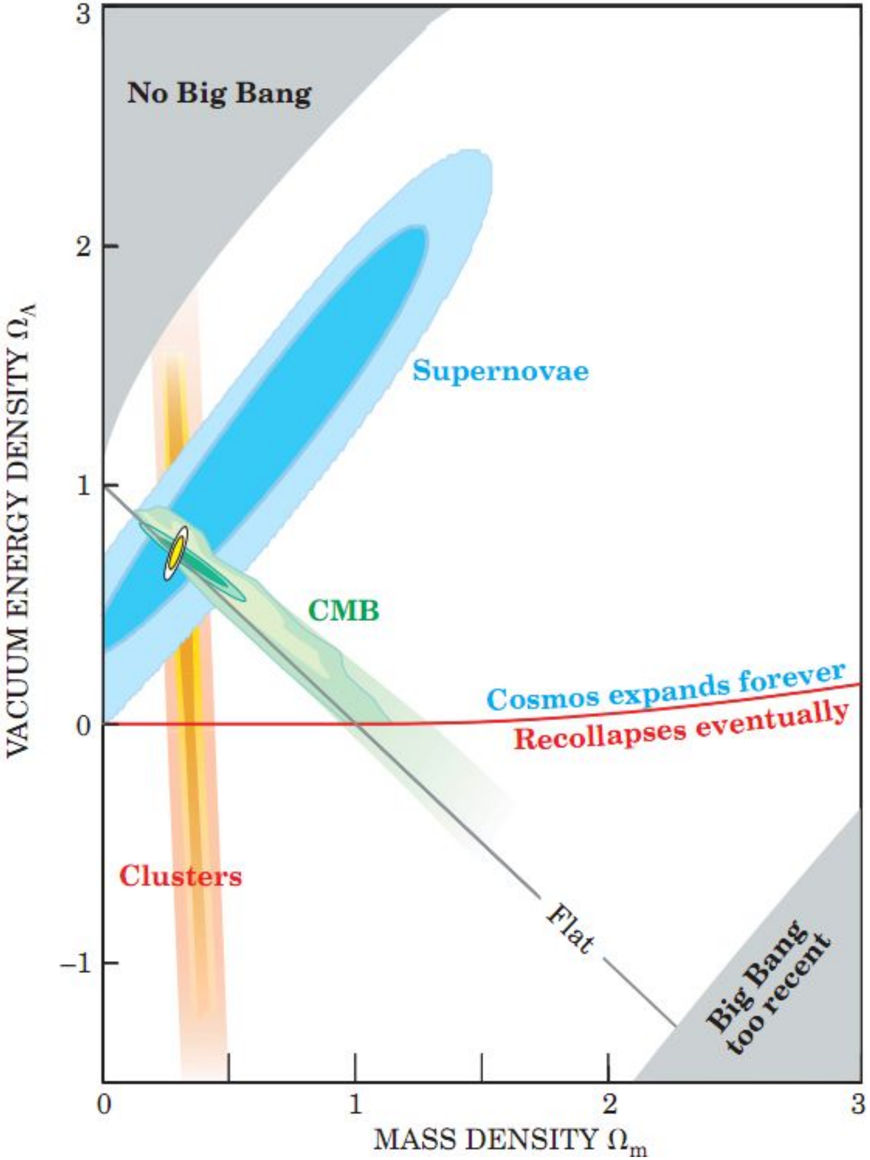
Anisotropy observations

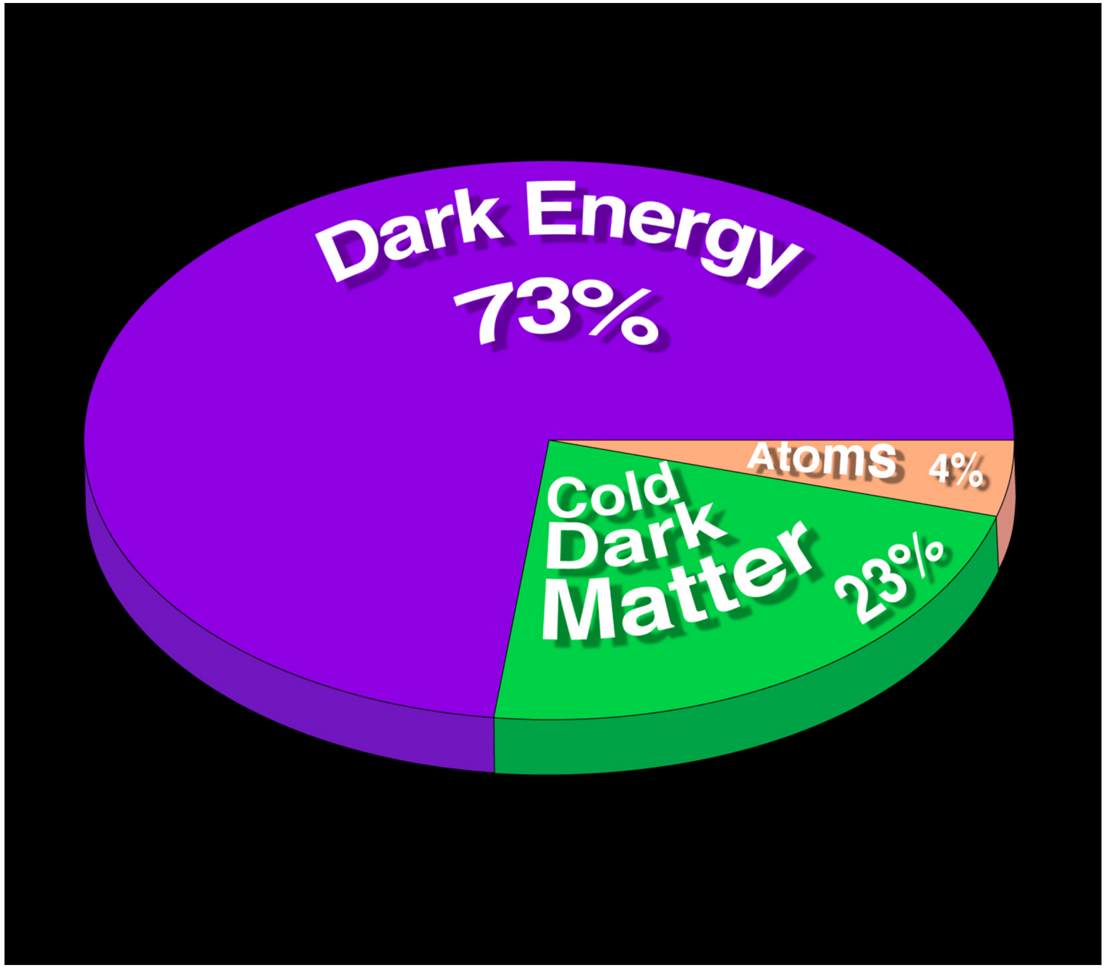


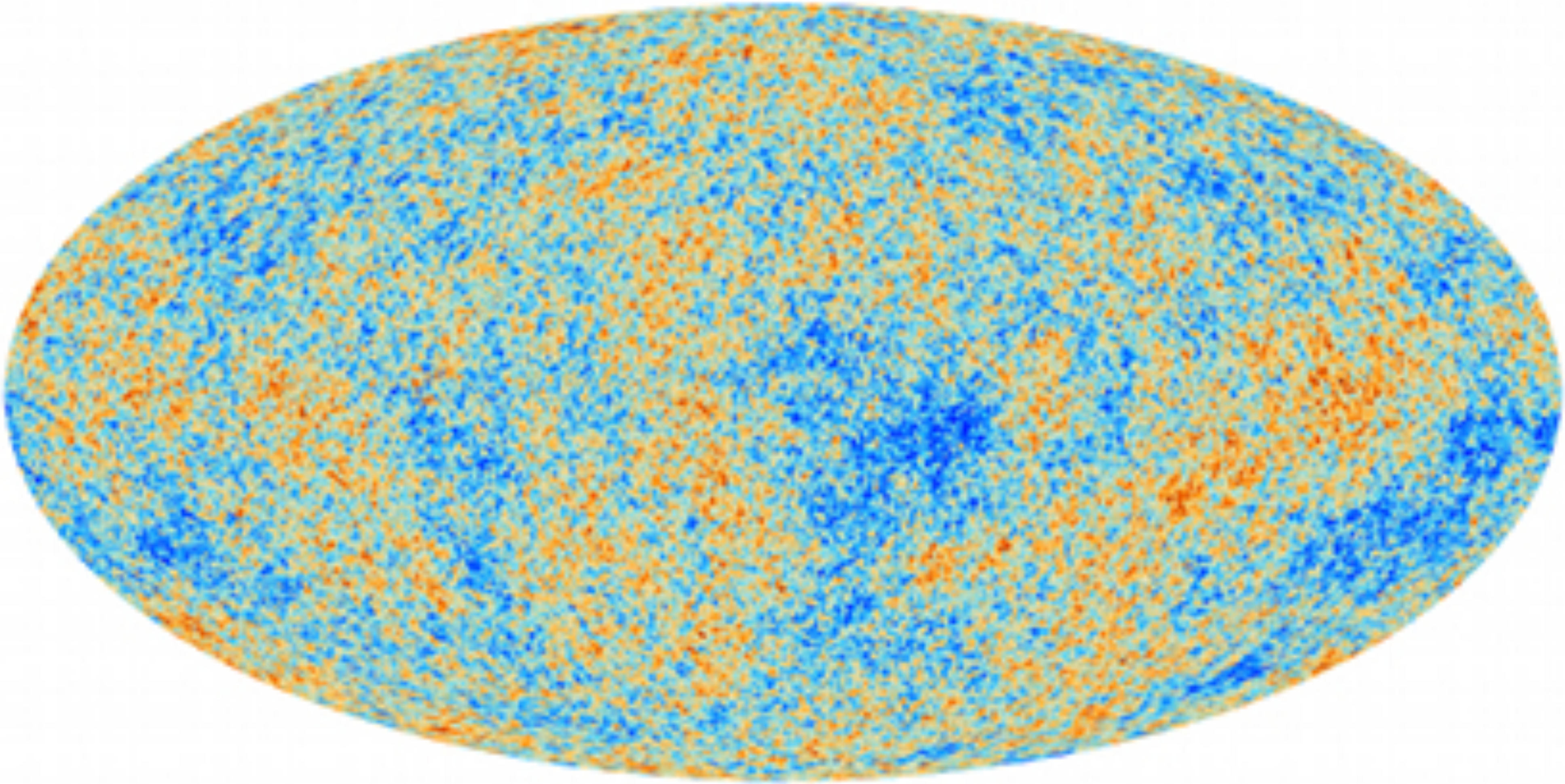
Evidence for Dark Energy from large-scale structure (clusters of galaxies)

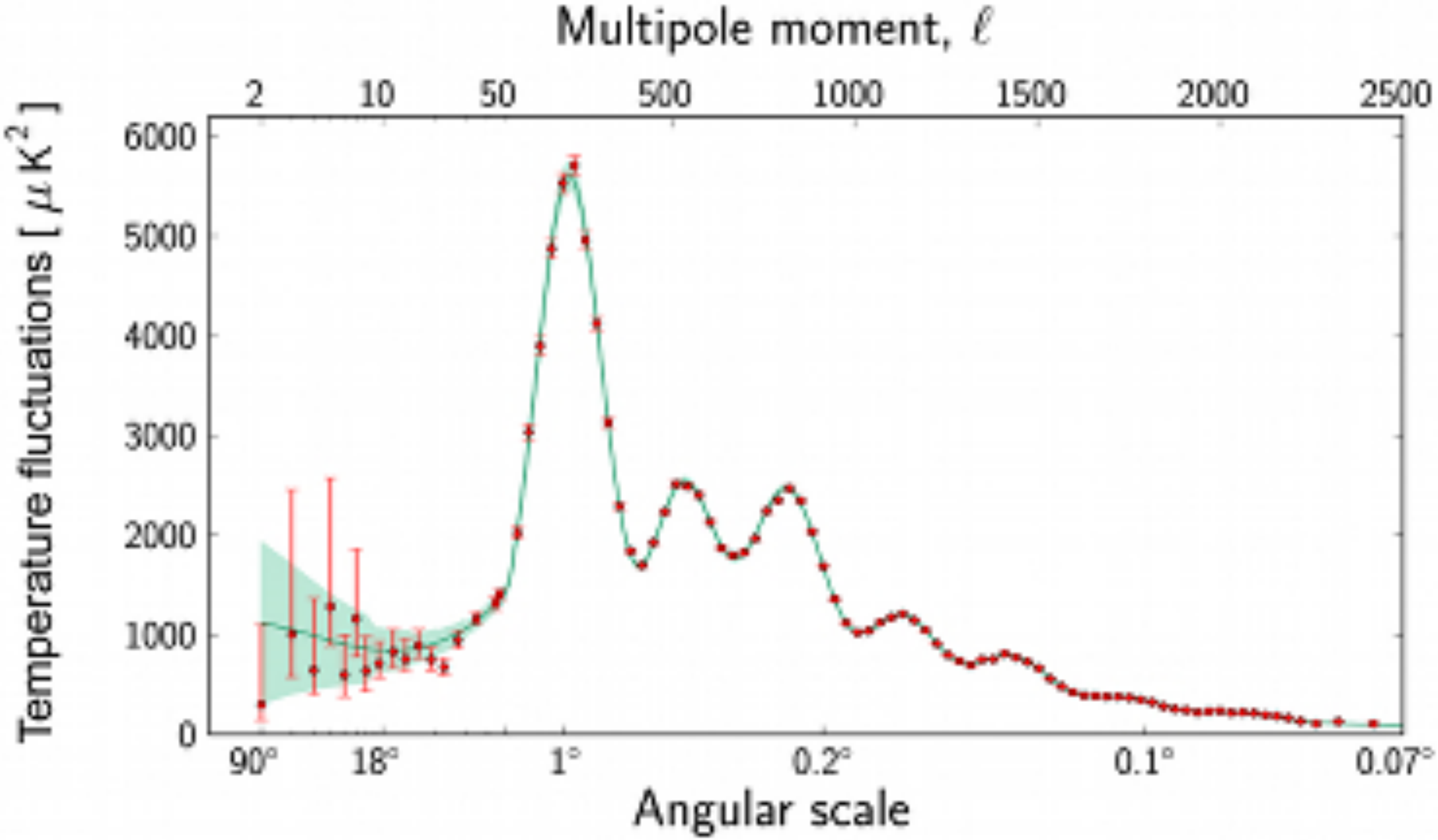
- Counting and weighing clusters of galaxies can infer the matter energy density in the universe
- The matter energy density found is usually around ~0.3 the critical density
- CMB best fit model has a total energy density of ~1, so another ~0.7 is required
- The same ~0.7 is required from combining supernovae data and CMB constraints

Contents of the universe (from observations up to last year)

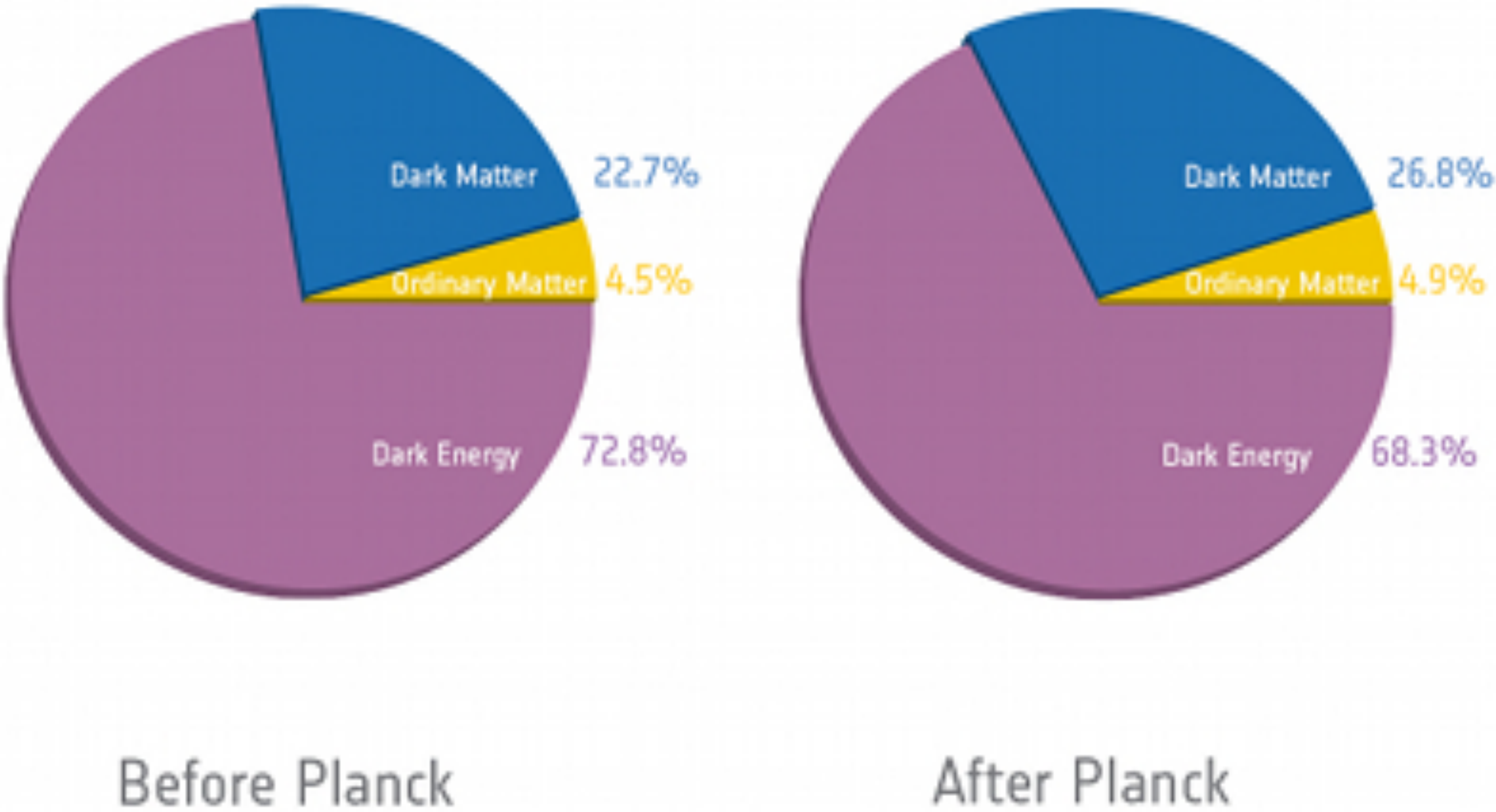








Multipole moment, ℓ
2
10
50
500
1000
1500
2000
2500
Temperature fluctuations [μK^2]
6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale


Dark Matter
22.7%
Ordinary Matter
4.5%
Dark Energy
72.8%
Before Planck
Dark Matter
26.8%
Ordinary Matter
4.9%
Dark Energy
68.3%
After Planck