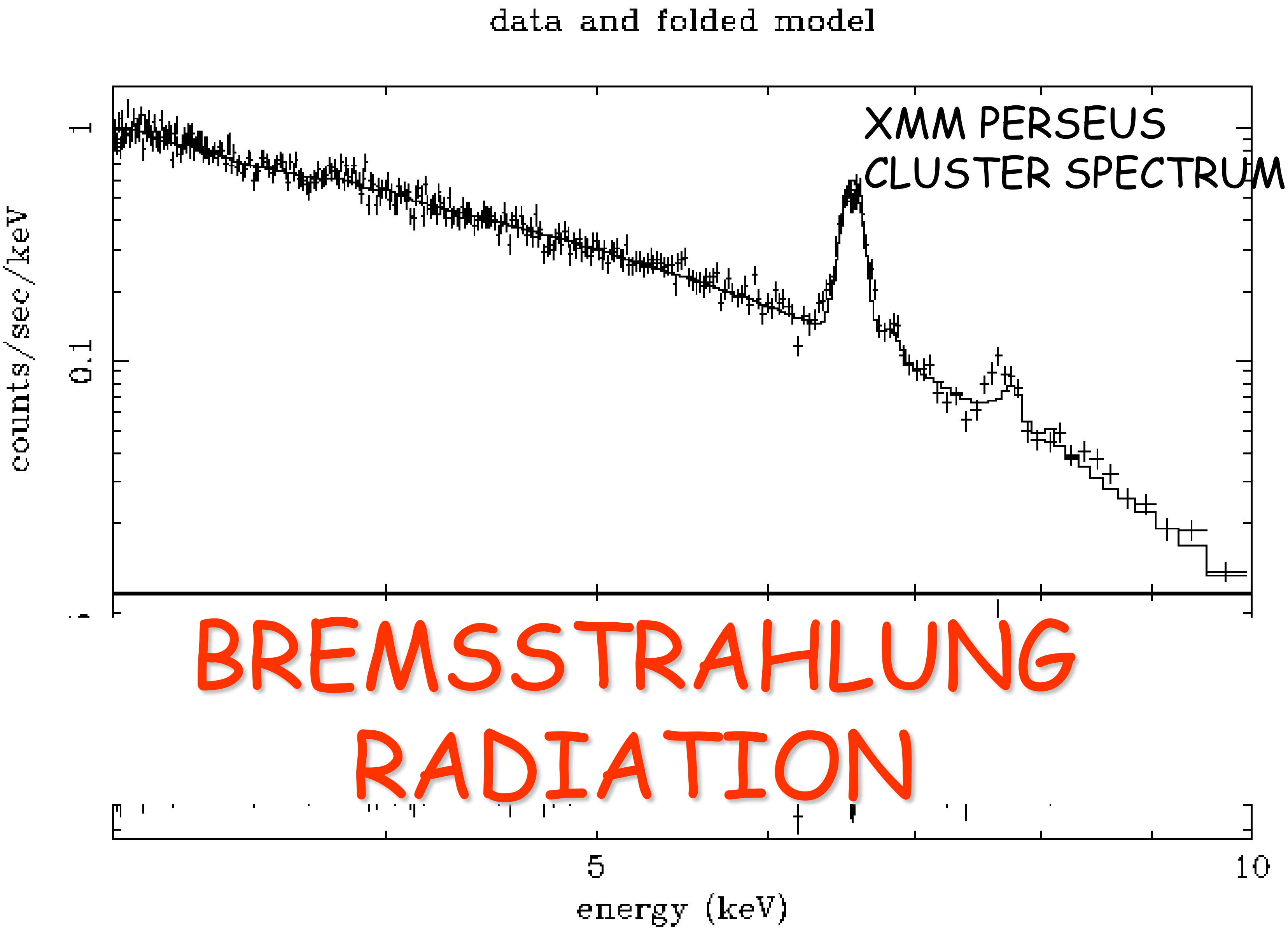
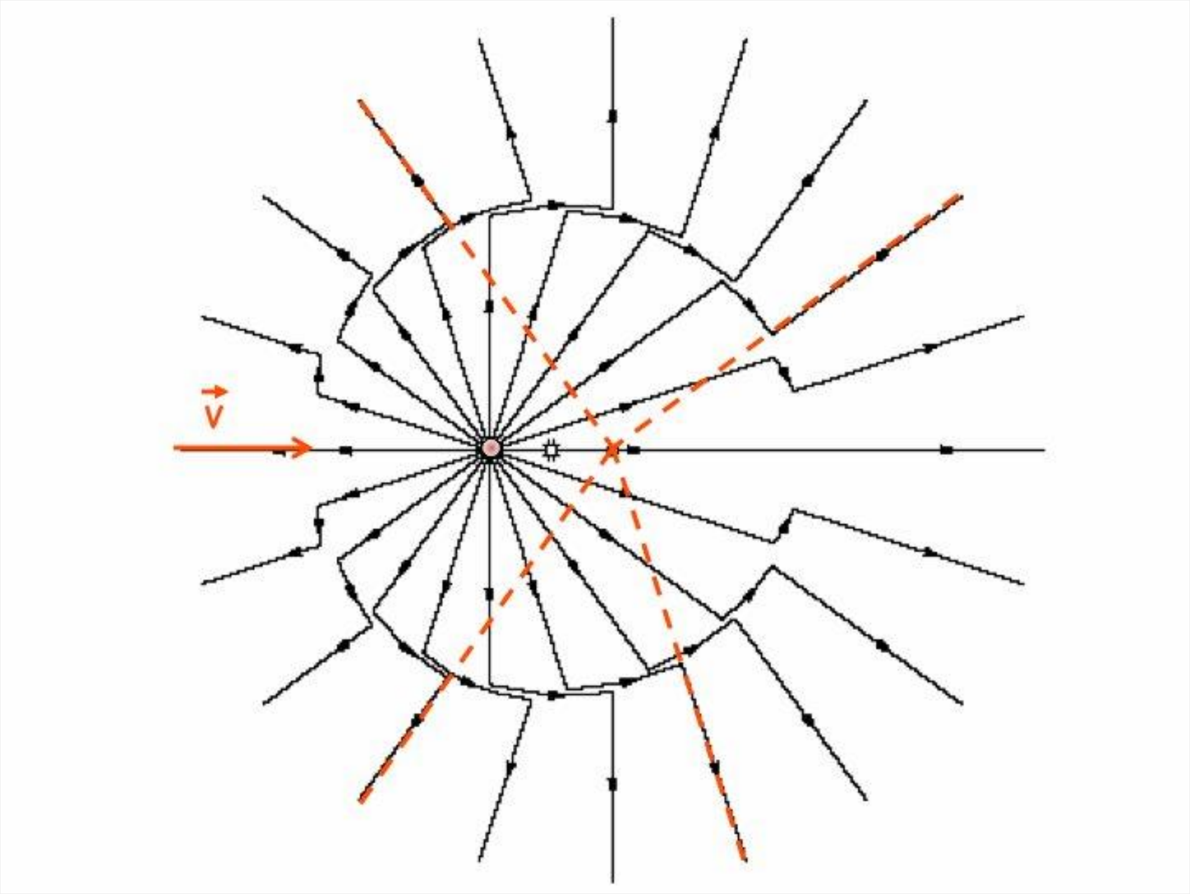

data and folded model
XMM PERSEUS
CLUSTER SPECTRUM
counts/sec/keV
1
0.1
5
10
energy (keV)
BREMSSTRAHLUNG
RADIATION

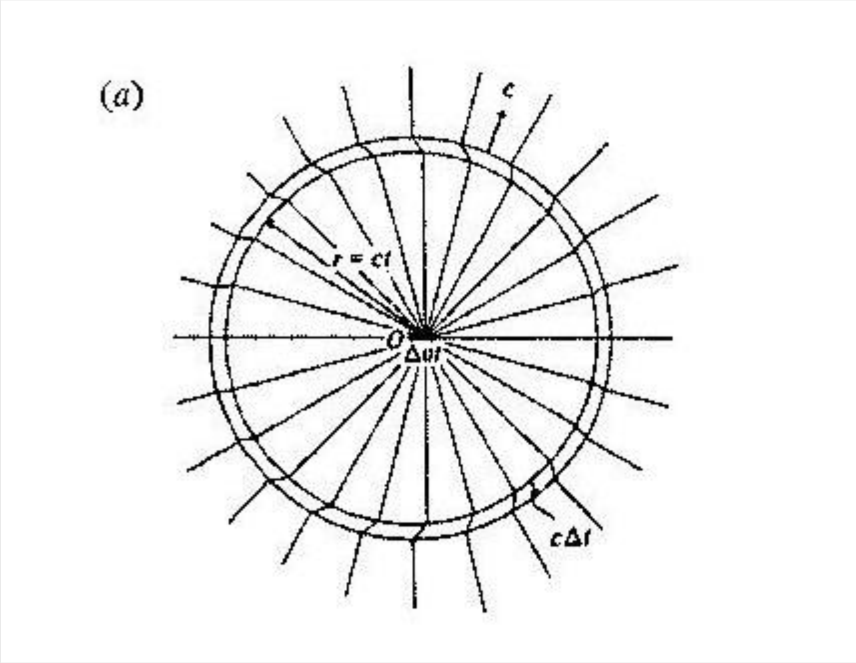
OUTLINE OF THE LESSON

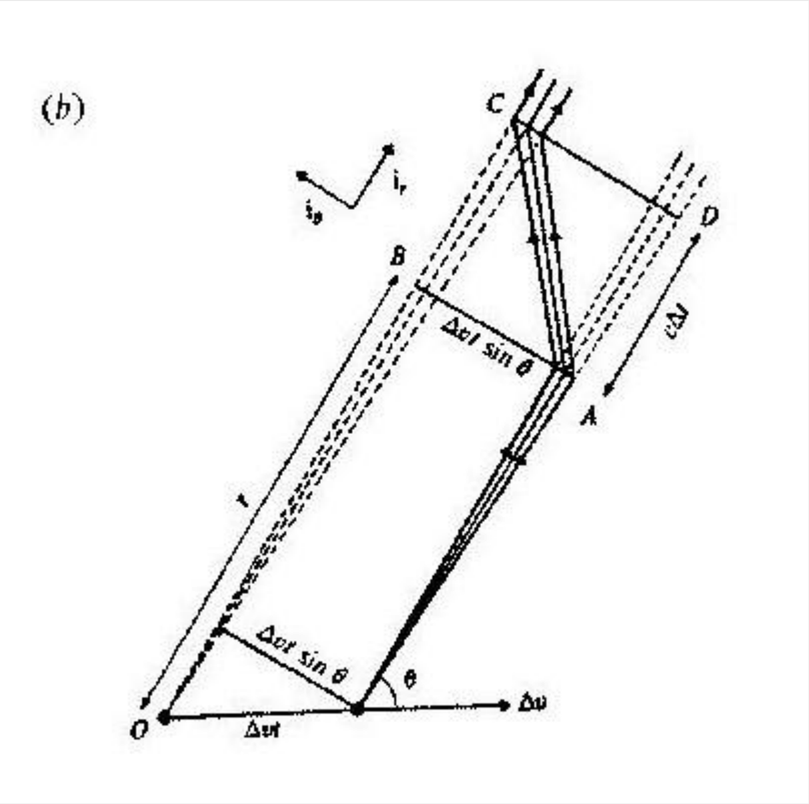
- RADIATION FROM AN ACCELERATED CHARGE (LARMOR FORMULA)
- RADIATION EMITTED BY A SINGLE ELECTRON DURING ITS NEAR COLLISION WITH A PROTON AND RELATIONSHIP BETWEEN ELECTRON SPEED, IMPACT PARAMETER AND FREQUENCY OF THE EMITTED RADIATION
- RADIATION EMITTED BY A DISTRIBUTION OF ELECTRON
- APPLICATION TO GALAXY CLUSTERS: WHAT PHYSICAL QUANTITIES CAN WE DERIVE ?

RADIATION FROM AN ACCELERATED CHARGE: EURISTIC DERIVATION

(Thomson 1903)







$$\frac{E_\theta}{E_r} = \frac{\Delta v t \sin\theta}{c\Delta t} \quad \Rightarrow \quad E_\theta = \frac{q|\vec{a}|\sin\theta}{c^2 r}$$

key r^{-1} dependence of acceleration field: it will allow the transport of energy, i.e. radiation

LARMOR'S FORMULA

The outflow is energy is calculate through the Poynting vector

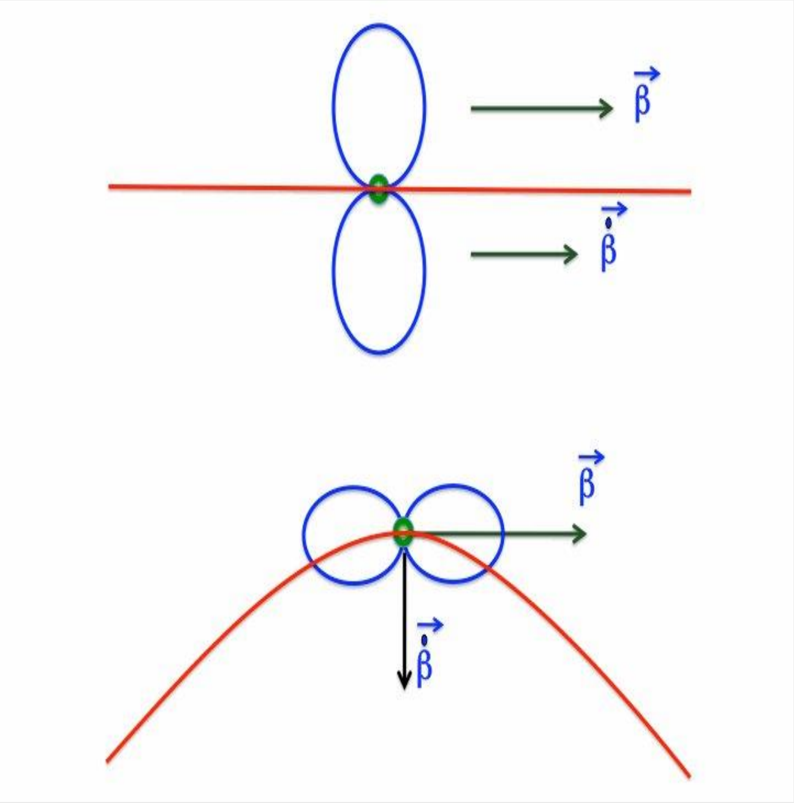
$$S = \frac{c}{4\pi} E_{rad}^2 = \frac{c}{4\pi} \frac{q^2 a^2}{r^2 c^4} \sin^2 \theta$$

$$\frac{dW}{dt d\Omega} = \frac{q^2 a^2}{4\pi c^3} \sin^2 \theta$$

Multiply for the area $r^2 d\Omega$

$$\boxed{P = \frac{dW}{dt} = \frac{2q^2}{3c^3} a^2}$$

LARMOR'S FORMULA

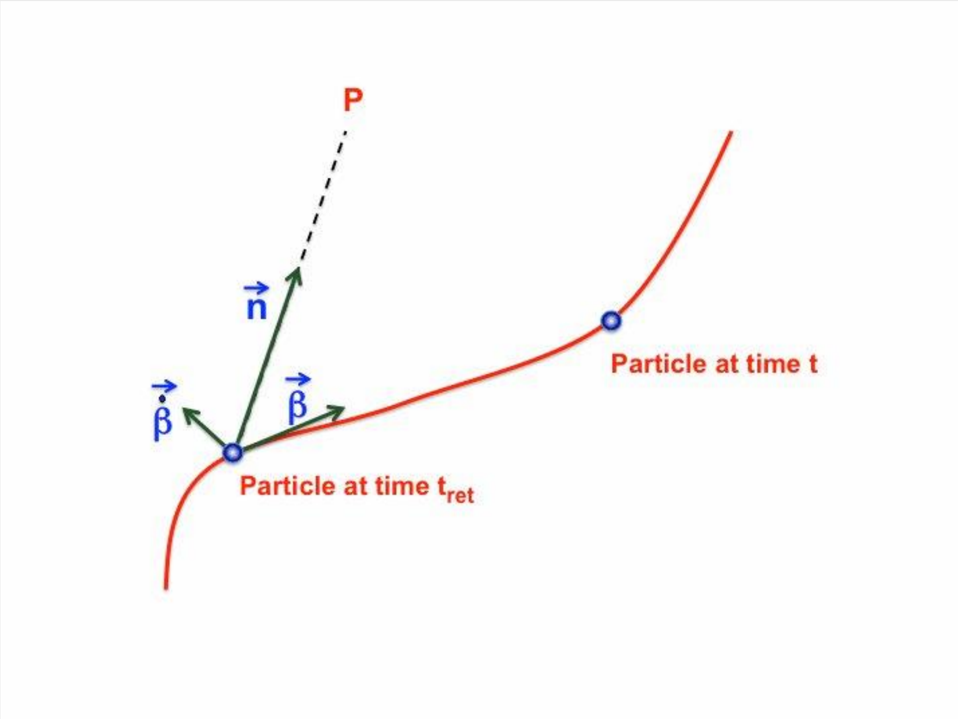


- Power radiates is proportional to the square of the charge and the square of the acceleration
- Characteristic dipole pattern proportional to $\sin^2\theta$: no radiation emitted in the direction of acceleration and the maximum is emitted perpendicular to the acceleration
- The radiation is polarized: if the particle accelerated along a line, we will have a 100% polarization

RADIATION FROM AN ACCELERATED CHARGE: RIGOROUS DERIVATION

Fully relativistic, exploiting the fact that P = dW/dt is a relativistic invariant. It starts from Maxwell's equations using the scalar and vector potentials at retarded times (Liénard-Wiechart potentials)

If we want the fileds at point r and time t we first must determine the "retarded" position and time of the particle r_{ret} and t_{ret}



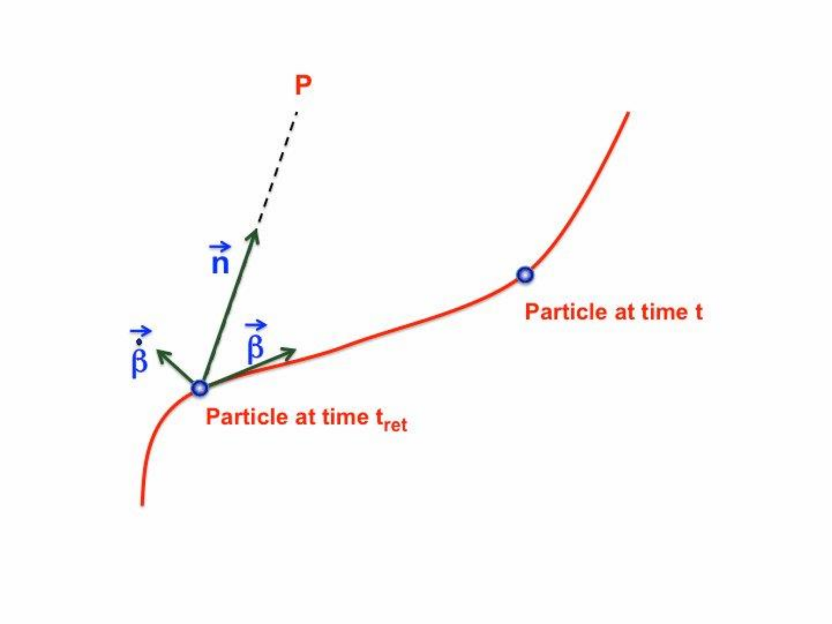
$$t_{ret} = t - \frac{R(t_{ret})}{c}$$

$$\vec{\beta} = \frac{\vec{v}}{c}$$

$$k = 1 - \vec{n} \cdot \vec{\beta}$$

$$\vec{E} = q\left[\frac{(\vec{n}-\vec{\beta})(1-\beta^2)}{kr^2}\right]_{t_{ret}} + \frac{q}{c}\left\{\frac{\vec{n}\times\left[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{k^3 r}\right\}_{t_{ret}}$$

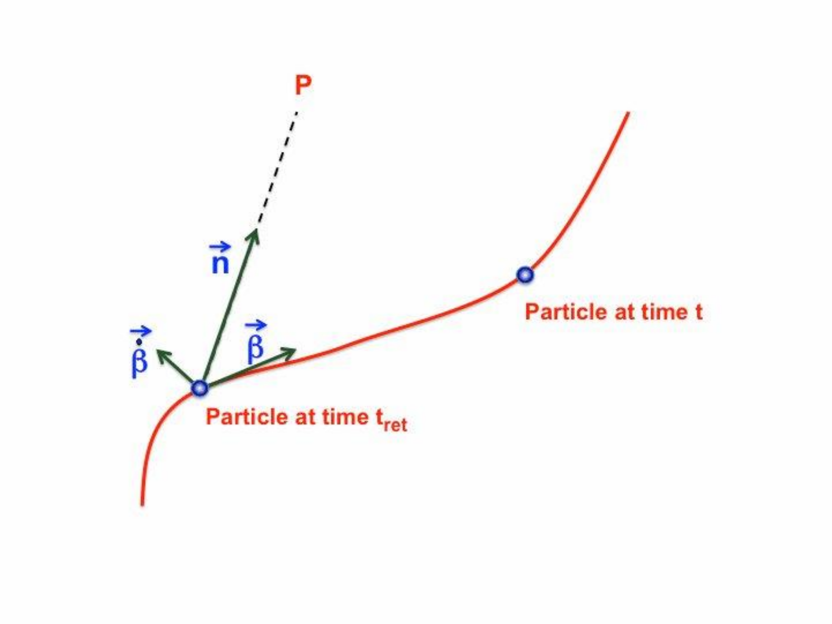
$$\vec{B} = \vec{n} \times \vec{E}$$



$$\boxed{\vec{E} = q\left[\frac{(\vec{n}-\vec{\beta})(1-\beta^2)}{kr^2}\right]_{t_{ret}}} + \frac{q}{c}\left\{\frac{\vec{n}\times\left[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{k^3 r}\right\}_{t_{ret}}$$

Velocity field: reduces to electrostatic when ß=0

It does not transport energy: SxA proportional to $r^{-2} \rightarrow 0$



$$\vec{E} = q\left[\frac{(\vec{n}-\vec{\beta})(1-\beta^2)}{kr^2}\right]_{t_{ret}} + \boxed{\frac{q}{c}\left\{\frac{\vec{n}\times\left[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{k^3 r}\right\}_{t_{ret}}}$$

Acceleration field: it transports energy because SxA does not go to zero

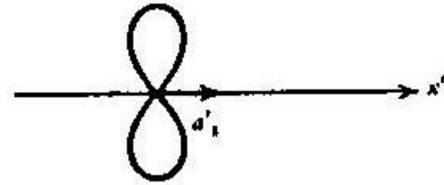
$$\vec{E}_{rad} = \frac{q}{c}\left\{\frac{\vec{n}\times\left[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{k^3 r}\right\}_{t_{ret}}$$

In the non-relativistic case $k = 1 - \vec{n}\cdot\vec{\beta} \approx 1$

$$\left|\vec{\beta}\right| << 1 \qquad \vec{n}-\vec{\beta} \approx \vec{n}$$

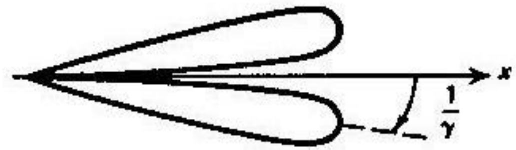
$$\left|\vec{E}_{rad}\right| = \frac{q}{c}\left\{\frac{\vec{n}\times\left[\vec{n}\times\dot{\vec{\beta}}\right]}{r}\right\}_{t_{ret}} = \frac{qa}{rc^2}\sin\theta$$

Which is the expression we already found in the euristic derivation



(a)

Figure 4.11a Dipole radiation pattern for particle at rest.

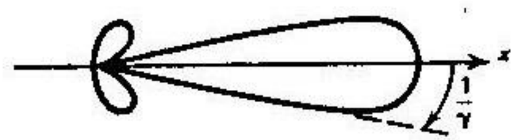


(b)

Figure 4.11b Angular distribution of radiation emitted by a particle with parallel acceleration and velocity.

(c)

Figure 4.11c Same as a.



(d)

Figure 4.11d Angular distribution of radiation emitted by a particle with perpendicular acceleration and velocity.

THE RADIATION SPECTRUM

The spectrum of radiation depends on the time variation of the electric field

There's no meaning in a spectrum at a precise instant of time, you can know only of a spectrum during a sufficiently long time Δt and still you can only define the spectrum to within a frequency resolution $\Delta\omega\Delta t > 1$

You use Fourier analysis and the properties of Fourier transforms to work out the spectral properties of the radiation

$$\hat{E}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E(t)e^{i\omega t}dt$$

Fourier transform of E(t)
Contains all the information about the frequency behavior of E(t)

THE RADIATION SPECTRUM

$$\frac{dW}{dA} = \frac{c}{4\pi}\int_{-\infty}^{+\infty} E^2(t)dt$$

Total energy per unit area in terms of the Poynting vector

$$\int_{-\infty}^{+\infty} E^2(t)dt = 2\pi\int_{-\infty}^{+\infty} \left|\hat{E}(\omega)\right|^2 d\omega$$

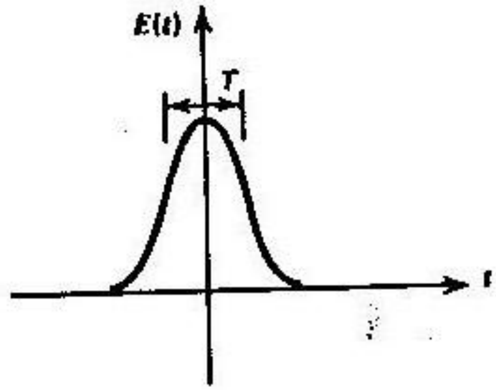
From Parseval's theorem

$$\frac{dW}{dA} = c\int_{0}^{+\infty} \left|\hat{E}(\omega)\right|^2 d\omega$$

From simmetry properties of E(t) which is real so negative frequencies can be eliminated multiplying by 2

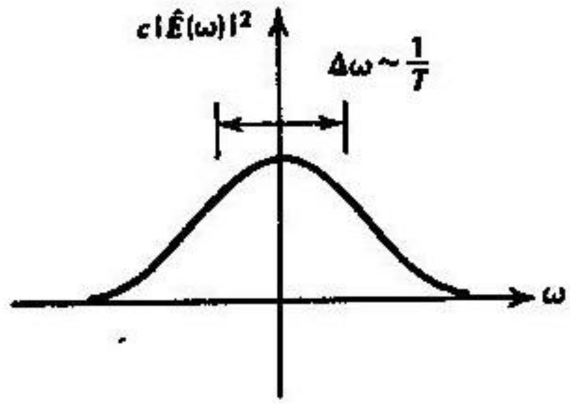
$$\boxed{\frac{dW}{dA\,d\omega} = c\left|\hat{E}(\omega)\right|^2}$$

Energy per unit area per unit frequency



(a)

Figure 2.1a Electric field of a pulse of duration T.



(b)

Figure 2.1b Power spectrum for a.

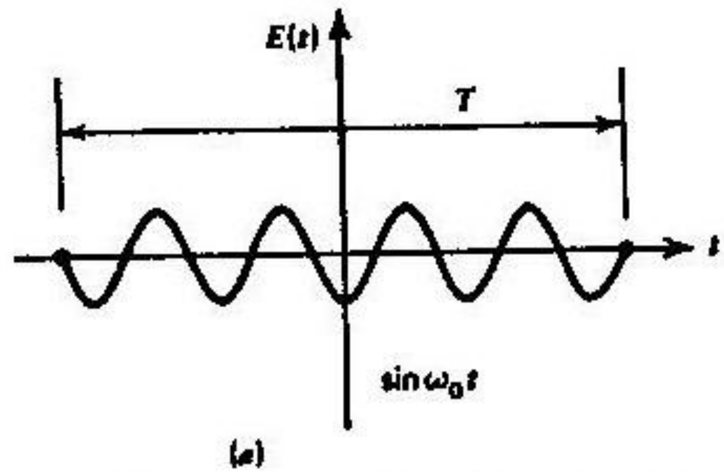


Figure 2.2a Electric field of a sinusoidal pulse of frequency ω_0 and duration T.

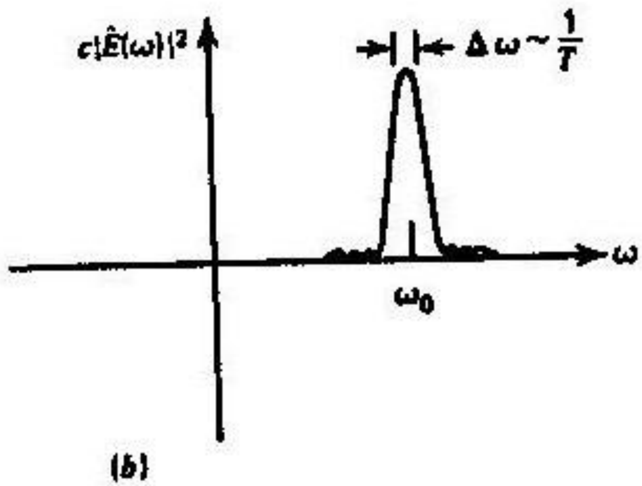


Figure 2.2b Power spectrum for a.

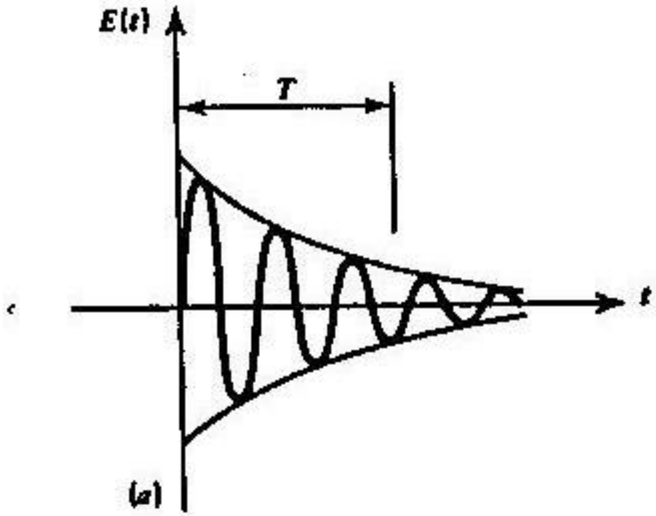


Figure 2.3a *Electric field of a damped sinusoid of the form* $\exp(-t/T)\sin\omega_0 t$.

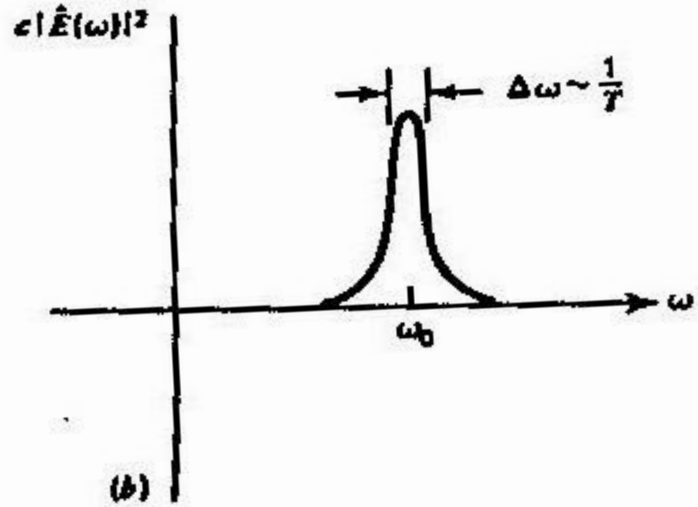


Figure 2.3b *Power spectrum for a.*

DIPOLE APPROXIMATION

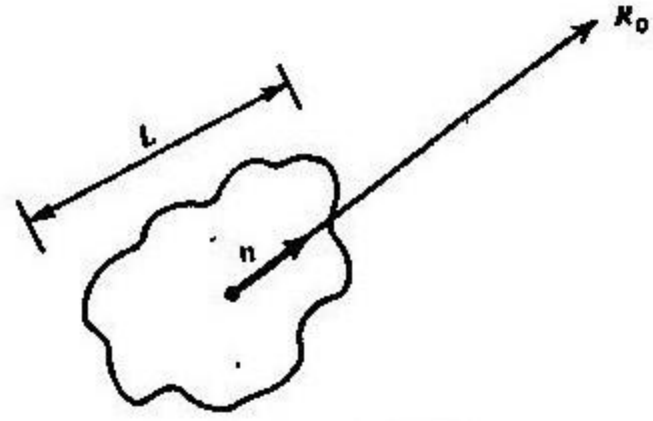


Figure 3.4 Radiation from a medium of size L.

Consider a system with many particles with positions r_i velocities v_i in principle retarded times will differ for each particle.

It is possible to ignore this difficulty in some situations: τ is the typical time scale for changes, then if $\tau >> L/c$ then the differences are negligible

DIPOLE APPROXIMATION

The condition $\tau >> L/c$ translates into, if $\nu \approx 1/\tau$, $c/\nu >> L$ which means $\lambda >> L$ size of the system small compared to wavelength.

Another way is to consider as l the characteristic scale of the particle's orbit, then $\tau \approx l/v$ and so $v/c \ll l/L$ equivalent to the non relativistic condition $v \ll c$

$$\vec{d} = \sum_i q_i \vec{r}_i$$ Dipole moment of the distribution of charges

$$\frac{dP}{d\Omega} = \frac{\ddot{\vec{d}}}{4\pi c} \sin^2\theta \qquad P = \frac{2\ddot{d}^2}{3c^3}$$

DIPOLE APPROXIMATION

Spectrum of radiation in the dipole approximation

$$d(t) = \int_{-\infty}^{+\infty} e^{-i\omega t} \hat{d}(\omega) d\omega$$

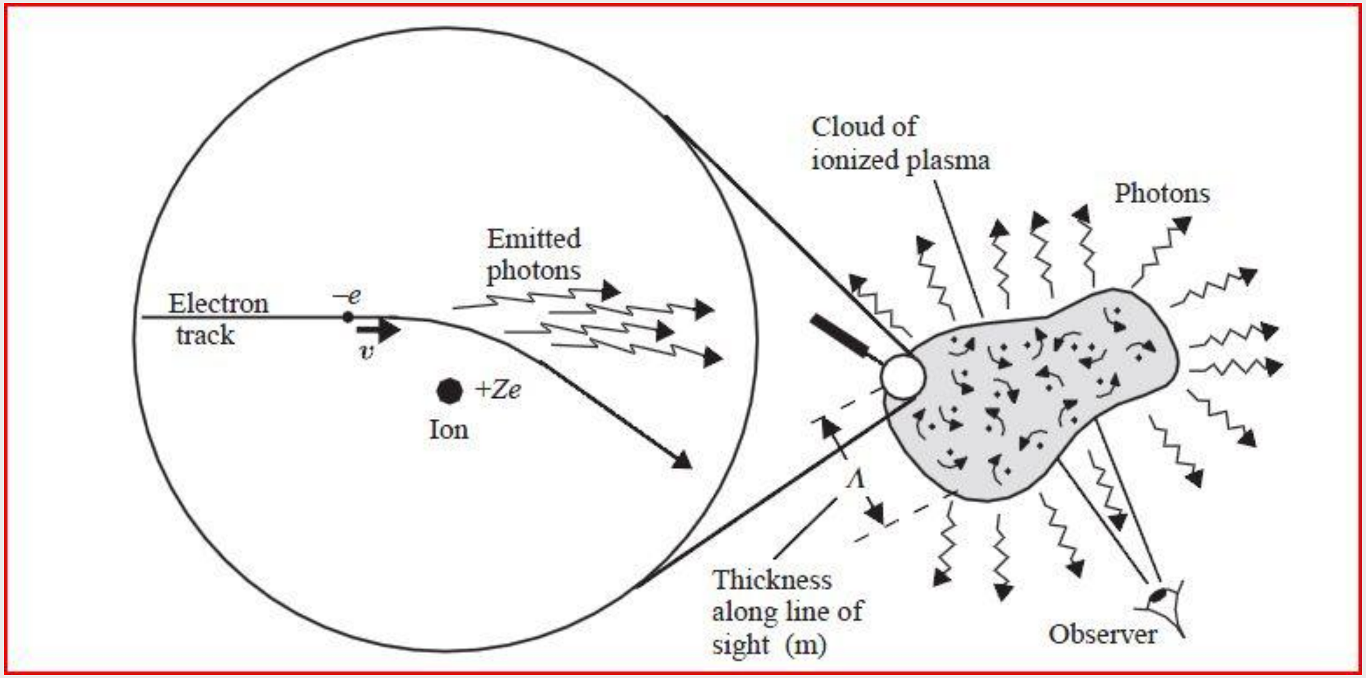
Fourier transform of d(t)

$$\ddot{d}(t) = -\int_{-\infty}^{+\infty} \omega^2 \hat{d}(\omega) e^{-i\omega t} d\omega \quad \hat{E}(\omega) = -\frac{1}{c^2 r} \omega^2 \hat{d}(\omega) \sin\theta$$

$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3} \left| \hat{d}(\omega) \right|^4$$

Interesting property of dipole radiation that the emitted spectrum is related directly to the frequencies of oscillation of the dipole moment

BREMSSTRAHLUNG



Radiation due to the acceleration of a charge in the Coloumb field of another charge (also free-free emission)
Classical treatment then quantum correction (Gaunt factor) to the classical formula
Collisions of like-particles in the dipole approximation is zero because dipole proportional to the constant center of mass

BREMSSTRAHLUNG IN A NUTSHELL

- Electron of mass m_e with impact parameter b and velocity v in the Coloumb field of the proton
- The acceleration of the electron is $a \approx (q^2/ m_e b^2)$ and lasts for a time b/v
- This encounter will result in a emission of energy $W \approx (q^2 a^2/c^3)(b/v) \approx (q^6/c^3 m_e^2 b^3 v) \approx (q^6 n_i/c^3 m_e^2 v)$ as $b \approx n_i^{-1/3}$
- The total energy radiated per unit volume will be $n_e W$
- Because each collision lasts for a time (b/v) there will be little radiation at frequencies greater than (v/b). For frequencies lower than (v/b) we may take the energy emitted per unit frequency as nearly constant
- In the case of plasma in thermal equilibrium $v \approx (k_B T/m_e)^{1/2}$

BREMSSTRAHLUNG IN A NUTSHELL

$$j_\omega = \frac{dW}{d\omega dt dV} \cong \left(\frac{q^6}{m_e^2 c^3}\right)\left(\frac{m_e}{k_B T}\right)^{1/2} n_e n_i \propto n^2 T^{-1/2}$$

• Bremsstrahlung spectrum is flat up to kT/h and it will fall rapidly beyond that frequency, simply because and electron with typical energy kT cannot emit photons with higher energy

$$j = \frac{dW}{dt dV} \approx j_\omega \times \frac{kT}{h} \propto n^2 T^{1/2}$$

EMISSION FROM SINGLE SPEED ELECTRONS

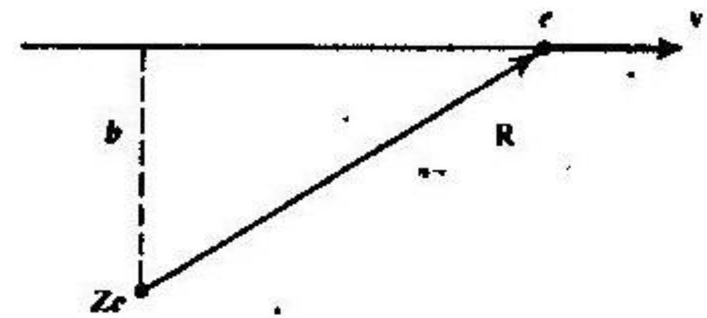


Figure 5.1 An electron of charge e moving past an ion of charge Ze.

Small-angle scattering regime: the electron moves rapidly enough that the deviation from a straight line is negligible

$$\vec{d} = -e\vec{r} \qquad \ddot{\vec{d}} = -e\dot{\vec{v}}$$

$$-\omega^2 \hat{d}(\omega) = -\frac{e}{2\pi}\int_{-\infty}^{+\infty} \dot{v} e^{i\omega t}\, dt$$

EMISSION FROM SINGLE SPEED ELECTRONS

$$\tau = \frac{b}{v}$$ Collision time

$$\hat{d}(\omega) \approx \frac{e^2}{2\pi\omega^2}\Delta v \qquad \omega\tau << 1 \qquad\qquad \hat{d}(\omega) \approx 0 \qquad \omega\tau >> 1$$

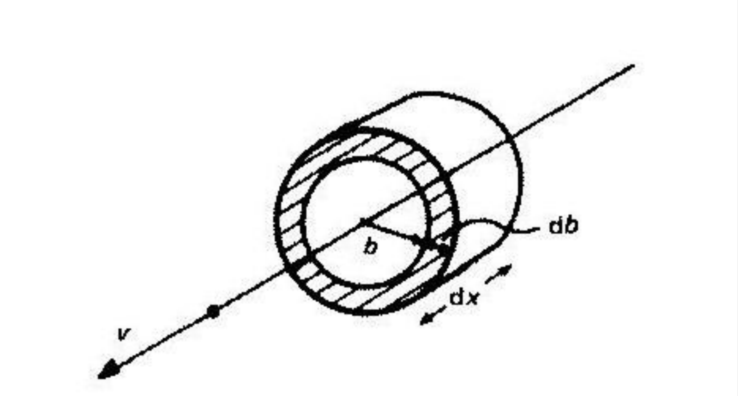
To estimate Δv we realize that the change in velocity is predomiinantly normal to the path

$$\Delta v = \frac{Ze^2}{m}\int_{-\infty}^{+\infty}\frac{b}{(b^2+v^2t^2)^{3/2}}dt = \frac{2Ze^2}{mbv}$$

EMISSION FROM SINGLE SPEED ELECTRONS

$$\frac{dW(b,v)}{d\omega} = \frac{8Z^2e^6}{3\pi c^3 m^2}\frac{1}{v^2b^2}$$

Total spectrum for a medium with ion density n_i, electron density n_e, and for a fixed speed v. The flux of electrons incident on one ion is $n_e v$ and the element of area is $2\pi bdb$



$$\frac{dW}{d\omega dV dt} = n_e n_i 2\pi v \int_{b_{\min}}^{b_{\max}} \frac{dW(b,v)}{d\omega} bdb$$

EMISSION FROM SINGLE SPEED ELECTRONS

$$\frac{dW}{d\omega dVdt} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

$b_{max} = v/\omega$ whereas for b_{min} we can have a classical limit where all the maximum potential energy is converted into kinetic energy or quantistic when the indetermination principle takes action. To be general the Gaunt factor is introduced

$$\frac{dW}{d\omega dVdt} = \frac{16\pi e^6}{3\sqrt{3}c^3m^2v} n_e n_i Z^2 g_{ff}(v,\omega)$$

THERMAL BREMSSTRAHLUNG

We need to average over a distribution of velocities: the ones astrophysically relevant are power-laws and thermal (Maxwellian) distribution

$$dP \propto \exp\left(-\frac{mv^2}{2kT}\right)d^3v \propto \exp\left(-\frac{mv^2}{2kT}\right)4\pi v^2 dv$$

$$\left\langle \frac{dW(T,\omega)}{dVdtd\omega} \right\rangle \propto n_e n_i \boxed{\frac{e^{-\frac{h\omega}{kT}}}{\sqrt{T}}}\,\bar{g}_{ff}(\omega)$$

THERMAL BREMSSTRAHLUNG

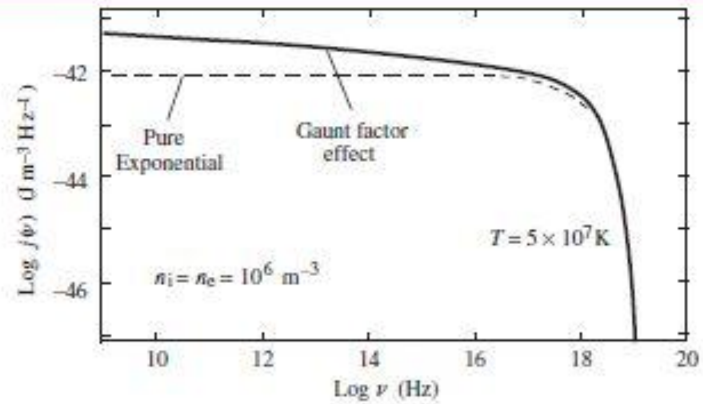


Fig. 5.5: Theoretical continuum thermal bremsstrahlung spectrum. The volume emissivity (37) is plotted from radio to x-ray frequencies on a log-log plot with the Gaunt factor (38) included. The specific intensity $I(\nu, T)$ would have the same form. Note the gradual rise toward low frequencies due to the Gaunt factor. We assume a hydrogen plasma ($Z = 1$) of temperature $T = 5 \times 10^7$ K with number densities $n_i = n_e = 10^6$ m^{-3}.

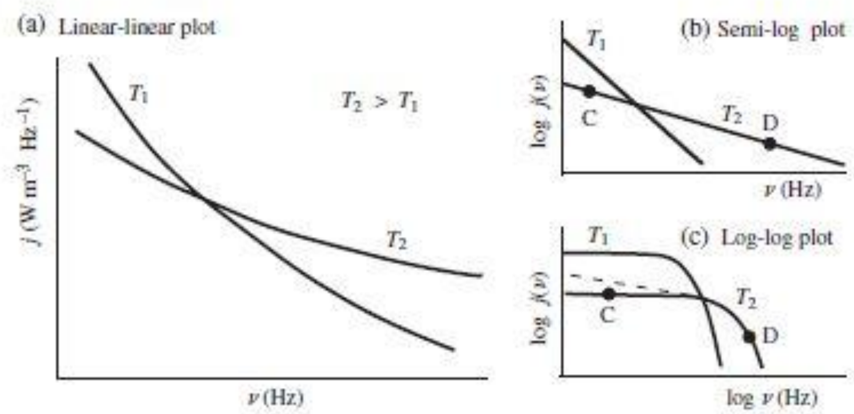


Fig. 5.6: Thermal bremsstrahlung spectra (as pure exponentials) on linear-linear, semilog, and log-log plots for two sources with the same ion and electron densities but differing temperatures, $T_2 > T_1$. Measurement of the specific intensities at two frequencies (e.g., at C and D) permits one to solve for the temperature T of the plasma as well as for the emission measure $\langle n_e^2 \rangle_{av} \Lambda$. [From H. Bradt, *Astronomy Methods*, Cambridge, 2004, Fig. 11.3, with permission]

Clusters of Galaxies

Why study Clusters

- Clusters are the largest structures in the Universe to have clearly decoupled from the Hubble flow, they carry important cosmological information
- Physical conditions in clusters are unlike anywhere else. They allow us to explore physical phenomena under unique conditions.

Galaxy Clusters for beginners

Overview

- Basic properties of Galaxy Clusters
- Zero order model for the Intra Cluster Medium
- Global measurements of T and Z
- Spatially Resolved Spectroscopy

Setting the context

Basic properties of the ICM

- The ICM is tenuous, typical densities 10^{-4} to a few 10^{-2} cm^{-3} .
- These are very low densities,
  the tail lobes of the earth magnetosphere have densities of 10^{-2} cm^{-3} .

Setting the context

Basic properties of the ICM

- The ICM is hot, temperatures are in the range of 10^7 to 10^8 K (1-10 keV)
- highly ionized: H, He completely ionized heavier elements partially ionized
- Chemically enriched, heavy elements such as O, Si and Fe are present in almost solar proportions

Setting the context

Basic properties of the ICM

Weakly magnetized typically 0.1 to a few μGauss

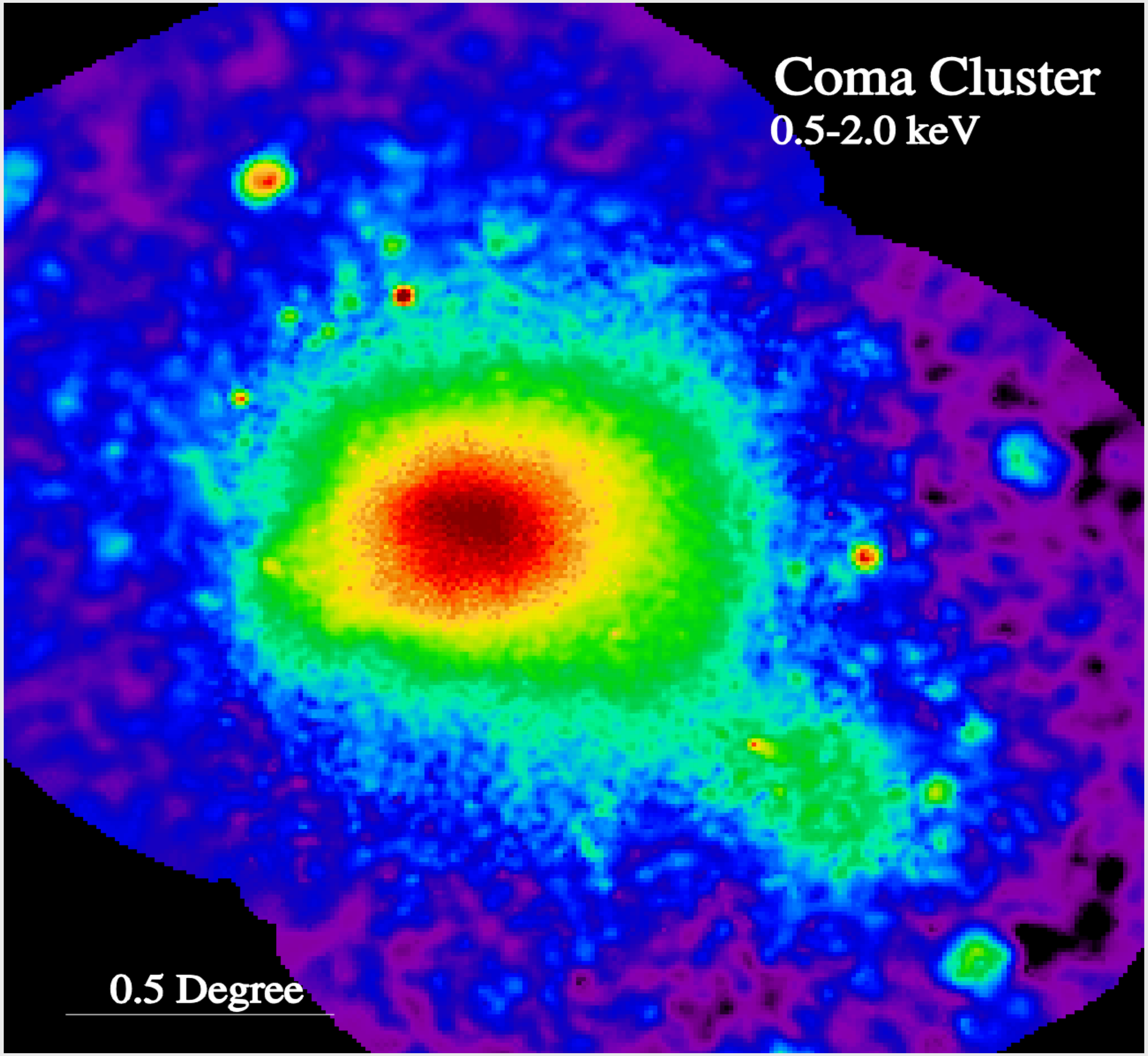
Galactic magnetic field	10 μGauss
Solar Wind	50 μGauss
Interstellar molecular cloud	1 mGauss
Earth's field at ground level	1 Gauss
Solar surface field	1-5 Gauss
Massive star (pre supernova)	100 Gauss
Sun spot field	1000 Gauss

Setting the context

Basic properties of the ICM

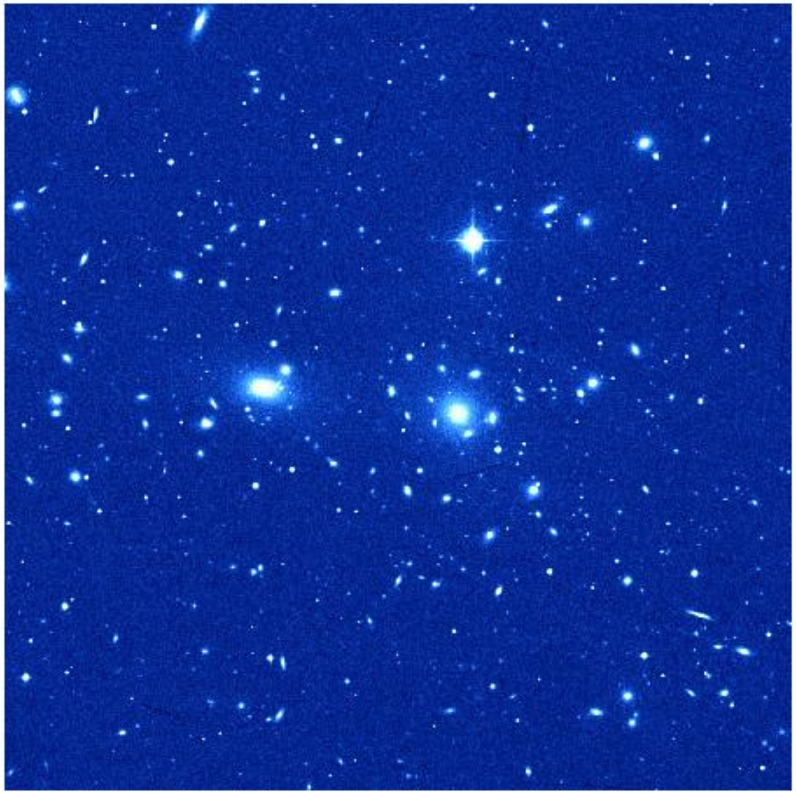
The X-ray emission from clusters is extended.

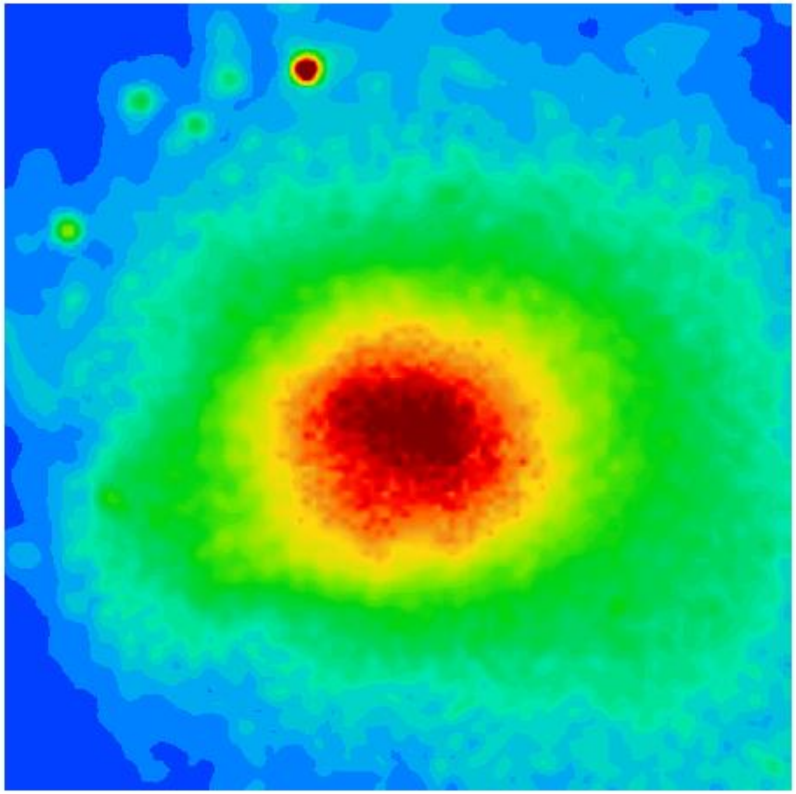
Setting the context



X-Ray Imaging

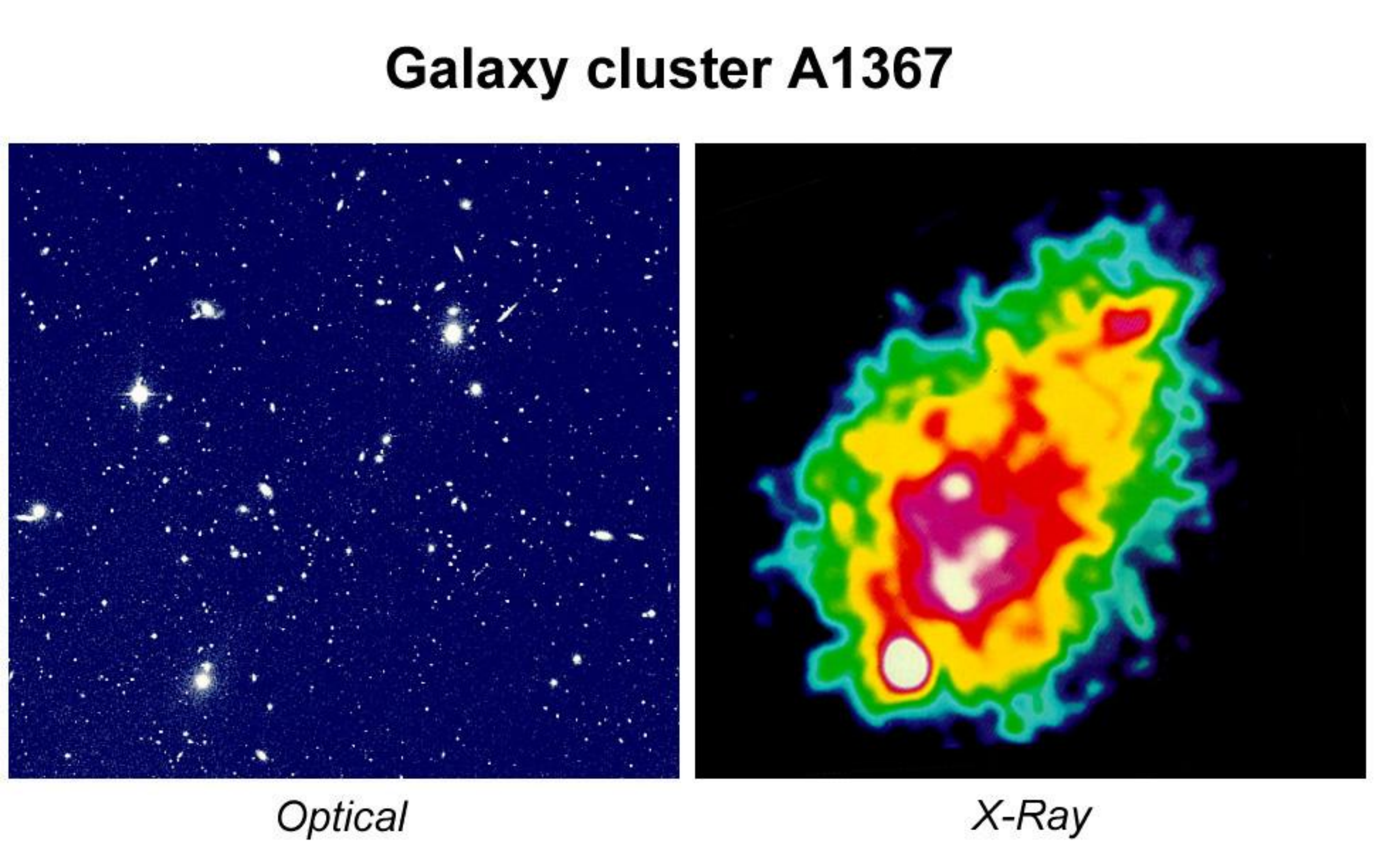
X-rays and optical light show us a different picture

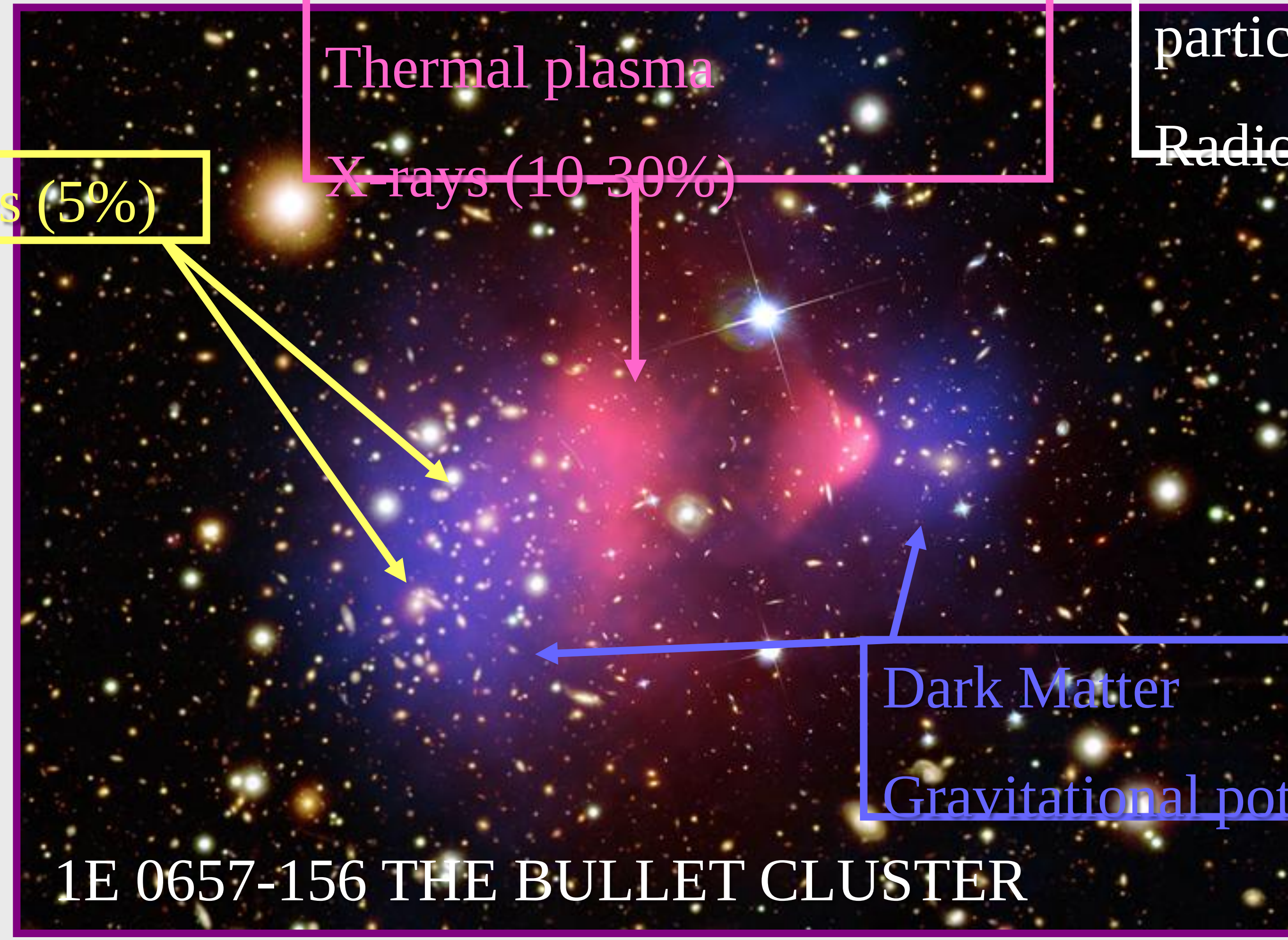




X-Ray Imaging

X-rays and optical light show us a different picture





Galaxy clusters are the most massive ($M \sim 10^{14}$-10^{15} M_{SUN}) objects in the Universe

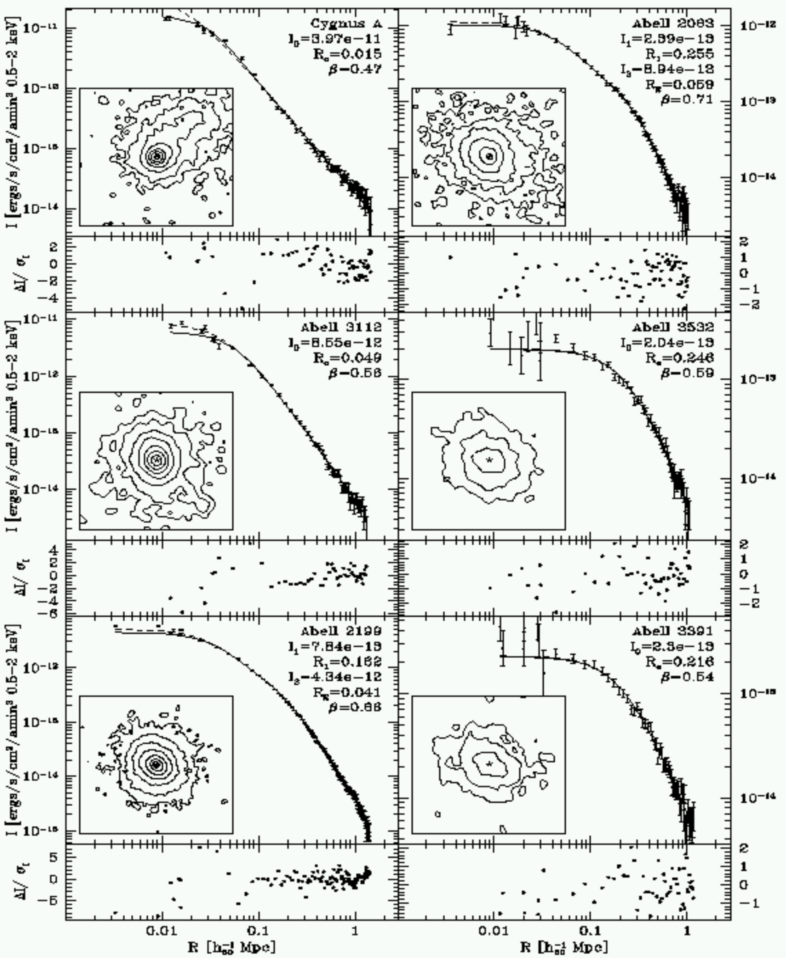
Properties of groups and clusters

	CLUSTERS	GROUPS/POOR CLUSTERS
L_X (erg/s)	10^{43} - 10^{45}	$10^{41.5}$ - 10^{43}
kT_X (keV)	2 – 15	≤ 2
N gal	100-1000	5 – 100
σ_v (km/s)	500-1200 (median 750)	200 – 500
M_{tot} (< 1.5 Mpc)	10^{14} – 5 x 10^{15}	$10^{12.5}$ - 2 x 10^{14}
Number Density	10^{-5} – 10^{-6} Mpc^{-3}	10^{-3} – 10^{-5} Mpc^{-3}

Groups and poor clusters provide a natural and continuous extension to lower mass, size, luminosity and richness of rich, massive and rare clusters

BHACALL 1999

X-Ray Imaging



Mohr et al. (1999)

- Central regions feature approx. constant surface brightness
- In outer regions the surface brightness falls off as a power-law with index approx. 3
- Emission is traced out to 1-2 Mpc from the core

Timescales & other fundamentals

Cooling timescale

The ICM cools by emitting radiation

$$\boxed{t_{cool} \approx \frac{u}{\varepsilon} \propto n_p^{-1} T_g^{1/2}}$$

$$\boxed{t_{cool} \approx 8.5 \times 10^{10}\, yr \left(\frac{n_p}{10^{-3}\, cm^{-3}} \right)^{-1} \left(\frac{T_g}{10^8\, K} \right)^{1/2}}$$

Except for the innermost regions where n_p is high, gas cools on timescales > Hubble time

In first approx. we may consider the ICM as a stationary ball of hot plasma

Heating

- No major on-going heating of the gas is necessary (the cooling is very slow)
- The ultimate origin of the bulk of the thermal energy of the ICM is the gravitational energy lost by the gas as it falls into the cluster's potential well

$$\boxed{T_g \approx \frac{GM}{r}}$$

- The temperature of the ICM is related to the depth of the potential well and to the total mass of the cluster

- The ICM is highly ionized

  $T_g \sim 10^7 - 10^8$ $\Rightarrow$ H, He completely ionized
- Coulomb interactions are the dominant mechanism for collisions, for $T_e = T_i$ the mean free path is:

$$\boxed{\lambda_e = \lambda_i \approx 23kpc\left(\frac{T_g}{10^8 K}\right)^2 \cdot \left(\frac{n_p}{10^{-3} cm^{-3}}\right)^{-1}}$$

$$\boxed{\lambda_e = \lambda_i << R_{cluster} \approx 1Mpc}$$

ICM can be treated as a fluid, satisfying hydro-dynamical equations

Timescale for a sound-wave to cross the cluster

$$\boxed{v_s \approx \left(\frac{p}{\rho}\right)^{1/2} \propto T_g^{1/2}}$$

$$\boxed{t_s \approx 6.6\times10^8\,yr\left(\frac{D}{Mpc}\right)\cdot\left(\frac{T_g}{10^8\,K}\right)^{-1/2}}$$

$$\boxed{t_s << t_{cool}} \qquad \boxed{t_s << t_{age}}$$

The ICM is in hydrostatic equilibrium

Radiation process

Continuum emission is dominated by thermal bremsstrahlung

$$\boxed{\varepsilon_\nu \propto n_g^2 \cdot T_g^{-1/2} \cdot \exp\left(-\frac{h\nu}{kT_g}\right)}$$

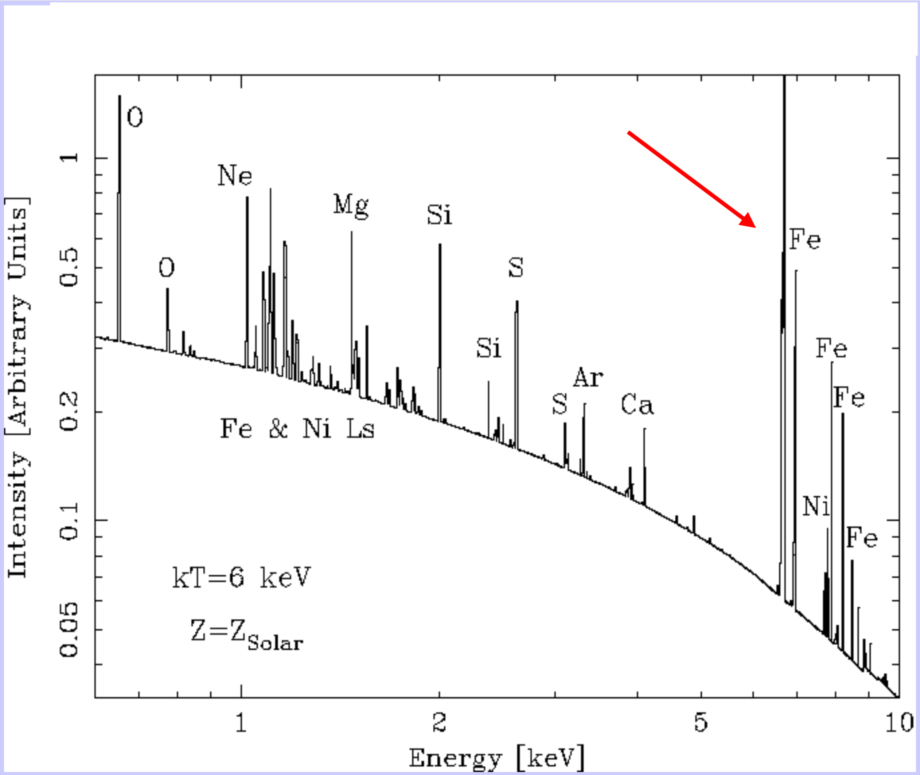
spectral shape $\rightarrow T_g$

spectrum normal. $\rightarrow n_g$

Optically thin: $n_g \, \sigma_T \, L \approx 10^{-2} \; 6.65\text{x}10^{-25} \; 3.18\text{x}10^{24} \ll 1$

Line emission

The Fe K_α complex ~ 7 keV is dominated by emission from He-like (6.7 keV) and H-like (6.9 keV), the ratio of lines intensity depends upon the ionization state of the gas which is a function of the gas temperature.



Summary of basic properties

- A hot, tenuous and weakly magnetized plasma (ICM) rests in the potential well of galaxy clusters
- The temperature of the ICM is related to the total mass of the cluster
- The ICM is enriched and heavily ionized
- It dissipates energy at a very slow rate by emitting X-rays by thermal bremsstrahlung
- It can be treated as a fluid in hydro-static equilibrium
- The pressure associated with the weak B field does not drive gas dynamics

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