

Musica e Matematica

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Musica e Matematica

- Il ritmo: un esempio (non così) banale
- La formazione delle scale musicali

*Perché le note
sono sette?*



- Musica – Scienza - Evoluzione

Qualche concetto di base...

- Orecchio assoluto / relativo



- Intervalli e melodie



- Frequenza

- Suoni “alti” (acuti) e “bassi” (gravi)

Frequenza

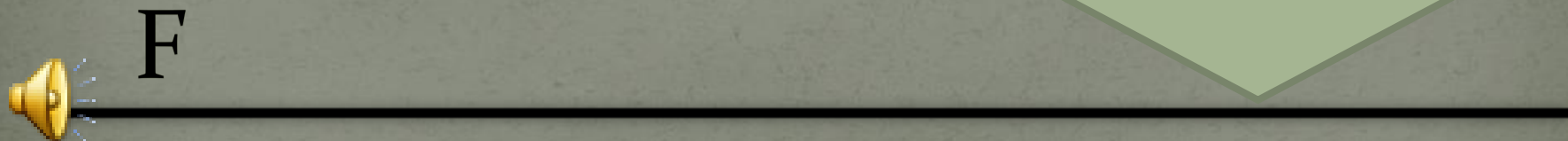
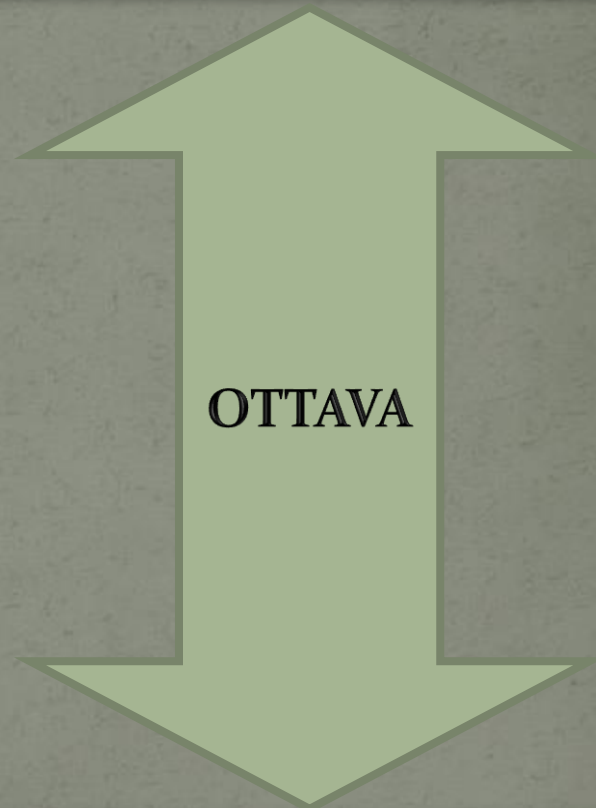
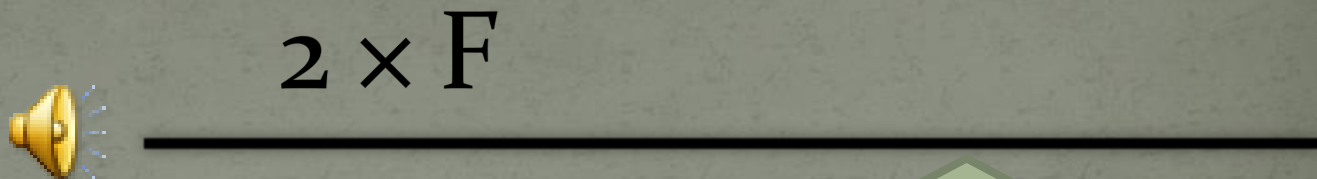
- Una vibrazione al secondo = 1 Hertz = 1 Hz
- Suoni udibili hanno frequenza tra circa 16 Hz e 20,000 Hz

20 Hz

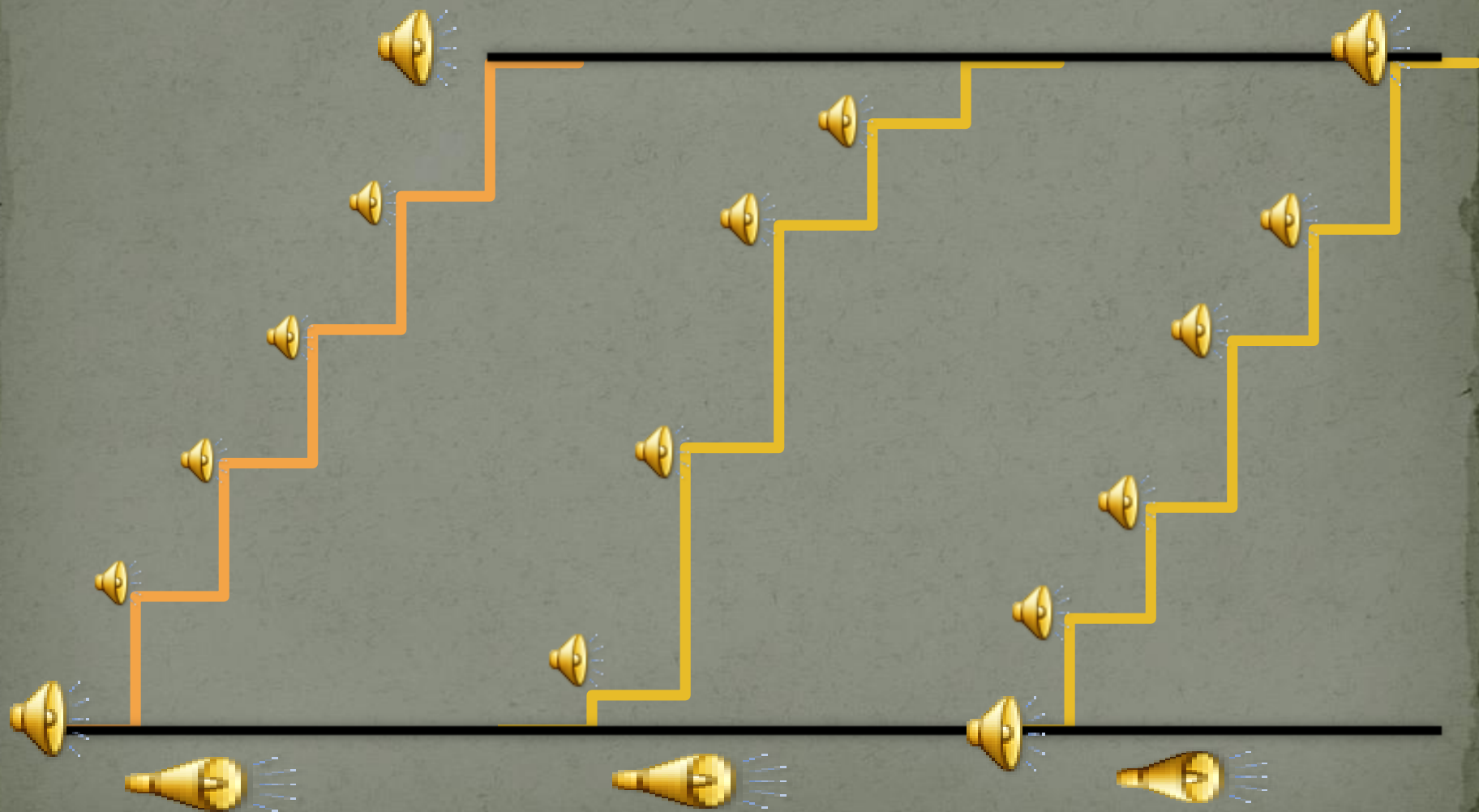
4000 Hz



Un intervallo particolare



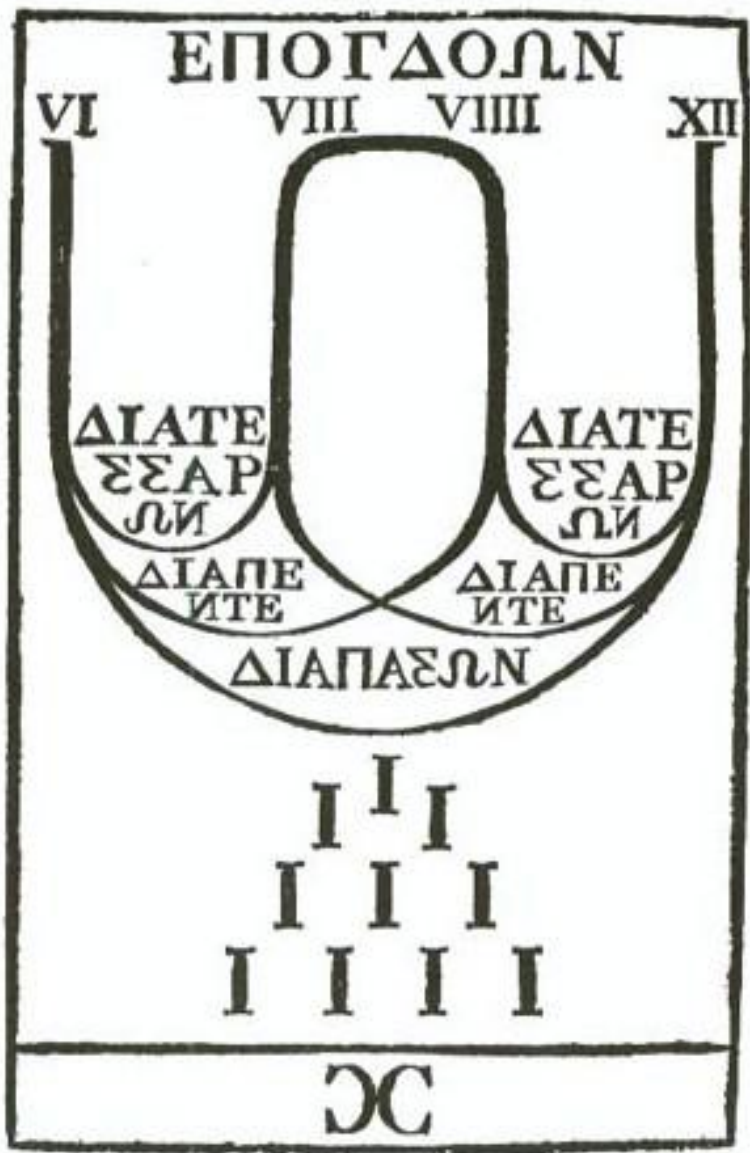
Come scegliere cinque note ?



Raffaello Sanzio - La scuola di Atene (1511)

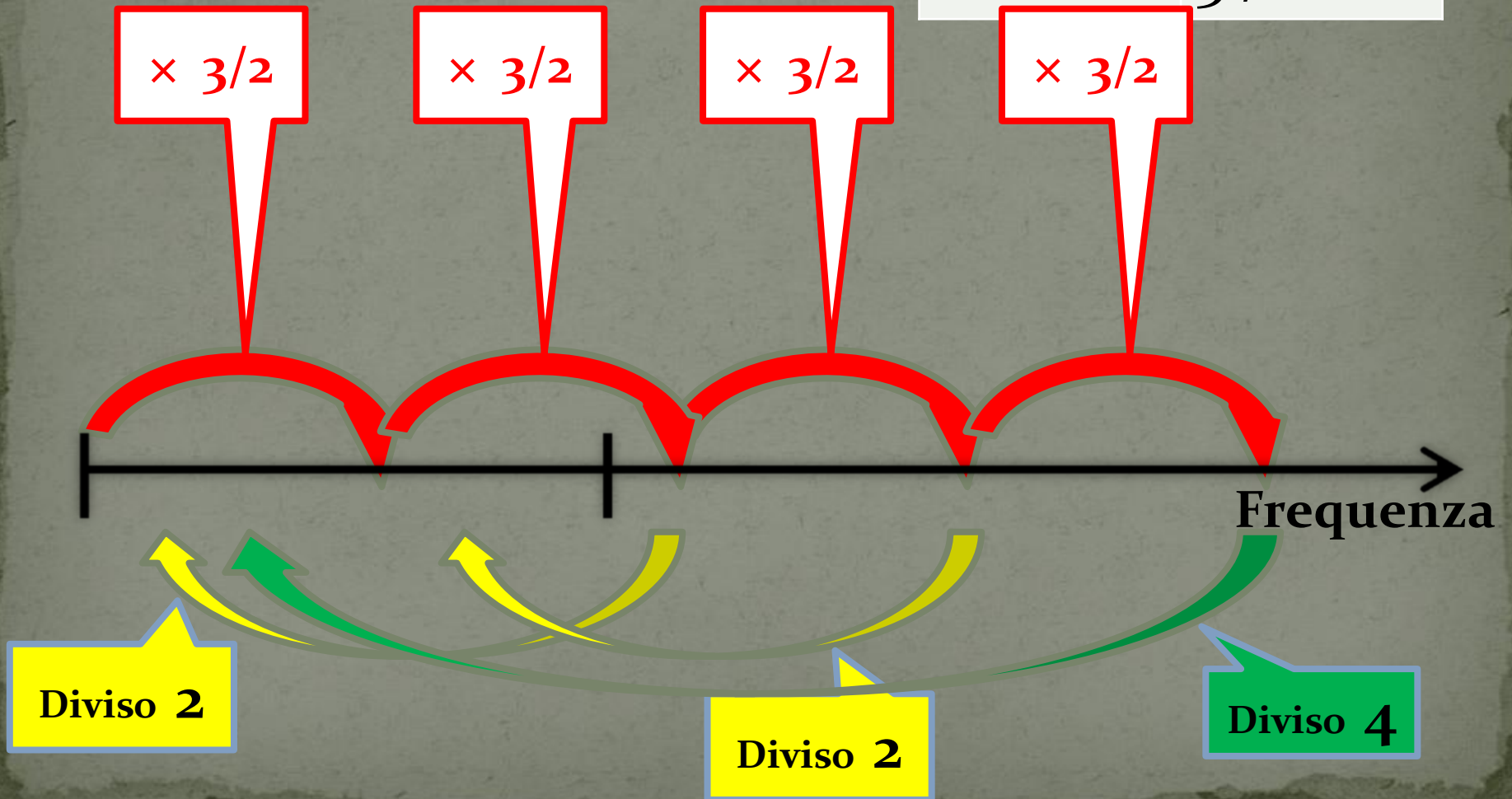




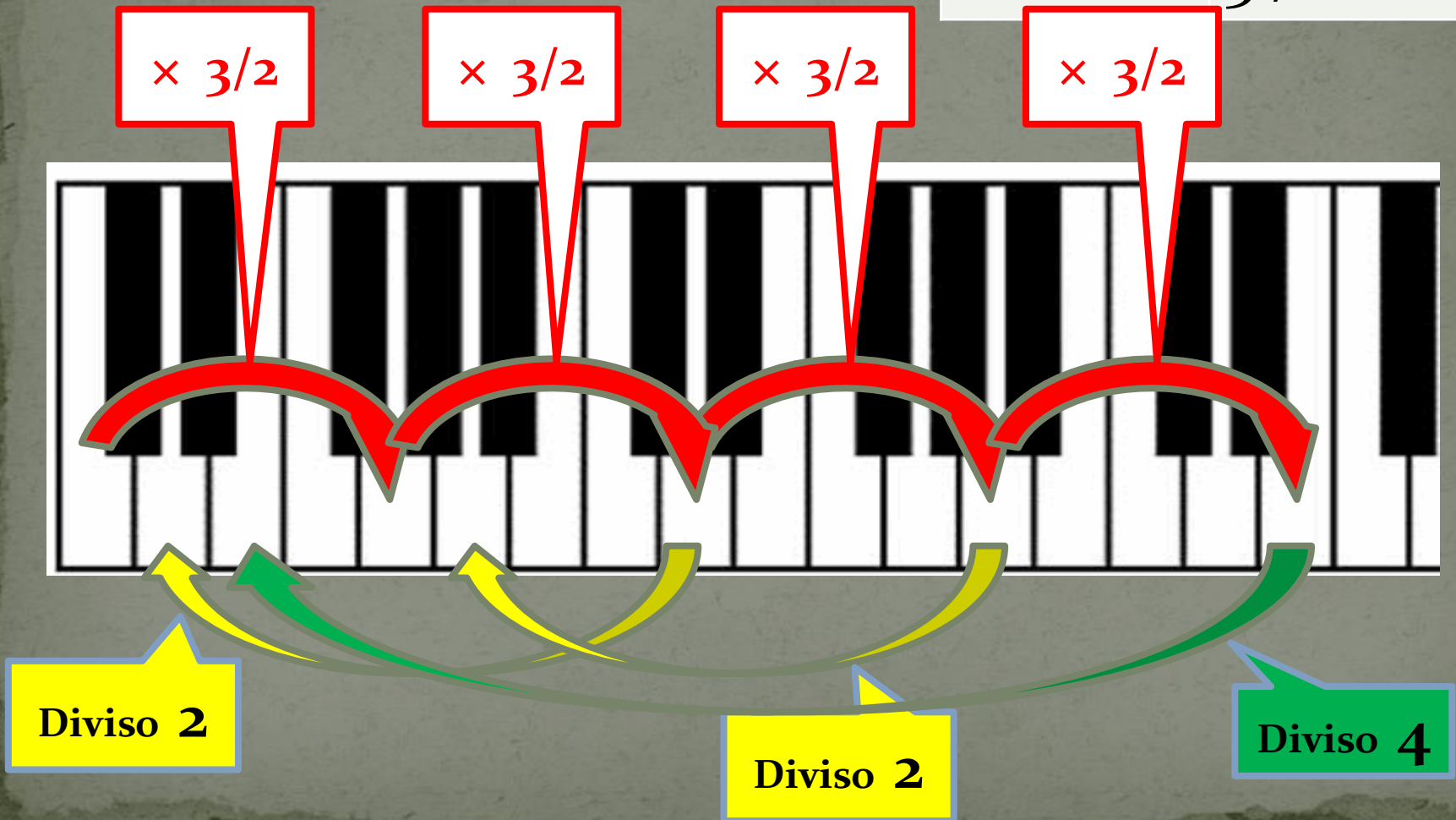


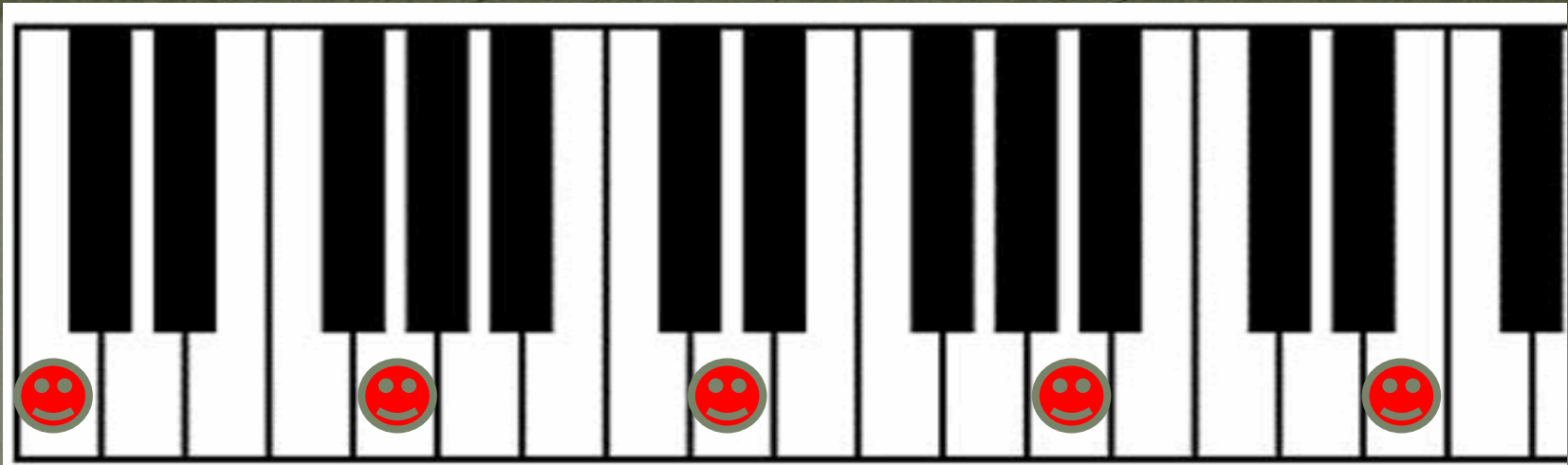
INTERVALLI CONSONANTI	Rapporto tra le frequenze
OTTAVA	2 / 1
QUINTA	3 / 2
QUARTA	4 / 3

INTERVALLO	Rapporto tra le frequenze
OTTAVA	2 / 1
QUINTA	3 / 2



INTERVALLO	Rapporto tra le frequenze
OTTAVA	2 / 1
QUINTA	3 / 2





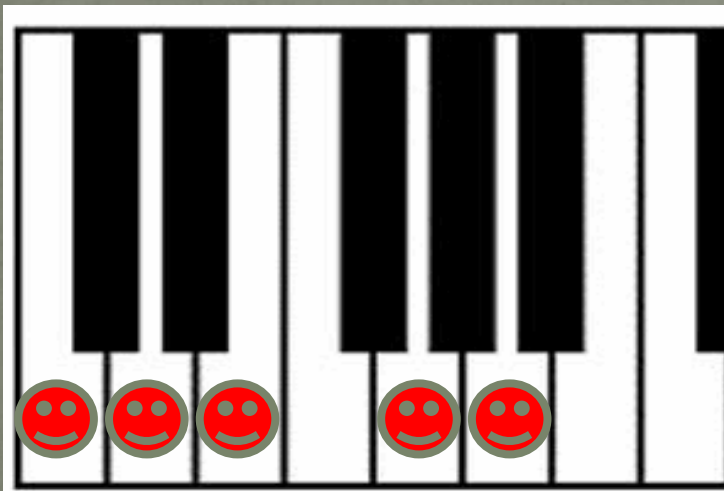
Do

Sol

Re

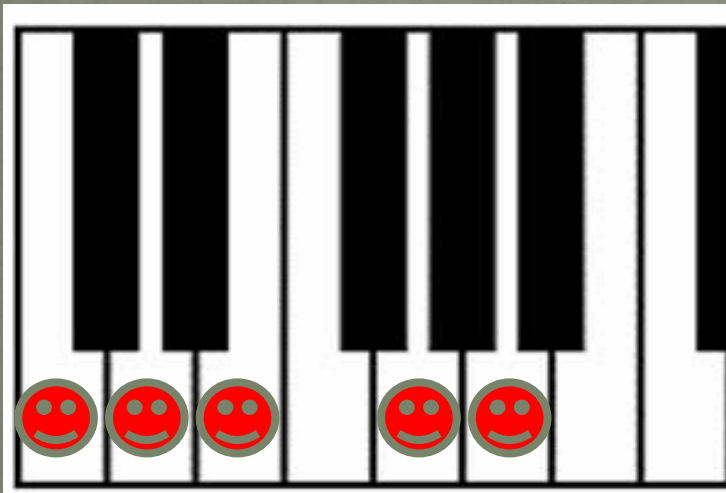
La

Mi



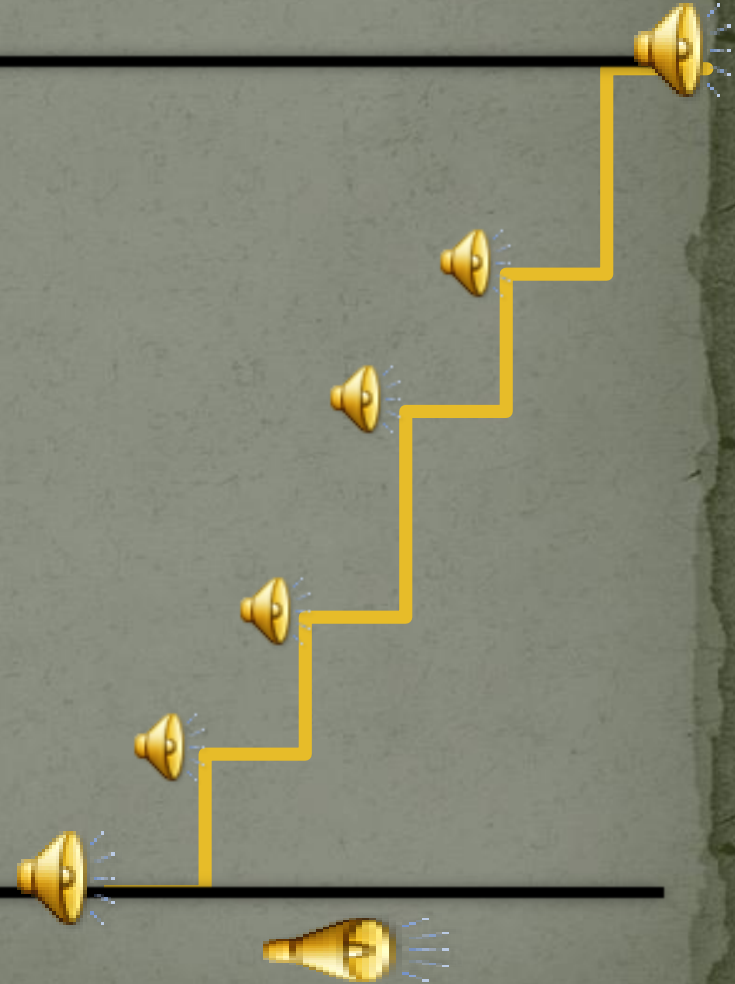
Do Re Mi

Sol La

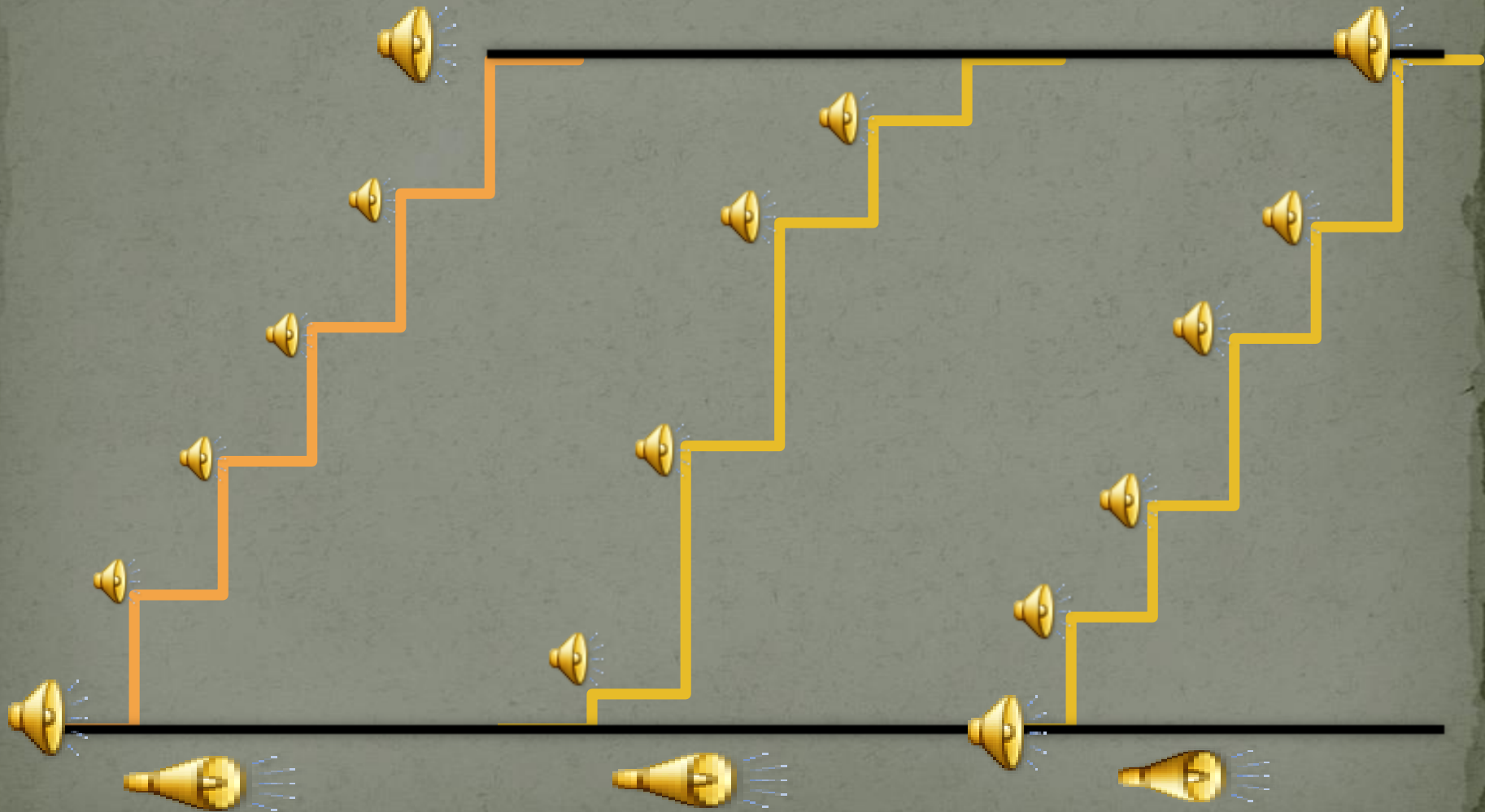


Do Re Mi

Sol La



Patterns...



Scala Pitagorica

FA #	$(3/2)^6$: 8 =	729 / 512 ≈ 1,424
SI	$(3/2)^5$: 4 =	243 / 128 ≈ 1,898
MI	$(3/2)^4$: 4 =	81 / 64 ≈ 1,265
LA	$(3/2)^3$: 2 =	27 / 16 = 1,6875
RE	$(3/2)^2$: 2 =	9 / 8 = 1,125
SOL	3/2	=	3 / 2 = 1,5
DO	1	=	1 = 1

Scala Pitagorica

FA #	$(3/2)^6 : 8 =$	$729 / 512$	$\approx 1,424$
SI	$(3/2)^5 : 4 =$	$243 / 128$	$\approx 1,898$
MI	$(3/2)^4 : 4 =$	$81 / 64$	$\approx 1,265$
LA	$(3/2)^3 : 2 =$	$27 / 16$	$= 1,6875$
RE	$(3/2)^2 : 2 =$	$9 / 8$	$= 1,125$
SOL	$3/2 =$	$3 / 2$	$= 1,5$
DO	1 =	1	= 1
FA	$2/3 \times 2 =$	$4 / 3$	$\approx 1,33$
SI b	$(2/3)^2 \times 4 =$	$16 / 9$	$\approx 1,78$
MI b	$(2/3)^3 \times 4 =$	$32 / 27$	$\approx 1,185$
LA b	$(2/3)^4 \times 8 =$	$128 / 81$	$\approx 1,58$
RE b	$(2/3)^5 \times 8 =$	$256 / 243$	$\approx 1,05$
SOL b	$(2/3)^6 \times 16 =$	$1024 / 729$	$\approx 1,41$

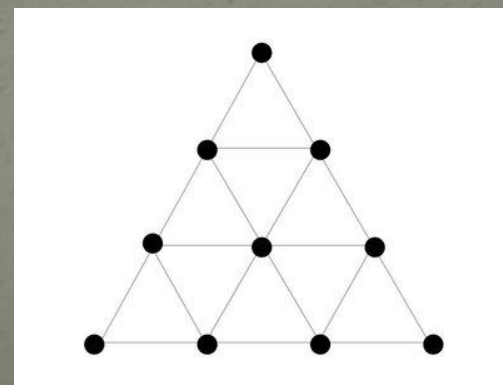
	DO	RE	MI	FA	SOL	LA	SI	DO
PITAGORICA	1	9/8	81/64	4/3	3/2	27/16	243/128	2
								

Tutte le QUINTE
sono perfettamente
intonate secondo
un rapporto $3/2$



INTERVALLI CONSONANTI	Rapporto tra le frequenze
OTTAVA	$2 / 1$
QUINTA	$3 / 2$
QUARTA	$4 / 3$
TERZA MAGG.	$5 / 4$
TERZA MINORE	$6 / 5$

TETRAKTYS



	DO	RE	MI	FA	SOL	LA	SI	DO
PITAGORICA	1	9/8	81/64	4/3	3/2	27/16	243/128	2
								

Tutte le **QUINTE**
sono perfettamente
intonate secondo
un rapporto $3/2$

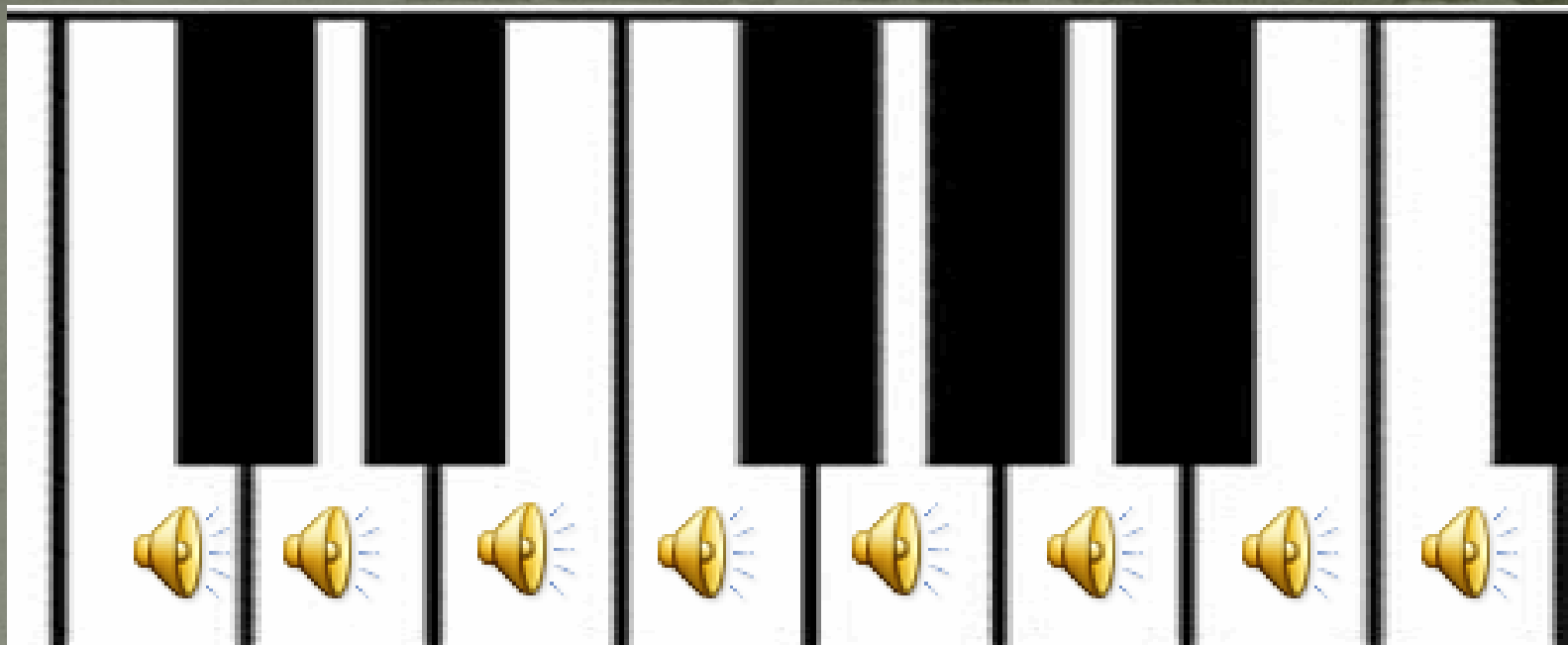
**... ma le TERZE
sono stonate !**

















INTERVALLI CONSONANTI	Rapporto tra le frequenze
OTTAVA	2 / 1
QUINTA	3 / 2
QUARTA	4 / 3
TERZA MAGGIORE	5 / 4
TERZA MINORE	6 / 5

	DO	RE	MI	FA	SOL	LA	SI	DO
PITAGORICA	1 	9/8 	81/64 	4/3 	3/2 	27/16 	243/128 	2 
NATURALE	1 	9/8 	5/4 	4/3 	3/2 	5/3 	15/8 	2 

Problema del
“temperamento”

INTERVALLI CONSONANTI	Rapporto tra le frequenze
OTTAVA	2 / 1
QUINTA	3 / 2
QUARTA	4 / 3
TERZA MAGG.	5 / 4
TERZA MINORE	6 / 5



	DO	RE	MI	FA	SOL	LA	SI	DO
PITAGORICA	1 	9/8 	81/64 	4/3 	3/2 	27/16 	243/128 	2 
NATURALE	1 	9/8 	5/4 	4/3 	3/2 	5/3 	15/8 	2 

Scala temperamento equabile

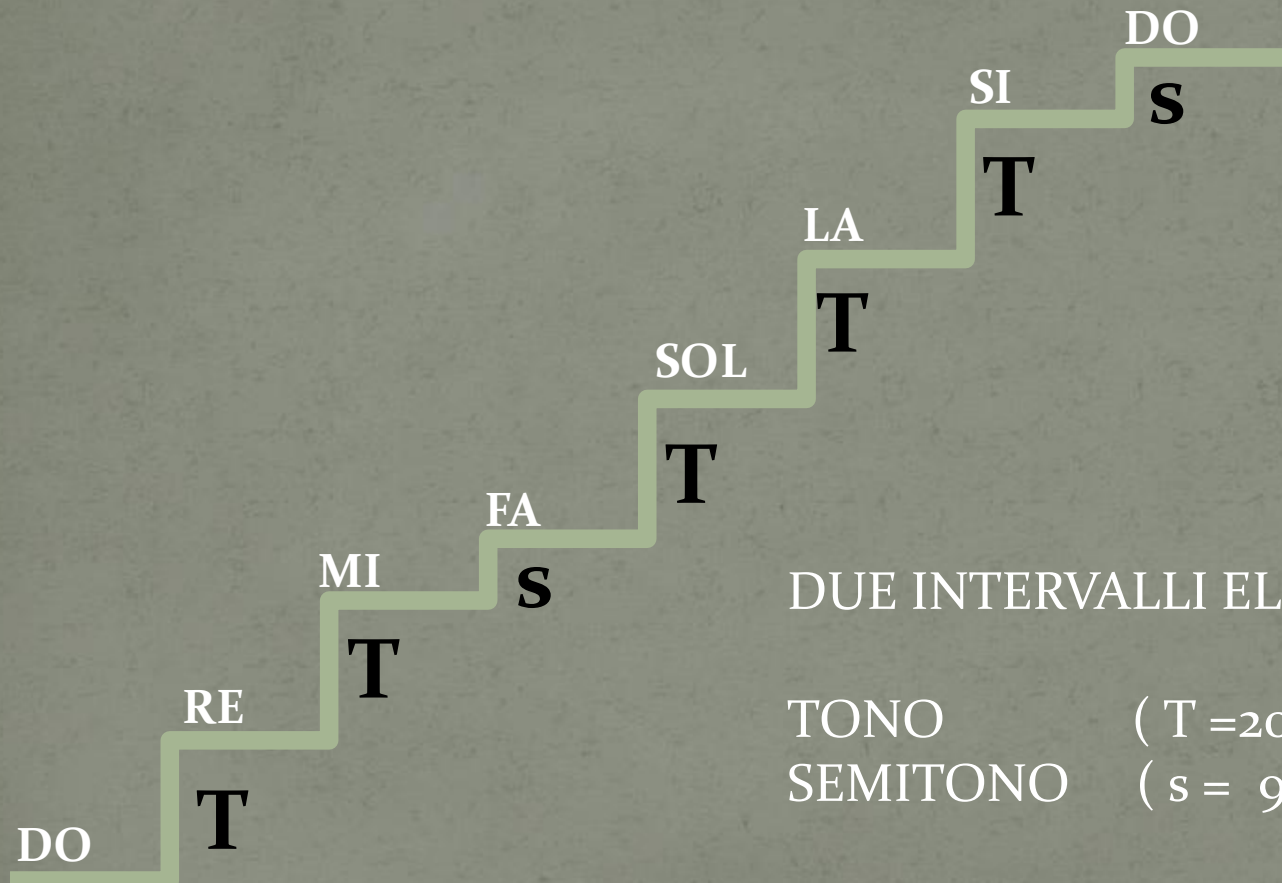


DUE INTERVALLI ELEMENTARI:

TONO	(T = 200 cent)	$2^{(1/6)}$
SEMITONO	(s = 100 cent)	$2^{(1/12)}$

TTsTTTs

Scala pitagorica



DUE INTERVALLI ELEMENTARI:

TONO	(T = 204 cent)	9/8
SEMITONO	(s = 90 cent)	256/243

TTsTTTs

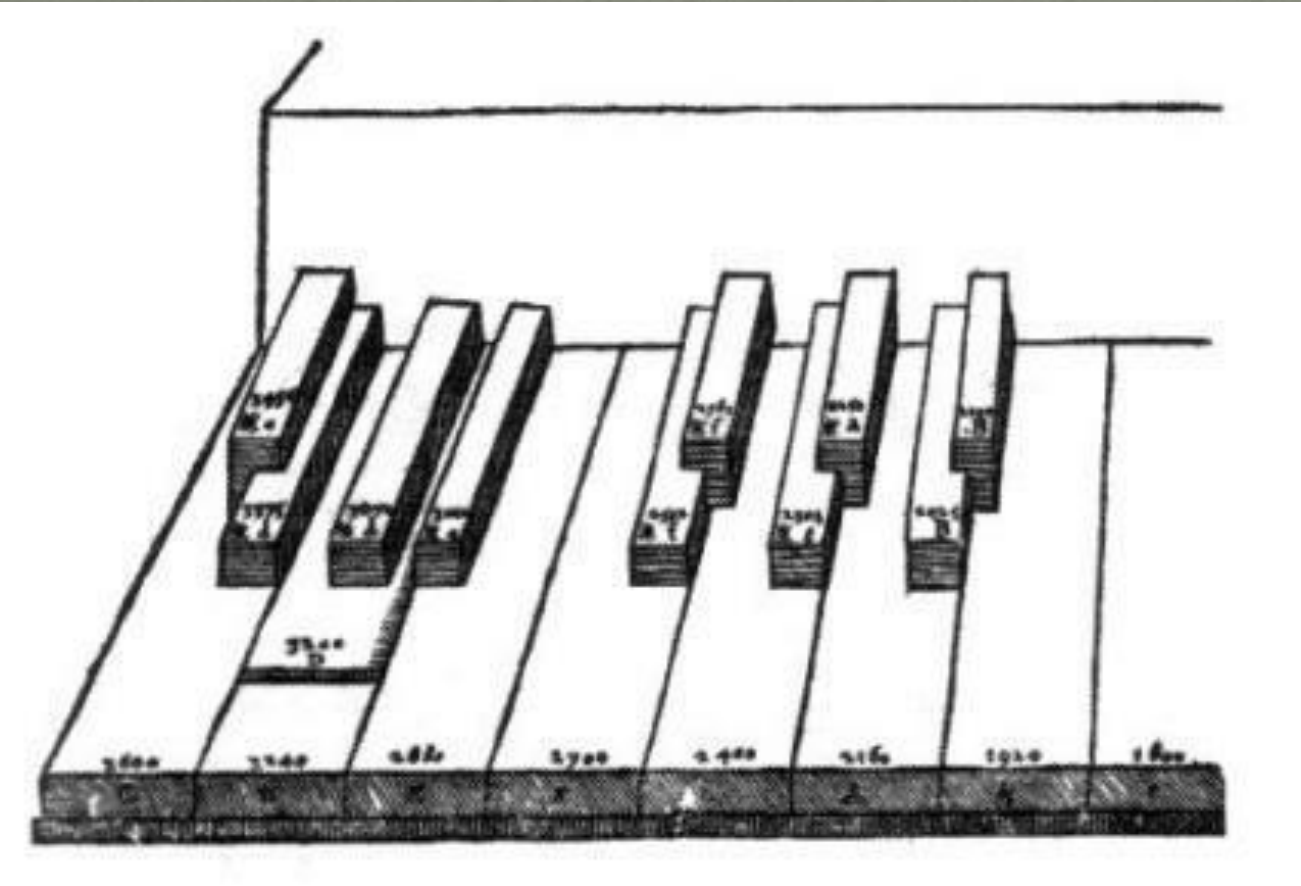
Scala naturale

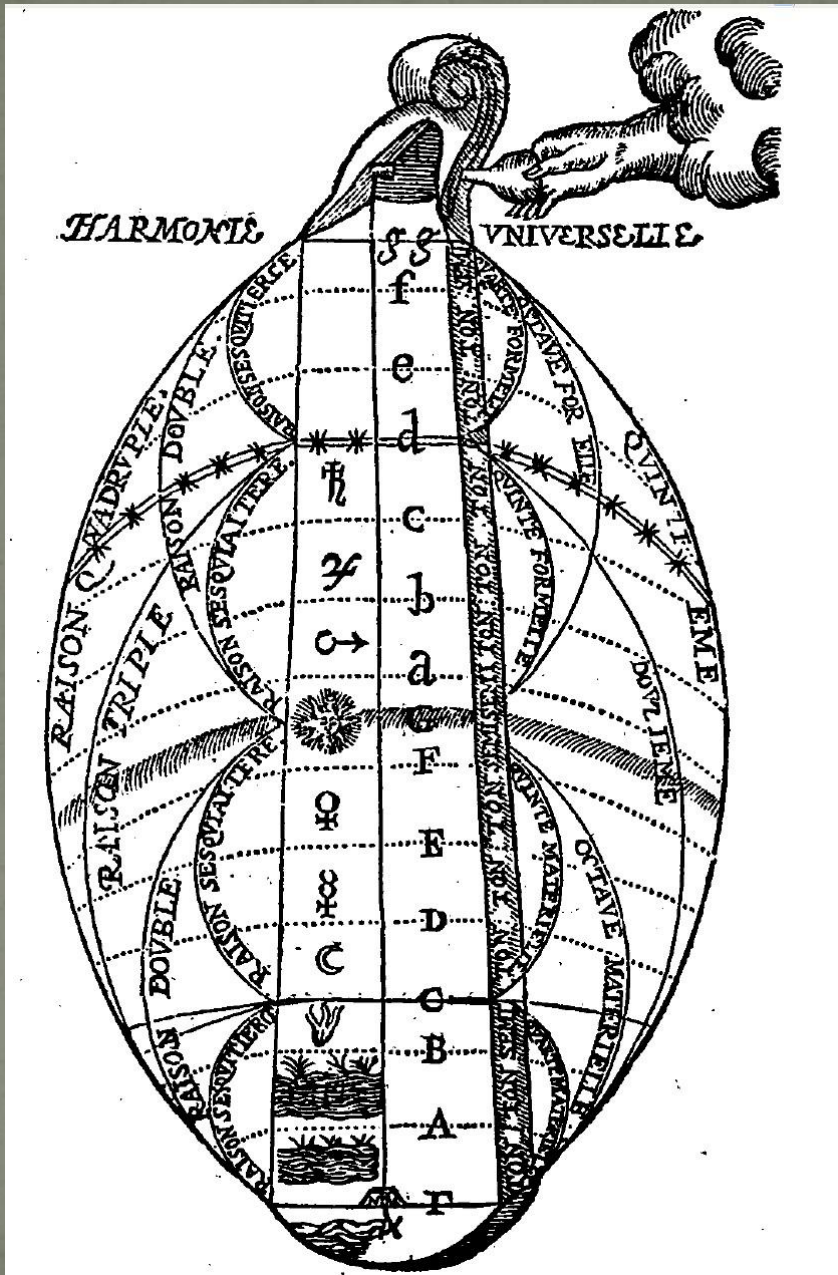


TRE INTERVALLI ELEMENTARI:

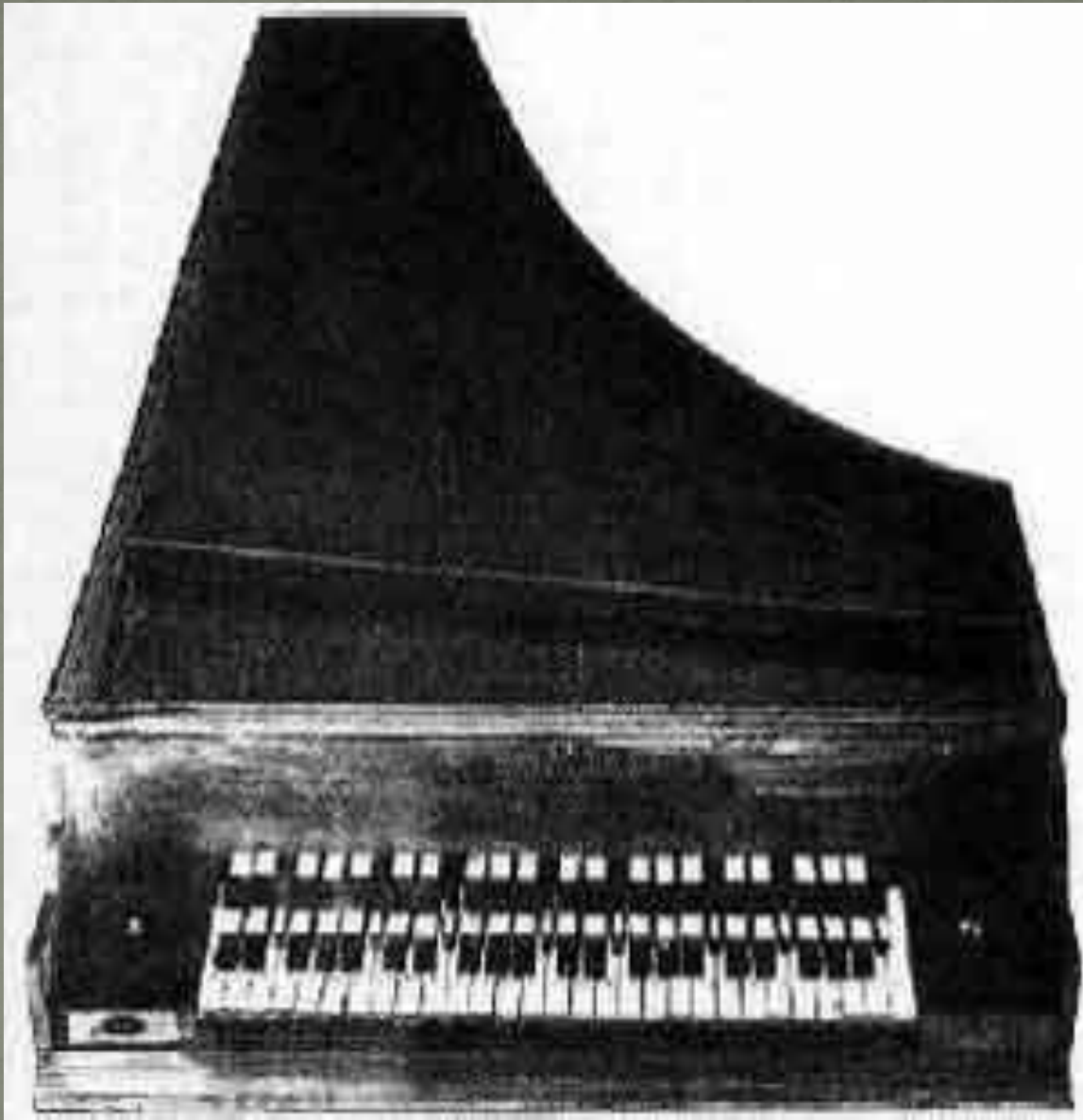
TONO MAGGIORE	(M=204 cent)	9/8
TONO MINORE	(m= 182 cent)	10/9
SEMITONO	(S = 112 cent)	16/15

M m S M m M S





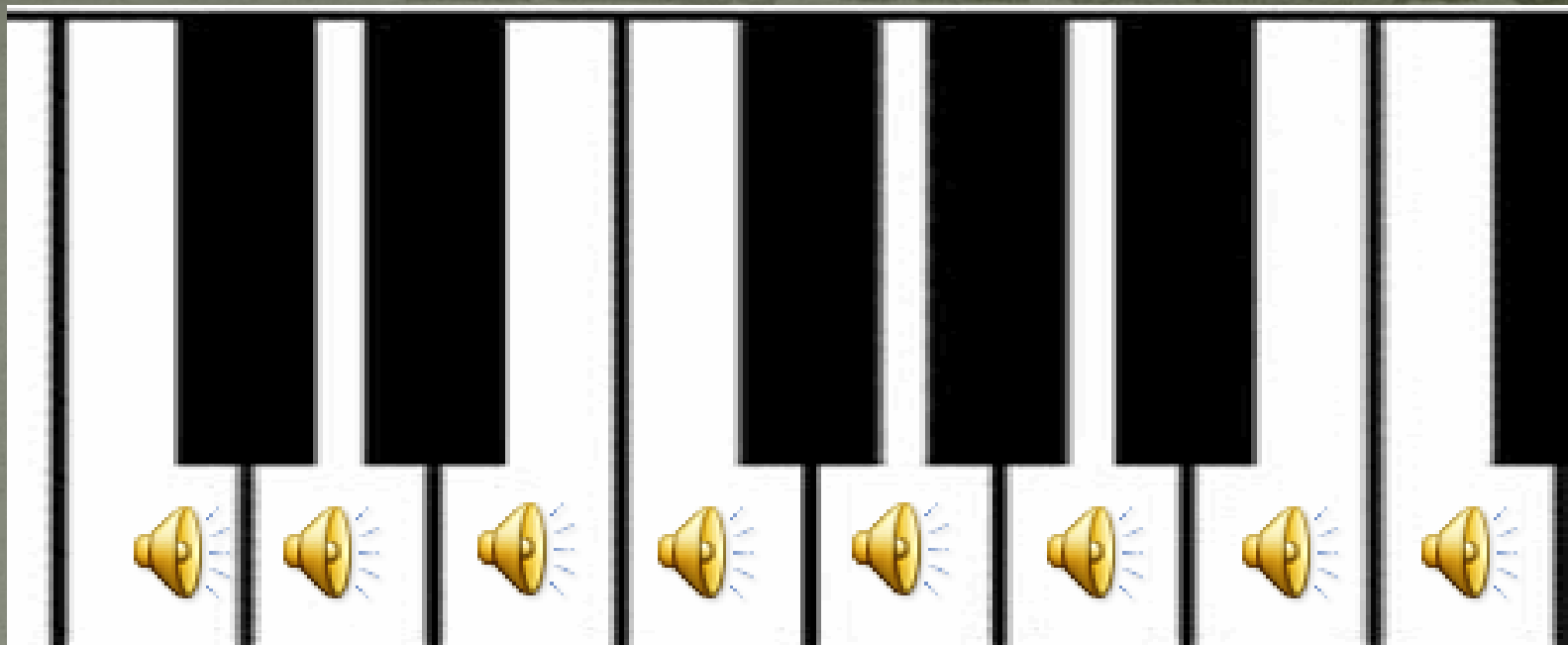
Robert Fludd
1574-1637



















Clavicordo di Trasuntino (1606)



Armonium di Bosanquet (1876)



	DO	RE	MI	FA	SOL	LA	SI	DO
PITAGORICA	1 	9/8 	81/64 	4/3 	3/2 	27/16 	243/128 	2 
NATURALE	1 	9/8 	5/4 	4/3 	3/2 	5/3 	15/8 	2 

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- Il ritmo: un esempio (non così) banale
- La formazione delle scale musicali
- **Musica – Scienza - Evoluzione**