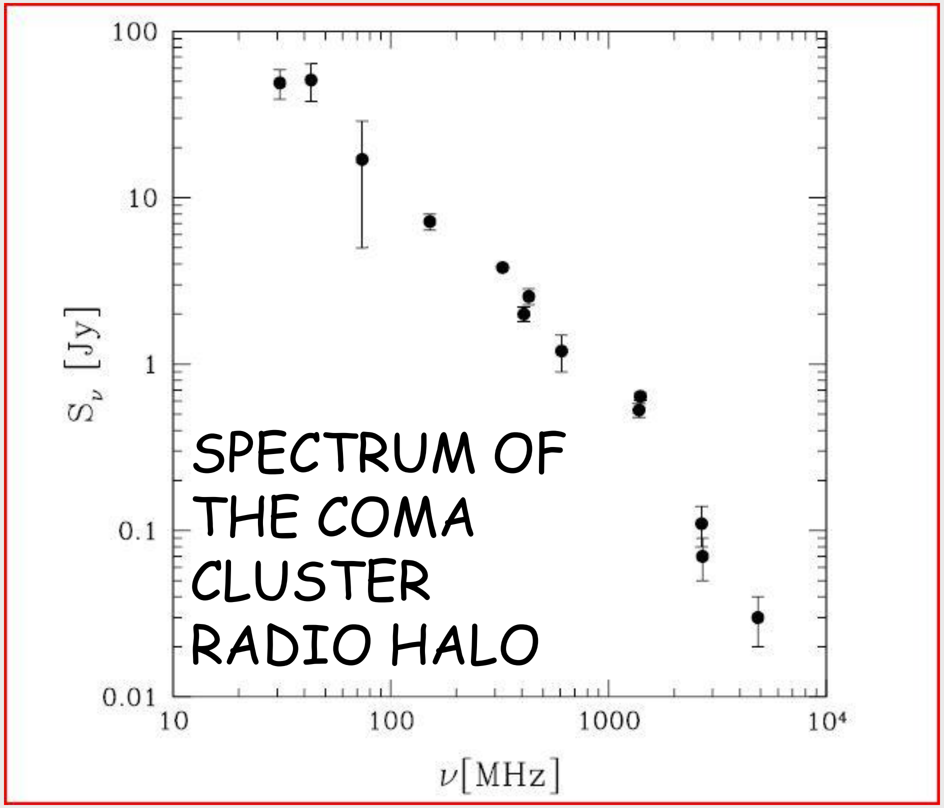
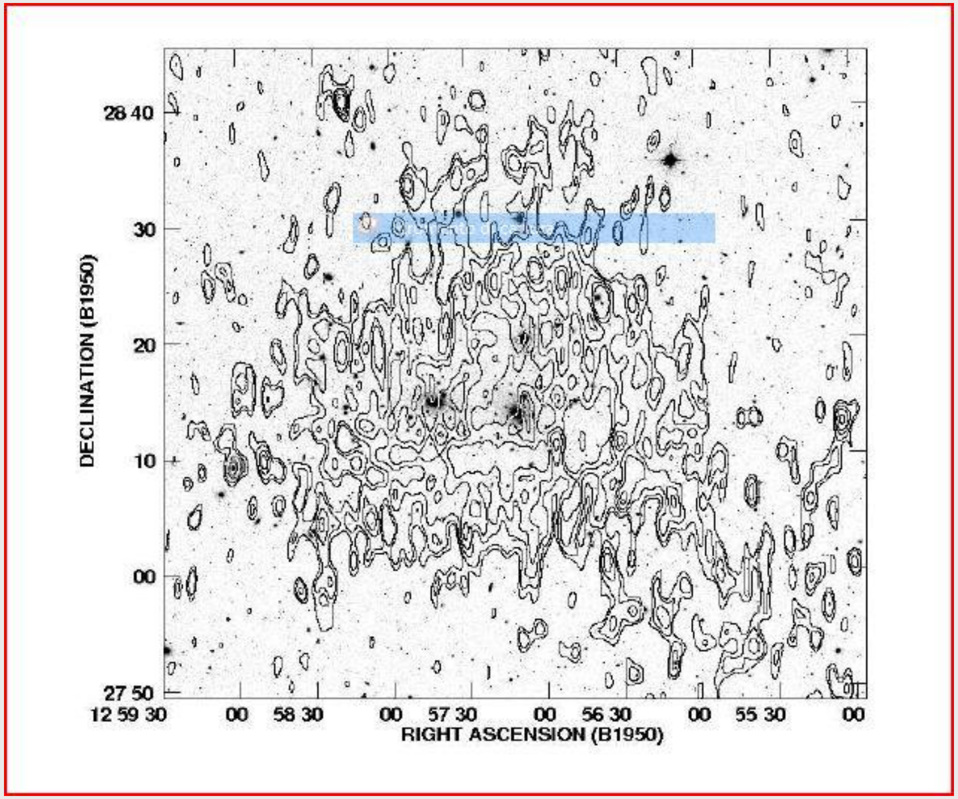


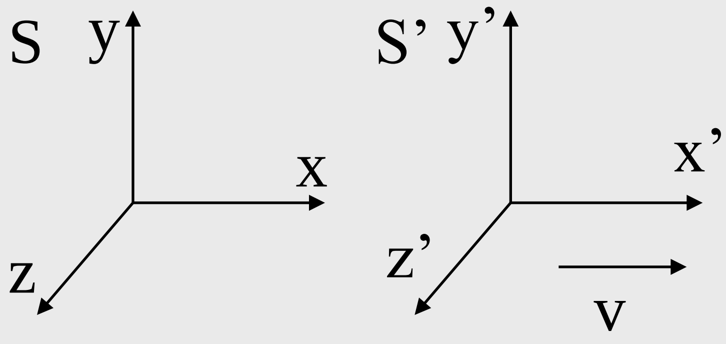


SYNCHROTRON RADIATION

OUTLINE OF THE LESSON

- REMINDER SPECIAL RELATIVITY: BEAMING, RELATIVISTIC LARMOR FORMULA
- CYCLOTRON EMISSION
- SYNCHROTRON POWER AND SPECTRUM EMITTED BY A SINGLE ELECTRON
- SPECTRUM FROM A DISTRIBUTION OF ELECTRONS AND POLARIZATION
- EXTENDED RADIO EMISSION IN GALAXY CLUSTERS

LORENTZ TRANSFORMATIONS



$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y \quad z' = z$$

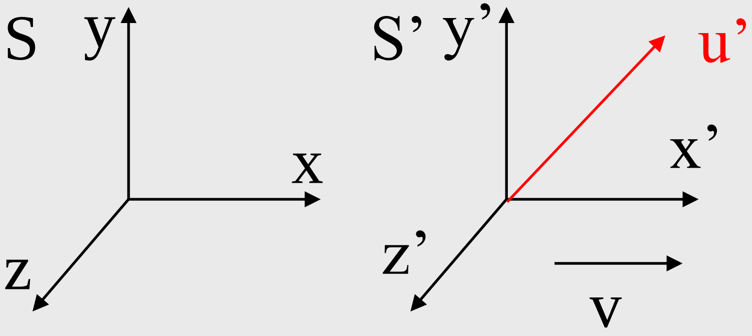
$$t' = \frac{t - v/c^2 x}{\sqrt{1 - v^2/c^2}}$$

$$x = \frac{x' + vt}{\sqrt{1 - v^2/c^2}}$$

$$y = y' \quad z = z'$$

$$t = \frac{t' + v/c^2 x}{\sqrt{1 - v^2/c^2}}$$

LORENTZ TRANSFORMATION OF VELOCITIES



$$dx = \gamma(dx' + v\,dt') \qquad \gamma = \left(1 - v^2/c^2\right)^{-1/2}$$

$$dt = \gamma(dt' + v/c^2\,dx')$$

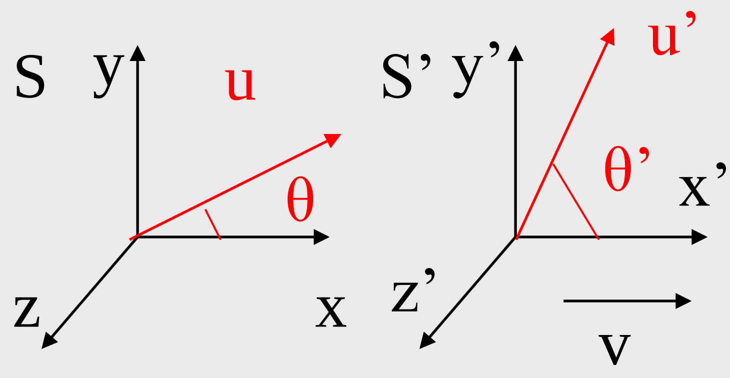
$$u_x = \frac{dx}{dt} = \frac{u'_x + v}{1 + v u'_x/c^2}$$

$$u_x = \frac{dx}{dt} = \frac{u'_x + v}{1 + v u'_x/c^2}$$

$$u_y = \frac{u'_y}{\gamma(1 + v u'_x/c^2)}$$

$$u_z = \frac{u'_z}{\gamma(1 + v u'_x/c^2)}$$

RELATIVISTIC BEAMING



$$u_{\parallel} = \frac{u'_{\parallel} + v}{(1 + v u'_{\parallel} / c^2)} \qquad u_{\perp} = \frac{u'_{\perp}}{\gamma (1 + v u'_{\parallel} / c^2)} \qquad \tan\theta = \frac{u_{\perp}}{u_{\parallel}} = \frac{u' \sin\theta'}{\gamma (u' \cos\theta' + v)}$$

If photons u'=c $\quad \tan\theta = \dfrac{\sin\theta'}{\gamma(\cos\theta' + v)} \qquad \sin\theta = \dfrac{\sin\theta'}{\gamma(1 + v\cos\theta')}$

If $\quad \gamma >> 1 \qquad \boxed{\theta \approx \dfrac{1}{\gamma}}$

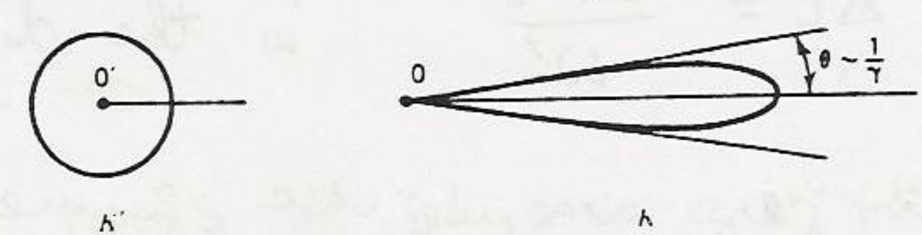
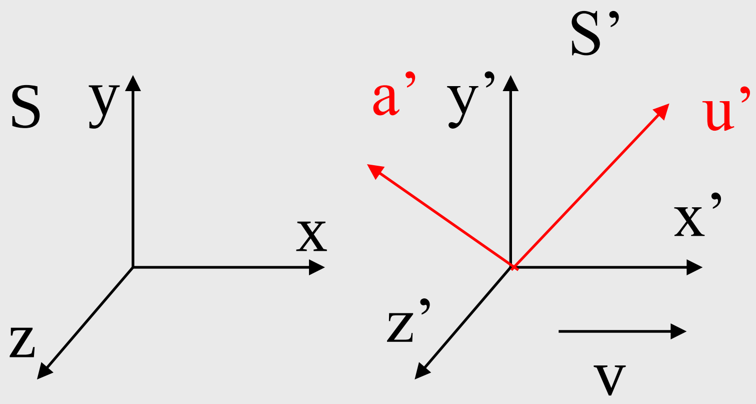


Figure 4.3 Relativistic beaming of radiation emitted isotropically in the rest frame K'.

LORENTZ TRANSFORMATION OF ACCELERATIONS



$$du_x = \gamma^{-2}\kappa^{-2}du'_x \qquad \kappa = 1 + vu'_x/c^2$$

$$dt = \gamma(dt' + v/c^2 dx') = \gamma\kappa dt'$$

$$a_x = \frac{du_x}{dt} = \frac{a'_x}{\gamma^3\kappa^3}$$

$$du_y = \gamma^{-1}\kappa^{-2}(\kappa du'_y - vu'_y/c^2 du'_x)$$

$$a_y = \frac{du_y}{dt} = \gamma^{-2}\kappa^{-3}\left(\kappa a'_y - \frac{vu'_y}{c^2}a'_x\right)$$

$$a_x = \frac{du_x}{dt} = \frac{a'_x}{\gamma^3\kappa^3}$$

$$a_y = \gamma^{-2}\kappa^{-3}\left(\kappa a'_y - \frac{vu'_y}{c^2}a'_x\right)$$

$$a_z = \gamma^{-2}\kappa^{-3}\left(\kappa a'_z - \frac{vu'_z}{c^2}a'_x\right)$$

RELATIVISTIC LARMOR'S FORMULA

In a instantaneous rest frame K' the particle has zero velocity

$$a'_{\parallel} = \gamma^3 a_{\parallel} \qquad a'_{\perp} = \gamma^2 a_{\perp}$$

$$P = \frac{dE}{dt} = \frac{2q^2}{3c^3}(a'^2_{\perp} + a'^2_{\parallel}) = \boxed{\frac{2q^2}{3c^3}\gamma^4(a'^2_{\perp} + \gamma^2 a'^2_{\parallel})}$$

Total emitted power (subtle: emitted and received might not be the same in extreme cases, angle transformation !) is Lorentz invariant so the power can be valuated in any frame

CYCLOTRON EMISSION

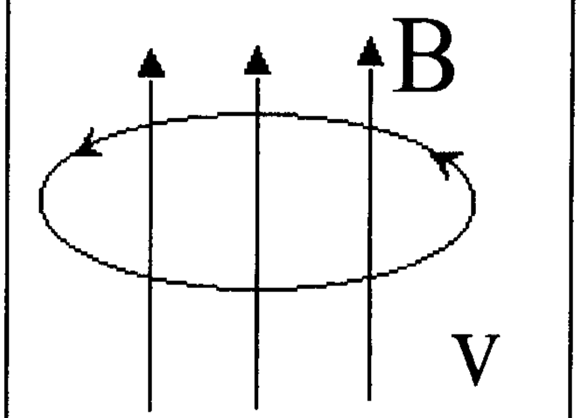


Figure 5.1: As a particle moves in a magnetic field, it has a circular orbit.

Charged particle q in a magnetic field B in the non-relativistic limit
Balance of centrifugal force against Lorentz force

$$m\frac{v^2}{r} = \frac{qBv}{c}$$

Charged particle emit a narrow line at the cyclotron frequency

$$\nu_{cyc} = \frac{qB}{2\pi mc} = 2.8B\ MHz = 11.6\frac{B}{10^{12}}keV$$

B in Gauss e for an electron

CYCLOTRON EMISSION

- As in any other emission process, also cyclotron **absorption** is possible ⇒ cyclotron absorption lines
- In practice, a cyclotron line is **broadened** by nonuniformity in the magnetic field, by collisional broadening, and (for energetic particles) by relativistic effects.

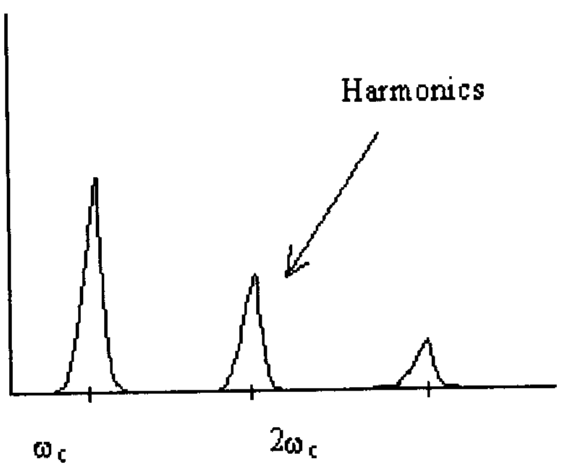
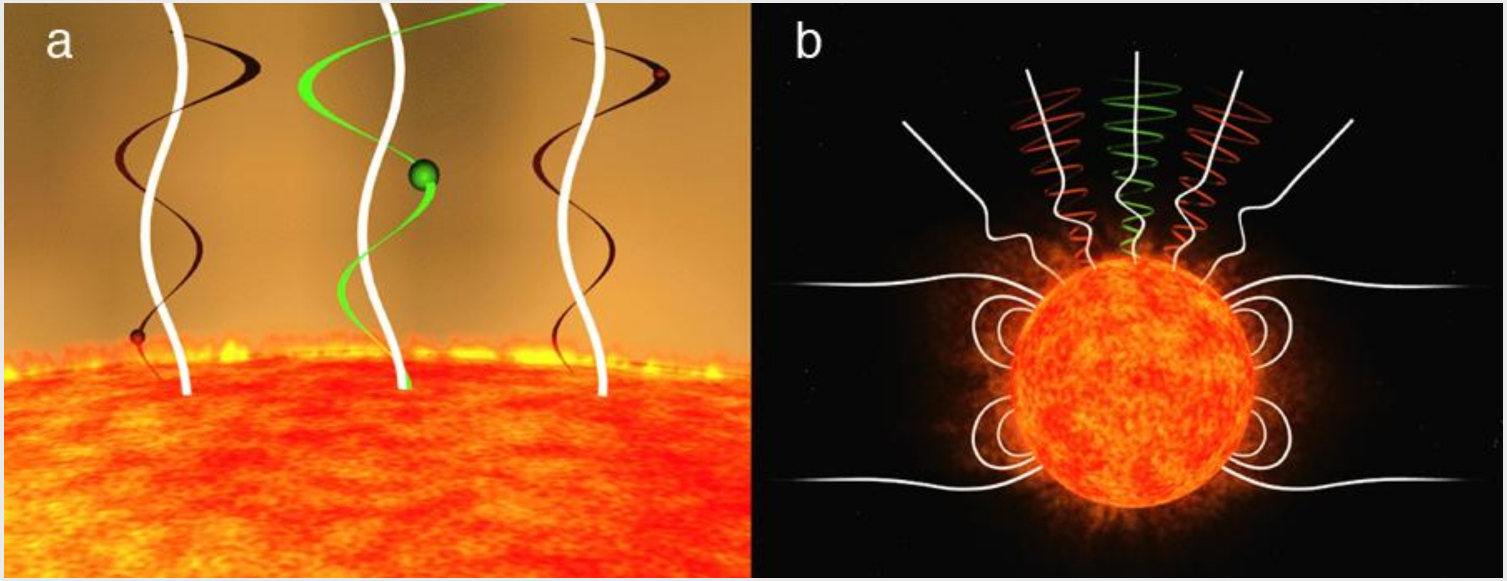


Figure 5.2: A cyclotron emission spectrum consists of a spike at the fundamental frequency νc, and harmonics at integral factors higher.

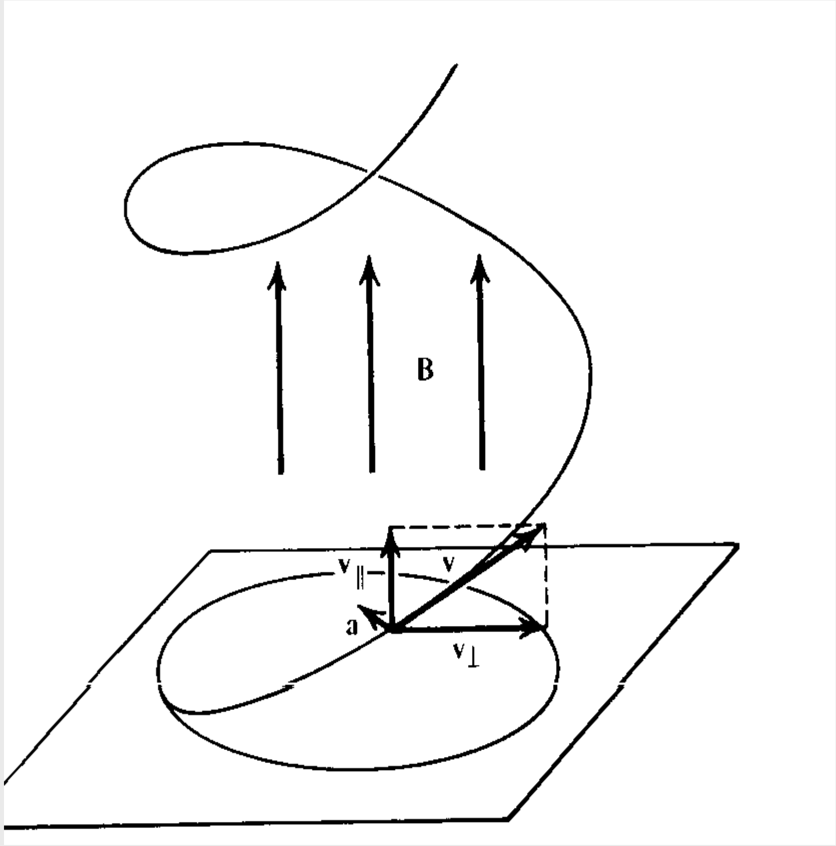
If acceleration is not a perfect sinusoid, emission also at multiples of cyclotron frequency: **harmonics**

CYCLOTRON EMISSION

- Solar Astronomy
- Planets (Jupiter in particular)
- Magnetic stars (white dwarfs and neutron stars)



SYNCHROTRON EMISSION



Charged particle in a magnetic field B in the relativistic limit

$$\frac{d}{dt}(\gamma m\vec{v}) = \frac{q}{c}\vec{v}\times\vec{B}$$

$$\frac{d}{dt}(\gamma mc^2) = q\vec{v}\cdot\vec{E} = 0$$

$$r_L = \frac{v_\perp^2}{a_\perp} = \frac{\gamma mc^2\beta\sin\alpha}{eB}$$

$$\nu_B = \frac{eB}{2\pi\gamma mc} = \frac{\nu_L}{\gamma}$$

The energy of the particle doesn't change (besides emission of radiation over a orbit). The Lorentz force does not work. A fundamental frequency is the giration frequency ν_B

SYNCHROTRON POWER

$$a_{\perp} = \frac{evB\sin\alpha}{\gamma m_e} \qquad P = \frac{2q^2}{3c^3}\gamma^4 \frac{q^2B^2v^2\sin^2\alpha}{\gamma^2 m^2{}_e c^2}$$

$$U_B{}^2 = \frac{B^2}{8\pi} \qquad r_e = \frac{e^2}{m_e c^2} \qquad \sigma_T = \frac{8\pi}{3} r_e^2 \qquad \beta = \frac{v}{c}$$

Energy density magnetic field

Electron classical radius

Thomson cross section

For an isotropic distribution of velocities you average over angles a (pitch angle)

$$\frac{\int \sin^2\alpha \, d\Omega}{4\pi} = \frac{2\pi}{4\pi}\int_0^{\pi} \sin^3\alpha \, d\vartheta = \frac{1}{2}\frac{1}{12}\left[\cos(3x) - 9\cos x\right]_0^{\pi} = \frac{16}{24} = \frac{2}{3}$$

SYNCHROTRON POWER

$$P = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B$$

You will find a similar expression for Inverse Compton emission. The analogy is that the electron sees a variable electro-magnetic field (photons) with typical frequency $h\nu_B$

$$\tau = \frac{E}{P} = \frac{\gamma m_e c^2}{4/3\sigma_T c U_B \gamma^2} = \frac{7.75\times 10^8 s}{\gamma^2 B^2} = \frac{24.6\ yr}{\gamma B^2}$$

Synchrotron lifetime is in many sources smaller than the age of the source meaning continuous injection or reacceleration

SYNCHROTRON SPECTRUM: A QUALITATIVE DISCUSSION

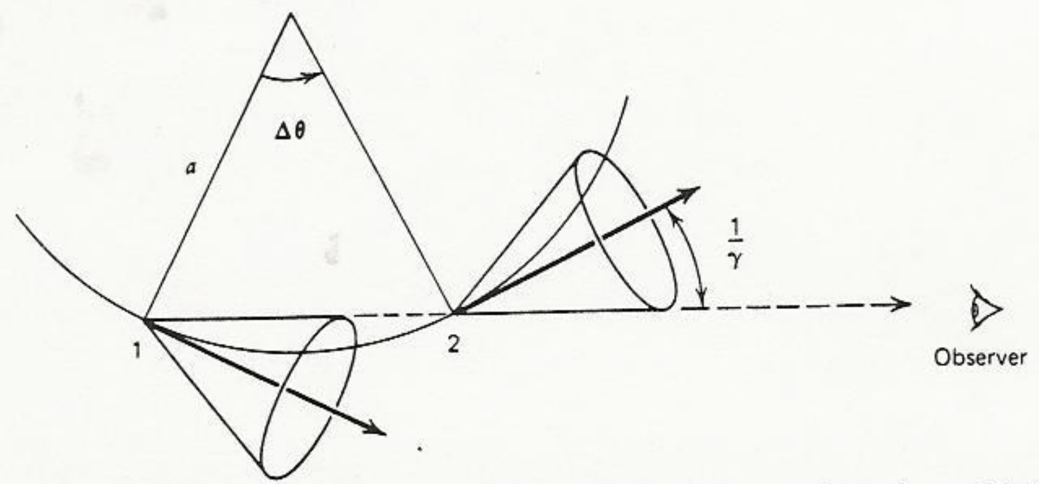
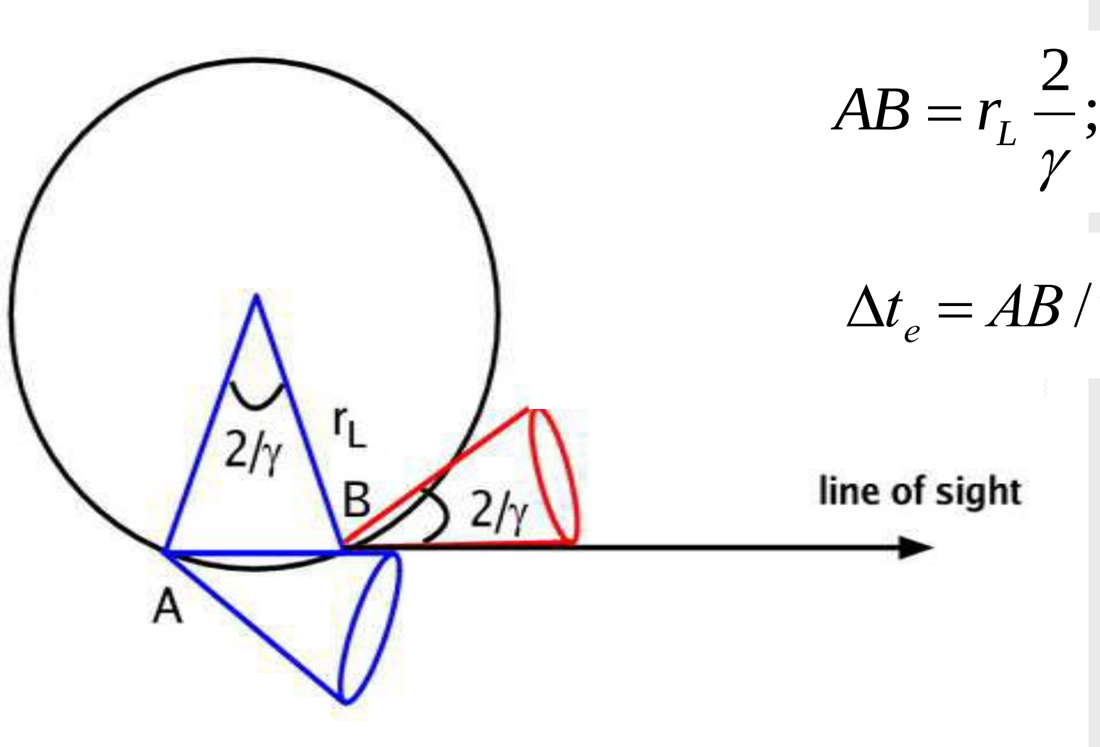


Figure 6.2 Emission cones at various points of an accelerated particle's trajectory.

There is a typical frequency associated with the synchrotron process and it is not the inverse of the revolution period (as for cyclotron) but it is related to the fraction of time for each orbit during which the observer receives some radiation

SYNCHROTRON SPECTRUM: A QUALITATIVE DISCUSSION



$$AB = r_L \frac{2}{\gamma}; \; r_L = \frac{\gamma v_\perp}{\omega_{cycl}} = \frac{\gamma v m_e c}{eB}$$

$$\Delta t_e = AB / v = \frac{2 m_e c}{eB}$$

$$\Delta t_A = \Delta t_e - AB/c = \Delta t_e - \Delta t_e \frac{v}{c} = \Delta t_e (1-\beta) = \Delta t_e \frac{1-\beta^2}{1+\beta} \sim \frac{\Delta t_e}{2\gamma^2} = \frac{m_e c}{eB\gamma^2}$$

Recalling the gyro angular frequency $\omega_B = \dfrac{eB}{\gamma m_e c} \Rightarrow \Delta t_A \approx \dfrac{1}{\gamma^3 \omega_B}$

SYNCHROTRON SPECTRUM: A QUALITATIVE DISCUSSION

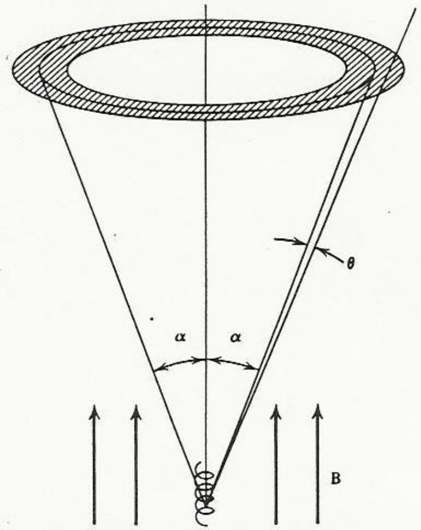
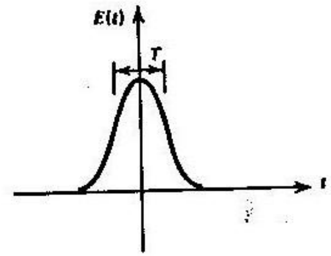


Figure 6.5 Synchrotron emission from a particle with pitch angle α. Radiation is confined to the shaded solid angle.

$$\Delta t_A \approx \frac{2\pi}{\gamma^3 \nu_B \sin\alpha}$$

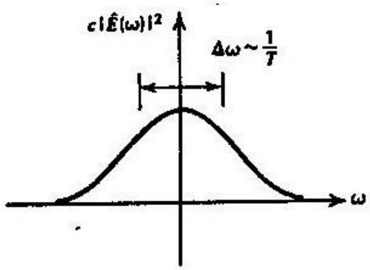
$$\nu_C = \frac{3}{4\pi}\gamma^3 \omega_B \sin\alpha$$

The pulse of the electric field E(t) has this typical width
So you can expect a broad spectrum extending to frequencies of the order ν_c before falling away



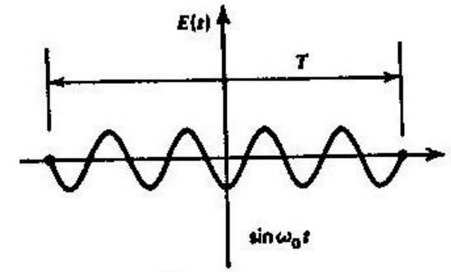
(a)

Figure 2.1a   Electric field of a pulse of duration T.



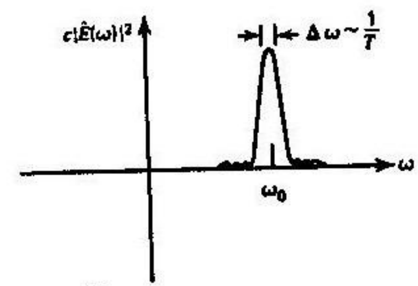
(b)

Figure 2.1b   Power spectrum for a.



(a)

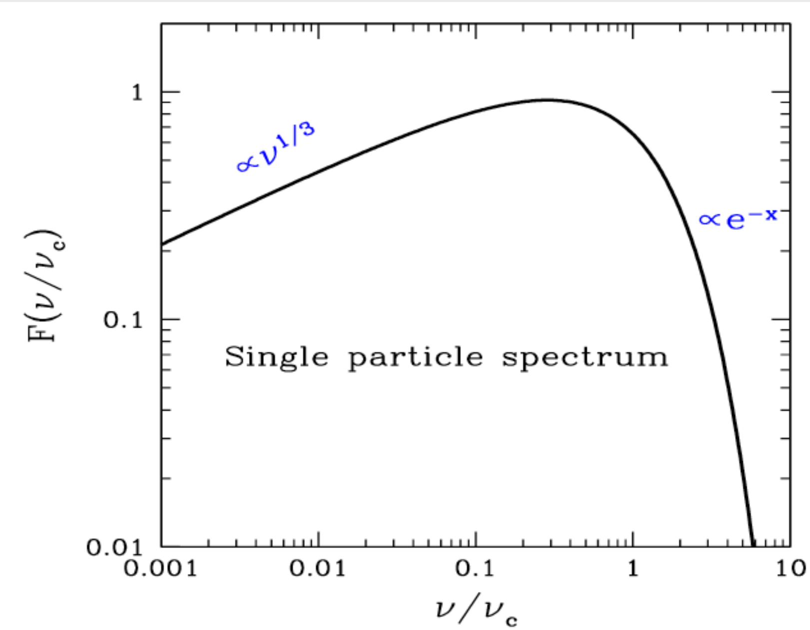
Figure 2.2a   Electric field of a sinusoidal pulse of frequency ω_0 and duration T.

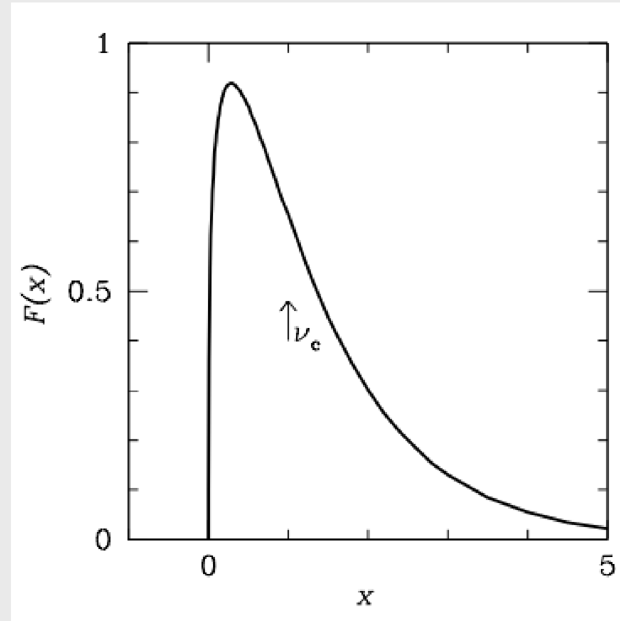


(b)

Figure 2.2b   Power spectrum for a.

SYNCHROTRON SPECTRUM

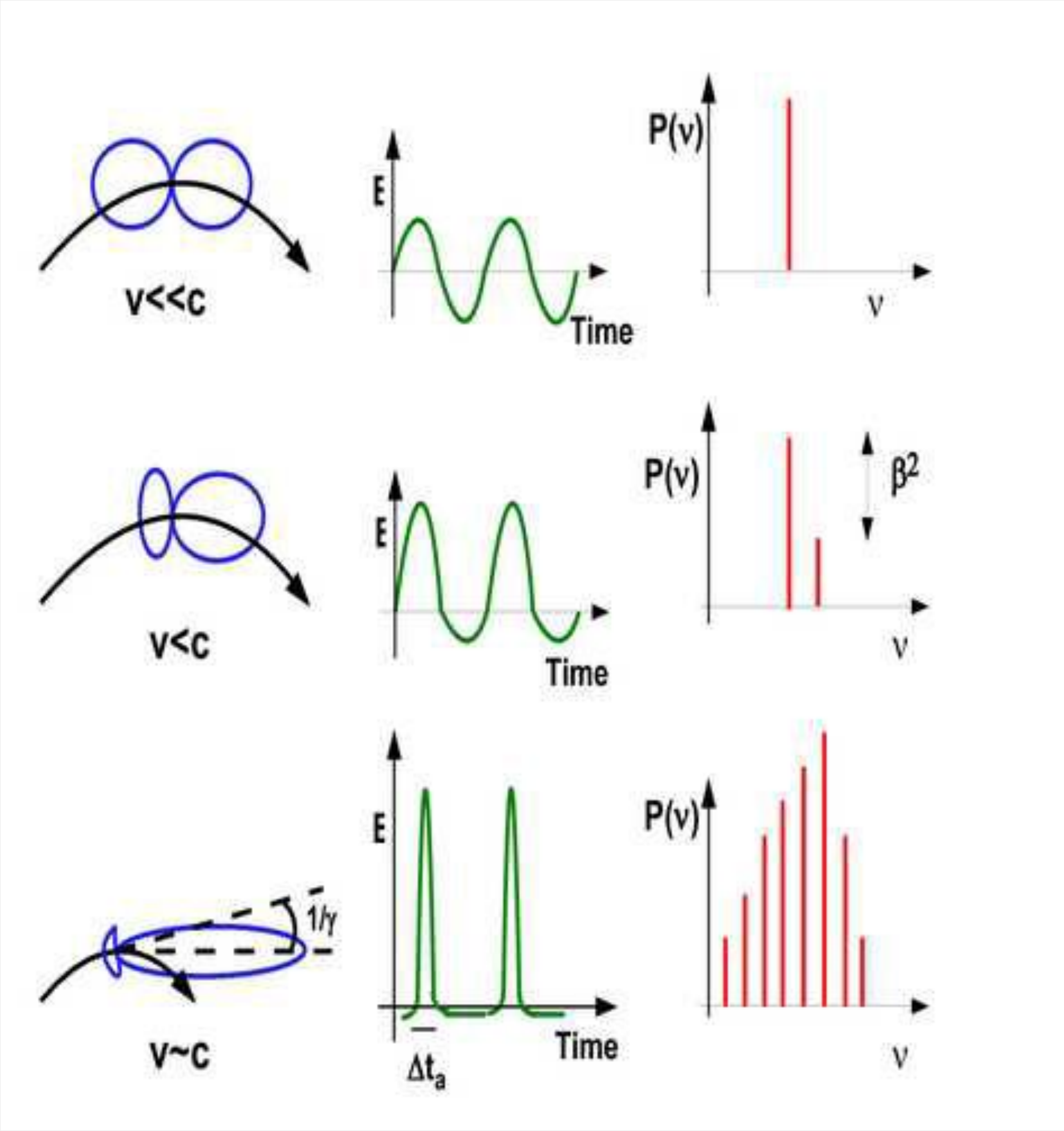




$$P(\omega) = \frac{\sqrt{3}}{2\pi} \frac{e^3 B \sin\alpha}{mc^2} F\left(\frac{\omega}{\omega_C}\right)$$

With detailed calculation this is the spectrum emitted by a single electron. F is a function with a peak at 0.29 ω_c

FROM CYCLOTRON TO SYNCHROTRON



The observer sees a **sinusoidal** (in time) electric field E(t).

Increasing the velocity, the pattern becomes **asymmetric** and the second harmonic appears.

Emission concentrated in the time $\Delta t_A \Rightarrow$ Fourier transformation of E(t) contains **many harmonics**, and the power is concentrated in harmonics with $\nu \sim 1/\Delta t_A$.

SPECTRUM POWER LAW DISTRIBUTION

Observed spectra for synchrotron sources are power laws

$$J(\nu) \propto \nu^{-\alpha}$$

α is the spectral index

A number of processes yields power law energy distribution for particles, in particular at high energies, i.e. shock acceleration

$$N(E)dE \propto kE^{-p}dE$$

p is the particle index

The synchrotron spectrum is sharply peaked, much narrower than the width of the power law electron energy spectrum, so we can simply approximate that an electron of energy E radiates its energy at the critical frequency

$$\nu_C = \gamma^2 \nu_L = \left(\frac{E}{m_e c^2} \right)^2 \nu_L$$

SPECTRUM POWER LAW DISTRIBUTION

$$J(\nu)d\nu = \left(-\frac{dE}{dt}\right)N(E)dE$$

Energy radiated in the range v to v+dv can be attributed to electrons with energies in the range E to E+dE

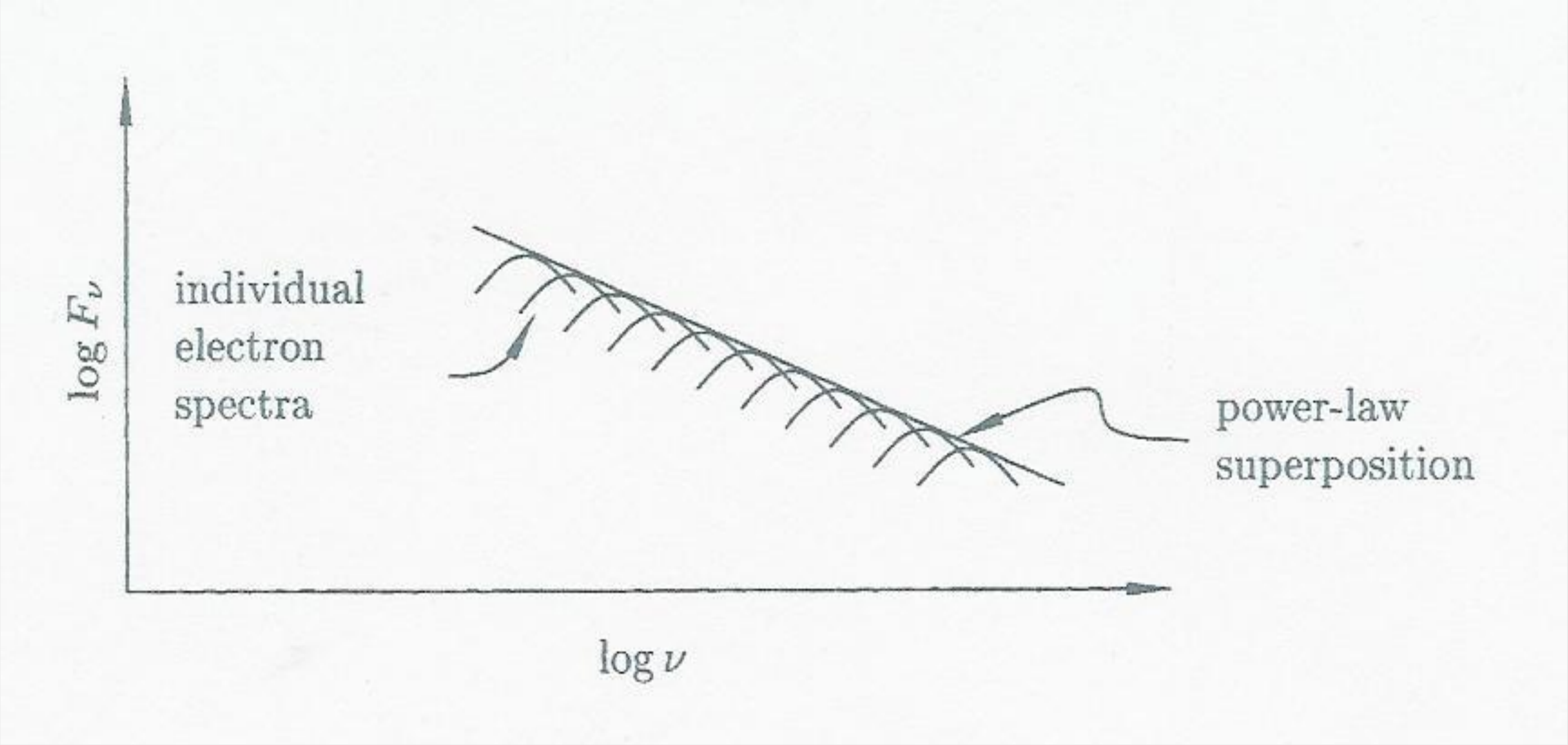
$$E = \left(\frac{\nu}{\nu_L}\right)^{1/2} m_e c^2 \quad dE = \frac{m_e c^2}{2\nu_L^{-1/2}}\nu^{-1/2}d\nu \quad -\frac{dE}{dt} = \frac{4}{3}\sigma_T c\left(\frac{E}{m_e c^2}\right)\beta^2 \frac{B^2}{8\pi}$$

$$J(\nu) = const\, B^{(p+1)/2}\nu^{-(p-1)/2}$$

$$\boxed{\alpha = (p-1)/2}$$

The emitted spectrum is determined by the slope of the electron energy spectrum, rather than by the shape of the emission spectrum of a single electron

SPECTRUM POWER LAW DISTRIBUTION

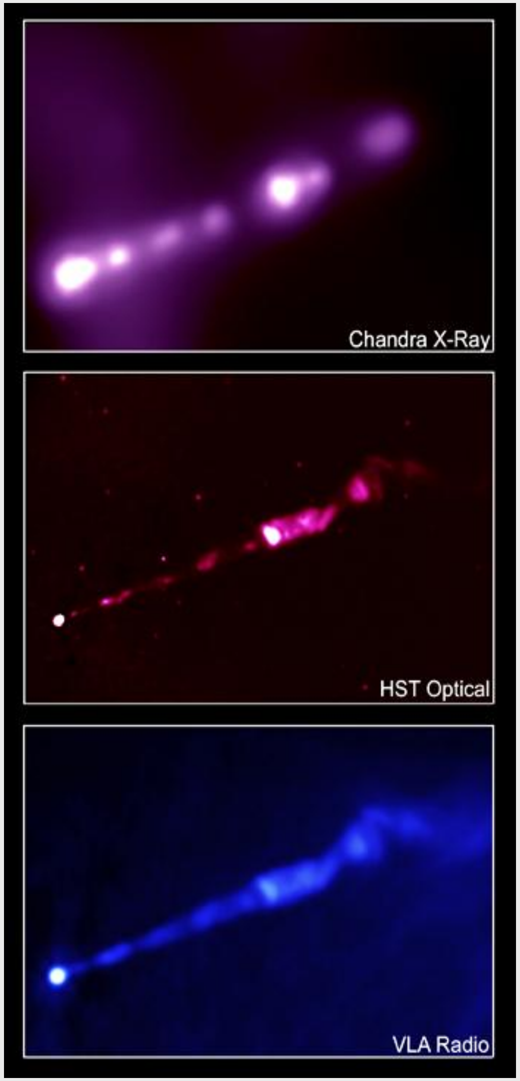


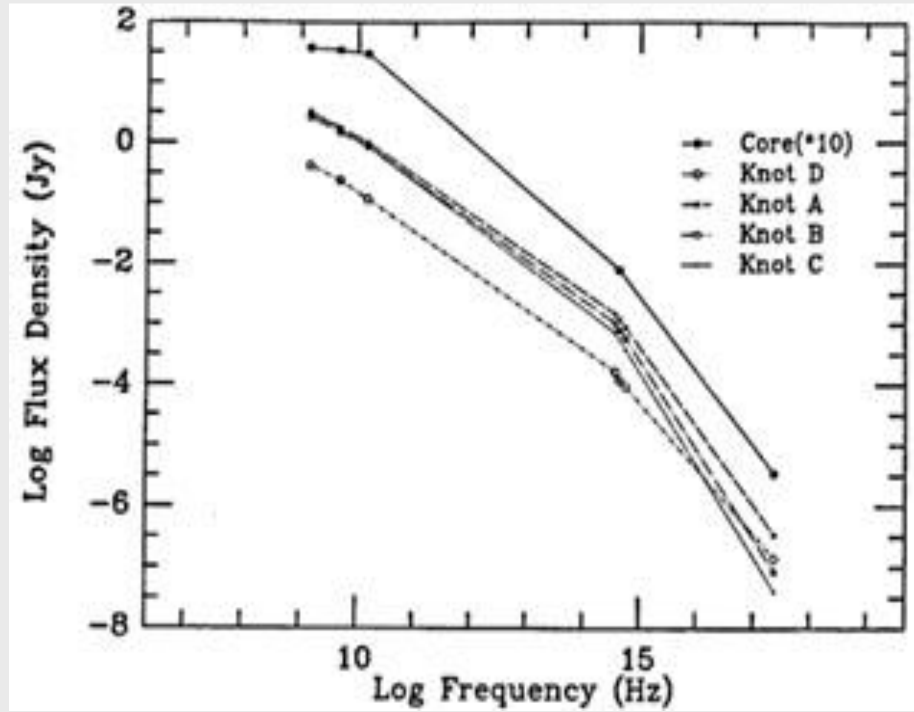
POLARIZATION

$$\Pi = \frac{p+1}{p+7/3}$$

An essential feature of synchrotron radiation is that it is **polarized**. The degree of polarization can be very high, i.e. for p=3 it is 75%.
However sometimes it is difficult to achieve this high level of polarization due to disordered magnetic fields.

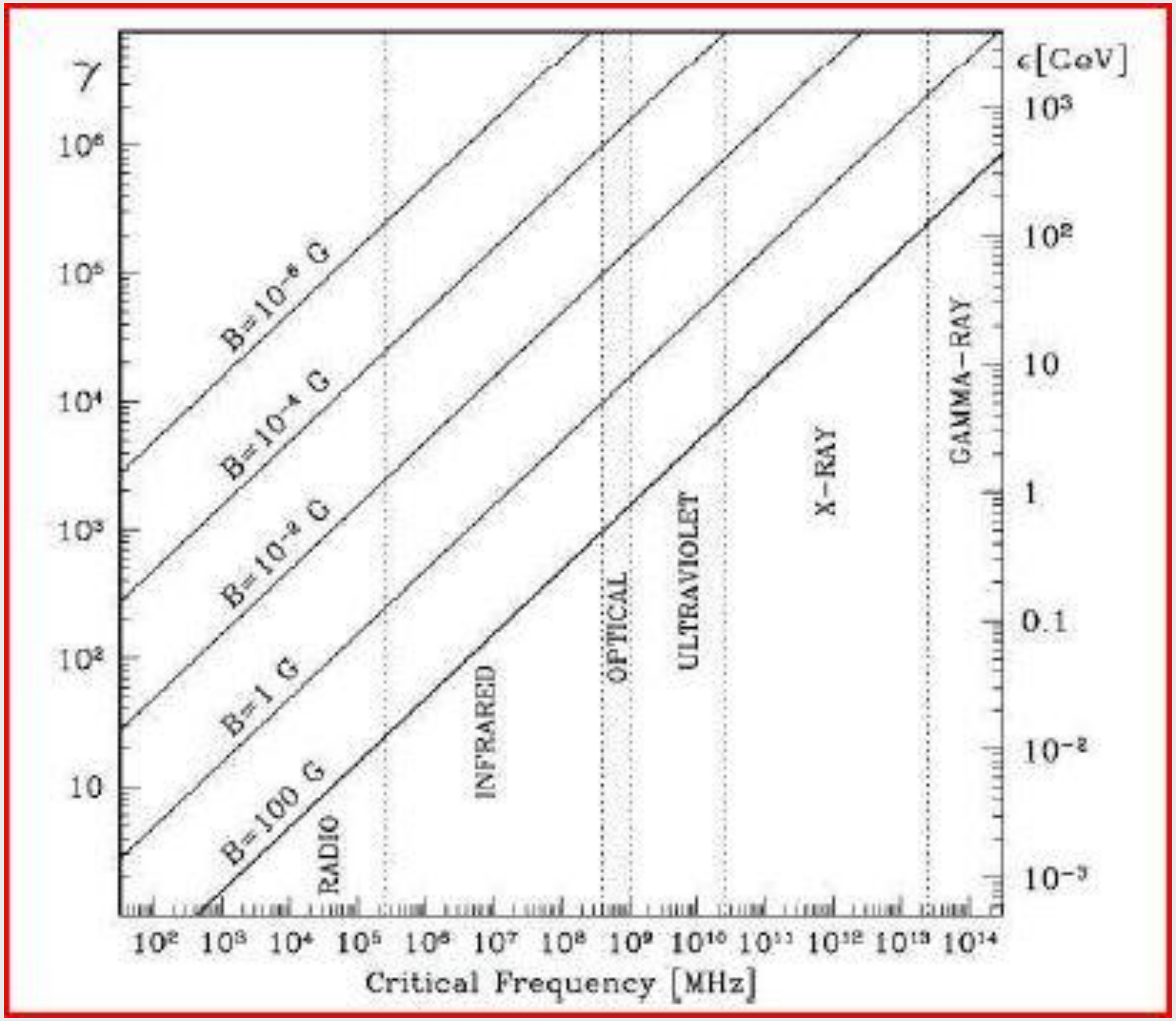
RELATIVISTIC JETS



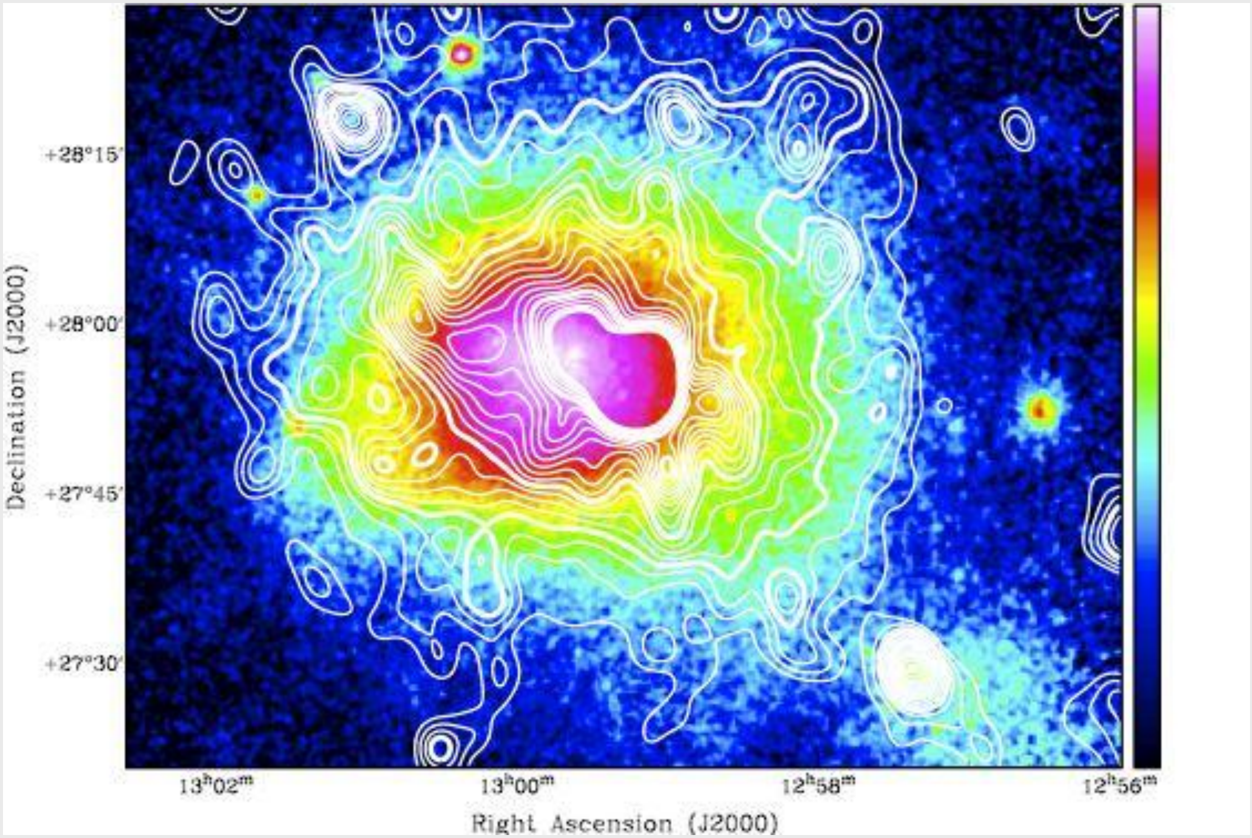


M87 jet: morphology is the same at radio, optical annd X-ray wavelength. Power law spectrum, steepening due to losses, polarized. Only one side jet is visible.

EXTENDED RADIO EMISSION IN CLUSTERS



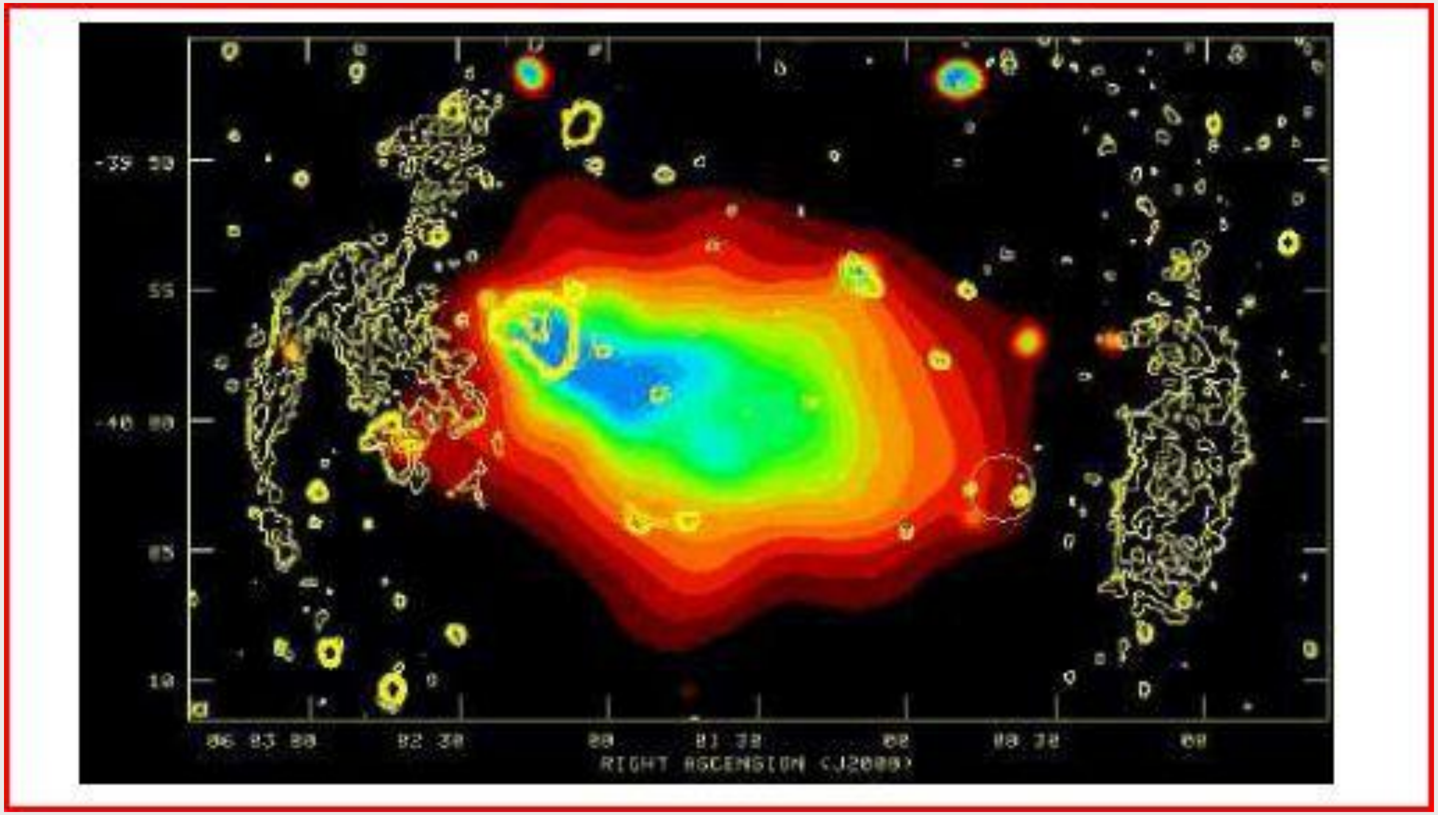
EXTENDED RADIO EMISSION IN CLUSTERS



Coma cluster

Radio halos fill the volume of the clusters, Mpc extension. Given the short synchrotron lifetime electrons can't simply diffuse. No detected polarization

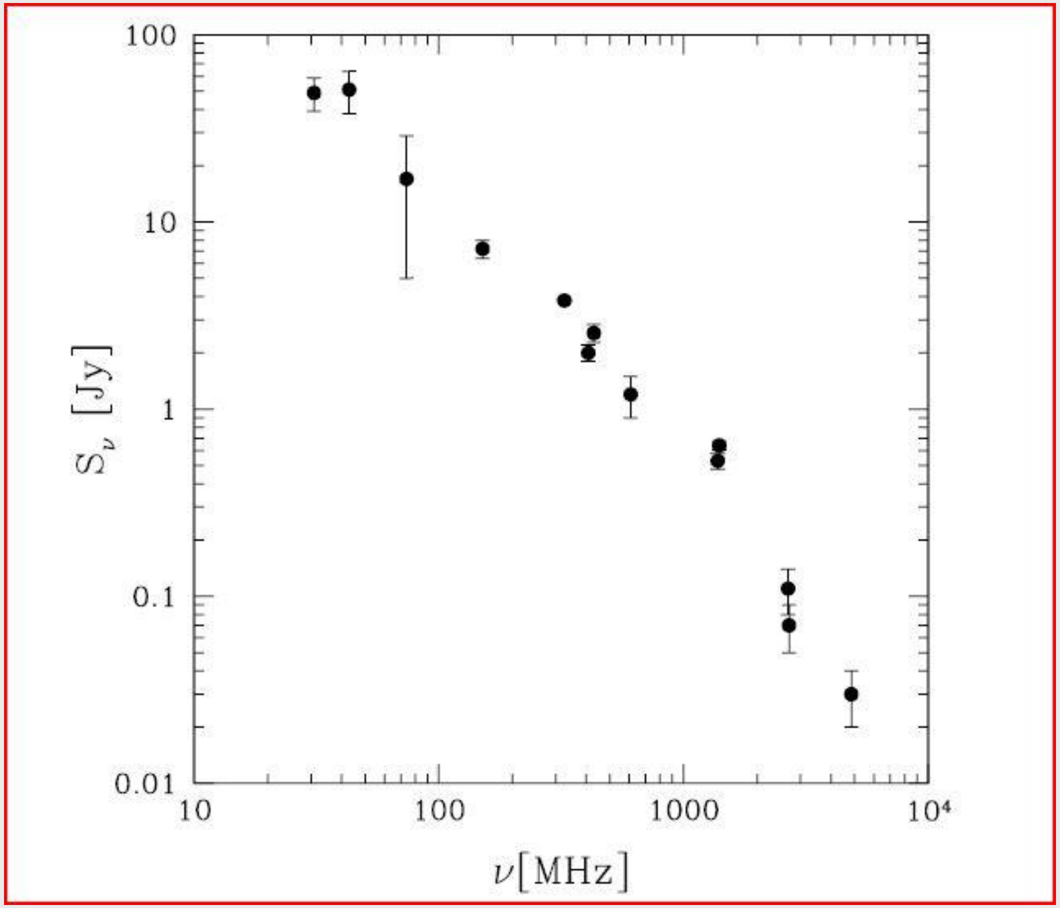
EXTENDED RADIO EMISSION IN CLUSTERS



A3376

Radio relics linear structures of Mpc size, usually at the outskirts of the diffuse extended emission. 20%-50% detected polarization

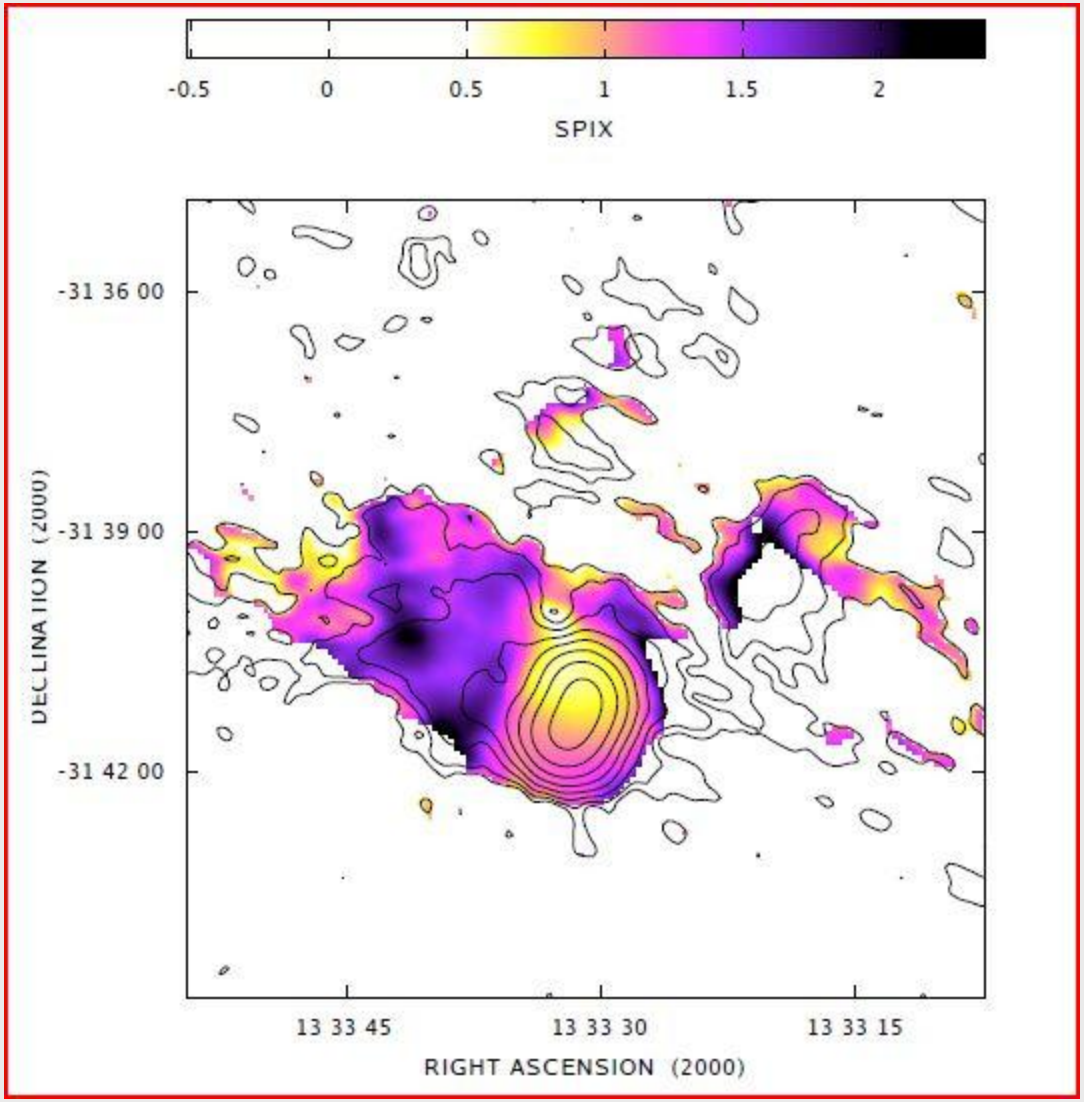
SPECTRA OF RADIO HALOS



COMA CLUSTER
RADIO HALO
SPECTRUM

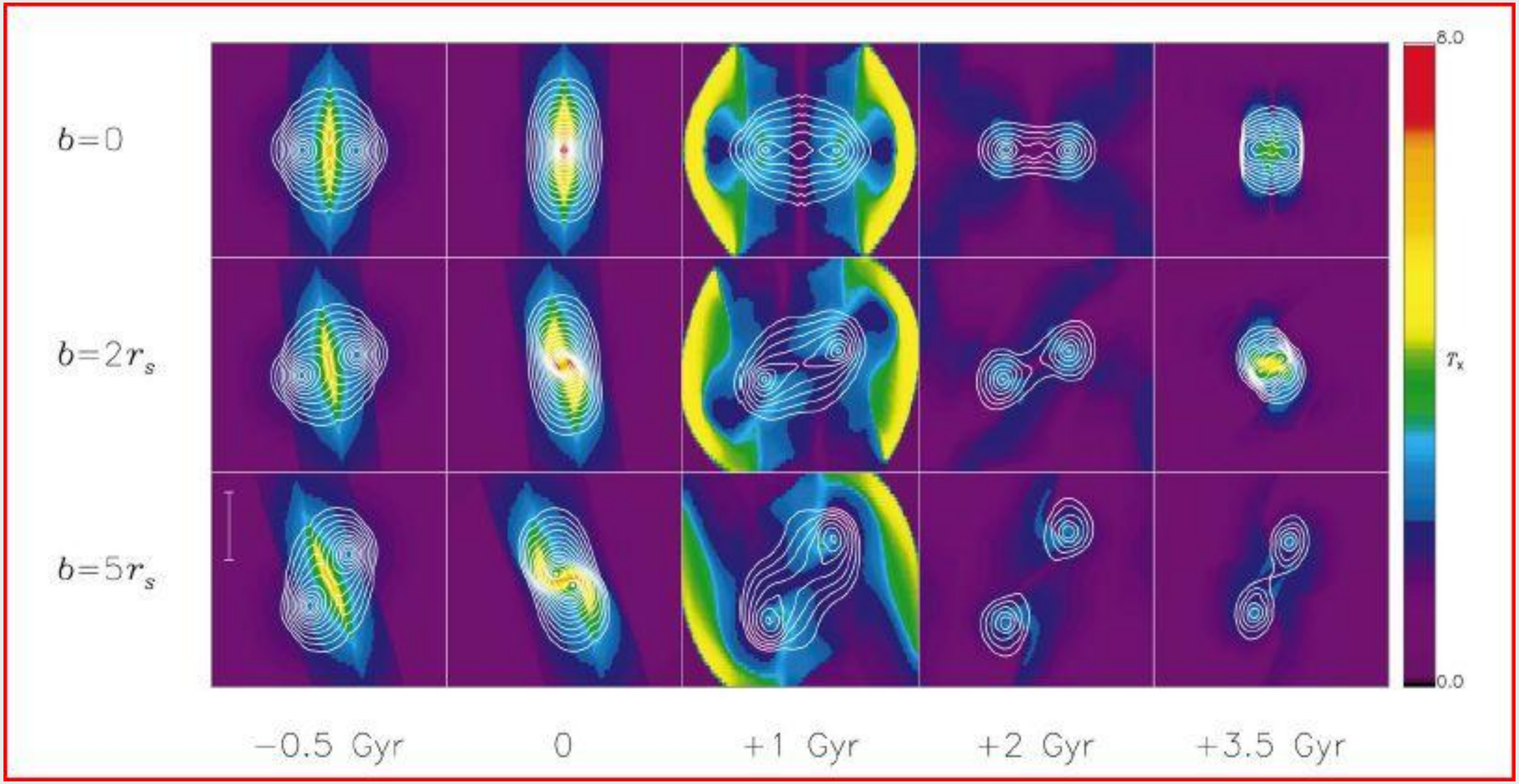
Power law spectra with α ≥ 1, in the best studied case spectral maps show spectrum steepening in the outskirts (energy losses)

SPECTRA OF RADIO HALOS



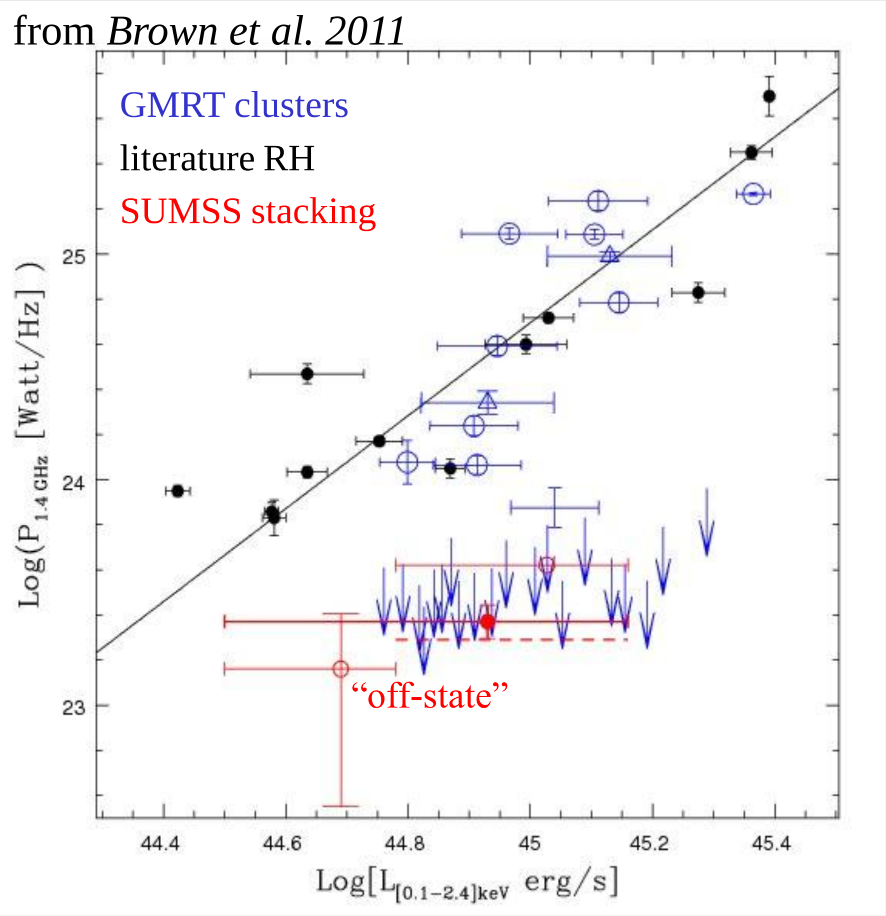
A3562 RADIO SPECTRAL MAP

CONNECTION WITH CLUSTER MERGERS



Radio emission related to mergers: shocks accelerate seed electrons at relics, turbulence is responsible for radio halo emission

CONNECTION WITH CLUSTER MERGERS



Radio halo bimodality of clusters: only luminous massive mergers have radio halos (strong support for primary models, i.e. reaccelerated electrons, not electrons produces in p-p collisions, i.e. "secondary models).

Per domande/informazioni il mio sito web
http://www.iasf-milano.inaf.it/~gasta/personal.html
Posta elettronica: gasta@iasf-milano.inaf.it